Maximizing Energy Efficiency in Wireless Networks with a Minimum Average Throughput Requirement

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Abstract—Given the growing concern over energy consumption and associated global warming, green communication is becoming more and more important. Lots of efforts have been put into investigating energy efficiency based design in wireless systems. Unfortunately, the maximum energy efficiency of a point-to-point link is normally achieved when the transmit power approaches zero, which, however, is not desirable in practical systems due to the low achievable data rate. In this paper, we consider the energy efficiency optimization with a practical power consumption model, where, besides a maximum transmit power constraint, we set a rate constraint \( r_0 \) to guarantee an average throughput requirement \( (R_{th}) \). Due to the possible outage in transmission, \( r_0 \) is in general different from \( R_{th} \), and determining \( r_0 \) for a given \( R_{th} \) is not trivial. We shall derive a closed-form solution for the optimal transmit power that maximizes energy efficiency. We will also demonstrate that a carefully selected rate constraint \( r_0 \) can guarantee the required average throughput \( R_{th} \) and provide the freedom to achieve different tradeoffs between energy efficiency and average throughput.

Keywords: Green communications, energy efficiency, power model.

I. INTRODUCTION

Given the increasing concern over global warming, green communication is attracting more and more attention. The conventional design approaches may not be applicable to green communications and should be revisited. For instance, the conventional design of a wireless communication system falls into two major categories: 1) maximizing the system throughput with a maximum transmit power constraint; and 2) minimizing the transmit power for a given rate constraint. We shall refer to the above two methods as the maximum-throughput (MaxTH) and minimum-transmit-power (MinTP) approaches, respectively. However, both of these two approaches cannot guarantee an optimal energy efficiency. Design approaches that directly optimize energy efficiency should therefore be adopted.

The idea of maximizing energy efficiency (MaxEE) in wireless communications has been proposed for a long time [1]. The definition of energy efficiency is quite intuitive. It is the ratio between the amount of transmitted data and the total energy consumption. Therefore, the objective of MaxEE is to utilize the least amount of energy to transmit a fixed amount of data. The maximization of energy efficiency with the instantaneous CSI (channel states information) at the transmitter was considered in [2] and [3]. Chong and Jorswieck [4] considered the maximization of energy efficiency over a long-time transmission period based on statistical CSI, where the constant transmit power is used over the whole transmission. This work was generalized in [5] where the optimal transmit power for each time slot is adaptively determined based on the current CSI. The tradeoff between energy efficiency and spectral efficiency was discussed in [6] and [7].

Unfortunately, the transmit power that maximizes energy efficiency may not guarantee a satisfactory average throughput. For instance, the optimal energy efficiency for a point-to-point link, if only considering the signal transmit power, is achieved when the transmit power approaches zero [1], [8]. This indicates that, to achieve the maximum energy efficiency, we should transmit at a very low rate per degree of freedom, which cannot meet the throughput requirement in practical bandlimited systems. Therefore, it is important to find a way to guarantee the required average throughput and furthermore, to control the tradeoff between energy efficiency and the achievable average throughput.

In this paper, we consider maximizing energy efficiency with a practical linear power model, which is widely employed to model the power consumption behavior of both base stations and mobile terminals. To guarantee a given average throughput requirement \( (R_{th}) \), we add a minimum instantaneous rate constraint \( (r_0) \) to the optimization problem, and determine the optimal transmit power that maximizes the energy efficiency. Because of outage, the achieved throughput will be different from the rate constraint. As a result, we need to select \( r_0 \) for a given \( R_{th} \). Note that adjusting \( r_0 \) can provide different tradeoffs between throughput and energy efficiency. We further apply the optimization results to some practical devices, such as macro base stations and sensor nodes. It is shown that, by adding the minimum rate constraint, we can indeed increase the average throughput and achieve a desired tradeoff between throughput and energy efficiency.

