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A Signal of Hydrodynamical Transverse Flow in High Energy Hadronic Collisions

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It is shown that the hadronization temperature and the transverse velocity can be determined simultaneously by $\langle p_t \rangle$ for pions and kaons produced in the central rapidity region. The data suggest that the temperature is nearly constant while the transverse velocity grows with the incident energy.

Recently the relativistic fluid models[1)~3)] for high energy hadronic collisions are intensively studied because of the interest on the formation and the hadronization of quark-gluon plasma. In such models the hadronization is assumed to take place at certain temperature T_h, which is implied by lattice-QCD calculations.[4)] The existence of the hadronization temperature is expected also in old statistical fire-ball models.[5),6)]

In such fluid or thermodynamical models, it is interesting to study the effect of the surface motion at the hadronization. The authors pointed out that the difference of the average p_t between pion and kaon produced in CERN-SPS collider at $\sqrt{s}=540$ GeV is naturally explained by the transverse flow of the fluid.[7)] Because of the transverse flow, the heavier produced particles get the greater additional p_t, which is in accordance with the data.

In this paper we show that the temperature and the transverse velocity at the hadronization are determined by $\langle p_t \rangle$ of π and K, and that the temperature is nearly constant while the transverse velocity grows with the incident energy. Although our calculation is based on a model, the conclusion is not changed by the details of the model and is independent of the nature of the fluid; that is, one can imagine the quark-gluon fluid or the hadronic one.

The assumptions for the model are the following:

a) The shape of the fluid is cylindrically symmetric with respect to the collision axis.

b) All the hadrons are produced from the surface of the fluid at the same temperature T_h and their energy spectrum is given by the Boltzmann distribution in the local rest frame of the fluid.

Then the probability $\rho^*(\varepsilon^*)$ that the hadron has the energy ε^* in the local rest frame of the fluid is given as

$$\rho^*(\varepsilon^*)=\varepsilon^*\exp(-\varepsilon^*/T_h)/c\,, \tag{1}$$

where $c=4\pi T_h m^2 K_2(m/T_h)$ and m is the mass of the hadron. The normalization is taken as $\int dy^* d^2\boldsymbol{p}_t{}^* \rho^*(\varepsilon^*)=1$, where y^* is the rapidity of the produced particles and the asterisk (*) means the quantity in the local rest frame of the fluid.

c) The rapidity distribution of the fluid is constant at least in the central region. That is, the distribution function is given as

$$D(\eta)=dn/d\eta=\text{const}\,, \qquad \text{in the central region}\,, \tag{2}$$

where η is the rapidity of the fluid.

d) The transverse velocity $\boldsymbol{v}_t$ of the fluid surface is independent of η in the central region.

Although the last two assumptions seem to be rather restrictive, in practice we can get the similar results as long as the distribution function $D(\eta)$ has mild behavior.

From the above, one can write the particle density as

$$\frac{1}{\pi}\frac{dn}{dydp_t{}^2}=\int d\eta D(\eta)\int\frac{d\theta}{2\pi}\rho^*(\varepsilon^*)\,, \tag{3}$$

where θ is the angle between the transverse momentum $\boldsymbol{p}_t$ of the hadron and the transverse velocity $\boldsymbol{v}_t$ of the fluid surface where the hadron is produced. By the Lorentz transformation, ε^* in Eq. (3) can be expressed as

$$\varepsilon^*=\gamma_t m_t\cosh(y-\eta)-v_t\gamma_t p_t\cos\theta\,, \tag{4}$$

where $m_t=(p^2+m^2)^{1/2}$ and $\gamma_t=(1-v_t{}^2)^{-1/2}$.

