# When Does it Take a Nixon to Go to China - Errata and Further Results*

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**Abstract**

This note points out and corrects an algebraic error that occured in the paper "When does it take a Nixon to go to China?" by Cukierman and Tommasi (CT). The correction leads to tighter restictions on the range of shocks for which the political equilibrium in the paper is valid. But, for this range, the conditions for policy reversals are similar to those descibed in the paper. Since the modified equilibrium requires several, possibly conflicting, restrictions on the model's parameters this range may, in principle, be empty. However the note shows that there exists a dense set of parameters for which this range is non empty and that, within it, the broad conclusions of CT, including comparative statics and the discussion of credibility in section IV apply. The tighter restrictions also lead to new comparative statics results. For example an increase in office motivation, and a decrease in electoral uncertainty or in party polarization widen the range of policy reversals.

In a paper in this review Alex Cukierman and Mariano Tommasi (1998) (hereinafter CT) identify circumstances under which a right wing policy is more likely to be implemented by a left wing incumbent party or, more generally, circumstances under which a given policy is more

*We are indebted to Allan Drazen for a private communication in which he produces a counterexample suggesting that the equilibrium solution in the original article does not hold for **all** possible shock realizations. This note was triggered by Drazen's counterexample. It presents a modified analysis that explicitly takes into consideration the additional restrictions required to support the equilibrium solution in the original article.

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likely to be implemented by the unlikely party than the likely party for that policy. As in the paper we refer to such situations as "reversals". CT show, for a two party political system, that the policy chosen by an incumbent party may be nearer to the center or further away from it than the party's ideal point. These two possibilities arise because the choice of policy triggers two effects. One is the traditional Hotelling effect that pulls the incumbent's policy towards the center. The other is due to the fact that voters utilize the incumbent's policy as a signal for a utility relevant state of nature about which the incumbent has better information. This signaling function of policy may induce the incumbent party to diverge away from the center in comparison to the party's ideal point.

Depending on parameter values the first or the second factor dominates the choice of policy. The case in which policy ultimately converges somewhat towards the center is labeled "case 1" and the case in which it tends to move away from the party's ideal point in the opposite direction is labeled "case 2". CT note that reversals are possible only in the first case and proceed to analyze it in more detail. In this case the solutions for policy choices under a left wing incumbent (LWI) and a right wing incumbent (RWI) are given respectively by

$$x_L = B_L + \frac{1}{2}(\gamma + \epsilon_L) \equiv B_L + \frac{1}{2}\mu_L, \quad x_R = B_R + \frac{1}{2}(\gamma + \epsilon_R) \equiv B_R + \frac{1}{2}\mu_R, \tag{1}$$

where $B_L$ and $B_R$ are defined in equations (21) and (22) of CT, $x_j$, $j = L, R$, is the policy chosen by incumbent $j$, $\gamma$ is a state of nature shock that induces a unidirectional change in the preferred policies of all voters and $\epsilon_j$, $j = L, R$, is an intra-party shock that affects the ideal policy of party $j$. Both shocks have zero means, are normally distributed with variances $\sigma_\gamma^2$ and $\sigma_\epsilon^2$ respectively and are statistically independent. The incumbent party knows the precise realizations of each of the two shocks but voters know only the distributions of the shocks and observe the incumbent's party policy. CT show that case 1 arises when $\sigma_\gamma^2 < \sigma_\epsilon^2$ and derive the solutions for policy choices in this case under the assumption that, when in office, either party converges somewhat towards the center. That is,

$$x_L > I_L \equiv c_L + \epsilon_L + \gamma, \quad x_R < I_R \equiv c_R + \epsilon_R + \gamma, \tag{2}$$

where $I_L$ and $I_R$ are the (stochastic) ideal policies of the two parties and $c_L$ and $c_R$ are the deterministic components of those ideal policies.

However, as can be seen from a comparison of equations (1) and (2), the inequalities in equation (2) do not hold for all possible realizations of the sums, $\mu_L$ and $\mu_R$ of the two shocks. CT neglect the fact that, although the inequalities in equation (2) hold for some range of shocks' realizations they do not hold for **all** such realizations. Thus, the solutions in equation (1) are correct only as long as the shock realizations are such that the inequalities in equation (2) are satisfied.

