Virtual MIMO-based Single Frequency Network

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Abstract—In this paper, we introduce virtual multiple input multiple output (MIMO) techniques into single frequency network (SFN) for providing multimedia multicast/broadcast. Virtual MIMO-based SFN can achieve the full spatial diversity inherent in SFN and greatly improve the reliability of transmission without sacrificing much bandwidth. Furthermore we provide a general method for evaluating the bit error rate (BER) performance of virtual MIMO-based SFN. By this method, we derive an approximate formula for the average BER of M-ary quadrature amplitude modulation (M-QAM). We also present simulations to confirm the formula and compare virtual MIMO-based SFN with traditional SFN.

I. INTRODUCTION

Single frequency network (SFN) is a powerful and efficient means for providing multimedia multicast/broadcast. All transmitters simultaneously transmit the same signals on the same frequency in SFN. In contrast to traditional broadcast systems, SFN can offer great coverage and high reliability at a low cost [1]. In contrast to traditional cellular systems, SFN has high spectrum efficiency and does not need complex handoff operation [2]. Because of these advantages, SFN has been employed or adopted in many wireless systems such as video broadcast system-terrestrial (DVB-T) [3] and forward link only (FLO) [2].

However, SFN is very different from the other wireless networks and may be impaired by multipath propagation resulting from the structure of SFN. The SFN structure induces two kinds of multipath propagation: the natural multipath propagation due to scattering from the objects in SFN, and the artificial multipath propagation due to the identical signals from multiple transmitters. Multipath propagation leads to delay spread and deep fading. Delay spread causes inter-symbols interference (ISI), and deep fading causes very low instantaneous received signal-to-noise ratio (SNR). So multipath propagation severely degrades the performance of SFN. In order to combat multipath propagation, orthogonal frequency division multiplexing (OFDM) has been used as the modulation technique in SFN [3]. OFDM is useless for mitigating deep fading, though it can completely eliminate ISI by extending symbol period and adding cyclic prefix. Forward error correction (FEC) is an effective and widely applied technique for alleviating the effect of deep fading in SFN. But, FEC based on adding redundant data wastes much wireless bandwidth.

Recently, the use of transmit diversity has been suggested as a new approach to reducing the effect of deep fading in SFN. Transmit diversity comes from the fact that a receiver is able to get independent signals from several transmitters in SFN, which have a low probability of experiencing deep fading simultaneously. The first transmit diversity scheme in SFN was proposed in [4], in which a very simple transmit diversity was provided by adding a distinct phase offset to the subcarrier components of each transmitter. In [5], a receiving scheme was used to separate the signals with different incident angles by using a beamformer. In [6] and [7], a new transmit diversity scheme was proposed based on the frame structure of the time domain synchronous - orthogonal frequency division multiplexing (TDS-OFDM) system.

In this paper, we introduce virtual MIMO (also called distributed MIMO in some literature) techniques into SFN. The idea of virtual MIMO-based SFN is to synchronously control multiple transmitters to emulate a distributed antenna array, which transmits multicast/broadcast data encoded by orthogonal space-time block coding. Compared with the above transmit diversity schemes, virtual MIMO-based SFN can efficiently achieve the full spatial diversity inherent in SFN and greatly improve the reliability of transmission without sacrificing much bandwidth. Though virtual MIMO is not a new concept, most of the previous work on virtual MIMO has been restricted to ad hoc networks or sensor networks where every mobile node has only a single antenna because of limitation of cost, size and power. In these networks, virtual MIMO is created by transmitting message from a source node to its destination node both directly and through relay, thus significant performance gains can be obtained [8] [9].

In this study, we propose a new decoding algorithm for orthogonal space-time block code. Furthermore we develop a general method for evaluating the BER performance of virtual MIMO-based SFN. By the method, we derive an approximate formula for the average bit error rate (BER) of M-ary quadrature amplitude modulation (M-QAM). In our analysis, the effect of distance is taken into consideration. In addition, we present simulations to confirm the formula and compare virtual MIMO-based SFN with traditional SFN.

