Network Code Division Multiplexing for Wireless Relay Networks

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Abstract—In this paper, we investigate the performance of a wireless relay network with multiple transmission sessions, in which multiple groups of source nodes communicate with their respective destination nodes via a shared wireless relay network. A multiple transmission session model with network code division multiplexing (NCDM) scheme is proposed to remove the inter-session interference at each destination. The fundamental idea of the NCDM scheme takes advantage of the property of $G \cdot H^T = 0$ of the low-density generator matrix (LDGM) codes. Based on the analysis of the NCDM scheme, we investigate the relationship among the equivalent received signal vector, the number of sessions and the column weight of the generator matrix. New code design criteria for the construction of the generator matrix is proposed. We further evaluate the multiple transmission session model with the proposed NCDM scheme in terms of throughput and complexity. Our evaluation demonstrates that the proposed scheme not only has a linear computational complexity, but also shows a similar error performance in the AWGN case and a considerable throughput improvement compared with its counterpart, which is referred to as a serial session scheme, where groups of source nodes communicate with their respective destinations in a time division manner.

Index Terms—Distributed network coding, low density generator matrix codes, parallel session model, network code division multiplexing, code design criteria.

I. INTRODUCTION

Distributed coding is a special channel coding strategy developed for cooperative communication networks [1], [2]. It has also been applied in the design of conventional channel codes, forming such as distributed turbo codes [3], distributed space-time codes [4] and distributed low-density parity-check (LDPC) codes [5]. In the above distributed coding schemes, the transmission reliability over point-to-point wireless communication channels has been efficiently improved.

The distributed coding schemes discussed above [1]–[5] are developed for small-scale unicast relay networks, in which messages are sent from a single source to a single destination through single/multi-hop relay nodes. In large-scale wireless sensor networks (WSNs), a large number of sensor nodes are often deployed to gather information from the surrounding environment. The information gathered from these sensor nodes, which are referred to as source nodes, are delivered then to a common destination through other sensor nodes, serving as relay nodes. As the source and the relay nodes are in different spatial locations, their signals can be combined at the destination node to achieve spatial diversity. Clearly, if a destination node receives replicas of the transmitted signal via multiple relay nodes, its signal will have higher spatial diversity and better performance [6].

Despite the potential benefits of relay nodes, they may consume considerable amounts of radio spectrum and energy, and cause further spectrum congestion and interference if not used properly. Therefore, for dense large-scale wireless networks, we need a novel approach to reduce bandwidth consumption of relay nodes while harnessing their benefits.

A novel approach that minimizes the bandwidth consumption is network coding (NC) [7]. Network coding is introduced as a generalization of routing. In routing, the relay nodes simply store and forward the received packets to the destination. In the NC scheme, the relay nodes are allowed to encode the packets received from multiple source nodes and send the encoded packets instead. Coding operations enable the relay nodes to compress the information, and whenever possible, to reduce the number of transmissions and bandwidth consumption. Prior work shows that network coding can achieve the multi-cast network capacity by making linear combinations of packets they received [7]–[9]. The idea of NC can be easily extended to wireless networks. A considerable amount of research that has been dedicated to exploiting NC on cooperative communication networks have been done, some of them focus on the research at the physical layer [2], [10]–[13], while others are for the network layer [14]–[16]. The idea of NC can be further applied to WSNs, where multiple source nodes communicate with a common destination through multi-hop relay nodes [17]. In [18], extrinsic information transfer (EXIT) charts are employed to design the irregular low-density
generator matrix (LDGM) codes [19] based on a single session model. By a single session, we refer to a group of source nodes transmitting to a common destination node, while multiple sessions refer to multiple groups of source nodes transmitting to multiple respective destination nodes.

In this paper, for the applications where at each destination node, only the information from its own session is required, we propose to design novel network coding schemes for wireless networks. In wireless networks, multiple relays are shared among multiple sessions. We assume that at each destination node, the information from other sessions are not allowed to be decoded due to the security reasons. This assumption is realistic in the application scenario in smart meter networks, where multiple sensors, measuring water, gas and energy consumptions, communicate with multiple utility control centers [20]. The information from various utilities, such as for water, gas and energy consumptions, cannot be shared. The bandwidth consumption will be reduced by sharing dedicated relay stations among various sessions and the use of novel network coding mechanisms, referred to as network code division multiplexing (NCDM). The NCDM scheme can provide security protection on physical layer. Traditionally, the research of the security techniques are focused on the key-based enciphering technique on upper layers [21]. However, in wireless network, it requires a large number of computational resources to deal with the encryption key. We will consider the security protection problem from the physical layer [22]. Since the physical layer security techniques can operate essentially independently from the upper layers, the NCDM scheme can be used to enhance the security of the data transmission. A common set of relays, as opposed to separate relay nodes for each session, can save spectrum, energy, communication infrastructure and bring additional spatial diversity. The signals from multiple sessions will be mixed in a controlled manner at the relay nodes to cooperatively produce a distributed network code. As inter-session interference is introduced in the cooperative process [23], novel network codes, with a special algebraic structure, enabling interference removal at the destinations and high coding gains, will be designed. Transmission of each session will be assisted by all relay nodes resulting in a maximum spatial diversity.

Particularly, the signal processing methods at the relays dominate the overall system performance. On the one hand, if all relays belonging to different transmission sessions are cooperative with each other, a maximum spatial diversity can be potentially achieved. On the other hand, inter-session interference is minimized if signals, belonging to different sessions, are transmitted from relays in an orthogonal manner. A usual way to remove interference is dividing the relay transmission time into \( L \) relay periods, corresponding to \( L \) sessions in a wireless relay network. One relay period contains \( N \) time slots, where \( N \) is the number of relays in the network. All relays process and transmit data for the \( i \)-th session in a relay period \( i (i = 1, \cdots, L) \). In this way, data from different sessions are separated by time division multiplexing and there will be no interference in the transmitted signals from multiple sessions. For convenience, in the following, we refer to the above described model as a serial session model. In this model, however, transmission in separate relay periods may result in a lower throughput or more spectrum consumption, compared to the transmission of all signals from multiple sessions in one relay period.

To mitigate interference while maximizing the spectrum efficiency, we propose to combine data from different sessions to form a network code, and forward such coded signals to the destinations, within one single relay period. In contrast to the serial session model, we refer to our proposed scheme as a parallel session model. The combined operation of data from different sessions in parallel session model will cause inter-session interference at the destinations. By using the proposed NCDM scheme at the destination nodes by orthogonalizing the equivalent sub-channels of different sessions, inter-session interference among multiple sessions will be minimized. Furthermore, we will maximize the coding gain of these network codes, by optimizing the network code topology. Please note that the NCDM scheme is different from the interference alignment approach given in [24]–[26]. The key idea of the interference alignment is to construct precoding vectors at the transmitters, and concentrate the interference into a partial signal space while the desired signal becomes free of interference at each receiver [26]. The NCDM scheme does not use a precoding operation at the transmitters, it only deals with the inter-session interference at the destinations. The NCDM scheme is also different from the aligned interference neutralization schemes, which is proposed in [27] for the relay-aided networks. In the aligned interference neutralization scheme, for the design of the beamforming vectors, the channel state information (CSI) must be known at both the transmitters and the receivers or at the receivers. Also, it requires full connection between the source and the relay nodes, and between the relay and the destination nodes. While for the NCDM scheme, the CSI is only required at the receivers. The situations that no full connection exists between the source and the relay nodes, and between the relay and the destination nodes are also considered in the NCDM scheme.

In this paper, we investigate the impact of the number of sessions on bit error rate (BER) performance, and examine the scalability of the proposed NCDM scheme when the number of nodes increases. In the analysis of the proposed NCDM process, we focus on the relationship among the information and the noise parts of the equivalent received signal vector, the number of sessions and the column weight of the formed generator matrix. We analyze the reasons which cause the degradation of the BER performance of the parallel session model during the NCDM process and investigate possible approaches to solving these problems. The theoretical analysis is followed by the derivation of the code design criteria to guide the selection of source nodes at each relay node. By implementing the proposed code design criteria, a desired code performance can be achieved. It is shown that the proposed NCDM scheme has a linear computational complexity of \( N \) and \( L \), and a similar error performance as a serial session scenario, while achieving a considerable throughput improvement compared with the serial session model.