The value of $\langle p_t\rangle$ at fixed y is, then, given by

$$\langle p_t\rangle=\frac{\displaystyle\int dp_t{}^2 p_t\frac{dn}{dydp_t{}^2}}{\displaystyle\frac{dn}{dy}}\,. \tag{5}$$

Before giving the results of the numerical calculation, we briefly discuss the model dependence of the results. The average transverse mass $\langle m_t\rangle$, which approximately gives the average p_t as $\langle p_t\rangle\simeq(\langle m_t\rangle^2-m^2)^{1/2}$, can be calculated as

$$\begin{aligned}\langle m_t\rangle&=\langle\varepsilon\rangle|_{y=0}\\&=\frac{1}{\left.\dfrac{dn}{dy}\right|_{y=0}}\int dy^*\pi dp_t{}^{*2}\int d\eta D(\eta)\int(d\theta^*/2\pi)\varepsilon\rho^*(\varepsilon^*)\delta(y-\eta-y^*)|_{y=0}\\&=\int dy^*\pi dp_t{}^{*2}\rho^*(\varepsilon^*)\gamma_t m_t{}^*\\&=\gamma_t\langle m_t{}^*\rangle\,,\end{aligned} \tag{6}$$

where we used the equation

$$\varepsilon=\gamma_t m_t{}^*\cosh y+v_t\gamma_t p_t{}^*\cos\theta^*\,. \tag{7}$$

Equation (6) shows the effect of the transverse flow in a transparent way. It should be noted that Eq. (6) holds approximately when the function $D(\eta)$ has mild behavior as compared with the y^* dependence of $\rho^*(\varepsilon^*)$. Since $\rho^*(\varepsilon^*)$ has a sharp peak at $y^*=0$, this condition may be satisfied in any model that can reproduce the

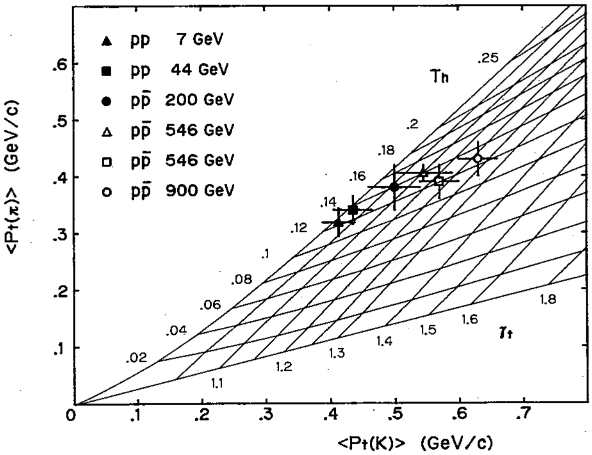

Fig. 1. T_h and γ_t dependence of $\langle p_t \rangle$ for π and K given by Eq. (5). The experimental data$^{8)\sim13)}$ suggest that the hadronization temperature T_h is constant while the transverse flow becomes faster with the incident energy. The values of $\langle p_t \rangle$ for π at 200 GeV and 900 GeV are estimated from the data of $\langle p_t \rangle$ for charged particles,$^{13)}$ $\langle p_t \rangle$ for K and K/π ratio.$^{9)}$

central plateau in dn/dy.

In Fig. 1 we show the results of Eq. (5) for π and K as a two-dimensional plot, which clearly shows the T_h and γ_t dependence of $\langle p_t \rangle$. From the experimental data$^{8)\sim13)}$ of $\langle p_t \rangle$ also shown in the figure, one can see the following: (a) The hadronization temperature T_h seems to be independent of the incident energy, and its value is between 120 and 140 MeV. (b) Below ISR energies the transverse flow is negligibly small, i.e., $\gamma_t \simeq 1$, but it seems to grow rapidly with the incident energy. (c) Without the transverse flow the SPS data cannot be explained, as long as one assumes that π and K are produced at the same temperature.

If the data of $\langle p_t \rangle$ for π and K in the same experiment are available, using Fig. 1, one can determine T_h and γ_t simultaneously. This figure may become more important when the data at higher energy are added and the data are given with more accuracy.

In Fig. 2 we compare our results with the data of $\langle p_t \rangle$ for π and K at various energies, where the constant T_h and energy dependent γ_t are set as

$$T_h = 124\,\text{MeV}\,,$$

$$\gamma_t = 1 + 0.00185\,\log^2(s/s_0)\,, \quad s_0 = (5\,\text{GeV})^2\,, \tag{8}$$

so as to fit the data for π. The results for K shown in Fig. 2 are in good agreement with the data as expected from Fig. 1.

