

# The annihilation decay $B_c^- \rightarrow \eta' l^- \bar{\nu}$

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## Abstract

We first investigate the semileptonic annihilation decay  $B_c^- \rightarrow \eta' l^- \bar{\nu}$  in QCD. We find the  $\eta'$  momentum distribution is peaked within its small recoil region due to the loop effects. The branching ratio is estimated to be  $Br(B_c^- \rightarrow \eta' l^- \bar{\nu}) = 1.6 \times 10^{-4}$  for  $l = \mu, e$ , which is accessible at CERN LHC.

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# 1 Introduction

Being consisted of two different heavy flavors, the bottom-charmed meson  $B_c$  have many fascinating properties, which have motivated extensive studies in the literature. Its productions [1], spectroscopy [2, 3] and decays [4, 5] could be estimated to certain accuracy and provid windows for probing both strong and weak interactions.

The recent observation of  $B_c$  in 1.8 TeV  $p\bar{p}$  collisions using CDF detector at the Fermilab Tevatron has confirmed its existence in nature with mass  $M_{B_c} = 6.40 \pm 0.39 \pm 0.13$  GeV and lifetime  $\tau_{B_c} = 0.46_{-0.16}^{+0.18} \pm 0.03$  ps, which agree with the theoretical predictions [1, 5]. Further detailed experimental studies will be performed at Tevatron Run II and CERN large Hadron Collider(LHC). Especially, at LHC with the luminosity  $\mathcal{L} = 10^{34} \text{cm}^{-2} \text{s}^{-1}$  and  $\sqrt{s} = 14$  TeV, the number of  $B_c^\pm$  events is expected to be about  $10^8 \sim 10^{10}$  per year, so that some  $B_c$  rare decays of interests could be studied.

In this paper, we would like to present the first investigation on the annihilation decays  $B_c^- \rightarrow \eta' l^- \bar{\nu}$  in QCD. In this process,  $b$  and  $\bar{c}$  annihilate to leptons pair and meanwhile emit two gluons to form  $\eta'$ . Due to OZI suppression, its decay width should be two or three orders of magnitude lower than the dominant semileptonic decays induced by  $b \rightarrow c$  weak current. So it belongs to rare decays. However it is still sizable at LHC.

Compared with the pure leptonic decays  $B_c \rightarrow l\bar{\nu}_l$ , the suppression factor is  $\alpha_s^4$  in  $\Gamma(B_c^- \rightarrow \eta' l^- \bar{\nu})$  instead of the helicity suppression factor  $\frac{m_l^2}{m_{B_c}^2}$ . Considering the three bodies phase space much smaller than two bodies phase space, one could expect that the decay width  $\Gamma(B_c^- \rightarrow \eta' l^- \bar{\nu})$  may be the same order as  $\Gamma(B_c \rightarrow \mu\bar{\nu}_\mu)$ , anyhow it would be much larger than  $\Gamma(B_c \rightarrow e\bar{\nu}_e)$ . As we know from the radiative  $J/\Psi$  decays, the coupling  $g^* g^* \rightarrow \eta'$  is much larger then  $g^* g^* \rightarrow \eta, \pi^0$ . For example, the ratio  $BR(J/\Psi \rightarrow \gamma\eta')/BR(J/\Psi \rightarrow \gamma\pi^0)$  is as large as  $10^2$ . In the following, the branching ratio is found to be  $Br(B_c^- \rightarrow \eta' l^- \bar{\nu}) \sim 10^{-4}$  for  $l = \mu, e$ . It is of interests that the common start point suppression factor  $|k_{\eta'}|$  arising from the phase space integration  $d^3k_{\eta'}$  is canceled by the loop functions, so, the distribution  $dBr(B_c \rightarrow \eta' l\bar{\nu})/dE_{\eta'}$  is peaked within the small recoil region of  $\eta'$ . This feather makes the decay recognizable from the mean experimental background  $B_u^- \rightarrow \eta' l\bar{\nu}$ .

This paper is organized as followings. In section 2, we give the details of the calculation of the amplitude and phase space integration. Section 3 is devoted to numerical results and discussions.