The remainder of the paper is organized as follows. Section II presents the system model and briefly introduces the
NCDM scheme and the distributed LDGM codes from the system point of view. Section III introduces the proposed NCDM scheme for the parallel session model. Section IV analyzes the relationship among the equivalent received signal nodes, the number of sessions and the column weight of the generator matrix. On the basis of the analytical results, code design criteria are derived for the parallel session model. Section V evaluates the performance of the NCDM scheme in terms of the BER, throughput and complexity. Finally, the conclusions are drawn in Section VI.

II. SYSTEM MODEL

We consider a wireless network containing $L$ transmission sessions, shown in Fig. 1. In each session, a group of source nodes communicate with their common destination node through a group of relay nodes. We use $\varphi_i$ to denote the $i$th session and assume that $\varphi_i$ has $S_i$ source nodes, where $i \in \{1, 2, \cdots, L\}$. In total, there are $M = \sum_{i=1}^{L} S_i$ source nodes in the $L$ sessions. Denote the destination node of session $\varphi_i$ by $D_i$. We assume that the network consists of $N$ relay nodes which are shared among all $L$ sessions. The $j$th relay node is denoted by $R_j$, $j \in \{1, 2, \cdots, N\}$.

Fig. 1. A network graph used to describe the system model.

As it is described in Section I, the model where groups of source nodes in different sessions communicate with their respective destinations is referred to as a serial session model. In the serial session model, transmission in separate relay periods may result in a lower throughput or more spectrum consumption, compared to the transmission of all signals from multiple sessions in one relay period. To mitigate interference while maximizing the spectrum efficiency, we propose to combine data from different sessions to form a network code, and forward such coded signals to the destinations, within one single relay period. We refer to this model as a parallel session model.

For the parallel session model, the data transmission from the source nodes to their destination nodes is carried out in two phases: a broadcast phase and a relay phase. We take the operations of the source nodes in session $\varphi_i$ in the broadcast phase, the $j$th relay node in the relay phase, and the destination node $D_i$, as an example to illustrate the data delivery process. The system diagram is shown in Fig. 2.

- **Broadcast phase.** In this phase, all the source nodes broadcast their data packets to all the relay and the destinations. Each data packet is composed of three parts: an information data part, a Cyclic Redundancy Check (CRC) part and a header. The indices of the source nodes and the session to which the source node belongs are contained in the header. The information data parts of all data packets have the same length, as well as the CRC parts and the headers. In case the length of the information data part of one source node is shorter, we just add “0” to the end of the data to make it the same length as that in other source nodes. The performance will not be affected by this operation. A certain MAC layer protocol, e.g., Time-Division Multiple Access (TDMA), Carrier Sense Multiple Access (CSMA), or Orthogonal Frequency-Division Multiple Access (OFDMA) [28], [29], is assumed to be employed. This ensures that the source node transmissions do not cause interference to each other. The transmitted data packet from each source node in session $\varphi_i$ is a vector of size $\ell \times 1$. Thus, the data packets from session $\varphi_i$ form a data matrix $m_i$ with size $\ell \times S_i$. We consider both additive white Gaussian noise (AWGN) and Rayleigh fading channels. We model channel coefficient $\alpha_{r,i}$ as a zero-mean, circularly symmetric complex Gaussian random variable with a unit variance. In this work, we assume binary phase shift keying (BPSK) modulation for all the transmitted data.

- **Relay phase.** In this phase, each relay node first selects a number $d_r$ with\(^1\) probability $\Omega_{d_r}$ according to the degree distribution $\Omega(x) = \sum \Omega_{d_r} x^{d_r}$. Then each relay node listens to the transmission from the source nodes, decodes the received data packets from the source nodes, checks

\(^1\)After applying the network code design criteria, which is proposed in Section IV-D, a set of column weights for the generator matrix, denoted by $\{d_r\}$, can be obtained. In our work, we only consider $\Omega_{d_r} = 1$. Thus, the set $\{d_r\}$ only contains one value, $d_r$. The indices of the selected $d_r$ data corresponding to the $d_r$ positions of non-zero elements in a column of the generator matrix. If irregular LDGM code is considered, each column of the generator matrix will has different column weight. Then, $d_r$ data will be selected at each relay node with probability $\Omega_{d_r}$, where $0 < \Omega_{d_r} < 1$ and $\sum \Omega_{d_r} = 1$. The network coding operation at relay nodes is similar to random linear network coding but with some constraints, i.e., the network coding operation must satisfy the parameters setting of the LDGM codes, for example, the selection of column weight of the generator matrix should follow the code design criteria.
the correctness of the decoded packets by using CRC and puts the correctly decoded packets into its buffer. Since each relay node only performs hard decision decoding and CRC check, the decoding process is very fast and the delay due to the decoding at each relay node can be neglected. After that, each relay node selects uniformly at random a number of $d_r$ data packets from its buffer to perform network coding by using linear combinations of the packets in the field of GF(2) [30]. Since the number $d_r$ is much smaller than the total number of source nodes in the transmission sessions, i.e., $d_r \ll M$, the probability of the case that the number of data packets in a relay node’s buffer is smaller than $d_r$ is negligibly small and we can always assume that the network coding process is achievable. In case the number of data packets in the buffer of a relay node is less than $d_r$, then all data packets in the buffer will be selected for network coding. Then at each relay node, a network-coded data packet is formed. The connection information of the source and the relay nodes contained in the header of each network-coded data packet. The number of selected source nodes is relatively small to reflect the fact of low density for the formed generator matrix. The length of the header in each data packet is much shorter than the information data part, therefore, the throughput loss due to the header can be neglected. All the network-coded data packets have the same length $\ell$. Finally, all the relay nodes broadcast their network-coded data packets to the destination nodes using a TDMA, CSMA or OFDMA MAC layer protocol.

At the destination nodes, data packets received from the source and the relay nodes in the broadcast and relay phases, respectively, are organized into a data matrix of size $\ell \times (M + N)$. We consider a quasi-static Rayleigh fading channel. Without loss of generality, we consider the $i$th row of the received data matrix. At the destination node $D_i$, the received signal can be expressed as $r_i = \alpha_i \otimes \sqrt{E_b} (J - 2\beta) + n_i$, where $r_i$ is a row vector of size $\ell \times (M + N)$, $\alpha_i$ is the fading coefficient vector of size $1 \times (M + N)$. The fading coefficients remain constant over the length of one data packet but change independently between adjacent packets. Thus, we can assume perfect CSI at the destination nodes, $\otimes$ represents the element-wise multiplication of the vector. For example, there are two vectors $a = [a_1, \ldots, a_i, \ldots, a_N]$ and $b = [b_1, \ldots, b_i, \ldots, b_N]$, $a \otimes b = [a_1b_1, \ldots, a_ib_i, \ldots, a_Nb_N]$. $E_b$ is the average transmitted energy per bit. $J$ is a $1 \times (M + N)$ vector with all elements equal to 1. The binary sequence $\beta$ is a random row of the data matrix $[m_1, \ldots, m_i, \ldots, m_L] \otimes G_i$, where $\otimes$ represents the multiplication operation over the field of GF(2). $[m_1, \ldots, m_i, \ldots, m_L]$ represents the information matrix of the $L$ sessions of size $\ell \times M$. $G_i$ is the corresponding LDGM generator matrix seen from $D_i$ of size $M \times (M + N)$.

Thus, $\beta$ is of size $1 \times (M + N)$. The parameter $n_i = [n_{i,1}, n_{i,2}, \ldots, n_{i,M+N}]$ is the additive white Gaussian noise with a zero mean and a double sided power spectral density of $N_0/2$. Next, each destination node implements the NCDM scheme to remove the interference from the source nodes of other sessions. After using the NCDM scheme, each destination node only receives the data packets from the source nodes of its own session. The particular description of the NCDM scheme will be given in Section III.

Note that since all the connection information are contained in the header of each data packet, the destination nodes know how the network coded data packets are formed at the relay nodes and can correspondingly replicate the code graph and perform message-passing decoding.

![Fig. 3. The bipartite graph.](image)

At each destination node, after implementing the NCDM scheme, a regular systematic LDGM code is obtained by combining the broadcast information data packets from the broadcast phase and the corresponding network-coded data packets from the relay phase, as shown in Fig. 3. The LDGM code formed at the $i$th destination node $D_i$ consists of $S_i$ systematic symbols $V_i, i \in \{1, 2, \ldots, S_i\}$, corresponding to the $S_i$ source nodes from $\varphi_i$, and $N$ coded symbols, corresponding to the $N$ relay nodes. $\{C_i\}$ are the check nodes associated with the syndromes. The decoding can be implemented by the Belief Propagation (BP) algorithm [31] based on the soft information of the received data packets.