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The rest of the paper is organized as follows. In Section II, we describe the virtual MIMO-based SFN system model. In Section III, we provide a general method to analyze the BER performance of virtual MIMO-based SFN. In Section IV, we present simulations and discuss the results. In Section V, we conclude this paper.

II. SYSTEM MODEL

As shown in Fig. 1, the given coverage area (like a city) of a virtual MIMO-based SFN is divided into trigonal cells, and the transmitters are located at the vertexes of each cell. According to the positions, all transmitters are classified into three groups which are labeled as Tx1, Tx2 and Tx3, respectively.

Fig. 2 shows the block diagram of virtual MIMO-based SFN. The transmitters in one group act as an antenna in virtual MIMO system and send identical signals. The structure of virtual MIMO-based SFN guarantees that a receiver can get signals from the closest three transmitters which, respectively, belong to different groups when it moves in the coverage area. So the proposed virtual MIMO-based SFN is equivalent to the MIMO system with three transmit antennas and one receive antenna.

It is assumed that OFDM utilizes N subcarriers. The symbol period is extended N times because OFDM modulation multiplexes the original symbol stream into N parallel subcarriers. As a result, OFDM modulation transforms a frequency-selective fading channel to multiple flat fading subchannels. If the channel from the ith (i= 1, 2, 3) transmitter group to a receiver has Li independent propagation paths, then the channel impulse response in time domain at time \( \tau \) can be modeled as

\[
h_i(\tau) = \sum_{l=1}^{L_i} \alpha_{i}(l) \delta(\tau - \tau_{i}(l)) \quad i = 1, 2, 3
\]

where \( \alpha_{i}(l) \) and \( \tau_{i}(l) \) represent the complex attenuation and the propagation delay associated with the \( l \)th path between the \( i \)th transmitter group and the receiver, respectively. The attenuations and the delays are random variables. But for slow fading, it is assumed that the attenuations and delays are constant during a frame and vary from one frame to another.

For OFDM modulation with enough cyclic prefix and proper sample timing, the frequency response of the channel at the \( n \)th (\( n = 1, \ldots, N \)) subcarrier is given by

\[
H_i(n) = \sum_{l=1}^{L_i} \alpha_{i}(l) \exp(-j2\pi n \tau_{i}(l)/T)
\]

where \( T \) is the OFDM symbol period. The delays only introduce a phase shift to the desired symbol component when cyclic prefix is larger than the delays. So the delays of signals from the closest three transmitters do not cause interference for a receiver, if enough cyclic prefix is used in virtual MIMO-based SFN.

In the space-time encoder, the modulated symbols are first arranged into blocks. Each block can be written in a 3×N matrix form as

\[
X = \begin{bmatrix}
X_1(1) & X_1(2) & \cdots & X_1(N) \\
X_2(1) & X_2(2) & \cdots & X_2(N) \\
X_3(1) & X_3(2) & \cdots & X_3(N)
\end{bmatrix}
\]

Then, the encoder maps a block to its associated codeword according to space-time block code shown in [10]. Each codeword can be expressed as a 4×3N matrix

\[
C = [C_1 \ C_2 \ C_3]
\]

where the \( n \)th (\( n = 1, \ldots, N \)) columns of \( C_1, C_2 \) and \( C_3 \) are given, respectively, by

\[
C_1(n) = [c_{11}(n) \ c_{12}(n) \ c_{13}(n) \ c_{14}(n)]^T
\]

\[
= [X_1(n) \ X_2(n) \ X_3(n) \ 0]^T
\]

\[
C_2(n) = [c_{21}(n) \ c_{22}(n) \ c_{23}(n) \ c_{24}(n)]^T
\]

\[
= [-X_2(n) \ X_1(n) \ 0 \ -X_3(n)]^T
\]

\[
C_3(n) = [c_{31}(n) \ c_{32}(n) \ c_{33}(n) \ c_{34}(n)]^T
\]

\[
= [-X_3(n) \ 0 \ X_1(n) \ X_2(n)]^T
\]

where \( (.)^T \) denotes transpose. \( C(n) \) (i=1,2,3) is assigned to the \( i \)th transmitter group and transmitted over the \( n \)th subcarrier in four consecutive OFDM symbol periods.