II. SYSTEM MODEL AND PERFORMANCE METRIC

The MaxTH and MinTP approaches have their respective application environments. However, neither of them guarantees the most efficient way to utilize energy. Instead, energy efficiency [1] was proposed as a powerful metric in evaluating the efficiency of energy consumption in a communication system. The definition of energy efficiency is given by [9]

\[
\eta_{EE} = \frac{\text{Total amount of data delivered}}{\text{Total energy consumed}},
\]

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where the unit is bits per Joule. Accordingly, the energy efficiency of a transmission over $T$ time slots is given by

$$
\eta_{EE} = \frac{\sum_{i=1}^{T} R_i}{\sum_{i=1}^{T} P_t^i},
$$

(2)

where $R_i$ and $P_t^i$ denote the rate and transmit power at the $i$th time slot, respectively. Note that the above definition includes several different cases. Specifically, $T = 1$ corresponds to the additive white Gaussian noise (AWGN) channel, or the transmission over a fixed channel realization; while $T \to \infty$ can be utilized to evaluate the case over ergodic channels where $\eta_{EE}$ becomes the ratio between the average throughput and the average power consumption. Unfortunately, the optimization for a general $T > 1$ requires future CSI, and thus is impractical. In the following, we will optimize the energy efficiency with only current CSI (i.e., $T = 1$).

Consider a point-to-point wireless communication system with channel gain $g$. The achievable throughput of this link is given by $r(P_t) = \log_2 \left( 1 + \frac{g}{\sigma_n^2} P_t \right)$, where $\sigma_n^2$ denotes the variance of the additive white Gaussian noise and $P_t$ represents the transmit power. The bandwidth-normalized energy efficiency for a given $g$ can be expressed as

$$
\eta_{EE} \triangleq \frac{\log_2 \left( 1 + \frac{g}{\sigma_n^2} P_t \right)}{P_t},
$$

(3)

where the unit is bits/Hz/J.

In practice, the transmit power $P_t$ is only a part of the total power consumption $P_{\text{total}}$. In this paper, we adopt a linear power model $P_{\text{total}} = \alpha P_t + \beta$, which is widely utilized in the literature and standards organizations [10], [11]. Here, $\alpha$ is determined by the transmit efficiency of the amplifier and $\beta$ represents the power utilized for other purposes such as cooling, battery backup, etc. This linear power model can be utilized to model the power consumption of both base stations [10] and mobile devices [11]. By taking the new power model into consideration, we redefine the energy efficiency as

$$
\eta_{EE} \triangleq \frac{\log_2 \left( 1 + \frac{g}{\sigma_n^2} P_t \right)}{\alpha P_t + \beta} = \frac{1}{\alpha} \log_2 \left( 1 + \frac{g}{\sigma_n^2} P_t \right) - \frac{\beta}{\alpha}.
$$

(4)

III. OPTIMIZATION WITH RATE AND POWER CONSTRAINTS

The definition of energy efficiency, though insightful, has two major issues. Firstly, it can be observed from Eq. (4) that when $\beta \to 0$, the optimal power for MaxEE is $P_t \to 0$ [4], [8], which causes a very low throughput. Thus, to guarantee a reasonable throughput, a minimum average throughput requirement is needed. Secondly, the maximum transmit power is normally limited in a practical system, which calls for a maximum transmit power constraint. Therefore, we propose to maximize the energy efficiency with both a minimum average throughput requirement ($R_{th}$) and a maximum transmit power constraint ($P_{\text{max}}$). Thus, the optimization of energy efficiency for a given channel $g$ can be formulated as

$$
\max_{P_t} \eta_{EE} = \frac{1}{\alpha} \log_2 \left( 1 + \frac{g}{\sigma_n^2} P_t \right)
$$

$s.t. \ 0 \leq P_t \leq P_{\text{max}}$

$$
\log_2 \left( 1 + \frac{g}{\sigma_n^2} P_t \right) \geq r_0(R_{th}),
$$

(5)

where the average throughput requirement is satisfied through a rate constraint $r_0$. Note that, although the minimum rate constraint $r_0$ in (5) is independent of $g$, it will be in general different from $R_{th}$. This is due to the outage as explained in the next subsection. We will later show how to determine $r_0$ from a given $R_{th}$. For simplicity, we shall denote $r_0(R_{th})$ as $r_0$.

