Independent Component Analysis Based Semi-Blind I/Q Imbalance Compensation for MIMO OFDM Systems

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Abstract—We propose a novel semi-blind compensation scheme for both frequency-dependent and frequency-independent I/Q imbalance based on independent component analysis (ICA) in multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) systems, where ICA is applied to compensate for I/Q imbalance and equalize the received signal jointly, without any spectral overhead. A reference signal is embedded in the transmitted signal with little power consumption and no spectral overhead introduced, to enable ambiguity elimination for the ICA output signal at the receiver. Moreover, channel interpolation is incorporated with layered space frequency equalization (LSFE) to enhance the system performance. Simulation results show that the proposed implicit compensation scheme can not only provide a better bit error rate (BER) performance and a higher bandwidth efficiency than the previous training based I/Q imbalance compensation method, but also outperform the ideal case with perfect channel state information (CSI) and no I/Q imbalance, due to additional frequency diversity.

Index Terms—I/Q imbalance, independent component analysis (ICA), orthogonal frequency division multiplexing (OFDM), multiple-input multiple-output (MIMO).

I. INTRODUCTION

I/Q imbalance is one of the typical radio frequency (RF) circuit impairments in direct conversion architecture (DCA) for multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) wireless communication systems [1]–[3]. There are two types of I/Q imbalance: 1) frequency independent I/Q imbalance, which is caused by non-ideal local oscillators and is constant over the whole signal bandwidth; 2) frequency dependent I/Q imbalance, which is due to the component mismatching in I and Q branches and is naturally frequency selective. Compensation approaches for I/Q imbalance have been studied in the literature [2]–[11]. A least squares approach as well as a pre-distorter were developed in [2] for the compensation in the digital domain. In [3], a compensation scheme based on a modified periodic pilot (MPP) was proposed to remove the effect of I/Q imbalance in the time domain. An improved approach based on the generalized periodic pilot (GPP) was then proposed to reduce the computational complexity [5]. A hybrid time-domain and frequency-domain compensation scheme was proposed in [6]. However, the above approaches [3]–[6] would enhance the noise power during the interference cancellation. And more importantly, these approaches do not exploit the frequency diversity induced by I/Q imbalance.

For MIMO systems, Zou et al. [7] proposed both training based and blind compensation approaches for I/Q imbalance at the transmitter and the receiver. However, these approaches can only be used in a MIMO system with space time block coding (STBC), and only the impairment of frequency-independent I/Q imbalance was considered. A two-step compensation approach was proposed in [8], using preamble and adaptive filtering. However, preamble reduces the spectral efficiency of the system due to the training overhead.

Blind compensation approaches have the advantage of increased spectral efficiency without extra bandwidth for training. A blind multichannel deconvolution algorithm and the equivalent adaptive separation via independence were proposed respectively in [9] and [10] to reject image signal based on a general frequency-dependent signal model. In [11], two blind I/Q imbalance compensation approaches were proposed by exploiting the circularity of the baseband signal. Nevertheless, the above blind compensation approaches tackled only the I/Q imbalance at the receiver.

In this letter, we propose a novel semi-blind compensation scheme for I/Q imbalance in a MIMO OFDM system using quadrature phase shift keying (QPSK) modulation, with the aid of independent component analysis (ICA). Our work is different in the following aspects. First, the proposed semi-blind compensation scheme mitigates frequency-independent and frequency-dependent I/Q imbalance at both the transmitter and the receiver. Second, I/Q imbalance is compensated for implicitly via semi-blind equalization to exploit the frequency diversity gain induced by I/Q imbalance, while there was an explicit process of I/Q imbalance compensation in the previous works [3]–[6]. Accordingly, ICA [12], an efficient higher order statistics (HOS) based blind source separation technique, is applied to compensate for I/Q imbalance and equalize the received signal in the frequency domain jointly, by exploiting the algebraic structure and the statistic characteristics of the received signal. A reference signal is embedded in the transmitted signal with very low power to enable resolution of the ambiguity in the ICA output signal. Unlike the training based method [3], the proposed scheme does not cause any loss of transmission rate, and change of the total transmission power. To further improve the performance, the ICA based compensation approach is incorporated with channel interpolation and layered space frequency equalization (LSFE). Simulation results show that the proposed semi-blind implicit compensation scheme not only can provide a better
bit error rate (BER) performance and a higher bandwidth efficiency than the training based MPP method [3], due to additional frequency diversity, but also can outperform the ideal case with perfect channel state information (CSI) and no I/Q imbalance, in the tested scenarios.

The system model is presented in Section II. The ICA based semi-blind I/Q imbalance compensation approach is proposed in Section III. Channel interpolation and LSFE are incorporated with ICA in Section IV. Section V provides a complexity analysis. Simulation results are shown in Section VI. Section VII draws the conclusions.