For comparison we also show in Fig. 2 the result without the transverse flow, i.e., $\gamma_t = 1$, where $\langle p_t \rangle$ for π is fitted with energy-dependent T_h, and then $\langle p_t \rangle$ for K is calculated. As seen in the figure, the difference of $\langle p_t \rangle$ between π and K cannot be reproduced. Further, without the flow, the difference should vanish at high T_h, but the data show the opposite tendency.

Using Eq. (3), we have calculated the p_t-distributions for mesons and baryons in SPS and ISR energy regions. All the results are consistent with the experimental p_t-distributions for $p_t \lesssim 2\,\text{GeV}/c$.

As for baryons, however, the reported values of $\langle p_t \rangle$ do not always coincide with our calculated values. This is mainly because of the lack of the low p_t data. The reported $\langle p_t \rangle$ values are calculated by fitting the data in the form $\exp(-bp_t)$ or others, so the values depend on the form of the fitting function. The "experimental" $\langle p_t \rangle$ values become greater if one takes, for example, the Boltzmann-type function, $m_t \exp(-m_t/T)$, instead of the form $\exp(-bp_t)$.

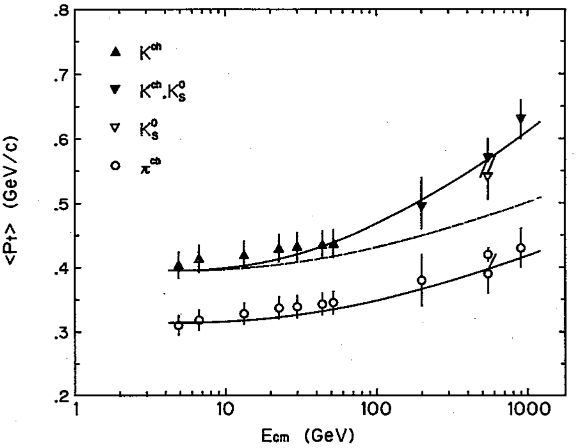

Fig. 2. The energy dependence of $\langle p_t \rangle$ for π and K in p-p and $\bar{p}$-p collisions. The solid lines show the results of Eq. (5) with the parameters given by Eq. (8). The dashed line is the results without the transverse flow. The data are from Refs. 8)~13). The values of $\langle p_t \rangle$ for π at 200 GeV and 900 GeV are estimated as in Fig. 1.

As for the SPS data,[11] reported $\langle p_t \rangle$ values for Λ, Ξ and $\bar{p}$ are 0.62 ± 0.08, 1.1 ± 0.2 and 0.66 ± 0.05 GeV/c, while our calculation gives 0.913, 1.03 and 0.812 GeV/c, respectively. The discrepancy for Λ and $\bar{p}$ may be partly because of the leading particle effect, which suppress the $\langle p_t \rangle$ value, but is not taken into account in our calculation.

For ISR energies, our model gives reasonable p_t-distributions for Λ and $\bar{p}$, although the transverse flow has a very little effect. The reported values of $\langle p_t \rangle$ for $\bar{p}$ are around 0.5 GeV/c,[8] while the calculation gives about 0.55 GeV/c in these energy regions. Though the leading particle effect is not present for $\bar{p}$ in p-p collisions, there are ambiguities from the poor statistics and the fitting procedure.

At present, therefore, we cannot conclude anything definitely for baryons. However, the p_t-distribution or $\langle p_t \rangle$ of baryons above SPS energies with good statistics gives a good test for our picture, because the transverse flow effect to the $\langle p_t \rangle$ is more clearly seen for heavier particles.

In conclusion, the data of $\langle p_t \rangle$ for π and K suggest that hadronization temperature is nearly constant while the transverse velocity grows with the incident energy. The more precise data of $\langle p_t \rangle$ for baryons and the data at higher energies would give definite proof for the flow.

The particle dependence of $\langle p_t \rangle$ may be derived from other pictures; for example, different hadron is produced at different temperature. There seems, however, little foundation for such pictures.

On the viewpoint of the fluid model, it is important to study whether the fluid is quark-gluon plasma or hadronic one, and how to estimate the initial parameters of the fluid, since the transverse flow is determined by the initial energy density.[14]

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