A. The outage event

The minimum rate constraint and the maximum power constraint are not always consistent with each other. Specifically, for the case

$$
g < g_0 \triangleq \frac{(2^{r_0} - 1)\sigma_n^2}{P_{\text{max}}},
$$

(6)

the maximum transmit power is not able to support the minimum rate constraint, i.e., $r(P_{\text{max}}) < r_0$. Given $g$ is available at the transmitter in our setting, the transmitter will decide not to transmit, i.e., to set $P_t = 0$, when $g < g_0$. We define this as an outage event. Note that our definition of outage is different from the conventional usage, where outage normally represents failure during transmission.

Due to outage, the achieved average throughput may be lower than $r_0$. Hence, setting $r_0$ equal to $R_{th}$ cannot guarantee the average throughput requirement. We need to carefully select the value of $r_0$ based on different system parameters, which will be illustrated in Section IV. Next, we will determine the optimal transmit power.

B. Optimal transmit power

By taking the derivative of $\eta_{EE}$ with respect to $P_t$ and setting the derivative equal to 0, we can get the only critical point as

$$
P_0 = \frac{\beta}{\alpha} - \frac{\sigma_n^2}{g} \left( \int e^{\frac{1}{\alpha} \frac{g}{\sigma_n^2} P_t} \left[ \log_2 \left( 1 + \frac{g}{\sigma_n^2} P_t \right) \right] - \frac{\sigma_n^2}{g} \right),
$$

(7)

where $W(\cdot)$ is the lambert W function. Given that the numerator and denominator of the first term on the right hand side of (4) are concave and convex functions, respectively, we know that energy efficiency is a quasi-concave function with respect to the transmit power $P_t$. As a result, if we define the minimum transmit power that can satisfy the minimum rate constraint as $P_{\text{min}} \triangleq \frac{(2^{r_0} - 1)\sigma_n^2}{g}$, then the optimal transmit power can be summarized as

$$
P_t^* = \begin{cases} 
P_{\text{min}} \text{ when } P_0 \leq P_{\text{min}} \\
\frac{\beta}{\alpha} \text{ when } P_{\text{min}} < P_0 < P_{\text{max}} \\
P_{\text{max}} \text{ when } P_0 \geq P_{\text{max}}
\end{cases}
$$

(8)
Next, we will specify detailed conditions for $P_t^*$ to take different values. This will provide insights on the impact of different system parameters and can be used for performance evaluation in the next section. Due to space limitation, the derivation is omitted. The results are as follows.

- **Case 1**: If $\frac{\beta}{P_{\text{max}}} > \frac{2^\frac{\beta}{\alpha} \ln 2}{2^\frac{\beta}{\alpha} - 1} - 1$, then the optimal transmit power is given by
  \[
  P_t^* = \begin{cases} 
  0 & g < g_0 \\
  P_{\text{max}} & g_0 \leq g < g_1 \\
  P_0 & g \geq g_1 
  \end{cases}
  \] (9)

  where $g_1$ is the threshold value given by
  \[
  g_1 \triangleq -\frac{\sigma_n^2}{P_{\text{max}}^2} \left( 1 + \frac{\beta}{\alpha} \right) \ln \left( 1 + \frac{\beta}{\alpha} \right) \frac{1}{W} \left( -\frac{1}{e} e^{-\frac{\beta}{\alpha} \frac{P_{\text{max}}}{P_0} \ln 2} \right) - \frac{\sigma_n^2}{P_{\text{max}}^2}. \] (10)

  For this case, $P_t^*$ can only take value $P_{\text{max}}$ or $P_0$.

- **Case 2**: If $\frac{\beta}{P_{\text{max}}} < \frac{2^\frac{\beta}{\alpha} \ln 2}{2^\frac{\beta}{\alpha} - 1} - 1$, then the optimal transmit power is given by
  \[
  P_t^* = \begin{cases} 
  0 & g < g_0 \\
  P_{\text{min}} & g_0 \leq g < g_2 \\
  P_0 & g \geq g_2 
  \end{cases}
  \] (11)

  where $g_2$ is the threshold value given by
  \[
  g_2 \triangleq \frac{2^\frac{\beta}{\alpha} \ln 2 - (2^\frac{\beta}{\alpha} - 1)}{\frac{\beta}{\alpha} / \sigma_n^2}. \] (12)

  For this case, $P_t^*$ can only take value $P_{\text{min}}$ or $P_0$.