### III. THE NCDM SCHEME

In this section, we introduce the NCDM scheme for the parallel session model, which can be used to remove the inter-session interference at each destination. The fundamental idea behind the process takes advantage of the property of $G \otimes H^T = 0$. Here $G$ and $H$ are the generator matrix and the parity check matrix of the LDGM codes, respectively. $H^T$ represents the transpose of the matrix $H$. Throughout the paper, we denote the LDGM generator matrix seen from the $t$th destination node $D_t$ by $G_t$, and the corresponding parity check matrix by $H_t$. In order to remove the inter-session interference, $G_t$ can be split into $L$ separate parts, so that

$$G_t = \left[ (G_{t,1})^T \cdots (G_{t,L})^T \cdots (G_{t,L})^T \right]^T, \quad (1)$$

where $G_t$ is of size $M \times (M + N)$. $G_{t,j}$ is the $j$th submatrix of $G_t$ of size $S_t \times (M + N)$, $t \in \{1, 2, \ldots, L\}$, $S_t$ is the number of source nodes in the $t$th session. Since $G_t$ is a systematic matrix, it can be written as

$$G_t = [I_M \ P_t], \quad (2)$$

where $I_M$ represents an $M \times M$ identity matrix. $P_t$ is an $M \times N$ parity part matrix.
The parity part matrix $P_i$ is given by

$$P_i = \left( [P_i^1]^T \cdots [P_i^j]^T \cdots [P_i^N]^T \right)^T,$$

where $P_i^j$ is an $S_i \times N$ parity part matrix of $G_i^j$. The parity-check matrix $H_i^j$ corresponding to the generator matrix $G_i^j$ is

$$H_i^j = \begin{bmatrix} I_{S_i} & 0_{S_i \times S_j} \otimes P_i^j & 0_{S_i \times N} \\ 0_{S_j \times S_i} & I_{S_j} \otimes P_i^j & 0_{S_j \times N} \end{bmatrix},$$

where $A_t = \sum_{t=1}^{L-1} S_t$, $(2 \leq t \leq L)$ and $A_t = 0$, when $t = 1$. $B_t = \sum_{t'=t+1}^{L} S_{t'}$, $(1 \leq t \leq L-1)$ and $B_t = 0$, when $t = L$. $(P_i^j)^T$ is the transpose of the matrix $P_i^j$.

Similarly, the 1st and the Lth parity check matrices corresponding to $G_1^1$ and $G_1^L$ are respectively given by

$$H_1^1 = \begin{bmatrix} 0_{(M-S_1) \times S_1} & I_{M-S_1} & 0_{(M-S_1) \times N} \\ 0_{S_1 \times (M-S_1)} & I_{S_1} & 0_{S_1 \times N} \end{bmatrix}, \quad H_1^L = \begin{bmatrix} 0_{(M-S_L) \times S_L} & I_{M-S_L} & 0_{(M-S_L) \times N} \\ 0_{S_L \times (M-S_L)} & I_{S_L} & 0_{S_L \times N} \end{bmatrix}.$$

A. The NCDM Scheme for a Parallel Session Model with Two Sessions

When $L = 2$, we can express the generator matrix $G_i$ seen from the $D_i$, $i \in \{1, 2\}$, as

$$G_i = \begin{bmatrix} G_i^1 \\ G_i^2 \end{bmatrix} = \begin{bmatrix} I_{S_i} & 0_{S_i \times S_2} \otimes P_i^2 \\ 0_{S_2 \times S_i} & I_{S_2} \otimes P_i^2 \end{bmatrix},$$

where parity part matrices $P_i^1$ and $P_i^2$ are of sizes $S_1 \times N$ and $S_2 \times N$, respectively.

The parity check matrices of $G_i^1$ and $G_i^2$ are respectively given by

$$H_i^1 = \begin{bmatrix} 0_{(M-S_1) \times S_1} & I_{M-S_1} & 0_{(M-S_1) \times N} \\ 0_{S_1 \times (M-S_1)} & I_{S_1} & 0_{S_1 \times N} \end{bmatrix}, \quad H_i^2 = \begin{bmatrix} 0_{(M-S_2) \times S_2} & I_{M-S_2} & 0_{(M-S_2) \times N} \\ 0_{S_2 \times (M-S_2)} & I_{S_2} & 0_{S_2 \times N} \end{bmatrix}.$$

The generator matrix after using the NCDM scheme at the destination node $D_i$ is named as the equivalent generator matrix, denoted by $\tilde{G}_i$. The equivalent generator matrix seen from $D_i$ for the session $\varphi_i$ when $i = 1$, is then

$$\tilde{G}_1 = G_1 \otimes (H_1^1)^T = G_1^1 \otimes (H_1^1)^T = \begin{bmatrix} I_{S_1} & P_1^1 \\ 0_{S_1 \times S_2} & I_{S_2} \end{bmatrix}.$$ (10)

Let $[m_1 \ m_2]$ represent an arbitrary row of the data matrix $[m_1 \ m_2]$. At the destination node $D_1$, the equivalent received vector after using the NCDM scheme is

$$r_1 (H_1^1)^T = \begin{bmatrix} \alpha_1 & \otimes & \sqrt{E_b} (J - 2 [m_1 \ m_2] \otimes G_1^1) \end{bmatrix} (H_1^1)^T + n_1 (H_1^1)^T,$$ (11)

where $(H_1^1)^T$ is the transpose of the parity check matrix $H_1^2$. $\alpha_1$ is the fading coefficient vector of size $1 \times (M + N)$. $J$ is a row vector of size $1 \times (M + N)$ with all one elements. $\Psi_j$ represents the set formed by the indices of positions of all the non-zero elements in the $j$th column of $(H_1^1)^T$.

Note that (11) cannot be calculated properly, because $\alpha_1$ is the fading coefficient vector and $n_1$ is a noise vector, while $(H_1^1)^T$ is a binary matrix, and the multiplication of them is implemented over the complex field. To tackle this problem, in [33] we employed a soft processing algorithm at the destination nodes to process the soft value of the received symbols. The soft processing algorithm is as follows:

For a binary random variable $X \in \{ \pm 1 \}$, its LLR is defined as $\zeta(X) = \log \left( \frac{P(X = +1)}{P(X = -1)} \right)$. Subsequently, the LLR for a transmitted symbol $x_k$ at the destination node is given by $\zeta_k = \frac{x_k + r_k}{\alpha_{i,k}}$, where $\alpha_{i,k}$ is the fading coefficient, which is assumed to be known at the receiver, $r_k$ is the received symbol corresponding to the transmitted symbol $x_k$.

Once the LLR values are obtained for all the transmitted symbols, we then need to find the corresponding input LLR values for each check node. This can be done by multiplying the LLR values with each column of the matrix $(H_1^1)^T$. The obtained vectors of the LLR values after the multiplication are then the LLR values of the input symbols for the check nodes. Let $z$ be the output symbol of the $j$th check node. Then $z = \oplus \sum_{i \in \Psi_j} x_k$, where $\oplus$ represents the addition operation over the field of GF(2). The output LLR values of each check node can be computed by $L_z = 2 \tanh^{-1} \left( \Pi_{i \in \Psi_j} \tanh(\frac{\zeta_i}{2}) \right)$. $J_{z}$ is the $i$th element of $J$, $\beta_{z}$ is the $i$th element of $\beta$, and $\beta = [m_1 \ m_2] \otimes G_1^1$. When $\left( \oplus \sum_{i \in \Psi_j} J_{z} - 2 \right) \neq 0$ for arbitrary $j, j \in \{ 1, 2, \ldots, M + N - S_2 \}$, we can construct a coefficient $\alpha_{i}'$, and $\alpha_{i}'$ is of size $1 \times (M + N - S_2)$. Let $\alpha_{i,j}'$ be the $j$th element of $\alpha_{i}'$, and $\alpha_{i,1,j}' = \frac{\sum_{i \in \Psi_j} \alpha_{i,j} (J_{z} - 2 \beta_{z})}{\sum_{i \in \Psi_j} J_{z} - 2 \sum_{i \in \Psi_j} \beta_{z}}$. Then (11) can be written as

$$r_1 (H_1^1)^T = \alpha_1 \otimes \sqrt{E_b} (J \otimes (H_1^1)^T - 2 [m_1] \otimes \tilde{G}_1^1) + \alpha_1 (H_1^1)^T.$$

(12)

When $\left( \oplus \sum_{i \in \Psi_j} J_{z} - 2 \right) \sum_{i \in \Psi_j} \beta_{z} = 0$, it will cause the problem of performance degradation. We will analyze this case in Section IV.