## 2 Calculations

To order of  $\alpha_s^4$ , the process  $B_c^- \rightarrow \eta' l^- \bar{\nu}$  is described by 6 diagrams as in Fig.1. It is easy to understand that the heavy quarkonium like  $B_c$  bound state justifies perturbative QCD calculations of the decay. The method outlined years ago for  $J/\Psi \rightarrow \eta' \gamma$  decays by *Korner, et al.*, [7] is appropriate for the present case. However, we would like to adopt an effective Lagrangian approach to avoid introducing  $B_c$  meson wave function. At first, we begin with the sub-amplitude of  $(b\bar{c}) \rightarrow g_a^* g_b^* l \bar{\nu}$ .

$$\begin{aligned} \mathcal{M}(b\bar{c} \rightarrow g_a^* g_b^* l \bar{\nu}) = & \frac{G_F}{\sqrt{2}} V_{cb} g_s^2 Tr[T_a T_b] \bar{v}_c(p_c) \left[ \gamma_\mu (1 - \gamma_5) \frac{i}{\not{p}_b - \not{K} - m_b} \gamma_\beta \frac{i}{\not{p}_b - \not{k}_1 - m_b} \gamma_\alpha \right. \\ & + \gamma_\alpha \frac{i}{\not{k}_1 - \not{p}_c - m_c} \gamma_\beta \frac{i}{\not{K} - \not{p}_c - m_c} \gamma_\mu (1 - \gamma_5) + \gamma_\beta \frac{i}{-\not{p}_c + \not{k}_2 - m_c} \gamma_\mu (1 - \gamma_5) \frac{i}{\not{p}_b - \not{k}_1 - m_b} \gamma_\alpha \left. \right] u_b(p_b) \\ & \times \bar{l} \gamma^\mu (1 - \gamma_5) \nu_l + (\alpha \leftrightarrow \beta, k_1 \leftrightarrow k_2). \quad (1) \end{aligned}$$

Using the identity

$$\gamma^\mu \gamma^\alpha \gamma^\beta = \gamma^\mu g^{\alpha\beta} + \gamma^\beta g^{\mu\alpha} - \gamma^\alpha g^{\mu\beta} - i \epsilon^{\mu\alpha\beta\delta} \gamma_\delta \gamma_5 \quad (2)$$

and Dirac equation  $(\not{p} - m)u(p) = 0$ , after a bit of algebra, we arrive at an effective Lagrangian which has the form

$$\mathcal{A} = \frac{G_F}{\sqrt{2}} V_{cb} g_s^2 Tr[T_a T_b] \bar{c} \gamma_\delta (1 - \gamma_5) b \bar{l} \gamma_\mu (1 - \gamma_5) \nu_l \mathcal{F}^{\delta\mu\alpha\beta} \frac{1}{k_1^2} \frac{1}{k_2^2} \langle g_{a\alpha}^* g_{b\beta}^* | \eta' \rangle. \quad (3)$$

Then we can use the definition

$$\langle 0 | \bar{c} \gamma_\mu (1 - \gamma_5) b | B_c(P) \rangle = i f_{B_c} P_\mu \quad (4)$$

and the  $g_a^* g_b^* \rightarrow \eta'$  coupling

$$\langle g_a^* g_b^* | \eta' \rangle = g_s^2 \delta_{ab} \frac{A_{\eta'}}{k_1 \cdot k_2} \epsilon_{\alpha\beta mn} k_1^m k_2^n \quad (5)$$

which has been widely used in  $\eta'$  and pseudoscalar productions in heavy quarkonium decays and in high energy colliders [6]. Here the parameter  $A_{\eta'}$  is understood as a combination of  $SU(3)$  mixing angles and nonperturbative objects, and can be extracted from the decay  $J/\Psi \rightarrow \eta'\gamma$ . We obtain the total amplitude as