- **Case 3**: If $\frac{\beta}{P_{\text{max}}} = \frac{2^\frac{\beta}{\alpha} \ln 2}{2^\frac{\beta}{\alpha} - 1} - 1$, then the optimal transmit power is given by
  \[
  P_t^* = \begin{cases} 
  0 & g < g_0 \\
  P_0 & g \geq g_0 
  \end{cases}
  \] (13)

  For this case, $P_t^*$ will be between $P_{\text{min}}$ and $P_{\text{max}}$.

It can be observed that the optimal solution for the non-outage case can only take on a value from the set \{ $P_{\text{min}}, P_0, P_{\text{max}}$ \}, which corresponds to the transmit power for the minimum rate constraint point, the critical point, and the maximum transmit power point, respectively, as shown in Fig. 1. Next, we will consider the effects of the power model on the optimal transmit power.

1) $\beta \to \infty$: It can be observed from the above solution, that when $\beta$ is large, $P_t^*$ falls in Case 1. Specifically, it takes a value from \{ $0, P_0, P_{\text{max}}$ \}. This can be explained by looking at the relationship between $P_{\text{min}}, P_0$ and $P_{\text{max}}$ in Fig. 1. In particular, as $\beta \to \infty$, the point $-\frac{\beta}{\alpha}$ approaches $-\infty$. As a result, the critical point B, which corresponds to $P_0$, moves along the logarithmic curve towards infinity. By doing so, $P_0$ will move from the left side of $P_{\text{max}}$ to the right side of $P_{\text{max}}$. This justifies why $P_t^* \to P_{\text{max}}$ when $\beta \to \infty$.

2) $\beta \to 0$: If $\beta$ is small, $P_t^*$ falls in Case 2. Specifically, it takes a value from \{ $0, P_{\text{min}}, P_0$ \}. It can be observed from Fig. 1 that, as $\beta \to 0$, the point $-\frac{\beta}{\alpha}$ approaches 0 from the left side. As a result, the critical point B moves towards (0,0). In this case, $P_0$ will move from the right side of $P_{\text{min}}$ to the left side of $P_{\text{min}}$. This explains why as $\beta \to 0$, we have $P_t^* \to P_{\text{min}}$.

IV. ACHIEVING THE AVERAGE THROUGHPUT REQUIREMENT THROUHG $R_0$

In this section, we consider how to achieve the minimum average throughput requirement $R_0$ by setting $r_0$.

A. Energy efficiency and average throughput

We first determine the achievable energy efficiency and average throughput for a given $r_0$. Specifically, we consider an ergodic setting and the energy efficiency is given by

\[
\tilde{\eta}_{\text{EE}} = \frac{1}{\alpha} \frac{R}{P_t + \beta / \alpha}. \] (14)

The average throughput $\tilde{R}$ and average transmit power $\tilde{P}_t$ can be determined for the three different cases in the solution of $P_t^*$.

For Case 1, we can obtain

\[
\tilde{R}^{C_1} = \int_{g_0}^{g_1} \log_2 \left( 1 + \frac{x}{\sigma_n^2} P_{\text{max}} \right) f(x) dx \] (15)

\[
+ \log_2(e) \int_{g_1}^{\infty} \left( 1 + W \left[ \frac{1}{e} \left( \frac{x}{\sigma_n^2} \beta - 1 \right) \right] \right) f(x) dx
\]

\[
\tilde{P}_t^{C_1} = \int_{g_0}^{g_1} f(x) dx \] (16)

where $f(x)$ is the pdf of the channel gain $g$.

For Case 2, we can get

\[
\tilde{R}^{C_2} = r_0 \int_{g_2}^{g_0} f(x) dx + \log_2(e)
\]

\[
\times \int_{g_2}^{\infty} \left( 1 + W \left[ \frac{1}{e} \left( \frac{x}{\sigma_n^2} \beta - 1 \right) \right] \right) f(x) dx
\]

\[
\times \left( 1 + \left( \frac{x}{\sigma_n^2} \beta - 1 \right) \right) f(x) dx
\]
\[
\bar{P}_t^{C_2} = \int^{y_2}_{y_0} \frac{2^{y_2} - 1}{y_2} \sigma_n^2 f(x) dx 
\]

\[
+ \int^{\infty}_{y_2} \left[ \frac{\beta/\alpha - \sigma_n^2}{\frac{y_2}{\frac{\beta}{\alpha} - 1}} - \frac{\sigma_n^2}{x} \right] f(x) dx.
\]