The purpose of this note is to incorporate those neglected restrictions into the CT framework and to delineate how this alters, if at all, their analysis and results. From equations (1) and (2), the solutions in equation (1) satisfy the conditions in equation (2) if and only if

$$\mu_L < 2(B_L - c_L) \equiv q_L, \quad \mu_R > 2(B_R - c_R) \equiv q_R, \tag{3}$$

implying that the solutions in equation (1) apply only in the range of shocks specified in equation (3). We shall therefore limit our discussion to this range and present conditions that lead to reversals within this range.[1] Using equation (1) the bounds on $\mu_L$ and $\mu_R$ in equation (3) can be translated into bounds on $x_L$ and $x_R$. This yields the equivalent conditions

$$x_L < 2B_L - c_L \equiv x_L^c, \quad x_R > 2B_R - c_R \equiv x_R^c. \tag{4}$$

Thus, the solutions in equation (1) apply for **both** incumbents for policies, $x$, in the range

$$x_R^c < x < x_L^c. \tag{5}$$

Analogous restrictions have to be imposed in case 2 in which, since $\sigma_\gamma^2 > \sigma_\epsilon^2$, both incumbents diverge away from their ideal policies towards more extreme policies. These restrictions

[1] Analysis not reported here suggests that outside this range of shocks, when it exists, the rational expectations equilibrium takes a different form than the one presented in the text.

Note that, limiting the discussion to the range of shock realizations (and therefore of policies chosen) for which the equilibrium in the text applies does not impose any restrictions on the distributions of $\gamma$ and of $\epsilon_j$ , $j = L, R$, whose supports remain unbounded. But the equilibrium analysis and statements in the text apply only in the range of shocks and corresponding policy realizations defined respectively by equations (3) and (5) of the text.

are given by

$$x_L > 2B_L^2 - c_L \equiv x_L^{c2},\ \ x_R < 2B_R^2 - c_R \equiv x_R^{c2}, \tag{6}$$

where $B_L^2$ is given by the second formula in equation (21) of CT and $B_R^2$ is the counterpart of $B_L^2$ for a RWI. Hence, for case 2, the solutions in equation (1) and in the paper apply for **both** incumbents for policies in the range

$$x_L^{c2} < x < x_R^{c2}. \tag{7}$$

In this range, since the probability of reelection of a type $j$ incumbent ($j = L, R$) goes down as his policy shifts to an uncharacteristic range of policies, there are no reversals (details appear on page 187 of CT). We therefore focus from now on, on the case $\sigma_\gamma^2 < \sigma_\epsilon^2$ (case 1) in which there might be reversals and, following CT, assume for simplicity that the political system is symmetric around zero. That is,

$$c_R = -c_L > 0,\ \ \overline{c} = -\underline{c} > 0, \tag{8}$$

where $\overline{c}$ and $\underline{c}$ are the upper and lower bounds of the uniform distribution that characterizes electoral uncertainty. This specification implies that the center of the political spectrum is located at zero. Symmetry also implies that

$$x_L = -B + \frac{1}{2}\mu_L,\ \ x_R = B + \frac{1}{2}\mu_R,\ \ x_L^c = -x_R^c = c_R - 2B,\ \ q_L = -q_R = 2(c_R - B), \tag{9}$$

where

$$B \equiv (1+\rho)\overline{c} + \rho c_R - (1-\rho)\frac{h}{2},\ \ and\ \ \rho \equiv \frac{\sigma_\gamma^2}{\sigma_\epsilon^2} < 1\ \ for\ case\ 1. \tag{10}$$

where $h$ is a positive parameter that measures the degree of office motivation of parties. For reasons elaborated in CT the solutions in equation (1) apply provided their Assumption 1 holds (p. 185). This assumption requires that voters always expect a right wing challenger, if elected, to deliver a more rightist policy than a LWI and a left wing challenger, if elected, to deliver a more leftist policy than the RWI. The following lemma establishes conditions on the office motivation, $h$, of parties which assure that the range of policies in equation (5) is non-empty and that it satisfies Assumption 1 in CT.

**Lemma 1** *If the political system is symmetric in the sense specified in equation (8) and*

$$h_{xc} \equiv \frac{2}{1-\rho}\left[(1+\rho)\overline{c} + \rho c_R - \frac{1}{2}c_R\right] < h < \frac{2}{1-\rho}\left[(1+\rho)\overline{c} + \rho c_R\right] \equiv h_B \tag{11}$$

*then*

*(i) The range of policies in equation (5) is non-empty,* $x^c_L$ *is positive,* $x^c_R$ *is negative and they are positioned at equal distances from the center of the political spectrum* ($x^c_L = -x^c_R > 0$)*. The center of the political spectrum is located at zero.*

*(ii)* $B > 0$ *and the mean policy of a RWI is to the right of the center of the political spectrum while the mean policy of a LWI is to the left of the center. The mean policies of the two incumbents are located at equal distances from the center.*

*(iii) Assumption 1 of CT is satisfied for all policies in the range* ($x^c_R$, $x^c_L$)*.*

*(iv) Provided* $c_R < (1+\rho)\overline{c}$*, the conditional probability,* $P^j(x)$*, that incumbent* $j$*,* $j = L, R$*, is reelected given that he has chosen policy* $x$*, is strictly between zero and one in the range* ($x^c_R$, $x^c_L$)*. When policy is at the center of the political spectrum the probability of reelection of a RWI is equal to that of a LWI* ($P^L(0) = P^R(0)$)*.*

The proofs of the lemma as well as of all subsequent non immediate results appear in the appendix.