From (12) we can see that by using the NCDM scheme, destination node $D_1$ (and following a similar procedure, also $D_2$) only receives data from its own session. However, for the parallel session model with more than two sessions, directly multiplying generator matrix by the transpose of its submatrix parity-check matrix will lead to the interference calculation process stop. Therefore, the NCDM scheme proposed for the parallel session model with two sessions cannot be used directly in the parallel session model with more than two sessions. Next, we will introduce an effective method to adapt the NCDM scheme to the parallel session model with more than two sessions to minimize the inter-session interference.

B. The NCDM Scheme for a Parallel Session Model with More than Two Sessions

To obtain session $\varphi_i$'s information at the destination $D_i$, $i \in \{ 1, 2, \ldots, L \}$, the interference introduced from other
sessions to \( \varphi_i \) needs to be eliminated. This can be done by multiplying the generator matrix \( G_{\ell,i} \) by \((H_{\ell}^T)^T\). Using the property that \( G_{\ell,j}^T \odot (H_{\ell}^T)^T = 0, \forall \ell \in \{1, 2, \ldots, L\} \setminus i \), the interference from other sessions can be removed. However, different from the case with only two sessions, the parity check matrix \((H_{\ell}^T)^T\) is also presented to other submatrices of \( G_{\ell,j}, j \in \{1, 2, \ldots, L\} \setminus \ell \), as an undesirable by-product. For example, when \( t = 1 \), after carrying out the first session \( G_{1,i}'s \) interference cancelation process, the successive process of \( G_{2}^T \odot (H_{1}^T)^T \odot (H_{1}^T)^T \) is not equal to zero due to the contribution of \((H_{1}^T)^T\). As a result, the interference from the second session's information cannot be eliminated and the interference cancelation process stops. To solve this problem, in this paper, we deliberately introduce a term \((\tilde{H}_{1}^T)^{-1}\), so that the product of \((H_{1}^T)\) and \((\tilde{H}_{1}^T)^{-1}\) approximately equals to an identity matrix \( I \). The interference cancelation process continues until \( \varphi_i's \) information at \( D_{\ell} \) is obtained. \((\tilde{H}_{1}^T)\) is a matrix obtained by setting the linear dependent rows in \((H_{1}^T)^T\) to zeros and keeping other rows unchanged. \((\tilde{H}_{1}^T)\) and its inverse matrix are respectively expressed as follows:

\[
\begin{align*}
\tilde{H}_{1}^T &= \begin{bmatrix} I_{A_i} & 0_{A_i \times B_i} & 0_{A_i \times N} \\
0_{S_i \times A_i} & I_{S_i} & I_{S_i} \\
0_{B_i \times A_i} & I_{B_i} & I_{B_i} \\
0_{N \times A_i} & N_{B_i} & I_{N} \\
0_{N \times S_i} & N_{B_i} & I_{N} \end{bmatrix},
\end{align*}
\]

(13)

\[
\begin{align*}
\left(\tilde{H}_{1}^T\right)^{-1} &= \begin{bmatrix} I_{A_i} & 0_{A_i \times S_i} & 0_{A_i \times N} \\
0_{S_i \times A_i} & I_{S_i} & I_{S_i} \\
0_{B_i \times A_i} & I_{B_i} & I_{B_i} \\
0_{N \times A_i} & N_{B_i} & I_{N} \\
0_{N \times S_i} & N_{B_i} & I_{N} \end{bmatrix},
\end{align*}
\]

(14)

In the following, we denote by \( U_{Y_{1} \rightarrow Y_{2}} \) and \( U_{Y_{1} \setminus Y_{2}} \) the first \( Y \) and the last \( Y \) columns of a matrix \( U \), respectively. Let \( U_{Y_{1} \setminus Y_{2}} \) represent a part of the matrix \( U \) from the \( Y_{1} \)th column to the \( Y_{2} \)th column. When \( i \neq L \), a detailed description of the process to remove \( \varphi_i's \) interference is shown in the first part of the right hand side of (15). Since the first \( S_1 \) columns of \( G_{i} \) are zeros, excluding the first identity submatrix \( I_{S_1} \), within the process, \( G_{1}^T \odot P_{1}^i \) can be successfully eliminated when \( t \in \{2, \ldots, L\} \). \( P_{1}^i \) is the parity part matrix of \( G_{1,i} \), and \( P_{1}^i \) is of size \( S_1 \times N \).

\[
\begin{align*}
\begin{bmatrix}
G_{1}^T & (H_{1}^T)^T \\
G_{2}^T & (H_{1}^T)^T \\
G_{L}^T & (H_{1}^T)^T \\
\end{bmatrix} \\
\odot \left( \prod_{t \in \{2, \ldots, L\}} (H_{t}^T) \odot (\tilde{H}_{t}^T)^{-1} \right) \odot (H_{1}^T)^T
\end{align*}
\]

(15)

(15) shows how to keep the \( i \)th session \( \varphi_i \)’s information and remove other sessions’ information simultaneously. The equivalent generator matrix corresponds to the \( i \)th session \( \varphi_i \) after using the NCDM scheme at \( D_{\ell} \), can be expressed as

\[
\begin{align*}
\tilde{G}_{i}^i &= \begin{bmatrix} 0_{S_i \times A_i} & I_{S_i} & 0_{S_i \times (B_i - S_i)} \\
0_{S_{i-1} \times A_i} & I_{S_{i-1}} & I_{S_{i-1}} \\
0_{S_{i+1} \times A_i} & I_{S_{i+1}} & I_{S_{i+1}} \\
0_{N \times A_i} & N_{B_i} & I_{N} \end{bmatrix},
\end{align*}
\]

(16)

Let \( \Phi_i \) represent the term of

\[
\prod_{t \in \{1, \ldots, L-1\}} (H_{t}^T) \odot (\tilde{H}_{t}^T)^{-1} \odot (H_{1}^T)^T.
\]

In a matrix form, \( \Phi_i \) can be expressed as

\[
\Phi_i = \begin{bmatrix} 0_{S_1 \times A_1} & 0_{S_1 \times S_1} & 0_{S_1 \times (B_1 - S_1)} & 1_{(M - S_1) \text{ columns}} \\
0_{S_2 \times A_1} & 0_{S_2 \times S_1} & 0_{S_2 \times (B_1 - S_1)} & P_1^i \\
0_{S_3 \times A_1} & 0_{S_3 \times S_1} & 0_{S_3 \times (B_1 - S_1)} & P_1^{i-1} \\
\vdots & \vdots & \vdots & \vdots \\
0_{S_{L-1} \times A_1} & 0_{S_{L-1} \times S_1} & 0_{S_{L-1} \times (B_1 - S_1)} & P_1^1 \\
0_{S_L \times A_1} & 0_{S_L \times S_1} & 0_{S_L \times (B_1 - S_1)} & P_1^0 \\
0_{N \times A_1} & 0_{N \times S_1} & 0_{N \times (B_1 - S_1)} & I_N \end{bmatrix},
\]

(17)

where for the first \( M - S_L \) columns, all the elements are equal to zero except an identity submatrix \( I_{S_i} \) of size \( S_i \times S_i \), see (17). For the last \( N \) columns of \( \Phi_i \), the first \( L \) submatrices correspond to the parity part matrices of \( G_{1}^i, t \in \{1, \ldots, L\} \), which are given by (1). The last \( N \) rows form an identity matrix, as shown in (17).