Throughout the letter, we use superscripts $^*$, $T$ and $H$ to denote the complex conjugate, transpose, and complex conjugate transpose, respectively. Let $[\cdot]^i_k$ denote modulo-$K$, $[\cdot]^+$ pseudo-inverse, $\mathcal{F}$ Fourier transform, and sign($\cdot$) sign function, respectively. We use regular letters and bold letters to represent scalars and vectors/matrices. The “blackboard” letter denotes a combination of signals on subcarrier $k$ and its mirror subcarrier $k^*$, e.g., $\tilde{S}(k,i) = [\mathbf{S}^T(k,i), \mathbf{S}^T(k,i)^T]$, where the calligraphic letter represents a combination of real and imaginary parts of the signal, e.g., $S(k,i) = [\Re\mathcal{F}[\mathbf{S}^T(k,i)], \Im\mathcal{F}[\mathbf{S}^T(k,i)]]^T$. $\tilde{S}$ denotes the source signal $S$ with I/Q imbalance, while $S$, $\bar{S}$ and $\hat{S}$ denote the estimates of $S$ after ICA, ambiguity elimination and LSFE. $\bar{G}$ and $\hat{G}$ represent estimates of the equivalent mixing matrix before and after channel interpolation, respectively.

II. SYSTEM MODEL

We consider a MIMO OFDM spatial multiplexing system with $K$ subcarriers, $N_t$ transmit antennas and $N_r$ receive antennas, as depicted in Fig. 1. Let $\mathbf{D}(k,i) = [D_0(k,i), \ldots, D_{N_s-1}(k,i)]^T$ denote the unit variance source data vector of length $N_t$ on subcarrier $k$, where $k$ and $i$ are the subcarrier index and the OFDM symbol index, respectively. In order to enable elimination of ambiguity caused by blind compensation/equalization at the receiver, a reference data vector $\mathbf{D}_{ref}(k,i) = [D_{ref,0}(k,i), \ldots, D_{ref,N_s-1}(k,i)]^T$ is superimposed to the source data vector $\mathbf{D}(k,i)$. The reference data vector $\mathbf{D}_{ref}(k,i)$ is a unit variance random vector of length $N_t$, and is independent of the source data vector $\mathbf{D}(k,i)$, where the entries of $\mathbf{D}_{ref}(k,i)$ are independent and identically distributed (i.i.d) and have the same discrete probability distribution as the entries of $\mathbf{D}(k,i)$. The resulting transmit signal vector $\mathbf{S}(k,i)$ is given by

$$\mathbf{S}(k,i) = \frac{1}{\sqrt{1 + a^2}} \mathbf{D}(k,i) + a \mathbf{D}_{ref}(k,i)$$

where $\mathbf{S}(k,i) = [S_0(k,i), \ldots, S_{N_s-1}(k,i)]^T$ with $S_n(k,i)$ denoting the signal on subcarrier $k$ in the $i$-th symbol of the block transmitted by the $n$-th antenna in the frequency domain. The constant $a$ ($0 \leq a \leq 1$) in (1) gives a tradeoff on the transmit power allocation between the source data $\mathbf{D}(k,i)$ and the reference data $\mathbf{D}_{ref}(k,i)$. By properly choosing the value of $a$, the reference signal requires little power consumption. A cyclic prefix (CP) of length $L_{cp}$ is added at the beginning of each OFDM symbol, to avoid inter-symbol interference (ISI). The channel is assumed to be quasi-static fading, i.e., the CSI remains constant for a frame duration of $N_s$ OFDM symbols.

We consider the I/Q imbalance at both the transmitter and the receiver. At the transmitter, the frequency-independent I/Q imbalance for each source data stream $s_n(t,i)$ is characterized by the amplitude mismatch $\alpha_n$ and phase mismatch $\phi_n$ due to the up-conversion local oscillator signal mismatches between the I branch and Q branch. The frequency-dependent I/Q imbalance is described by two mismatch low-pass filters with frequency responses $X_{I,n}(f)$ and $X_{Q,n}(f)$ for the I and Q signal branches, respectively. Following [6] and [7], we define

$$x_{1,n}(t) = \frac{1}{2} F^{-1}\{X_{I,n}(f) + \alpha_n e^{j\phi_n} X_{Q,n}(f)\}$$

$$x_{2,n}(t) = \frac{1}{2} F^{-1}\{X_{I,n}(f) - \alpha_n e^{-j\phi_n} X_{Q,n}(f)\}$$