Since from equation (9), the expected value of policies chosen by a LWI and the expected value of policies chosen by a RWI are $(-B)$ and $B$ respectively, part (ii) of the lemma implies that, on average, right wing incumbents choose more rightist policies than their leftist counterparts. Part (iii) assures that the solutions for policy choices in equation (9) apply for all policies over the range ($-x^c_L$, $x^c_L$) or, equivalently, that they apply for all realizations of $\mu$ such that $-q_L < \mu < q_L$. An immediate corollary of part (iv) is that the conditional probability of reelection of a LWI is higher than that of a RWI when he chooses policies that are right of the center and the conditional probability of a RWI is higher than that of a LWI when he chooses policies that are left of the center.

# 1 Characterization of reversals in the range $(-x_L^c,\, x_L^c)$ and comparative statics

The main lesson from the preceding discussion is that the broad lines of the analysis in CT still hold provided $x$ is restricted to the range $(-x_L^c,\, x_L^c)$ and, correspondingly, the realization of the stochastic sum of the shocks, $\mu$, is restricted to the range $(-q_L, q_L)$. We shall refer to it in the sequel as "the relevant range". In this range the analytical expressions for policy choices and for the public's expectation of $\gamma$ are identical to those in CT. The main difference is that we now have to find conditions for reversals only within the relevant range, $(-x_L^c,\, x_L^c)$. When this range contains the range $(\overline{x}, \underline{x})$, which is specified in proposition 1 of CT, there are reversals within the relevant range.[2] In this case most of the analysis in section III of CT, including in particular Figure 2, apply for appropriate ranges of policies and of parameters. (This figure is reproduced here with the appropriate bounds and related adjustments as Figure 1). But, since there is no characterization of equilibrium outside the relevant range no statements about the existence or non existence of reversals outside it can be made. In particular, part 3 of proposition 1 of CT need not hold and additional conditions have to be imposed for part 2 of the proposition to be true. On the other hand additional results now become visible. An appropriately modified version of proposition 1 of CT follows

**Modified and expanded version of proposition 1 of CT:** *If*

$$\sigma_{\varepsilon 2}^2 \equiv \frac{4}{1+\rho} \frac{2B(-2B+c_R)}{\ln\left[\frac{A+d(-2B+c_R)}{A-d(-2B+c_R)}\right]} < \sigma_\varepsilon^2 < \frac{4}{1+\rho}\frac{A}{d}B \equiv \sigma_{\varepsilon 1}^2, \tag{12}$$

*(where $d \equiv \frac{1-\rho}{1+\rho}$ and $A$ is defined on page 188 of CT) then the relevant range of policies $(-x_L^c,$ $x_L^c)$ can be partitioned in the following way*

*1. There is a central region* $(\overline{x}, \underline{x})$ such that $\overline{x} = -\underline{x} > 0$, *in which the conventional result obtains (policies to the left of the center of the political spectrum - - at zero - - are more*

[2] $\overline{x}$ and $\underline{x}$ (where $\overline{x} > \underline{x}$) partition the space of policies into two ranges; A center range, between $\overline{x}$ and $\underline{x}$, within which any given policy is more likely to be implemented by the likely party for that policy; A, non contiguous, relatively extreme range to the right of $\overline{x}$ and to the left of $\underline{x}$ that is characterized by policy reversals.

*likely to be implemented by a LWI and policies to the right of the center are more likely to be implemented by a RWI).*

*2. (Reversals only at sufficiently extreme policies) There is a region outside* $(\overline{x}, \underline{x})$, *but still within the relevant range, in which sufficiently left-wing policies* $(-x_L^c < x < \underline{x})$ *are more likely to be implemented by a RWI and sufficiently right-wing policies* $(\overline{x} < x < x_L^c)$ *are more likely to be implemented by a LWI.*

*3. The range of parameters defined by equation (12) is non-empty.*

*4. (Universal policy reversals) If* $\sigma_{\varepsilon 1}^2 < \sigma_\varepsilon^2$, *a LWI is more likely to implement* ***all*** *policies within the relevant range that are right of the center and a RWI is more likely to implement* ***all*** *policies within this range that are left of the center.*

*5. (No reversals) If* $\sigma_\varepsilon^2 < \sigma_{\varepsilon 2}^2$, *there are no policy reversals within the relevant range.*