Similarly, we can determine the results for Case 3 as
\[
\bar{R}_t^{C_3} = \log_2 \left( e \right) \int^{y_2}_{y_0} \left( 1 + W \left[ \frac{1}{e} \left( \frac{x}{\sigma_n^2} - \frac{\beta}{\alpha} - 1 \right) \right] \right) f(x) dx
\]

\[
\bar{P}_t^{C_3} = \int^{\infty}_{y_0} \left[ \frac{\beta/\alpha - \sigma_n^2}{\frac{y_2}{\frac{\beta}{\alpha} - 1}} - \frac{\sigma_n^2}{x} \right] f(x) dx.
\]

### B. Achieving \( R_{th} \) by setting \( r_0 \)

In this subsection, we investigate whether there is a \( r_0 \) which can guarantee that the achievable average throughput is greater or equal to \( R_{th} \).

First, let \( y_0^0 \) denote the solution of the equation \( \frac{\beta/\alpha}{\sigma_n^2} = 2^{\alpha_0} \ln \frac{2}{\ln 2} - 1 \). Then Cases 1 and 2 correspond to the conditions \( y_0 < y_0^0 \) and \( y_0 > y_0^0 \), respectively. We then consider these two cases separately. In the following discussion, we will assume Rayleigh fading with distribution \( f(x) = e^{-\frac{x}{\sigma}} \), where \( L \) accounts for the path loss effects.

For Case 1 \((0 \leq y_0 < y_0^0)\), we have
\[
\frac{d\bar{R}_t^{C_1}}{dr_0} = -r_0 \frac{2^{\alpha_0} \ln 2}{\bar{R}_t^{C_1}} e^{-\frac{(2^{\alpha_0} - 1)\sigma_n^2}{r_0 \max}} \leq 0.
\]

Thus the average throughput decreases when we increase \( r_0 \), i.e., the maximum average throughput is obtained at \( r_0 = 0 \). This means that setting a nonzero \( r_0 \) is not desirable for this case, as it will reduce the average throughput. Note that \( r_0 = 0 \) does not mean zero throughput, and the achievable average throughput can be calculated by (15).

For Case 2 \((r_0 > y_0^0)\), by taking derivative of \( \bar{R} \) with respect to \( r_0 \), we get
\[
\frac{d\bar{R}_t^{C_2}}{dr_0} = Le^{-\frac{y_2}{\sigma_n^2}} - Le^{-\frac{y_2}{\sigma_n^2}} + r_0 g_2 e^{-\frac{y_2}{\sigma_n^2}} - r_0 g_1 e^{-\frac{y_2}{\sigma_n^2}} - \log_2 \left( e \right) \left[ 1 + W \left[ \frac{1}{e} \left( \frac{2^{\alpha_0} y_0 \ln 2 - 2^{\alpha_0}}{2^{\alpha_0}} \right) \right] \right] g' \leq 0
\]

where \( g_1' = \frac{2^{\alpha_0} \ln 2}{\bar{R}_t^{C_1}} \) and \( g_2' = \frac{2^{\alpha_0} \ln 2}{\bar{R}_t^{C_1}} \) are the derivatives of \( g_1 \) and \( g_2 \) with respect to \( r_0 \).

1. If \( \frac{d\bar{R}_t^{C_2}}{dr_0} \leq 0 \), then the average throughput will be a decreasing function of \( r_0 \) for all \( r_0 \geq 0 \), and setting \( r_0 > 0 \) will not increase the average throughput. Therefore, the maximum average throughput is obtained at \( r_0 = 0 \). If the average throughput at \( r_0 = 0 \) is no smaller than \( R_{th} \), then we can find a \( r_0 \) to guarantee the throughput requirement.
2. If there exists \( r_0 \) satisfying \( \frac{d\bar{R}_t^{C_2}}{dr_0} > 0 \), there must be at least one critical point \( r_0^* \) with \( \frac{d\bar{R}_t^{C_2}}{dr_0} \bigg|_{r_0=r_0^*} = 0 \). As \( \lim_{r_0 \to \infty} \frac{d\bar{R}_t^{C_2}}{dr_0} = 0^- \), thus, the local maximum average throughput \( \bar{R}_t^{C_2} \bigg|_{r_0=r_0^*} \) can be achieved at \( r_0^* \), where \( r_0^* \) is the solution of
\[
\frac{d\bar{R}_t^{C_2}}{dr_0} = 0, \quad \frac{d^2\bar{R}_t^{C_2}}{dr_0^2} < 0.
\]