for each transmit antenna $n$. Then, define $x_{1,n} = [x_{1,n,0}, \ldots, x_{1,n,L_{cp}-1}]^T$ and $x_{2,n} = [x_{2,n,0}, \ldots, x_{2,n,L_{cp}-1}]^T$ as the coefficients vectors of the equivalent finite impulse response (FIR) filters for $x_{1,n}(t)$ and $x_{2,n}(t)$ with $L_{x_1}$ and $L_{x_2}$ denoting the lengths of $x_{1,n}$ and $x_{2,n}$, respectively. The equivalent transmit symbol vector $\mathbf{S}(k,i)$ on subcarrier $k$ is given by

$$\mathbf{S}(k,i) = \mathbf{X}_1(k)\mathbf{S}(k,i) + \mathbf{X}_2(k)\mathbf{S}^*(k,i)$$

where $\mathbf{X}_1(k)$ and $\mathbf{X}_2(k)$ are the diagonal matrices $\mathbf{X}_1(k) = \text{diag}\{X_{1,0}(k), \ldots, X_{1,N_t-1}(k)\}$ and $\mathbf{X}_2(k) = \text{diag}\{X_{2,0}(k), \ldots, X_{2,N_t-1}(k)\}$ are the frequency responses of $x_{1,n}$ and $x_{2,n}$, respectively. Let $\beta_n$ and $\gamma_n$ denote respectively the amplitude mismatch and phase mismatch at the RF front-end of receive antenna $n$. Let $Y_{1,m}(f)$ and $Y_{Q,m}(f)$ denote the frequency responses of the I/Q branch filters at receive antenna $m$. We define [6], [7]

$$y_{1,m}(t) = \frac{1}{2} F^{-1}\{Y_{1,m}(f) + \beta_m e^{-j\psi_n} Y_{Q,m}(f)\}$$

$$y_{2,m}(t) = \frac{1}{2} F^{-1}\{Y_{1,m}(f) - \beta_m e^{j\psi_n} Y_{Q,m}(f)\}$$

Then, $y_{1,m} = [y_{1,m,0}, \ldots, y_{1,m,L_{y_1}-1}]$ and $y_{2,m} = [y_{2,m,0}, \ldots, y_{2,m,L_{y_2}-1}]$ are the corresponding discrete-time representations of $y_{1,m}(t)$ and $y_{2,m}(t)$ with $L_{y_1}$ and $L_{y_2}$ denoting the lengths of $y_{1,m}$ and $y_{2,m}$, respectively. Assume that the CP length $L_{cp}$ is no less than the total length of the transmitter filter, receiver filter, and channel, i.e. $L_{cp} \geq \max\{L_{x_1}, L_{x_2}\} + \max\{L_{y_1}, L_{y_2}\}$, where $L_e$ is the maximum channel memory. The received OFDM symbol $\mathbf{R}(k,i) = [R_0(k,i), \ldots, R_{N_r-1}(k,i)]^T$ on subcarrier $k$ is given by

$$\mathbf{R}(k,i) = \mathbf{Y}_1(k)\mathbf{H}(k)\mathbf{S}(k,i) + \mathbf{Y}_2(k)\mathbf{H}^*(k)\mathbf{S}^*(k,i) + \mathbf{Z}(k,i)$$

where $\mathbf{H}(k)$ is the $(N_r \times N_t)$ channel frequency response matrix on subcarrier $k$. $\mathbf{Z}(k,i)$ is the complex-value-added white Gaussian noise (AWGN) vector whose entries have zero mean and variance $\sigma^2$. The diagonal matrices $\mathbf{Y}_1(k) = \text{diag}\{Y_{1,0}(k), \ldots, Y_{1,N_t-1}(k)\}$ and $\mathbf{Y}_2(k) = \text{diag}\{Y_{2,0}(k), \ldots, Y_{2,N_t-1}(k)\}$ account for I/Q imbalance on the receiver. Using (4), $\mathbf{R}(k,i)$ can be written as

$$\mathbf{R}(k,i) = \mathbf{G}_1(k)\mathbf{S}(k,i) + \mathbf{G}_2(k)\mathbf{S}^*(k,i) + \mathbf{Z}(k,i)$$
been proposed in the literature to eliminate this mirror interference. I/Q imbalance compensation approaches [3]–[6] have been used to exploit the frequency diversity. Moreover, they did not exploit the noise power. More importantly, they did not exploit the frequency diversity induced by I/Q imbalance.

Fig. 1. System model of the MIMO OFDM system with ICA based semi-blind receivers.

\[
\begin{align*}
G_1(k) &= Y_1(k)H(k)X_1(k) + Y_2(k)H^*(\hat{k})X_2^*(\hat{k}) \quad (8) \\
G_2(k) &= Y_1(k)H(k)X_2(k) + Y_2(k)H^*(\hat{k})X_1^*(\hat{k}) \quad (9)
\end{align*}
\]

According to (7), the received signal consists of the desired signal \(G_1(k)S(k,i)\), and the mirror interference \(G_2(k)S^*(\hat{k},i)\), which degrades the system performance severely. I/Q imbalance compensation approaches [3]–[6] have been proposed in the literature to eliminate this mirror interference. However, these approaches would enhance the noise power. More importantly, they did not exploit the frequency diversity induced by I/Q imbalance.