The modified proposition implies that, given the ratio $\rho \equiv \frac{\sigma_\gamma^2}{\sigma_\varepsilon^2}$, the range of policies for which there are reversals increases with $\sigma_\varepsilon^2$. When $\sigma_\varepsilon^2$ is smaller than the lower threshold, $\sigma_{\varepsilon 2}^2$, there are **no reversals** in the relevant range. When it is bounded between $\sigma_{\varepsilon 2}^2$ and $\sigma_{\varepsilon 1}^2$, **part** of the relevant range of policies is characterized by reversals. This case is the one illustrated graphically in Figure 1. When $\sigma_\varepsilon^2$ is larger than the upper threshold, $\sigma_{\varepsilon 1}^2$, **all** policies in the relevant range are characterized by reversals. Furthermore, it is shown in the third part of the appendix that

**Result 1:** In the intermediate case an increase in $\sigma_\varepsilon^2$, keeping $\rho$ constant, widens the range of policy reversals.[3]

The "intermediate case" is characterized by condition (12) above which assures that the range $(\overline{x}, \underline{x})$ is non degenerate and contained within the relevant range. A discussion of the underpinnings of Result 1 follows. Since $\rho$ is held constant in this experiment an increase in $\sigma_\varepsilon^2$ is equivalent to an increase in the variance, $\mathbf{V} = \frac{\sigma_\varepsilon^2(1+\rho)}{4}$, of the distribution of policy choices of the two incumbents, but in a way that does not affect the public's learning parameter, $\theta = \frac{\rho}{1+\rho}$. Intuitively, when, given $\rho$, $\sigma_\varepsilon^2$ increases there is an increase in the variance of the distributions of policy choices so that the two truncated normal distributions in panel A of Figure 1 become more spread out.[4] As illustrated by this figure a LWI is always more likely to propose a policy

[3] This result is an appropriately qualified restatement of Result 3 in CT.

[4] Following CT the distributions of policy choices are denoted by $Q^j(x)$, $j = L, R$.

that is left of center (at 0) and a RWI is always more likely to propose a policy that is right of center. But when the distributions of policy proposals become more spread out the relative difference between the probability that, say, a left wing policy will be proposed by a LWI in comparison to the probability that it will be proposed by a RWI shrinks leading to a widening of the range of policy reversals. This amplifies CT conclusion that policy reversals should be more common in countries characterized by "catchall" parties that comprise a wide spectrum of relatively heterogeneous constituencies in conjunction with high uncertainty about the state of nature.

In the experiment above the ratio, $\rho$, between the variance of the state of nature shock and of the intra-party shock is kept constant in order to separate the direct effect of increased uncertainty on the range of reversals from its effect through the public's learning process (via the learning parameter $\theta$). We turn now to the effect of an increase in $\rho$ on the range of reversals for a given variance, $V$, of the distribution of policy proposals. By keeping $V$ constant this complementary experiment isolates the effect of a change in the relative size of the variances via the public's learning process. It is shown in part 4 of the appendix that

**Result 2:** In the intermediate case an increase in $\rho$, keeping $V$ constant, reduces the range of policy reversals.[5]

This result is a consequence of two effects both of which operate via a change in the learning parameter. The increase in $\rho$, keeping $V$ the same, implies that the learning parameter, $\theta$, increases. This triggers two effects that reinforce each other. First, the increase in $\theta$ implies that, following a marginal move of the incumbent towards the center, the impact of the negative expectations effect on the probability of reelection is now larger in comparison to the positive Hotelling effect ($d$ goes down). As a consequence the range of policy reversals shrinks. In terms of panel B of Figure 1 the increase in $\rho$ implies that the absolute values of the slopes of the probability of reelection lines go down. Second, the increase in the learning parameter pulls the mean policies of the two incumbent types away from each other as they try to avoid the stronger adverse effect (via the stronger impact of policy on the expectation of the state of nature shock, $\gamma$) of a move towards the center on the probability of reelection. This raises the

[5] A precise definition and meaning of the "intermediate case" appear immediately following Result 1.

relative probability advantage of a given incumbent in proposing his typical range of policies leading to a reduction in the range of policy reversals. In terms of panel A of Figure 1 this corresponds to a shift of the distributions of policy proposals away from each other with their point of intersection anchored at the center of the political spectrum ($x = 0$).

**Result 3:** In the intermediate case an increase in office motivation, $h$, widens the range of policy reversals.

Although the formal proof of this result (in the appendix) is somewhat involved the intuition is relatively simple. An increase in office motivation pulls the distributions of policy proposals in panel A of Figure 1 towards each other. The reason is that $B$ is decreasing in $h$ so that the distributions of policy choices of the two incumbent types are nearer to each other when $B$ is smaller. This reduces the relative probability advantage of a given incumbent in proposing his typical policy range and widens the range of policy reversals.