Thus, if \( \bar{R}_t^{C_2} \bigg|_{r_0=r_0^*} > \bar{R}_t^{C_1} \bigg|_{r_0=0} \), then the average throughput can be increased by setting \( r_0 \), and we can compare the maximum achieved throughput with \( R_{th} \) to check whether the requirement can be obtained.

The above steps can be used to determine the region where setting a rate constraint \( r_0 \) can increase the average throughput. Furthermore, they can help check whether we can find a \( r_0 \) to guarantee the minimum average throughput requirement. The actual selection of \( r_0 \) will be based on the achieved energy efficiency and average throughput tradeoff curve, which will be demonstrated in the next section.

### V. Numerical Results

#### A. Effective region for practical devices

In this section, we evaluate the effects of setting a minimum rate constraint on increasing the average throughput by considering several practical devices in GSM networks and wireless sensor networks (WSN). Based on the results in (22), we first determine the region where setting a rate constraint can increase the average throughput, which is referred to as the effective region in Fig. 2. It can be observed that the effective region is a function of the pathloss.

Next, we will consider two practical devices in GSM and WSN networks and identify their positions in Fig. 2. Based on the parameters in [10], we determine \( \alpha \) and \( \beta \) for a GSM base station as 4.03 and 77.25, respectively. The path loss exponent is set to be \( \gamma = 4 \) for the exponential path loss model. The carrier frequency is assumed to be 950MHz, and the maximum transmit power for one base station is 10W. The noise variance is \( N_0B = kT_b \), where \( k \) is the Boltzman constant, \( T \) is
assumed to be 290K and the bandwidth is 200kHz. From Fig. 2, it can be observed that the parameters of the base station fall in the ineffective region, indicating that adding a minimum rate constraint will not help achieve a higher throughput.

We also consider a sensor node in a wireless sensor network, whose power model parameters are listed in [11]. Based on these parameters, we can determine $\alpha = 2.86$ and $\beta = 0.0982$. In the simulation, we set the maximum transmit power constraint as 500mW. Note that this is the maximum transmit power constraint and the average transmit power with MaxEE is much less than this. The transmission distance is 100m, the carrier frequency is 2.5GHz and the bandwidth is 10kHz. The path loss model is assumed to be the same as that for the GSM networks. It is observed from Fig. 2 that setting a proper rate constraint will increase the average throughput for the concerned sensor node.

B. Selecting $r_0$ for a given $R_{th}$

The achievable tradeoff between energy efficiency and average throughput for the proposed MaxEE approach is shown in Fig. 3 for the sensor node. It is shown that, by introducing the rate constraint, we can control the tradeoff between throughput and energy efficiency for different applications. In particular, for a given $R_{th}$, we can determine the maximum energy efficiency from the tradeoff curve in Fig. 3. Then, the minimum rate constraint $r_0$ that helps to achieve $R_{th}$ can be determined from the average throughput performance of MaxEE in Fig. 4. Note that the tradeoff curve in Fig. 3 will be different for different types of devices, and this tradeoff is different from the spectrum efficiency and energy efficiency tradeoff in [7], where no outage due to the transmit power/rate constraint was considered.

VI. CONCLUSIONS

In this paper, we considered the maximization of energy efficiency with both a minimum average throughput requirement and a maximum transmit power constraint. It was shown that the average throughput can be increased by adjusting the minimum rate constraint, where different tradeoffs between average throughput and energy efficiency can be obtained. Accordingly, we can achieve a given minimum average throughput requirement by selecting an appropriate minimum rate constraint. This was verified by applying the proposed optimization to practical macro base stations and sensor devices.

REFERENCES