III. ICA BASED SEMI-BLIND COMPENSATION FOR I/Q IMBALANCE

In order to improve the system performance by exploiting frequency diversity and to avoid enhancement of the noise power, we propose an ICA based semi-blind I/Q imbalance compensation approach, where I/Q imbalance is implicitly included in an equivalent MIMO system. The ICA based semi-blind equalization is then used to recover the source data in the equivalent system. Thus, I/Q imbalance compensation is embedded in the ICA based equalization. This is different from conventional I/Q imbalance compensation approaches [3]–[6], where there is an explicit process of I/Q imbalance compensation to suppress the interference introduced by I/Q imbalance. The advantage of the proposed implicit scheme over conventional approaches is that I/Q imbalance is exploited to achieve the frequency diversity, which results in performance improvement as shown by the simulation results in Section VI. After the ICA based equalization/compensation as described in Subsection III-A, ambiguity in the ICA output signal is resolved, as described in Subsection III-B.

A. I/Q Imbalance Compensation

The ICA based I/Q imbalance compensation for the received signal \(R(k,i)\) in (7) can be classified into two cases.

Case 1: For subcarriers \(1 \leq k \leq (K/2 - 1)\), due to the transmitter and receiver I/Q imbalance, the received signal \(R(k,i)\) is the superposition of the transmit signals on subcarrier \(k\) and its mirror subcarrier \(\hat{k} = K - k\). Thus, we have

\[
\begin{bmatrix}
R(k,i) \\
R^*(\hat{k},i)
\end{bmatrix} = \begin{bmatrix}
G_1(k) & G_2(k)
\end{bmatrix} \begin{bmatrix}
S(k,i)
\end{bmatrix} + \begin{bmatrix}
Z(k,i)
\end{bmatrix}. \quad (10)
\]

Define \(S(k,i) = [S^T(k,i), S^{*T}(\hat{k},i)]^T\), \(R(k,i) = [R^T(k,i), R^{*T}(\hat{k},i)]^T\), \(Z(k,i) = [Z^T(k,i), Z^{*T}(\hat{k},i)]^T\) and \(G(k) = [G_1(k), G_2(k)]\). Then (10) becomes

\[
\mathbb{R}(k,i) = G(k)S(k,i) + Z(k,i). \quad (11)
\]

According to (10) and (11), for the original I/Q imbalance embedded \((N_t \times N_r)\) OFDM system on subcarriers \(k = 1, \ldots, (K/2 - 1)\) and \(\hat{k} = (K/2 + 1), \ldots, (K - 1)\), we can derive an equivalent \((2N_t \times 2N_r)\) system of \((K/2 - 1)\) pairs of subcarriers without I/Q imbalance. The mixing matrix \(G(k)\) in (11) corresponds to the equivalent channel gain matrix. Therefore, I/Q compensation can be accomplished implicitly via equalization for the equivalent MIMO system on each pair of subcarriers. Since the transmit signal on subcarrier \(k\) is received on both subcarrier \(k\) and its mirror subcarrier \(\hat{k}\), due to I/Q imbalance, the equivalent \((2N_t \times 2N_r)\) system of \((K/2 - 1)\) pairs of subcarriers can obtain an additional frequency diversity [13] of up to 2, compared to an \((N_t \times N_r)\) system without I/Q imbalance.

In order to recover \(\hat{S}(k,i)\) from (11), we employ ICA [12], an efficient HOS based blind source separation technique without any extra bandwidth needed for training. ICA maximizes the independence of the output signal, where the independence is measured by non-Gaussianity. In (11), the received signal \(\mathbb{R}(k,i)\) is a linear mixture of input signal \(S(k,i)\). Since \(S(k,i)\) is i.i.d. and non-Gaussian, ICA can be applied on \(\mathbb{R}(k,i)\) to obtain the estimate of the source signal as \(\hat{S}(k,i) = [S^T(k,i), S^{*T}(\hat{k},i)]^T\). In this letter, we...
employ JADE [12, 14], a numerical ICA algorithm which has been applied to MIMO OFDM systems for blind equalization [15]–[17], to perform I/Q imbalance compensation and spatial equalization jointly. JADE is based on joint diagonalization of cumulant matrices of the received signal \( \mathbb{R}(k, i) \), and thus requires shorter data sequences for block processing than other ICA numerical algorithms. First, the received signal \( \mathbb{R}(k, i) \) is whitened. Then Jacobi algorithm [14] is used to obtain the estimates of source symbols by diagonalizing the cumulant matrices of the received signal \( \mathbb{R}(k, i) \) jointly. Notice that we use ICA to handle subcarriers 1 to \((K/2 - 1)\) only. Equation (11) with \( k \geq (K/2 + 1) \) is the mirror of itself with \( k \leq (K/2 - 1) \).