$$\begin{aligned}
\mathcal{M} = & \frac{G_F}{\sqrt{2}} V_{cb} g_s^4 Tr[T_a T_b] \delta_{ab} 4A_{\eta'} i f_{B_c} \bar{l} \gamma^\mu (1 - \gamma_5) \nu_l \\
& \frac{1}{2} \left[ \frac{2P_\mu k_\delta + 2P_\delta K_\mu - 2P \cdot K g_{\mu\delta} + 2i\epsilon_{\mu\rho\delta\sigma} P^\sigma K^\rho + 4M_{B_c} m_b g_{\mu\delta} - 4p_{b\mu} P_\delta}{K^2 - 2K \cdot p_b} \right. \\
& \times \int \frac{d^4 q}{(2\pi)^4} \left[ \frac{1}{k_1^2 - 2p_b \cdot k_1} \left( \frac{k_1^\delta}{k_1^2 k_2^2} - \frac{k_2^\delta}{k_2^2 k_1 \cdot k_2} \right) - \frac{1}{k_2^2 - 2p_b \cdot k_2} \left( \frac{k_1^\delta}{k_1^2 k_1 \cdot k_2} - \frac{k_2^\delta}{k_1^2 k_1^2} \right) \right] \\
& + \frac{2P_\mu k_\delta + 2P_\delta K_\mu - 2P \cdot K g_{\mu\delta} + 2i\epsilon_{\mu\rho\delta\sigma} P^\sigma K^\rho + 4M_{B_c} m_c g_{\mu\delta} - 4p_{c\mu} P_\delta}{K^2 - 2K \cdot p_c} \\
& \times \int \frac{d^4 q}{(2\pi)^4} \left[ \frac{-1}{k_1^2 - 2p_c \cdot k_1} \left( \frac{k_1^\delta}{k_1^2 k_2^2} - \frac{k_2^\delta}{k_2^2 k_1 \cdot k_2} \right) - \frac{1}{k_2^2 - 2p_c \cdot k_2} \left( \frac{k_1^\delta}{k_1^2 k_1 \cdot k_2} - \frac{k_2^\delta}{k_1^2 k_1^2} \right) \right] \\
& + \int \frac{d^4 q}{(2\pi)^4} \frac{2(k_{2\mu} k_1^2 P \cdot k_2 - k_{1\mu} k_2^2 P \cdot k_1)}{k_1^2 k_2^2 k_1 \cdot k_2} \left( \frac{1}{k_1^2 - 2k_1 \cdot p_b} \right) \Big], \tag{6}
\end{aligned}$$

where  $K = k_1 + k_2$  is the momentum of  $\eta'$ ,  $P$  is the momentum of  $B_c$  and  $q$  is the loop momentum with the relation  $2q = k_1 - k_2$ , and the factor  $\frac{1}{2}$  takes into account that both sub-amplitudes have already been symmetrized with respect to the two gluons.

For the heavy  $b$  and  $c$  quarks, it is reasonable to neglect the relative momentum of the quark constituents and their binding energy relative to their masses. In this nonrelativistic limit, the constituents are on mass shell and move together with the same velocity. It implies the following equations valid to good accuracy

$$M(B_c) = m_c + m_b, \quad p_{\bar{c}} = \frac{m_c}{M} P, \quad p_b = \frac{m_b}{M} P. \tag{7}$$

Hence, we have omitted the terms in eq(6) which are proportional to

$$\epsilon^{\mu\nu\alpha\beta} p_{b\mu} p_{\bar{c}\nu} \dots \tag{8}$$

It is straightforward to perform the loop integration in eq(6) using Dimensional Regularization. The amplitude is found to be

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{cb} g_s^4 Tr[T_a T_b] \delta_{ab} 4A_{\eta'} i f_{B_c} \frac{i}{16\pi^2} (P_\mu f_1 + K_\mu f_2) \bar{l} \gamma^\mu (1 - \gamma_5) \nu_l \tag{9}$$

with  $f_1, f_2$  defined by

$$\begin{aligned}
f_1 = & -4C_{11}(K, p_b - K, 0, 0, m_b) + 4C_{12}(K, p_b - K, 0, 0, m_b) \\
& -2C_{11}\left(\frac{K}{2}, \frac{K}{2} - p_b, 0, \frac{m_{\eta'}}{2}, m_b\right) - 2C_{12}\left(\frac{K}{2}, \frac{K}{2} - p_b, 0, \frac{m_{\eta'}}{2}, m_b\right) \\
& -4C_{11}(K, p_c - K, 0, 0, m_c) + 4C_{12}(K, p_c - K, 0, 0, m_c) \\
& +2C_{11}\left(\frac{K}{2}, \frac{K}{2} - p_c, 0, \frac{m_{\eta'}}{2}, m_c\right) + 2C_{12}\left(\frac{K}{2}, \frac{K}{2} - p_c, 0, \frac{m_{\eta'}}{2}, m_c\right) \\
& + \frac{2m_b}{m_c}C_{12}\left(\frac{K}{2}, p_b - K, 0, \frac{m_{\eta'}}{2}, m_b\right) - \frac{2m_c}{m_b}C_{12}\left(\frac{K}{2}, p_c - K, 0, \frac{m_{\eta'}}{2}, m_c\right) \\
& - \frac{2M(m_b - m_c)}{m_b m_c} \left( C_{12}\left(\frac{K}{2} - p_c, P - K, \frac{m_{\eta'}}{2}, m_c, m_b\right) \right. \\
& \left. - \frac{m_c}{M}C_{11}\left(\frac{K}{2} - p_c, P - K, 0, \frac{m_{\eta'}}{2}, m_c, m_b\right) \right), \tag{10}
\end{aligned}$$