At the receiver, after OFDM demodulation, the symbol vector received at the \( n \)th subcarrier can be expressed as

\[
Y(n) = [Y_1(n) \ Y_2(n) \ Y_3(n) \ Y_4(n)]^T
\]

\[
= \frac{1}{4} \sum_{i=1}^{4} C_i(n)H_i(n) + W(n)
\]

where \( Y_i(n) \) (i=1,2,3,4) denotes the symbol received at symbol period \( t_i \) and \( W(n)=[W_1(n) \ W_2(n) \ W_3(n) \ W_4(n)] \) denotes the additive white Gaussian noise vector at the \( n \)th subcarrier. Assuming that the channel state information is unknown at the transmitter but perfectly known at the receiver, the decoding algorithm is given by

\[
\hat{C} = \arg\min_{C} \sum_{i=1}^{4} \sum_{n=1}^{N} |Y_i(n) - \frac{1}{4} \sum_{j=1}^{4} \hat{c}_{j}(n)H_{j}(n)|^2
\]

The decoding algorithm chooses a codeword from a given codeword set based on maximum likelihood principle. Expanding (7) and using (5) we get
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performing separate decoding for each transmitted symbol.

Consequently, the decoding complexity increases linearly, regarded as constant. Thus, (8) can be simplified as

\[
\hat{\mathbf{X}} = \arg\min_{\hat{\mathbf{X}}} \sum_{n=1}^{N} \sum_{t=1}^{4} |Y_t(n)|^2
\]

\[
+ \sum_{i=1}^{3} (\mathbf{H}(n)|\hat{X}_i(n)|^2 - R_i(n)\hat{X}_i(n) - R_i(n)\hat{X}_i(n))
\]

where

\[
R_i(n) = H_i(n)Y_i(n) + H_2(n)Y_2(n) + H_3(n)Y_3(n)
\]

\[
R_i(n) = H_i(n)Y_i^2(n) - H_2(n)Y_2(n) + H_3(n)Y_3(n)
\]

\[
R_i(n) = H_i(n)Y_i^2(k) - H_2(n)Y_2(n) - H_3(n)Y_3(n)
\]

\[
\mathbf{H}(n) = \mathbf{H}(n)^T + |H_2(n)|^2 + |H_3(n)|^2
\]

In each decoding operation, \(Y_t(n), R_t(n)\) and \(\mathbf{H}(n)\) can be regarded as constant. Thus, (8) can be simplified as

\[
\hat{\mathbf{X}} = \arg\min_{\hat{\mathbf{X}}} \sum_{n=1}^{N} \sum_{t=1}^{4} |R_t(n) - \mathbf{H}(n)\hat{X}_t(n)|^2
\]

Because of independence of each component, the decoding algorithm (11) can be separated into independent parts as follow

\[
\hat{X}_t(n) = \arg\min_{\hat{X}_t(n)} |R_t(n) - \mathbf{H}(n)\hat{X}_t(n)|^2
\]

Consequently, the decoding complexity increases linearly, instead of exponentially, with the number of subcarriers by performing separate decoding for each transmitted symbol.

III. PERFORMANCE ANALYSIS

In this section, we convert the virtual MIMO channel from three transmitter groups to a receiver into an equivalent single-input single-output (SISO) channel. Then, the SISO channel is used to derive the average BER formula for M-QAM. We believe that the proposed method in this section is universal and can be applied to analyzing the BER performance of the other orthogonal space-time block codes in virtual MIMO-based SFN.

In the performance analysis, we assume that a receiver only can get signals from the closest three transmitters. The signals from other transmitters are ignored because of greatly fading due to the long distance from the receiver.