Result 1 of CT still holds for the intermediate case and is restated as Result 4.

**Result 4:** In the intermediate case an increase in electoral uncertainty (measured by $\overline{c}$), reduces the range of policy reversals.[6]

A weaker version of Result 2 of CT obtains now. It is summarized here as Result 5.

**Result 5:** In the intermediate case an increase in the degree of party polarization (measured by $c_R$) widens the normal range of no policy reversals.[7]

# 2 Concluding remarks

The main conclusion from the analysis of this note is that there exists a dense set of shock realizations and of parameter values for which the internal equilibrium solutions presented in CT are valid. The note provides a detailed road map of the required additional restrictions and of their implications for proposition 1 of CT and for their comparative statics results. Furthermore, subject to the restriction that the realizations of shocks are such that the policy

[6] The proof that the range $(\overline{x}, \underline{x})$ widens is identical to that of CT. In addition we now also have to verify what happens to the relevant range, $(-x_L^c, x_L^c)$, when $\overline{c}$ rises. It is easy to see from equation (9) and (10) that $\frac{\partial x_L^c}{\partial \overline{c}}$ is negative, so that $(-x_L^c, x_L^c)$ shrinks reinforcing the reduction in the range of reversals.

[7] The proof that the normal range, $(\overline{x}, \underline{x})$, widens is identical to the proof in CT. The reason that it is not possible to make now a definite statement about the range of reversals is that it is not possible to determine, in general, whether the relevant range, $(-x_L^c, x_L^c)$, widens or shrinks.

chosen by the incumbent party is in the relevant range, the discussion of credibility in section IV of CT remains the same.

Although they do not alter the broad message of the three comparative statics results in CT, the additional restrictions qualify and amplify those results. They also lead to two novel comparative statics results. One is that, an increase in office motivation widens the range of policy reversals. The other is that, given the variance of incumbent's policy choices, an increase in the variance of, welfare relevant, states of nature in relation to the variance of intra-party shocks reduces the range of policy reversals. Finally, in the presence of a sufficiently high degree of uncertainty about policy choices, the entire relevant range is characterized by reversals.

# 3 Appendix

## 3.1 Proof of Lemma 1

(i) The range of policies defined by equation (5) is non-empty if and only if $x_R^c < x_L^c$. From equation (9) $x_L^c = -x_R^c$. Hence, the range in equation (5) is non-empty if and only if $x_L^c = c_R - 2B > 0$. Using the definition of $B$ from equation (10) in this condition and rearranging, reveals that $x_L^c > 0$ if and only if $h > h_{xc}$.

(ii) Requiring that $B$ be positive, using the definition of $B$ from equation (10) and rearranging reveals that $B > 0$ if and only if $h < h_B$.

(iii) After some rearrangement, and using symmetry, condition (A1) in the appendix of CT implies that, for a LWI, Assumption 1 is satisfied if and only if

$$\mu_L < \frac{2(c_R + B)(1 + \rho)}{(1 - \rho)} \equiv \mu_L^c. \tag{13}$$

Since the range of policies considered is within $(x_R^c, x_L^c)$, $x$ is smaller than $x_L^c$ or, equivalently, $\mu_L$ is smaller than $q_L$. Hence, if it can be shown that $q_L$ is smaller than $\mu_L^c$ it will follow that the condition in equation (13) is satisfied completing the proof. Using the definition of $\mu_L^c$ (from equation (13)) and of $q_L$ (from equation (9)), and rearranging, it can be shown that $q_L < \mu_L^c$ if and only if

$$0 < \frac{4}{1+\rho}(\rho c_R + B) \tag{14}$$

which is always satisfied since all terms on the right hand side are positive. The proof for a RWI is analogous.

(iv) From equation (9) $x_L$ is a monotonically increasing function of $\mu_L$ and $x_R$ is a monotonically increasing function of $\mu_R$. Hence, it is possible to find the upper and lower bounds for $P^j(x)$, $j = L, R$, over the range of policies $(-x_L^c, x_L^c)$ by substituting the expressions for these limiting values from equation (9) and (10) into the expressions for $P^j(x)$, $j = L, R$, given at the bottom of page 188 in CT. The proof is completed by noting that the lowest values of both $P^L(x)$ and $P^R(x)$ are positive and that the condition $c_R < (1+\rho)\overline{c}$ implies that the highest values of these functions are strictly smaller than one. The proof that the conditional probability of reelection at the center of the political spectrum is the same for both a LWI and a RWI is immediate by substituting $x = 0$ into the expressions for $P^j(x)$, $j = L, R$, given at the bottom of page 188 in CT. QED