and

$$\begin{aligned}
f_2 = & \frac{-4Mm_b}{K^2 - 2p_b \cdot K} \left( 2C_{11}(K, p_b - K, 0, 0, m_b) - C_{12}(K, p_b - K, 0, 0, m_b) + C_{11}\left(\frac{K}{2}, \frac{K}{2} - p_b, 0, \frac{m_{\eta'}}{2}, m_b\right) \right) \\
& + \frac{4Mm_c}{K^2 - 2p_c \cdot K} \left( 2C_{11}(K, p_c - K, 0, 0, m_c) - C_{12}(K, p_c - K, 0, 0, m_c) + C_{11}\left(\frac{K}{2}, \frac{K}{2} - p_c, 0, \frac{m_{\eta'}}{2}, m_c\right) \right) \\
& + \frac{M}{m_c} \left( C_{11}\left(\frac{K}{2}, p_b - K, \frac{m_{\eta'}}{2}, 0, m_b\right) - 2C_{12}\left(\frac{K}{2}, p_b - K, \frac{m_{\eta'}}{2}, 0, m_b\right) + C_0\left(\frac{K}{2}, p_b - K, \frac{m_{\eta'}}{2}, 0, m_b\right) \right) \\
& - \frac{M}{m_b} \left( C_{11}\left(\frac{K}{2}, p_c - K, \frac{m_{\eta'}}{2}, 0, m_c\right) - 2C_{12}\left(\frac{K}{2}, p_c - K, \frac{m_{\eta'}}{2}, 0, m_c\right) + C_0\left(\frac{K}{2}, p_c - K, \frac{m_{\eta'}}{2}, 0, m_c\right) \right) \\
& - \frac{M(m_b - m_c)}{m_b m_c} \left( C_{11}\left(\frac{K}{2} - p_c, P - K, \frac{m_{\eta'}}{2}, m_c, m_b\right) - 2C_{12}\left(\frac{K}{2} - p_c, P - K, \frac{m_{\eta'}}{2}, m_c, m_b\right) \right. \\
& \left. + C_0\left(\frac{K}{2} - p_c, P - K, \frac{m_{\eta'}}{2}, m_c, m_b\right) \right) \tag{11}
\end{aligned}$$

The scalar loop functions and their definitions can be found in ref [8]. The divergences are canceled as they should be.

With eq(9), we get

$$\frac{dB_r(B_c^- \rightarrow \eta' l^- \bar{\nu})}{dE_{\eta'}} = \frac{1}{(2\pi)^5} \frac{1}{16M_{B_c}} 4\pi \sqrt{E_{\eta'}^2 - m_{\eta'}^2} \frac{16\pi}{3} M_{B_c}^2 (E_{\eta'}^2 - m_{\eta'}^2) |f_1 + f_2|^2 C^2 \tau_{B_c}, \tag{12}$$

where

$$C = \frac{8}{3} \alpha_s^2 f_{B_c} A_{\eta'} \frac{G_F}{\sqrt{2}} V_{cb}. \tag{13}$$

### 3 Numerical Results and Discussions

For numerical results, we would take  $\alpha_s = \alpha_s(M_{B_c}) = 0.2$ ,  $V_{cb} = 0.04$ ,  $A_{\eta'} = 0.2$  and  $\tau_{B_c} = 0.46ps$ . The decay constant  $f_{B_c}$  probes the strong(nonperturbative) QCD dynamics which bind  $b$  and  $\bar{c}$  quarks to form the bound state  $B_c$ . It is common wisdom to realize that the size of  $B_c$  would much larger than the size of  $B_{\bar{q}}$ . The size of  $B_c$  scales as  $1/m_c$ , but the size of  $B_{\bar{q}}$  scales as  $1/m_{\bar{q}}$  ( $q = u, d, s$ ). The compact size of  $B_c$  would enhance the importance of its annihilation decays and imply the decay constant  $f_{B_c}$  would much larger than  $f_B$  [3]. In nonrelativistic limit,  $f_{B_c}$  can be related to the value of its wave function at origin [10]. Using the nonrelativistic potential models, Echiten and Quigg [3] estimated

$$f_{B_c} = \begin{cases} 500 \text{ MeV} & \text{(Buchmüller-Tye potential [11])} \\ 512 \text{ MeV} & \text{(power law potential [12])} \\ 479 \text{ MeV} & \text{(logarithmic potential [13])} \\ 687 \text{ MeV} & \text{(cornell potential [14])} \end{cases} \quad (14)$$