Let \([m_1, \ldots, m_i, \ldots, m_L]') represent an arbitrary row of the data matrix \([m_1, \ldots, m_i, \ldots, m_L]' \odot G_{i} \). At destination node \( D_{\ell,i} \) after the NCDM scheme, we can obtain the equivalent received signal as

\[
r_i \Phi_i = \alpha_i \odot \sqrt{e_i} \prod_{J \cap \{1, \ldots, L\}} (J - 2m_1, \ldots, m_i, \ldots, m_L)' \odot G_{i} \Psi_j \]

(18)

where \( \alpha_i \) is the fading coefficient of size \( 1 \times (M + N) \). \( J' \) is the \( i \)'th element of \( J \). \( \beta \) is the \( i \)'th element of \( \beta \), and \( \beta_i = [m_1, \ldots, m_i, \ldots, m_L]' \odot G_{i} \). \( \Psi_j \) represents the set
formed by the indices of positions of all the non-zero elements in the jth column of Φi.

When \( \left( \oplus_{i \in \Psi} J_{j_i} \right) - 2 \left( \oplus_{i \in \Psi} \beta_{j_i} \right) \neq 0 \) for arbitrary \( j, j \in \{1, 2, \ldots, M + N - S_L\} \), we can construct a coefficient \( \alpha'_i \), and \( \alpha'_i \) is of size \( 1 \times (M + N - S_L) \). \( \alpha'_{i,j} \) is the jth element of \( \alpha'_i \), and \( \alpha'_{i,j} = \frac{1}{\sum_{i \in \Psi} \alpha_{i,j} J_{j_i} - 2 \beta_{j_i}} \).

Then (18) can be written as

\[
\rho_i \Phi_i = \alpha'_i \sqrt{E_b} \left( J \circ \Phi_i - 2 [m_i] \circ G_i^t \right) + n_i \Phi_i. \tag{19}
\]

When \( \left( \oplus_{i \in \Psi} J_{j_i} \right) = 2 \left( \oplus_{i \in \Psi} \beta_{j_i} \right) = 0 \), the performance becomes degraded. We will analyze this case in Section IV.

It can be seen from (19) that only session \( \varphi_i \)'s data packet \( [m_i] \) is included in the equivalent received signal vector \( \rho_i \Phi_i \).

When \( i = L \), the same conclusion can be obtained.

IV. PERFORMANCE ANALYSIS AND THE GENERATOR MATRIX DESIGN CRITERIA

In this section, we analyze the performance of the parallel session model with the proposed NCDM scheme at the destination nodes. At first, based on the equivalent received signal vector after implementing the NCDM scheme defined in (19), the impact of the proposed NCDM scheme on the information and noise parts of the received signal \( \rho_i \) is analyzed. Then the LDGM codes design criteria are introduced based on the analytical results.

It can be seen from (19) that the equivalent received signal vector \( \rho_i \Phi_i \) consists of two parts, the information part \( J \circ \Phi_i - 2 [m_i] \circ G_i^t \) and the noise part \( n_i \Phi_i \). Also, it can be seen from (19) that the equivalent received signal vector only contains the data packets from the ith session \( \varphi_i \), \([m_i]\) included in the equivalent received signal vector. A row vector \( J \circ \Phi_i \) is given by

\[
J \circ \Phi_i = J - \Delta, \tag{20}
\]

where \( J \) is a row vector of size \( 1 \times (M + N - S_L) \) with all elements equal to one. \( \Delta = [\Delta_1, \ldots, \Delta_k, \ldots, \Delta_{M+N-S_L}] \) is a row vector of size \( 1 \times (M + N - S_L) \), in which the element \( \Delta_k, k \in \{1, 2, \ldots, M + N - S_L\} \) is reciprocal of the corresponding element \( J \circ \Phi_i \) in the field of GF(2).

The noise part of (19), \( n_i \Phi_i \), is denoted by \( \tilde{n}_i \), \( \tilde{n}_i = [\tilde{n}_{i,1}, \ldots, \tilde{n}_{i,k}, \ldots, \tilde{n}_{i,M+N-S_L}] \). The relationship between \( n_i \) and \( \tilde{n}_i \) can be represented as

\[
\tilde{n}_i = n_i \Phi_i = [n_{i,1}, n_{i,2}, \ldots, n_{i,M+N}] \Phi_i. \tag{21}
\]

Based on (20) and (21), (19) can be rewritten as

\[
r_i \Phi_i = \alpha'_i \sqrt{E_b} \left( J \circ \Phi_i - 2 [m_i] \circ G_i^t \right) + \tilde{n}_i \Phi_i = \alpha'_i \sqrt{E_b} (J - \Delta - 2 [m_i] \circ G_i^t) + \tilde{n}_i. \tag{22}
\]

\[
= \alpha'_i \sqrt{E_b} (J - 2 [m_i] \circ G_i^t) + \tilde{n}_i - \alpha'_i \sqrt{E_b} \Delta. \tag{23}
\]

From (22), the kth value of \( r_i \Phi_i \), denoted by \( (r_i \Phi_i)_k \), is given by

\[
\begin{align*}
\left( (r_i \Phi_i)_k \right) & = \left\{ \begin{array}{ll}
\alpha'_{i,k} \sqrt{E_b} (1 - 2 [m_i] \circ G_i^t) + \tilde{n}_{i,k}, & \text{if } \Delta_k = 0, \\
\alpha'_{i,k} \sqrt{E_b} (1 - 2 [m_i] \circ G_i^t) + \tilde{n}_{i,k}, & \text{if } \Delta_k = 1,
\end{array} \right. \\
& \text{where } \alpha'_{i,k} \text{ is the } k \text{th element of } \alpha'_i, \alpha'_{i,k} = 1 \text{ for the AWGN channel, } (m_i) \circ G_i^t \text{ represents the } k \text{th element of } (m_i) \circ G_i^t.
\]

In the following, we first present the relationship between the column weight of the matrix \( \Phi_i \) with the number of sessions being \( L \) and column weight \( \rho \) of the generator matrix \( G_i^t \), \( t \in \{1, \ldots, L\} \).

A. Relationship between Column Weight of the Matrix \( \Phi_i \), the Number of Sessions \( L \) and Column Weight \( \rho \) of \( G_i^t \)

The number of sessions \( L \) of the system, together with the column weight \( \rho \) of the matrix \( G_i^t \) of the LDGM codes, determine the column weight of the matrix \( \Phi_i \), which affects the received signal vector in (19).

As shown in (20) and (21), \( \Delta \) and \( \tilde{n}_i \) are directly related to the column weight of \( \Phi_i \). According to (20), \( \Delta \) can be expressed as

\[
\Delta = \tilde{J} - J \circ \Phi_i. \tag{25}
\]

The nonzero submatrix of the first \( M - S_L \) columns in \( \tilde{J} \) is \([0_{S_L} \times A, I_{S_L}, 0_{S_L} \times (B_t - S_L)]\). The first \( M - S_L \) elements of \( \Delta \) are \([0_{L \times A}, J_{1 \times S_L}, 0_{L \times (B_t - S_L)}] \), where \( J_{1 \times S_L} \) represents a vector of size \( 1 \times S_L \) with all one elements. Now, let us move to the last \( N \) elements of \( \Delta \). As shown in (17), the column weight of \( \Phi_i \) equals to the sum of the column weights of the \( L - 1 \) parity part matrices \( P_i^t \) plus one, where \( t \in \{1, \ldots, L\} \). As \( P_i^t \) is a part of \( G_i^t \), we only need to focus on the column weight of \( G_i^t \). That is, the two terms \( \Delta \) and \( \tilde{n}_i \) given in (21) and (25), respectively, are related to the generator matrix of the LDGM codes. Here, we only consider regular LDGM codes, where the column weight \( \rho \) of \( G_i^t \) remains unchanged.

Let us first derive the relationship between the system parameter, i.e., the number of sessions \( L \), the LDGM codes' parameter, i.e., the column weight \( \rho \) and \( \Delta \). In the following, we discuss separately the situations that the parameters \( L \) and \( \rho \) are odd and/or even numbers.

- When the number of sessions \( L \) is an even number
  - If \( \rho \) is an odd number, then \((L - 1) \rho \) is also an odd number. As the column weight of \( \Phi_i \) equals to the sum of \( L - 1 \) columns' weight of parity-check part in \( G_i^t \) plus 1, the column weight of \( \Phi_i \) is even and the last \( N \) elements of the row vector \( J \circ \Phi_i \) are all zeros. According to (20) and (25), \( J \circ \Phi_i = J - \Delta, \tilde{J} \) is a row vector with all one elements. Thus \( \Delta \) is a

The generator matrix of systematic LDGM codes can be expressed as \( G = [I \ P] \), so the parity-check matrix \( H = [P^T \ I] \) and the transpose matrix of \( H \) is given by \( H^T = \begin{bmatrix} \mathbb{P}^T & \mathbb{I} \end{bmatrix} \). Thus, the column weight of \( H^T \) equals to that of \( P \) plus 1.
row vector with all elements equal to one. It follows that the $k$th element of $\Delta$, $\Delta_k$, can be expressed as $\Delta_k = 1$, if $\rho$ is an odd number.