In each encoding operation, a block of 3\times N complex symbols is encoded to generate 3N parallel coded symbol sequences of length 4, and these sequences are transmitted over N subcarriers by three transmitter groups simultaneously in four OFDM symbol periods. Hence the transmission rate is 3/4. For the nth \(n=1, \ldots, N\) subcarrier, \(C_1(n), C_2(n), \text{and } C_3(n)\) satisfy

\[
\begin{bmatrix}
C_1^T(n) \\
C_2^T(n) \\
C_3^T(n)
\end{bmatrix}
= \begin{bmatrix}
|X_1(n)|^2 + |X_2(n)|^2 + |X_3(n)|^2
\end{bmatrix}
\]

where \(\mathbf{I}_3\) denotes 3\times3 identity matrix, and \((\cdot)^H\) denotes complex conjugate transpose. So the full spatial diversity of 3 can be achieved.

\[
R_i(n) = H_i(n)X_i(n) + R_i(n)
\]

\[
R_2(n) = \mathbf{H}(n)X_2(n) + R_2(n)
\]

\[
R_3(n) = \mathbf{H}(n)X_3(n) + R_3(n)
\]

where

\[
\mathbf{W}_1(n) = H_1(n)W_1(n) + H_2(n)W_2(n) + H_3(n)W_3(n)
\]

\[
\mathbf{W}_2(n) = H_1(n)W_1(n) - H_3(n)W_3(n) + H_4(n)W_4(n)
\]

\[
\mathbf{W}_3(n) = H_1(n)W_1(n) - H_2(n)W_2(n) - H_3(n)W_3(n)
\]

\[
\mathbf{W}_i(n) = H_i(n)W_i(n) - H_i(n)W_i(n) - H_i(n)W_i(n)
\]

\[
\mathbf{W}_i(n) = H_i(n)W_i(n) - H_i(n)W_i(n) - H_i(n)W_i(n)
\]

\[
E(\mathbf{W}_i(n)|H_1(n), H_2(n), H_3(n)) = 0 (16)
\]

where \(W_0\) is the noise power density at the nth subcarrier. As (16) indicates, \(\mathbf{W}_i(n)\) is a zero mean and statistically

Fig.2 block diagram of virtual MIMO-based SFN
independent Gaussian variable with variance $\overline{H}(n)W_0$ for a
given estimation of the channel frequency responses. As a
result, $\overline{W}(n)$ can be treated as an additive white Gaussian
noise component at the nth subcarrier within each frame. The
random variable $\overline{X}_i(n)$ $(t = 1, 2, 3)$ is independent and
identically distributed because of the complete independence
of $\overline{W}(n)$. Therefore, the virtual MIMO channel at the nth
subcarrier is equivalent to the SISO channel represented as
$$R(n) = \overline{H}(n)X(n) + \overline{W}(n)$$
(17)
where $\overline{W}(n)$ denotes the additive white Gaussian noise with
variance $\overline{H}(n)W_0$.

The performance over the original virtual MIMO channel is
equal to that over the corresponding SISO channel. The
instantaneous received signal-to-noise power ratio (SNR) per
symbol over the equivalent SISO channel is given by
$$\gamma_i(n) = \frac{|\overline{H}(n)|^2 E_s}{\overline{H}(n)W_0} = \frac{E_s\overline{H}(n)}{W_0}$$
(18)
where $E_s$ is the transmitted energy per symbol at each
transmitter. Consider rectangular M-QAM with Gray bit-
mapping. The conditional BER for coherent demodulation can
be approximated as [11, table 6.1]
$$P_e(n|\gamma_1(n), \gamma_2(n), \gamma_3(n)) = Q\left(\frac{B\gamma_i(n)}{2\sin^2 \theta}\right)$$
(25)
where $\gamma_i(n)$ represents the received SNR per symbol at the nth
subcarrier from the Txi. From (21), (19) can be simplified as
$$P_e(n|\gamma_1(n), \gamma_2(n), \gamma_3(n)) = A Q\sqrt{B \sum_{i=1}^{3} \gamma_i(n)}$$
(22)
where $P_i$ is the average received SNR per symbol from the
Txi. $P_i$ can be modeled as
$$\overline{\gamma}_i = \overline{\gamma} - 10n \log_{10} d_i$$
(24)
where $\overline{\gamma}$ is the average received SNR per symbol at a reference
distance of one kilometer, $n$ is the path loss exponent, and $d_i$
is the distance in kilometer between the Txi and the receiver.