For numerical illustrations, we would take  $f_{B_c} = 500 \text{ MeV}$ . The  $\eta'$  momentum distribution is displayed in Fig.2. We find the  $\eta'$  momentum distribution is peaked within small recoiling region of  $\eta'$ . One can expand the scalar functions in  $f_1$  and  $f_2$  in terms of basic scalar functions  $B_0$  and  $C_0$  in [8] and find the factor  $\sqrt{E_{\eta'}^2 - m_{\eta'}^2}(E_{\eta'}^2 - m_{\eta'}^2)$  will be canceled by the loop function. For an example, we expand  $C_{11}(K, p_b - K, 0, 0, m_b)$  as

$$\begin{aligned} C_{11}(K, p_b - K, 0, 0, m_b) &= \frac{1}{2(K^2(p_b - K)^2 - (K \cdot (p_b - K))^2)} \quad (15) \\ &\times \left[ (p_b - K)^2 (B_0(p_b, 0, m_b) - B_0(p_b - K, 0, 0) - K^2 C_0(K, p_b - K, 0, 0, m_b)) + \dots \right] \\ &= -\frac{1}{2m_b^2(E_{\eta'}^2 - m_{\eta'}^2)} \times [\dots]. \end{aligned}$$

Therefore, the  $\eta'$  momentum distribution would behave as

$$\propto \frac{1}{\sqrt{E_{\eta'}^2 - m_{\eta'}^2}}, \quad (16)$$

when  $E_{\eta'}$  is small. The singularity at the start point of the distribution due to the factor in eq(15) is integratable and give finite decay width. Such peculiar property would make the

decay itself recognizable from its mean background  $B_u^- \rightarrow \eta' l \bar{\nu}$  at LHC, especially, when the decay chain  $\eta' \rightarrow \gamma\gamma$  could be used to reconstruct the events in data analysis.

The branching ratio is estimated to be

$$Br(B_c^- \rightarrow \eta' l \bar{\nu}) = 1.6 \times 10^{-4}, \quad (17)$$

which is accessible at LHC. We can extend the estimation to  $Br(B_c \rightarrow \pi^0 l \bar{\nu})$ , if the following relation is valid

$$\frac{Br(B_c \rightarrow \eta' l \bar{\nu})}{Br(B_c \rightarrow \pi^0 l \bar{\nu})} \simeq \frac{Br(J/\Psi \rightarrow \eta' \gamma)}{Br(J/\Psi \rightarrow \pi^0 \gamma)} \simeq 10^2, \quad (18)$$

which is the ratio square of the relative  $\eta'$  and  $\pi^0$  coupling strengths to gluons, when the phase space difference is small due to  $m_{\eta'}$  and  $m_{\pi^0}$ . In this way, we get

$$Br(B_c \rightarrow \pi^0 l \bar{\nu}) \simeq 1.5 \times 10^{-6}. \quad (19)$$

In conclusion, we have presented the first study on the semileptonic annihilation decays  $B_c^- \rightarrow \eta' l \bar{\nu}$ . The  $\eta'$  momentum distribution in the decay is found peaking within the small recoil region of  $\eta'$  because of the loop effects, which is different from the common tree level cases. The branching ratio is estimated to be few times larger than the pure leptonic decays  $B_c^- \rightarrow \mu \bar{\nu}$  [9] and  $10^4$  times larger than  $B_c^- \rightarrow e \bar{\nu}$ . With large samples to be obtained at LHC, the decay could be measured and might be used to extract  $B_c$  decay constant and/or to probe strong and weak interactions.