Case II: For subcarriers \( k = 0 \) and \( K/2 \), we have \( k = \tilde{k} \). \( \mathbb{R}(k, i) \) in (7) can be written as

\[
\mathbb{R}(k, i) = \mathbf{G}(k)\mathbf{S}(k, i) + \mathbf{Z}(k, i).
\]

In order to use ICA for I/Q imbalance compensation and spatial equalization, we divide the complex baseband signal \( \mathbb{S}(k, i) \) and \( \mathbb{R}(k, i) \) into real and imaginary parts as \( \mathbb{S}(k, i) = [\Re\{\mathbb{S}^T(k, i)\}, 3\Im\{\mathbb{S}^T(k, i)\}]^T \) and \( \mathbb{R}(k, i) = [\Re\{\mathbb{R}^T(k, i)\}, 3\Im\{\mathbb{R}^T(k, i)\}]^T \). Thus, (12) can be rewritten in the following form:

\[
\mathbb{R}(k, i) = \mathbb{G}(k)\mathbb{S}(k, i) + Z(k, i)
\]

where AWGN vector \( Z(k, i) = \begin{bmatrix} \Re\{\mathbb{Z}^T(k, i)\}, 3\Im\{\mathbb{Z}^T(k, i)\}\end{bmatrix}^T \) has real-valued entries with zero mean and variance \( \sigma^2/2 \). The mixing matrix \( \mathbb{G}(k) \) is given by

\[
\begin{bmatrix}
\Re\{\mathbf{G}_1(k)\} + \Re\{\mathbf{G}_2(k)\} & -\Im\{\mathbf{G}_1(k)\} + 3\Im\{\mathbf{G}_2(k)\} \\
\Im\{\mathbf{G}_1(k)\} + 3\Im\{\mathbf{G}_2(k)\} & \Re\{\mathbf{G}_1(k)\} - \Re\{\mathbf{G}_2(k)\}
\end{bmatrix}
\]

After ICA, we can obtain the estimates of source symbols using the combination

\[
\hat{\mathbb{S}}(k, i) = \begin{bmatrix}
\hat{\mathbb{S}}_0(k, i), \ldots, \hat{\mathbb{S}}_{2N_t-1}(k, i)
\end{bmatrix}^T
\]

Due to the intrinsic ambiguity of ICA models, the ICA output signal \( \hat{\mathbb{S}}(k, i) \) and \( \mathbb{S}(k, i) \) may have different scaling coefficients, permutations and phase shifts. Therefore, further processing is needed to reorder and scale the ICA output signal.

B. Ambiguity Elimination

Based on the different ICA models (11) and (13), elimination of ambiguity in \( \hat{\mathbb{S}}(k, i) \) and \( \tilde{\mathbb{S}}(k, i) \) can also be performed in two cases.

Case I: For subcarriers \( 1 \leq k \leq (K/2 - 1) \), the ICA output signal \( \hat{\mathbb{S}}(k, i) \) has the phase ambiguity and permutation ambiguity. The phase ambiguity in \( \hat{\mathbb{S}}(k, i) \) can be resolved by derotating each data stream as

\[
\hat{\mathbb{S}}_n(k, i) = \hat{\mathbb{S}}_n(k, i) [\alpha_n(k)/|\alpha_n(k)|]
\]

with the factor \( \alpha_n(k) = \frac{1}{N_s} \sum_{i=0}^{N_s-1} [\hat{\mathbb{S}}_n(k, i)]^4 \) for QPSK modulation [17].

Due to the superimposition (1), the transmitted symbols are correlated with the reference symbols. Define

\[
\rho_{\pi_n(k), n} = \frac{1}{N_s} \sum_{i=0}^{N_s-1} \hat{\mathbb{S}}_n(k, i) \mathbb{D}_{\text{ref}, n}^*(k, i)
\]

as the cross correlation between the detected stream \( \pi_n(k) \) and the reference signal stream \( n \) on subcarrier pair \( k \), where \( \mathbb{D}_{\text{ref}, n}^*(k, i) \) is the \( n \)-th entry of the vector \( \left[ \mathbb{D}_{\text{ref}, n}^*(k, i) \right]^T \). Based on [15], the permutation \( \pi(k) \) is obtained by maximizing the cost function

\[
\pi_{\text{opt}}(k) = \arg \max_{\pi(k)} \sum_{n=0}^{2N_t-1} |\rho_{\pi_n(k), n}|^2
\]

After reordering, the estimate of \( \mathbb{S}(k, i) \) is given by

\[
\tilde{\mathbb{S}}(k, i) = \begin{bmatrix} \Re\{\mathbb{B}(k)\}, \Im\{\mathbb{B}(k)\} \end{bmatrix}^T
\]

where the entries \( b_n(k) \) are given by

\[
b_n(k) = \left[ j e^{-j\frac{\pi}{4}} \text{sign}(\rho_{\pi_n(k), n}) \right]^{-1}
\]

for QPSK modulation.