## 3.2 Proof of modified version of proposition 1 in CT

1, 2 : Simple algebra establishes that the condition $\sigma_\varepsilon^2 < \sigma_{\varepsilon 1}^2$ is equivalent to the condition $Vd/B < A$ in proposition 1 of CT where $V \equiv \frac{\sigma_\varepsilon^2(1+\rho)}{4}$. As in that proposition, this condition assures that the function $F(x)$ possesses the shape drawn in Figure 2 of CT implying that there are reversals at sufficiently extreme policies and that there are no reversals at policies that are sufficiently close to the center of the political spectrum.[8] From page 188 of CT a sufficient condition for the existence of reversals in the relevant range is that

$$F(x_L^c) < 0. \tag{15}$$

CT show that $\overline{x} = -\underline{x} > 0$ and the first part of lemma 1 above states that $x_L^c = -x_R^c > 0$. Due to this double symmetry the condition in equation (15) is sufficient for the existence of reversals at sufficiently extreme right-wing policies, as well as at sufficiently extreme left-wing policies

[8] This figure is reproduced here, subject to appropriate restrictions on the range of policy choice, as Figure 1.

(since the explicit form of equation (15) in CT also implies $F(-x_L^c) = -F(x_L^c) > 0$). Using the explicit expression for $F(x)$ from page 188 of CT and rearranging, the inequality in equation (15) becomes equivalent to the condition $\sigma_{\varepsilon 2}^2 < \sigma_\varepsilon^2$. This completes the proof of parts 1 and 2.

3. The range of $\sigma_\varepsilon^2$ defined by equation (12) is non-empty if and only if $\sigma_{\varepsilon 2}^2 < \sigma_{\varepsilon 1}^2$. Using the definitions of those two bounds and rearranging, this condition can be shown to be equivalent to the condition

$$P(y) \equiv \ln \frac{y+1}{y-1} - \frac{2}{y} > 0, \quad y \equiv \frac{A}{dx_L^c}. \tag{16}$$

Since $dx_L^c > 0$ and $A > 0$ in the relevant range $y$ is positive in this range. In addition, since $P^R(x_L^c) = \frac{A - dx_L^c}{4\bar{c}} > 0$, $y$ is larger than one. It is easy to verify that the limit of $P(y)$ as $y$ tends to one from above is positive, that $P(y)$ tends to zero as $y$ tends to infinity and that, since $P'(y) = -\frac{2}{y^2(y^2-1)} < 0$, the function $P(y)$ is monotonically decreasing for all $1 < y < \infty$. It follows that $P(y)$ is positive for all $1 < y < \infty$ which completes the proof of part 3.

4. The condition $\sigma_\varepsilon^2 > \sigma_{\varepsilon 1}^2$ is equivalent to the condition $Vd/B > A$ in proposition 1 of CT which is equivalent, from part 4 of the appendix of CT, to

$$F'(0) < 0. \tag{17}$$

implying that the function $F(x)$ is decreasing at $x = 0$. The discussion on page 195 of CT implies that in this case $F'(x)$ has no real roots. Since $F'(x)$ is continuous it is either negative for all $x's$ or positive for all $x's$. Since it is negative at $x = 0$ it must, therefore, be negative at all $x's$. Hence $F(x)$ is decreasing at all $x's$. Since, as shown on page 195 of CT, $F(0) = 0$, $F(x) < 0$ for all positive $x's$ and $F(x) > 0$ for all negative $x's$, and since $x_L^c = -x_R^c > 0$, the entire relevant range of $x$ is characterized by policy reversals.

5. By the first two parts, since $\sigma_\varepsilon^2 < \sigma_{\varepsilon 2}^2 < \sigma_{\varepsilon 1}^2$, the shape of $F(x)$ is as drawn in panel D of CT. Some algebra shows that the condition $\sigma_\varepsilon^2 < \sigma_{\varepsilon 2}^2$ is equivalent to the condition $F(x_L^c) > 0$ implying, since $x_L^c > 0$, that a RWI is more likely to implement the policy at the upper bound of the relevant range from which it follows, in view of panel D of Figure 1, that $x_L^c < \overline{x}$. Symmetry implies that $x_R^c > \underline{x}$. Hence the entire relevant range is contained within the range $(\overline{x}, \underline{x})$ of no reversals. It follows that there are no reversals within the relevant range. QED