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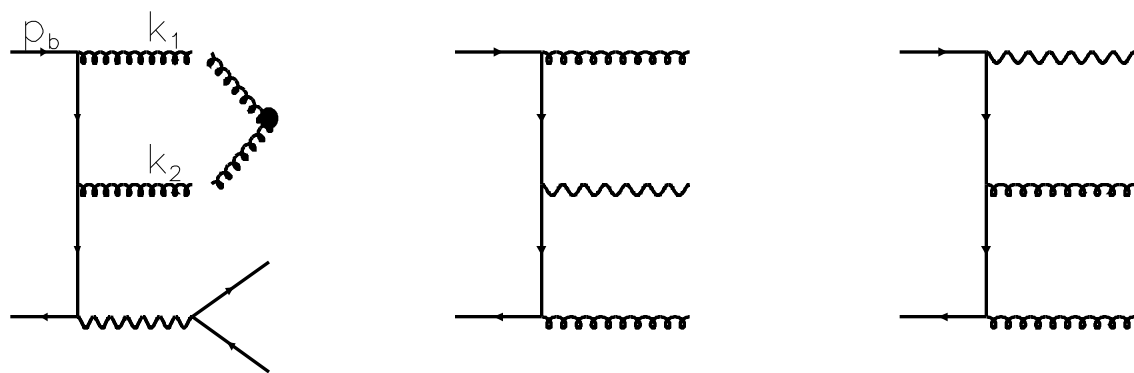
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## Figure Captions

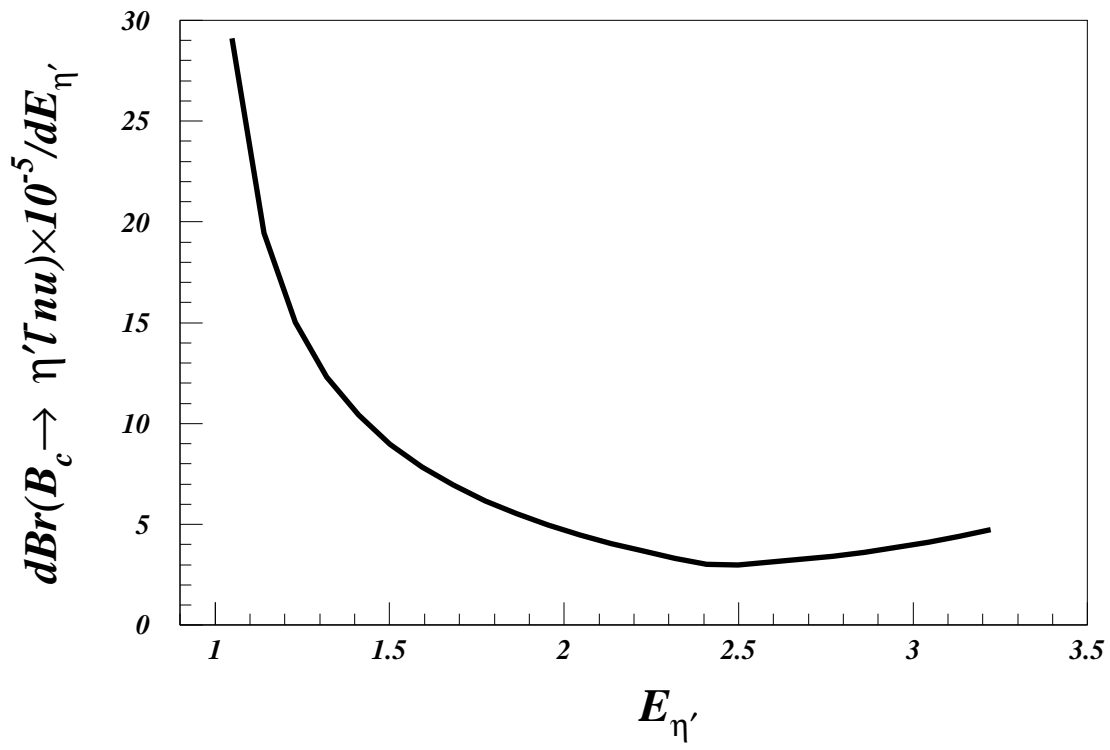
Figure 1: Diagrams for  $B_c^- \rightarrow \eta' l^- \bar{\nu}$  at the leading level. The blob represents  $\eta'$

Figure 2: The distribution of  $dBr(B_c^- \rightarrow \eta' l^- \bar{\nu})/dE_{\eta'}$  as a function of  $E_{\eta'}$



+ permutations

Fig. 1



**Fig.2**