Case II: For subcarriers \( k = 0 \) and \( K/2 \), all entries in the ICA model (13) are real. Therefore, the ICA output signal \( \hat{\mathbb{S}}(k, i) \) has the scaling, sign and permutation ambiguity, where the scaling ambiguity can be resolved based on the assumption that the variance of source symbols is unity, and the sign and permutation ambiguity can be resolved by reordering. Similar to Case I, we define

\[
\rho_{\pi_n(k), n} = \frac{1}{N_s} \sum_{i=0}^{N_s-1} \hat{\mathbb{S}}_n(k, i) \mathbb{D}_{\text{ref}, n}^*(k, i)
\]

and select the permutation corresponding to the largest \( |\rho_{\pi_n(k), n}| \) as the correct permutation. The estimated data \( \hat{\mathbb{S}}(k, i) \) is given by

\[
\tilde{\mathbb{S}}(k, i) = \begin{bmatrix} \Re\{\mathbb{B}(k)\}, \Im\{\mathbb{B}(k)\} \end{bmatrix}^T
\]

where the entries \( b_n(k) \) are given by

\[
b_n(k) = \text{sign}(\rho_{\pi_n(k), n}) \left| \frac{\rho_{\pi_n(k), n}}{|\rho_{\pi_n(k), n}|} \right|^{-1}
\]

for QPSK modulation. Normalization of \( \hat{\mathbb{S}}_n(k, i) = \hat{\mathbb{S}}_n(k, i)/E \left\{ |\hat{\mathbb{S}}_n(k, i)|^2 \right\} \) is then used to resolve the scaling ambiguity.

IV. CHANNEL INTERPOLATION AND LSFE

To refine the output of the ICA based compensation approach proposed in Section III, we incorporate ICA with channel interpolation [17] and LSFE [18]. Channel interpolation uses the correlation between adjacent subcarriers to correct imperfect source stream extraction that occurs on a small number of subcarriers. LSFE [18] is a nonlinear equalization approach and provides a better performance than conventional linear approaches including the zero forcing (ZF) and the minimum mean squared error (MMSE) based equalization methods.
A. Channel Interpolation

Channel interpolation is initialized with the least squares channel estimation from the ICA output signal in two cases.

Case I: For subcarriers $1 \leq k \leq (K/2 - 1)$, we obtain the initial least squares estimate of $\mathcal{G}(k)$ as

$$\tilde{\mathcal{G}}(k) = \mathbb{R}(k) [\tilde{S}(k)]^+ \quad (19)$$

where $\mathbb{R}(k) = [\mathbb{R}(k,0), \ldots, \mathbb{R}(k, N_r - 1)]$, and

$$\tilde{S}(k) = [S^T(k), S^{*T}(k)]^T$$

is the re-encoded signal from the hard estimate of the source data with $S(k) = [S(k,0), S(k,1), \ldots, S(k, N_r - 1)]$.

Case II: For subcarriers $k = 0$ and $K/2$, the initial least squares estimate of the mixing matrix $\mathcal{G}(k)$ is

$$\tilde{\mathcal{G}}(k) = \mathbb{R}(k) [\tilde{S}(k)]^+ \quad (20)$$

where $\mathbb{R}(k) = [\mathbb{R}(k,0), \mathbb{R}(k,1), \ldots, \mathbb{R}(k, N_r - 1)]$ is the received signal on subcarrier $k$, and

$$\tilde{S}(k) = [\tilde{g}_1(k), \tilde{g}_2(k), \ldots, \tilde{g}_{N_r}(k)]^T$$

is the re-encoded signal from the hard estimate of the source data. We divide the matrix $\mathcal{G}(k)$ into four sub-matrices

$$[\tilde{g}_{11}(k) \quad \tilde{g}_{12}(k)]$$

and

$$[\tilde{g}_{21}(k) \quad \tilde{g}_{22}(k)]$$

to obtain $\mathcal{G}_i(k)$ ($i = 1, 2$) as

$$\mathcal{G}_1(k) = \frac{1}{2} [\tilde{g}_{11}(k) + \tilde{g}_{22}(k)] + \frac{j}{2} [\tilde{g}_{21}(k) - \tilde{g}_{12}(k)]$$

$$\mathcal{G}_2(k) = \frac{1}{2} [\tilde{g}_{11}(k) - \tilde{g}_{12}(k)] + \frac{j}{2} [\tilde{g}_{21}(k) + \tilde{g}_{22}(k)] .$$