### 3.3 Proof of result 1

The proof follows from equation (24) of CT in conjunction with the fact that at both $\overline{x}$ and $\underline{x}$, $F'(x) < 0$ (see panel D in Figure 1) implying that the sign of $\frac{dx}{d\sigma_\epsilon^2}\Big|_{\rho=\text{constant}}$ is the same as the sign of $\frac{dF(x)}{d\sigma_\epsilon^2}\Big|_{\rho=\text{constant}}$. But $\frac{dF(x)}{d\sigma_\epsilon^2}\Big|_{\rho=\text{constant}} = -\frac{8B}{(1+\rho)\sigma_\varepsilon^4}x$ implying that $\frac{d\overline{x}}{d\sigma_\epsilon^2}\Big|_{\rho=\text{constant}} < 0$ and $\frac{d\underline{x}}{d\sigma_\epsilon^2}\Big|_{\rho=\text{constant}} > 0$ so that the range of no reversals shrinks. Inspection of equation (10) for $B$ reveals that, given $\rho$, $x_L^c = c_R - 2B$ does not depend on $\sigma_\varepsilon^2$ so that the relevant range, $(-x_L^c,$ $x_L^c)$, is not affected by the change in $\sigma_\varepsilon^2$. Hence the decrease in the range of no reversals implies that the range of policies characterized by reversals increases. QED

### 3.4 Proof of result 2

The proof follows from equation (24) of CT in conjunction with the fact that at both $\overline{x}$ and $\underline{x}$, $F'(x) < 0$, implying that the sign of $\frac{dx}{d\rho}\Big|_{V=\text{constant}}$ is the same as the sign of $\frac{dF(x)}{d\rho}\Big|_{V=\text{constant}}$. Differentiating the explicit expression for $F(x)$, given on page 188 of CT, with respect to $\rho$ and keeping $V$ constant, using the definitions of $A$, $B$ and $d$ given there we obtain after some algebra

$$\frac{dF(x)}{d\rho}\Big|_{V=\text{constant}} = 2x\left[\frac{\overline{c} + c_R + h/2}{V} + d^2\frac{2(\overline{c} + c_R) + h}{(A - dx)(A + dx)}\right]. \tag{18}$$

Note that $\overline{x}$ is positive (and $\underline{x}$ is negative) and contained within the relevant range and that, from part (iv) of lemma 1, the probabilities of reelection are internal within this range so that $(A - dx)(A + dx)$ is positive for $x = \underline{x}, \overline{x}$. Hence the term in the bracket of equation (18) is positive establishing that $\frac{d\overline{x}}{d\rho}\ |_{V=\text{constant}}$ is positive and that $\frac{d\underline{x}}{d\rho}\ |_{V=\text{constant}}$ is negative. This establishes that, given $V$, the range of no reversals widens when $\rho$ goes up. If the relevant range, $(-x_L^c$, $x_L^c)$, does not widen this also implies that the range of policy reversals shrinks. To complete the proof we need to show, therefore, that this is the case. Since $V = \frac{\sigma_\varepsilon^2}{4}(1+\rho)$ is kept constant an increase in $\rho$ has to be compensated by a decrease in $\sigma_\varepsilon^2$ so that $V$ is unaltered. But Result 1 implies that this range is not affected by a change in $\sigma_\varepsilon^2$. It remains to check how the relevant range is affected by the increase in $\rho$. Inspection of equations (9) and (10) reveals that $x_L^c$ is decreasing in $\rho$ implying that, taking all effects into consideration, the range of reversals shrinks when, holding $V$ the same, $\rho$ goes up. QED

### 3.5 Proof of result 3

To show that, for the intermediate case, the range of reversals widens when $h$ goes up we will show that $\frac{d\overline{x}}{dh}$ is negative, that $\frac{d\underline{x}}{dh}$ is positive and that the relevant range widens. Since $\frac{d\overline{x}}{dh} = -\frac{d\underline{x}}{dh}$ proof of the fact that $\frac{d\overline{x}}{dh} < 0$ also implies that $\frac{d\underline{x}}{dh} > 0$. Since $-\frac{1}{F(\overline{x})}$ is positive, and in view of equation (24) in CT, demonstration of the negativity of $\frac{dF(\overline{x})}{dh}$ suffices to also establish that $\frac{d\overline{x}}{dh} < 0$.

We proceed by differentiating the expression for $F(x)$ from page 188 of CT totally with respect to $h$. After some algebra this yields

$$\frac{dF(x)}{dh} = \frac{2xd}{\sigma_\varepsilon^2(A-dx)(A+dx)}\left[\rho d\sigma_\varepsilon^2 - 2(A^2 - d^2x^2)\right]. \tag{19}$$