- If $\rho$ is an even number, then $(L-1)\rho$ is also an even number, the column weight of $\Phi$, is an odd number. Thus the row vector $\mathbf{J} \odot \Phi$ has all elements equal to one. By using the above method, the value of $\Delta$ can be derived. It is a row vector with all elements equal to zero. Consequently, $\Delta_k$ can be expressed as $\Delta_k = 0$, if $\rho$ is even.

From the above descriptions, $\Delta_k$ can be generally expressed as

$$\Delta_k = \begin{cases} 1, & \text{if } L \text{ is an even number} \\ 0, & \text{if } L \text{ is an odd number} \end{cases}$$

(26)

- When the number of sessions $L$ is an odd number

No matter whether $\rho$ is an odd or even number, $(L-1)\rho$ is always an even number. It follows that the column weight of $\Phi$, equals to $(L-1)\rho + 1$, which is an odd number. Thus the row vector $\mathbf{J} \odot \Phi$, has all elements equal to one. Based on (20) and (25), $\mathbf{J} \odot \Phi = \mathbf{J} - \Delta$, $\Delta$ is a row vector with all elements equal to one. Consequently, $\Delta$ is a row vector with all elements equal to zero. Therefore, $\Delta_k$ can be expressed as

$$\Delta_k = 0, \text{ if } L \text{ is odd.}$$

(27)

B. The Impact of the Matrix $\Phi_i$ on the Noise Part of the Received Signal Vector

The number of sessions $L$ of the system and the column weight $\rho$ of the generator matrix of the formed LDGM code for each session determine the received signal vector through the column weight of the matrix $\Phi_i$. In this subsection, we first describe the impact of the matrix $\Phi_i$ on the noise part of the received signal vector. Then, we derive the equivalent received SNR based on the relationships described above. The equivalent received SNR can be used to compare the performance of the formed LDGM codes with various $L$ and $\rho$ values in the latter part of this section.

From (24), we can see that no matter what the value of $\Delta_k$ is, the noise part is always $\tilde{n}_{i,k}$. In other words, $\Phi_i$ always affects the noise part of the received signal vector as $\tilde{n}_i = n_i \Phi_i$.

The $k$th element of $\tilde{n}_i$ in (21) can be rewritten as

$$\tilde{n}_{i,k} = \sum_{q \in \Omega_k} n_{i,q},$$

(28)

where $\Omega_k$ denotes a set, in which the elements are the row indices of nonzero elements in the $k$th column of matrix $\Phi_i$. $\Omega_k$ has $(L-1)\rho + 1$ elements. $\tilde{n}_{i,k}$ equals to the sum of $(L-1)\rho + 1$ elements in $n_i$, where $n_i$ is the channel noise and $\tilde{n}_{i,k}$ is the $k$th element of $\tilde{n}_i$. It can be seen that when the value of $\rho$ or $L$ increases, the size of $\Omega_k$ gets larger and subsequently $\tilde{n}_{i,k}$ increases. The impact of $\rho$ or $L$ on the noise part $\tilde{n}_i$ in (21) increases.

Based on (23), the $k$th equivalent received signal $(r_i^k)$ can also be represented as

$$\begin{align*}
\alpha_i^k \sqrt{E_b} \left( 1 - 2 \left( |m_i| \odot \tilde{G}_i^k \right) \right) + \tilde{n}_{i,k}. \\
&\text{if } \Delta_k = 0 \\
\alpha_i^k \sqrt{E_b} \left( 1 - 2 \left( |m_i| \odot \tilde{G}_i^k \right) \right) + \tilde{n}_{i,k} - \alpha_i^k \sqrt{E_b}, \\
&\text{if } \Delta_k = 1
\end{align*}$$

(29)

To simplify the analytical expression, we assume that different sessions have the same number of source nodes, denoted by $M_e$ ($S_i = S_e$, $i \in \{1, 2, \cdots, L\}$). From (24), it can be seen that $\Phi_i$ generally affects the received signal vector, regardless of the value of $\Delta_k$. Let $P(\Delta_k = x, x \in \{0, 1\})$, be the probability of the event that $\Delta_k$ equals to $x$. From (29) and based on the relationship among the row vector $\Delta$, noise $\tilde{n}$, and column weight of $\tilde{G}_i^k$, an equivalent SNR, which is denoted by $\text{SNR}_e$, can be expressed as

$$\begin{align*}
\text{SNR} &= \frac{1}{M_e + N} \left\{ \sum_{k=0}^{M_e} \left| \alpha_i^k \sqrt{E_b} \right|^2 \right\} \\
&+ \sum_{k=M_e+1}^{M_e+N} \left[ P(\Delta_k = 0) \left| \alpha_i^k \sqrt{E_b} \right|^2 \frac{L - 1}{(L-1)\rho N_e + N_0} + P(\Delta_k = 1) \left| \alpha_i^k \sqrt{E_b} \right|^2 \right] \\
&+ \sum_{k=0}^{M_e} \left[ P(\Delta_k = 0) \left| \alpha_i^k \sqrt{E_b} \right|^2 \frac{L - 1}{(L-1)\rho N_e + N_0} + P(\Delta_k = 1) \left| \alpha_i^k \sqrt{E_b} \right|^2 \right]
\end{align*}$$

(30)

From (26) and (30), we can see that the value of $\text{SNR}_e$ is affected by the column weight $\rho$ of $\tilde{G}_i^k$ and the number of sessions $L$.

C. Unstable Zero Elements

In this part, we analyze the performance of systems with different number of sessions and with different column weights of the formed LDGM codes. The problem, unstable zero elements, which is caused by right multiplication of $\Phi_i$, is elaborated. The impacts of these unstable zero elements on the performance of the formed LDGM codes with different $L$ and $\rho$ values are analyzed, by using the equivalent received SNR derived in Section IV-B.

In Section IV-B, the relationship between the noise part in (21) and $\rho$ has been investigated. Here we investigate the relationship between the information part $\mathbf{J} \odot \Phi_i - 2|m_i| \odot \tilde{G}_i^k$ and $\rho$.

The information part, $\mathbf{J} \odot \Phi_i - 2|m_i| \odot \tilde{G}_i^k$, is denoted by $\mathbf{X}$, $\mathbf{X} = [x_1, \cdots, x_k, \cdots, x_{M_e+N-S_e}]$. From (20), we can see that $\mathbf{X}$ is a matrix of $\mathbf{J} \odot \Phi_i$, determined by $\mathbf{X}$. Similar to the previous subsection, we discuss the situations when $L$ and $\rho$ are even and/or odd numbers separately.

- When $L$ is an even number, as shown in (26), the value of each element in $\mathbf{X}$ depends on $\rho$. When $\rho$ is an odd number, $\Delta_k$ equals to 1; when $\rho$ is an even number, $\Delta_k$ equals to 0. Therefore, each element in $\mathbf{J} \odot \Phi_i$ has two possible values, 0 or 1. Moreover, $(|m_i| \odot \tilde{G}_i^k)$ may equal to 0 or 1. Thus, an element of $\mathbf{X}$, such as $x_k$, has four possible values, shown below

$$x_k = \begin{cases} +1, & \text{if } \Delta_k = 0, (|m_i| \odot \tilde{G}_i^k) = 0 \\
-1, & \text{if } \Delta_k = 0, (|m_i| \odot \tilde{G}_i^k) = 1 \\
0, & \text{if } \Delta_k = 1, (|m_i| \odot \tilde{G}_i^k) = 0 \\
-2, & \text{if } \Delta_k = 1, (|m_i| \odot \tilde{G}_i^k) = 1
\end{cases}$$

(31)
The $k$th equivalent received signal $\tilde{r}_k$ can be expressed as $\tilde{r}_k = x_k + \tilde{a}_k$. With BPSK demodulation rule, $\tilde{r}_k$ can be detected as either $+1$ or $-1$. Apparently, when $x_k \in \{+1, -1, -2\}$, $\tilde{r}_k$ can be easily detected as either $+1$ or $-1$. However, when $x_k = 0$, $\tilde{r}_k$ has a probability of $\frac{1}{2}$ to be correctly detected. Thus, the incorrectly detected values associated with zero elements introduce errors. We refer to these zero elements as the unstable zero elements.