The initial least squares estimates (20) and (19) are refined by channel interpolation [17], which is to use the frequency correlation for correction of a few erroneous initial estimates. For each of the single-input single-output (SISO) channel between transmit antenna $n$ ($n = 0, \ldots, N_t - 1$) and receive antenna $m$ ($m = 0, \ldots, N_r - 1$), we first obtain [19], [20]

$$\tilde{\mathbf{g}}_{i,m,n} = \mathbf{F}^T \mathbf{G}_{i,m,n} \quad (21)$$

where $\mathbf{G}_{i,m,n} = [\tilde{g}_{i,m,n}(0), \ldots, \tilde{g}_{i,m,n}(K - 1)]^T$ with $\tilde{g}_{i,m,n}(k)$ denoting the entry of the row $m$ and column $n$ of $\mathbf{G}(k)$ ($i = 1, 2$). $\mathbf{F}$ is the discrete Fourier transform (DFT) matrix of size $(K \times L_{cp})$. Equation (21) can be viewed as truncation of the time-domain estimates of the mixing matrices. The refined estimates of $\mathbf{G}_{i,m,n}$ ($i = 1, 2$) are obtained as [19], [20]

$$\tilde{\mathbf{G}}_{i,m,n} = \mathbf{F}^T \tilde{\mathbf{g}}_{i,m,n}. \quad (22)$$

B. LSFE

The refined estimates $\tilde{\mathbf{G}}_{i}(k)$ ($i = 1, 2$), which are obtained from channel interpolation, are inputted into LSFE [17], [18] to further improve the performance. For subcarriers $k = 0$ and $K/2$, we define $\tilde{\mathbf{G}}^{(i,:)}(k)$ as the $l$-th column of $\tilde{\mathbf{G}}(k)$, and

$$\mathbf{B}(k) = \left\{ \sum_l \tilde{\mathbf{G}}^{(i,:)}(k) \left[ \tilde{\mathbf{G}}^{(i,:)}(k) \right]^H + \frac{1}{2} \sigma^2 \mathbf{I}_{2N_r} \right\}^{-1/2} \quad (23)$$

where the summation is over all undetected streams. We combine the real parts and imaginary parts of the LSFE output signal [18]

$$\tilde{S}_n(k, i) = x_n^H(k) \mathbb{R}(k, i) \quad (24)$$

to obtain the soft estimates, where the equalizer vector is $x_n(k) = [\mathbf{B}(k)]^{-1} \tilde{\mathbf{G}}^{(i,:)}(k)$, and $n$ is the layer index with the lowest MSE for detection:

$$n = \arg \min_i \left\{ 1 - \left[ \tilde{\mathbf{G}}^{(i,:)}(k) \right]^H [\mathbf{B}(k)]^{-1} \tilde{\mathbf{G}}^{(i,:)}(k) \right\} .$$

The contribution of the detected stream $n$ is cancelled from the received signal after the decoding and hard decision. The above procedure is repeated until all data streams have been extracted.

For other subcarriers $1 \leq k \leq (K/2 - 1)$, we can follow the same procedure to obtain the estimates of source data symbols by replacing $\tilde{\mathbf{g}}(k), \mathbb{R}(k)$ and $\sigma^2/2$ in (23) - (25) with $\mathbf{G}(k), \mathbb{R}(k)$ and $\sigma^2$, respectively.

V. COMPLEXITY ANALYSIS

In this section, we investigate the complexity of the proposed ICA based semi-blind I/Q imbalance compensation scheme, in terms of the number of complex additions and multiplications. The complexity of the MPP approach [3] is also analyzed for comparison, where $P$ and $L$ denote the number of pilot symbols and the order of the FIR filter used for I/Q imbalance compensation, respectively. Table I shows that the proposed ICA based scheme has a higher complexity than the MPP structure [3], mainly due to the complexity of blind equalization and ambiguity elimination. However, since the complexity increases linearly with the number of subcarriers, the proposed approach is applicable to future wireless communication systems with a large number of subcarriers.