Since $\overline{x}$ is positive and since it is contained in the relevant range the first term on the right hand side evaluated at $\overline{x}$ is positive. Hence, it suffices to show that $\rho d\sigma_\varepsilon^2 - 2(A^2 - d^2x^2)$ is negative. The modified version of proposition 1 implies that, in the intermediate range, $\sigma_\varepsilon^2 < \sigma_{\varepsilon 1}^2$. From equation (12) this is equivalent to $\rho d\sigma_\varepsilon^2 < \frac{4\rho AB}{1+\rho}$. Adding and subtracting the last term to the term in brackets on the right hand side of equation (19) we obtain

$$\rho d\sigma_\varepsilon^2 - 2(A^2 - d^2x^2) = \left\{\rho d\sigma_\varepsilon^2 - \frac{4\rho AB}{1+\rho}\right\} + \left\{\frac{4\rho AB}{1+\rho} - 2(A^2 - d^2x^2)\right\} \equiv \left\{\rho d\sigma_\varepsilon^2 - \frac{4\rho AB}{1+\rho}\right\} + T(x). \tag{20}$$

We just saw that the first term in curly brackets on the right hand side of equation (20) is negative. The proof can therefore be completed by showing that the second term in curly brackets, denoted $T(x)$, is negative as well. This term is a quadratic in $x$ and, since the coefficient of $x^2$ is positive, it has a minimum. It will have two real roots if and only if there are some values of $x$ for which this quadratic is negative. Inspection of $T(x)$ suggests that in such a case the roots will be located at opposite and equal distances from zero and that the quadratic will attain a negatively valued minimum at $x = 0$. Hence, if we find a value of $x$ for which the quadratic is negative, the quadratic will be negative in the entire range between the two real roots. We will now show that when $x = x_L^c$ the value of the quadratic is negative, confirming the existence of two real roots as specified above. Furthermore, since $0 < \overline{x} < x_L^c$ this will imply that the quadratic is negative also at $\overline{x}$ completing the proof of the negativity of $\rho d\sigma_\varepsilon^2 - 2(A^2 - d^2x^2)$ at

$\overline{x}$.

From the definitions of $A$, $B$, $d$ and $x_L^c$ it can be verified that $A = (4B + h + x_L^c)d = (2B + h + c_R)d$. Evaluating $T(x)$ at $x_L^c$, using those identities and rearranging so that equal powers of $B$ are grouped together we obtain

$$T(x_L^c) = \frac{2d}{(1+\rho)}\{4\rho B^2 + [(6\rho - 4)h + (10\rho - 8)c_R]\, B - h(1-\rho)(h + 2c_R)\}. \tag{21}$$

This expression is a quadratic in $B$. Since the coefficient of the square term is positive it has a minimum, and provided we can show that there are values of $B$ for which it is negative, it has two real roots. From equation (10) $B$ is a monotonically decreasing function of $h$. Lemma 1 limits the range of possible values of $h$ to the range $(h_{xc}, h_B)$. Correspondingly the variation in $B$ is limited to the range between 0 and $\frac{c_R}{2}$ if $h_{xc} > 0$ and to the range between 0 and $\overline{c}(1+\rho) + \rho c_R < \frac{c_R}{2}$ if $h_{xc} < 0$ (note that the lower bound of $h$ in this case is zero). It is easily seen that at $h = h_B$, $T(x_L^c) = -2d^2(h_B + 2c_R)h_B$ which is negative. At $h = h_{xc}$ (assuming that $h_{xc} > 0$) we obtain

$$T(x_L^c) = -\frac{2d}{(1+\rho)}\left\{c_R^2 + 2(1+\rho)\overline{c}(h_{xc} + 3c_R)\right\},$$

which is also negative. At $h = 0$ (assuming that $h_{xc} < 0$) we obtain

$$T(x_L^c) = \frac{4d[(1+\rho)\overline{c} + \rho c_R]}{(1+\rho)}\left[\rho(1-\rho)h_{xc} + 6(\rho - \frac{1}{2})c_R - c_R\right]. \tag{22}$$

Note that $\rho < \frac{1}{2}$ is a necessary condition for $h_{xc} < 0$, implying that the second term in the brackets of (22) is negative, making the entire expression in (22) negative too.

In view of the shape of the quadratic in $B$ this implies that $T(x_L^c)$ is negative for all $h$'s in the permissible range: $(h_{xc}, h_B)$ if $h_{xc} > 0$ or $(0, h_B)$ if $h_{xc} < 0$. This establishes the negative sign of $\frac{dF(\overline{x})}{dh}$ and the positive sign of $\frac{dF(\underline{x})}{dh}$ for all the relevant range of $h$'s.

The proof is completed by showing that the relevant range, $(x_R^c,\, x_L^c)$, expands. From equation (9) $\frac{\partial x_L^c}{\partial h} = -\frac{\partial x_R^c}{\partial h} = 1 - \rho$ which is positive implying that the upper bound of the relevant range goes up and the lower bound goes down. QED

# 4 References

Cukierman Alex and Mariano Tommasi (1998), "When does it take a Nixon to go to China?", **American Economic Review**, 88, 180-197, March.