The unstable zero elements are introduced by right multiplication of $\Phi_i$ by the received signal vector. The probability of generating unstable zero elements can be expressed as

$$P(x_k = 0) = P(\Delta_k = 1) P\left(\left([m_i] \odot \tilde{G}_i\right)_k = 0\right),$$

(32)

where $P(\Delta_k = 1)$ denotes the probability of the event $\Delta_k = 1$. $P\left(\left([m_i] \odot \tilde{G}_i\right)_k = 0\right)$ denotes the probability of the event $\left([m_i] \odot \tilde{G}_i\right)_k = 0$. If $P(\Delta_k = 1) = 0$, then $P(x_k = 0) = 0$.

We assume that there are $N$ relay nodes in the network. If source nodes are selected randomly in the relay phase, then (32) can be rewritten as $P(x_k = 0) = \frac{1}{2} \sum_{i=0}^{N} \sum_{j=0}^{N} \frac{2}{M_i + N^t}$. These unstable zero elements, which are generated by right multiplication of $\Phi_i$ by the received signal vector, cause extra errors with probability $\frac{1}{2} P(x_k = 0)$. These errors would result in lower equivalent SNR and degrade the decoding performance.

- **When $L$ is an odd number**, $\Delta_k$ always equals to 0. From (31) we can see that $x_k$ is $+1$ or $-1$. Therefore, when $L$ is an odd number, $x_k$ can be decoded correctly with a high probability.

When the column weight of each generator matrix is an even number, the session has a better performance compared with the one whose generator matrix has an odd column weight.

- If the column weight of the matrix $G_i^t$ is an even number, the probability $P(\Delta_k = 1)$ in (32) equals to 0. Thus, in case of $P(x_k = 0) = 0$, no unstable zero elements are generated in the information part of the signal vector.

- If the column weight is an odd number, some unstable zero elements in the information part of the signal vector will be generated. These zero elements lead to the worse performance.

If the column weight of $G_i^t$ is a smaller value, the equivalent noise in (28) reduces, so the code performance is better than that with a bigger column weight.

As we discussed in (27), when $L$ is odd, $\Delta_0 = 0$. It means that no unstable zero elements are produced regardless of the value of $\rho$. In addition, the code with a smaller column weight has a better performance.

Based on the previous analysis, we can design network codes to achieve desired performance by properly selecting the code parameters according to the situation of the entire system. The code design criteria is summarized in the following.

D. Network Codes Design Criteria

As mentioned in (1), $G_i^t$ is the session $\varphi_i$’s generator matrix, seen from $D_i$. $H_i^t$ is the corresponding parity-check matrix of $G_i^t$. On the one hand, we should notice that when $L$ is an even number, it is necessary to make sure that after the NCDM scheme, the generator matrices have desired column weights. As indicated in (24), (26) and (27), the column weight $\rho$ of $G_i^t$ must be chosen to be an even number if $L$ is even. When $L$ is an odd number, the column weight of $G_i^t$ can be chosen as either an even or odd number, such that, $P(\Delta_k = 1) = 0$ and $P(x_k = 0) = 0$ do not produce unstable zeros elements. On the other hand, we need to select the column weight of $G_i^t$ as small as possible to reduce the effect from the noise part of (19). Based on the above explanation, to minimize a code error performance, the code design criteria can be summarized as

- Make sure that the column weight of each generator matrix $G_i^t$ is an even number if $L$ is an even number. If $L$ is an odd number, the column weight of each generator matrix $G_i^t$ can be either an even or odd number.

- Minimize the column weight of the generator matrix $G_i^t$, i.e., $\rho = 2$ when $L$ is an even number; $\rho = 2$ or $\rho = 3$ when $L$ is an odd number.

V. PERFORMANCE EVALUATION

In this section, we present simulation results for the proposed NCDM scheme. Due to the time-varying channels in the wireless network, within each transmission round, the LDGM code is generated on-the-fly to match the instantaneous network topology. The performance$^4$ of the designed code is based on the ensemble average performance. We consider a wireless network with 3 or 4 sessions, respectively. In the simulation, each session has 300 source nodes$^5$, sending data packets to its corresponding destination node via a common relay network. For convenience, we assume that the number of relay nodes equals the total number of source nodes in the investigated networks.

A. BER Performance

We assume that all source nodes have data packets to transmit. All the source-relay channels in the network are

$^4$In this paper, the effect from asynchronisation to the performance of both the serial session model and the parallel session model is not considered. Asynchronisation in distributed wireless network will cause collisions and idle periods among transmissions from different sensor nodes. Collisions cause the BER performance to be deteriorated for both the serial session model and the parallel session model. The throughput of both the two session models will decrease with the increasing of idle periods.

$^5$The simulation parameter, 300 source nodes in each transmission session, is arbitrarily chosen. In a practical network, the number of source nodes is determined by the topology of the network. We also set different values to the parameter according to the requirements in different applications. Note that there exists a trade-off between the end-to-end delay and the performance. Assume that the threshold of the end-to-end delay is $\Gamma$. $\ell_H$, $\ell_C$ and $\ell_H$ represent the length of the header, the CRC part, and the information data part of one data packet, respectively. When $\frac{\ell_H + \ell_C}{\ell_H} < \frac{1}{\lambda - \gamma}$, the additional time required for transmitting the information data in the parallel session model is tolerable. In general, the length of the information data part is much larger than the length of the header and the CRC part in one data packet, i.e., $\ell_H \gg \ell_H + \ell_C$. Thus, $\frac{\ell_H + \ell_C}{\ell_H}$ is a very small value. Therefore, we can come to the conclusion that in the general case, the end-to-end delay in the parallel session model is small and tolerable.
spatially independent and have equal transmitting power. By a serial session model, we refer to multiple groups of source nodes communicating with their respective destinations through a common relay network one by one. On the other hand, the parallel session model refers to multiple groups of source nodes communicating with their respective destinations through a common relay network simultaneously. In the presented figures, the label “no interference” corresponds to the serial session model. “NCDM” refers to the parallel session model.

Fig. 4 (a) presents the performance comparison of various column weights of $\mathbf{G}_i^L$ (i.e., $\rho = 2$ and $\rho = 3$), for an even number of sessions (i.e., $L = 4$). As we discussed in Section IV-C, when the number of sessions $L$ is an even number, if the column weight $\rho$ of $\mathbf{G}_i^L$ is an even number, only the noise part in (19) is affected. As a result, the BER performance gap between the serial session model and parallel session model is smaller than the one for the case when $\rho$ is an odd number. Note that when the column weight $\rho$ is an odd number, both the information part and the noise part in (19) are affected, and the BER performance gets worse. The observation is consistent with the theoretical analysis results.

Fig. 4 (b) presents the performance comparison of various column weights of $\mathbf{G}_i^L$, (i.e., $\rho = 2$ and $\rho = 3$), with an odd number of sessions (i.e., $L = 3$). As we discussed in Section IV-C, when the number of sessions is an odd number, only the noise part in (19) is affected. The smaller the column weight $\rho$ of $\mathbf{G}_i^L$ is, the closer the BER performance of the parallel session model with NCDM to that of the serial session model. From Fig. 4 (b) we can see that when $\rho = 2$, the BER performance of the parallel session model is very close to the one for the serial session model.

The idea of the repetition code is to repeat the message several times. We consider that there are $L = 3$ and 4 sessions in the network. After using the NCDM scheme, the code rate becomes $\frac{1}{3+1}$. Thus we compare the performance of our proposed scheme with the repetition code with parameter $d = L$, i.e., the messages are repeated $L$ times. From Fig. 4, it can be seen that the proposed system achieves a significant BER performance gain compared with the system with the repetition codes in a high SNR region.

For the Rayleigh fading channel, the BER performance gap between the serial session and parallel session model is much larger than that of the AWGN channel, as shown in Fig. 4. This is due to the information loss caused by LLR processing within the NCDM scheme. For the fading channel, the difference among the LLR values in the vector is much larger than that of the AWGN channel. The smaller LLR values result in worse BER performance gap between the serial session and parallel session model over the fading channel.