VI. SIMULATION RESULTS

In this section, the performance of the proposed ICA based semi-blind I/Q imbalance compensation approaches is evaluated by simulations. The MIMO OFDM system has $K = 128$ subcarriers, $N_t = 2$ transmit antennas and $N_r = 2$ receive antennas. The data rate is 16 Mbps with QPSK modulation. A CP of length $L_{cp} = 8$ is used. The Clarke’s block fading channel model [21] with the exponential power delay profile is employed, where the channel remains constant during a frame of $N_s = 200$ OFDM symbols, and the root mean square (RMS) delay spread is $\tau_{RMS} = 1.5$, normalized to the sampling time interval of $T_s = 0.22 \mu s$. The SNR is defined as the ratio of the received signal power including I/Q imbalance to the noise power, i.e., the ratio of the average power of signal $\mathbf{G}_1(k)\mathbf{S}(k,i) + \mathbf{G}_2(k)\mathbf{S}^*(k,i)$ to the average power of noise $\mathbf{Z}(k,i)$ in (7), for a fair comparison with the systems no I/Q imbalance. The superimposition constant is $a = 0.3$, resulting in a decrease of less than 0.4 dB in the effective SNR, much lower than the SNR loss due to precoding in [22]. The I/Q imbalance scenarios [3]–[6] at the transmitter and the receiver are summarized in Table II.

We illustrate the BER performance of the proposed ICA-only and ICA-LSFE compensation approaches. The BER performances of the MPP based compensation approaches [3], MMSE equalization and LSFE without I/Q imbalance compensation are shown for comparison. With MPP, only the I/Q imbalance at the receiver is considered, and the same pilot
sequence of 10 symbols is transmitted from each of the \( N_t \) antennas, resulting in a training overhead of 5\%. The order of FIR compensation filter for MPP is \( L = 5 \). We also include the results for two ideal cases with MMSE equalization and LSFE as benchmarks. The first ideal case has perfect CSI and no I/Q imbalance, while the second ideal case has perfect knowledge of mixing matrices \( G_1(k) \) and \( G_2(k) \) of the equivalent system.

The BER vs. SNR performance for the scenario of both frequency independent and frequency dependent I/Q imbalance is shown in Fig. 2 and Fig. 3 for MMSE and LSFE, respectively. Without I/Q imbalance compensation, there are error floors at BER = \( 2 \times 10^{-2} \) and BER = \( 3 \times 10^{-2} \) for the cases of MMSE equalization and LSFE, respectively. Compared to the training based MPP approaches, the proposed semi-blind compensation approaches achieve a better performance without spectral overhead. It is shown that the ICA-only approach outperforms the MPP-MMSE approach slightly, while the ICA-LSFE approach outperforms the MPP-LSFE approach significantly in the region of high SNR. The performance improvement of the ICA-LSFE approach over the MPP-LSFE approach is due to the additional frequency diversity introduced by I/Q imbalance, as discussed in Subsection III-A. In Fig. 3, the ICA-LSFE approach demonstrates a slope of around twice times as large of the slope of the MPP-LSFE approach, implying an additional diversity of order 2. The ICA-LSFE approach exploits the additional diversity in the equivalent system by interference cancellation. The achieved diversity increases after interfering symbols are cancelled out. Whereas the ICA-only approach does not benefit from the additional frequency diversity in the equivalent system, since it estimates the source symbols on subcarriers \( k \) and \( \bar{k} \) simultaneously. Therefore, in Fig. 2, where the ICA-only approach has a roughly equal slope compared to the MPP-MMSE approach.

Compared to the ideal cases of equalization with perfect CSI and no I/Q imbalance, the ICA-only approach obtains a BER performance similar to that of MMSE equalization with a gap of 2 dB at BER = \( 10^{-3} \) as shown in Fig. 2. While the ICA-LSFE approach outperforms LSFE with a gain of 5 dB at BER = \( 10^{-4} \) due to additional frequency diversity as shown in Fig. 3. Similarly, the gap is 2 dB at BER = \( 10^{-3} \) between the ICA-only approach and the ideal case of MMSE equalization with perfect knowledge of the matrices \( G_1(k) \) and \( G_2(k) \). Compared to the ideal case of LSFE with perfect knowledge of \( G_1(k) \) and \( G_2(k) \), the ICA-LSFE approach has the same diversity order and therefore the same slope, and is only right shifted by 2.5 dB, as shown in Fig. 3. The shift is caused by the ICA extraction errors which occur occasionally on a small number of subcarriers due to exploitation of HOS [16].

Fig. 4 shows the BER vs. SNR performance of the proposed ICA based approaches and the MPP based approaches in the
similar trends as in Fig. 2 and Fig. 3 can be observed.

scenario of frequency independent I/Q imbalance only, where similar trends as in Fig. 2 and Fig. 3 can be observed.

VII. CONCLUSIONS

We have proposed an ICA based semi-blind receiver structure to compensate for frequency-independent and frequency-dependent I/Q imbalance, as well as to perform equalization simultaneously. Thanks to the additional frequency diversity, the proposed implicit compensation scheme provides a better performance than the MPP based compensation method [3] without causing any loss of transmission rate and change of the total transmission power. In particular, by incorporating ICA with channel interpolation and LSFE, the achieved performance in the tested scenarios is better than the performance in the ideal cases with perfect CSI and no I/Q imbalance.

REFERENCES