B. System Throughput

The throughput performance of the parallel session model with the NCDM scheme over AWGN and Rayleigh fading channels are shown in Fig. 5. Let $\triangle t_b$ represent the transmission time for one bit. Here, the throughput is defined as the number of correctly delivered information bits by all the source nodes in $\triangle t_b$. $\triangle t$ represents one time slot used to transmit one data packet, and $\triangle t = L \triangle t_b$, $\ell$ is the length of one data packet. One transmission period $\triangle T$ is defined as the time required to transmit all the sources’ data packets and relays’ data packets in one round. Assume that the source nodes in the $i$th session $\varphi_i$ have $S_i$ data packets to transmit in one transmission period, and the number of correctly delivered data packets at $D_i$ is $B_i$ for the parallel session model and $C_i$ for the serial session model, respectively. Then, the number of transmitted data packets from all the sessions in one transmission period is $M = \sum_{i=1}^{L} S_i$, and the number of correctly delivered data packets at all the destination nodes is $B = \sum_{i=1}^{L} B_i$ for the parallel session model and $C = \sum_{i=1}^{L} C_i$ for the serial session model, respectively.

Assume that TDMA is employed. For the serial session model, the source nodes in transmission session $\varphi_i$ send their data packets to the relay nodes separately in a number of $S_i$ time slots. After all the source nodes in $\varphi_i$ sending their data packets, the relay nodes begin to decode the received data packets from $\varphi_i$, check the correctness of the decoded packets by using CRC and put the correctly decoded packets into their buffers. Then the relay nodes select a fix number
of data from their buffers and perform network coding to form network-coded data packets. Finally, the relay nodes send their network-coded data packets to the destination nodes in a number of $N$ time slots. The same operations between the transmission sessions and the relay nodes run alternately until all the relay nodes assist all the $L$ transmission sessions completing their transmissions. The total time slots for the data transmission from the source nodes to the destination nodes in the serial session model is $\sum_{i=1}^{L} S_i + LN$, i.e., $M + LN$.

For the parallel session model, the source nodes in all the $L$ transmission sessions send their date packets to the relay nodes in a number of $\sum_{i=1}^{L} S_i$ time slots. After all the source nodes sending their data packets, the relay nodes deal with the received data packets, perform network coding, and send the formed network-coded data packets to the destinations in a number of $N$ time slots. Thus, the total time slots for the date transmission from the source nodes to the destination nodes in the parallel session model is $\sum_{i=1}^{L} S_i + N$, i.e., $M + N$.

Let $\ell_H$ and $\ell_C$ represent the length of the header and the CRC part of one data packet, respectively. The throughput of the parallel session model with $L$ source group-destination pairs and the serial session scheme are calculated as follows:

$$\frac{\ell B - (\ell_H + \ell_C) B}{(\Delta T)_{\text{parallel session}}} = \frac{\ell \sum_{i=1}^{L} B_i - (\ell_H + \ell_C) \sum_{i=1}^{L} B_i}{\sum_{i=1}^{L} S_i \Delta t + N \Delta t} = \frac{(\ell - (\ell_H + \ell_C)) \sum_{i=1}^{L} B_i}{3N \Delta t} \sum_{i=1}^{L} S_i \Delta t + LN \Delta t} = \frac{1}{M + LN} \sum_{i=1}^{L} C_i \sum_{i=1}^{L} S_i \Delta t + N \Delta t} (33)$$

$$\frac{\ell C - (\ell_H + \ell_C) C}{(\Delta T)_{\text{serial session}}} = \frac{\ell \sum_{i=1}^{L} C_i - (\ell_H + \ell_C) \sum_{i=1}^{L} C_i}{\sum_{i=1}^{L} S_i \Delta t + LN \Delta t} = \frac{(\ell - (\ell_H + \ell_C)) \sum_{i=1}^{L} C_i}{\sum_{i=1}^{L} S_i \Delta t + LN \Delta t} = \frac{1}{M + LN} \sum_{i=1}^{L} C_i \sum_{i=1}^{L} S_i \Delta t + LN \Delta t}$$ (34)

In general, $\frac{\ell H + \ell C}{\ell}$ is a very small value. The effect from the overhead to throughput is very small.

It can be seen from Fig. 5 (a), the throughput for the parallel session model with the NCDM scheme significantly outperforms the serial session scheme. This is due to the fact that at each time unit, the serial session model only allows one source group to be relayed, while the parallel session model with the NCDM scheme allows all the sessions’ data packets to be relayed simultaneously. From (33) and (34) we can see, when $B = C$, the throughput of the parallel session model is $\frac{\ell + 1}{\ell}$ times that of the serial session model. In addition, we should notice that the BER performance of the parallel session model with the NCDM scheme approaches that of the serial session model over AWGN channels, as shown in Fig. 4.

Next, we compare the throughput of the NCDM scheme with a similar scheme, which use the parallel session model but make BP decoding without canceling any interference. In the similar scheme, at each destination node, though only the information from its own session is required, the information from all the $L$ transmission sessions are recovered. The comparison of the throughput of system with the NCDM scheme and the similar scheme is shown in Fig. 5 (b).

It can be seen from Fig. 5 (b), the throughput for the parallel session model with the NCDM scheme is worse than the similar scheme at low $E_b/N_0$ region. This is due to the fact that there are no interference problems among different sessions when using the similar scheme, while the parallel session model with the NCDM scheme has to deal with the inter-session interference. The throughput of the parallel session model with the NCDM scheme approaching to that of the similar scheme as the value of $E_b/N_0$ increases. At the region where $E_b/N_0 \geq 6 \text{dB}$ for the AWGN channels and $E_b/N_0 \geq 10 \text{dB}$ for the Rayleigh fading channels, the throughput of the parallel session model match that of the similar scheme. However, in the similar scheme, the information from all the sessions is recovered at all the destinations. Thus, though the similar scheme can achieve better throughput performance, it cannot satisfy the security requirement of the applications considered in this paper.
C. Complexity

For simplicity, in our investigation we assume that the number of source nodes \( M \) from all the sessions equals the number of relay nodes \( N \), i.e. \( M = N \). We further assume that the number of source nodes \( M_r \) in each session is the same, that is \( M_r = M \). We define the computational complexity as the total number of additions and multiplications. When \( L \gg 1 \), \( N \gg 1 \) and the column weight of the generator matrix \( G \), \( \rho \geq 2 \), the computational complexity for the NCDM scheme can be expressed by \((L-1)\rho N < L\rho N\).

Table I presents a summary of the complexity for the system models applying respectively repetition codes, the serial session model and the parallel session model with NCDM scheme. The system complexity includes the relaying complexity, the computational complexity of implementing interference cancelation by the proposed NCDM scheme and decoding complexity at the destination nodes.

As the decoding complexity of the three systems are the same, here, we only focus on discussing the computational complexity of implementing the proposed interference cancellation scheme. For the parallel session model with the proposed NCDM scheme, can be approximated as \( O(L\rho N) \). Compared with the serial session model, the additional computational complexity to carry out interference cancelation process is a linear function of \( N \) and \( L \).

VI. CONCLUSION

In this work, we investigated the distributed network coding schemes for a parallel session model with network code division multiplexing scheme. In parallel session model, multiple relays were shared among multiple sessions. We considered the inter-session interference problem of multicasting data packets from multiple groups of source nodes to their respective destination nodes via a common wireless relay network. We proposed a NCDM scheme for the parallel session model to remove the inter-session interference. In the system analysis, we found that the proposed NCDM scheme is able to minimize the inter-session interference at each destination. However, it also introduced some unstable zero elements and extra noise problems, which affect the formed code’s BER performance. Based on the analysis, we investigated the relationship among the equivalent received signal vector, the number of sessions and column weight of the generator matrix. A code design criteria for the generator matrix construction was proposed. Simulation results showed that by constructing the generator matrix of the LDGM code following the proposed code design criteria, the problems caused by the NCDM scheme can be managed effectively and the BER performance can be improved significantly.

We further evaluated the proposed NCDM scheme in terms of the system throughput and complexity. From the evaluation we can see that the throughput of the parallel session model with the proposed NCDM scheme is \( \frac{L}{L+1} \) times that of the serial session model. Our complexity evaluation showed that the proposed NCDM scheme for a parallel session model has a linear computational complexity in the number of relay nodes \( N \) and the number of sessions \( L \).

REFERENCES

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