By Graig’s formula for the Gaussian Q-function [11], we can get the conditional BER at the nth subcarrier in the
product form as
$$P_e(n|\gamma_1(n), \gamma_2(n), \gamma_3(n)) = \frac{A}{\pi} \int_0^\infty \prod_{i=1}^{3} \exp\left(-\frac{B\gamma_i(n)}{2\sin^2 \theta}\right) d\theta$$
(25)
where $\gamma_i(n)$ and $\gamma_j(n)$, the average BER at the nth subcarrier can be written as
$$P_e(n|\gamma_1(n), \gamma_2(n), \gamma_3(n)) = \frac{A}{\pi} \int_0^\infty \prod_{i=1}^{3} \exp\left(-\frac{B\gamma_i(n)}{2\sin^2 \theta}\right) d\theta$$
(26)
By adopting the change of variable $y=1/\sin^2 \theta$, the average
BER at the nth subcarrier can be rewritten as
$$P_e(n|\gamma_1(n), \gamma_2(n), \gamma_3(n)) = \frac{A}{\pi} \int_0^1 \prod_{i=1}^{3} \exp\left(-\frac{B\gamma_i(n)}{2}\right) dy$$
(27)
Substituting $x$ for $\sqrt{1-y}$ in (27) we get
$$P_e(n|\gamma_1(n), \gamma_2(n), \gamma_3(n)) = \frac{A}{\pi} \int_0^1 \prod_{i=1}^{3} \exp\left(-\frac{B\gamma_i(n)}{2}\right) dy$$
(28)
So the average BER of the OFDM system can be expressed as
$$P_e = \frac{1}{N} \sum_{i=1}^{N} P_e(n|\gamma_1(n), \gamma_2(n), \gamma_3(n))$$
(29)
Since the integrand in (29) is a simple function, we can derive a
closed-form expression for the average BER of QAM from (29).

IV. SIMULATION AND DISCUSSION

In order to illustrate the above theoretical results and
compare virtual MIMO-based SFN with traditional SFN, we
present simulations in this section. Then we discuss the results.

The simulations are built on the following setting. The
OFDM symbol rate is 1200 symbols/second and 4096
subcarriers are used. The cyclic prefix length is 400. The
modulation employed is 16-QAM. The path loss exponent $n$
is 3.5. In the simulations of traditional SFN, the (4, 3) cyclic
code is employed, and the depth of interleaving is 4096. In the
simulations of virtual MIMO-based SFN, interleaving and
FEC are not used.

We consider two places within a cell. As shown in Fig. 1,
the place A is at the center of the cell, and the place B is at the
edge with one kilometer from Txl. The average BER
performance with the cell edge length of five kilometer and
three kilometer is depicted in Fig. 3 and Fig. 4, respectively.
The theoretical results of virtual MIMO-based SFN
From Fig. 3 and 4, we observe the following properties:

1) The theoretical results of virtual MIMO-based SFN in section III are consistent with the corresponding simulation results, especially at high SNR.

2) Virtual MIMO-based SFN obviously improves the robustness of multicast/broadcast compared with traditional SFN. In traditional SFN, all transmitters send the same symbols, so the symbols from different transmitters cannot be separated by the receiver, and therefore the potential spatial diversity is ignored. By orthogonal space-time block code, virtual MIMO-based SFN can achieve the full spatial diversity inherent in SFN and greatly enhance the reliability of transmission.

3) The error rate curves of virtual MIMO-based SFN have a steeper slope than those of traditional SFN, especially at high SNR. This is due to the fact that diversity gain has much more influence on error probability than coding gain as SNR per symbol increases. The error probability at high SNR is dominated by deep fading, and that virtual MIMO-based SFN can effectively mitigate the effect of deep fading by introducing properly designed correlation into the symbols from different transmitters.

4) Cell size has a significant impact on the error probability of virtual MIMO-based SFN. We find that the BER of virtual MIMO-based SFN obviously decreases with shrinking the size of a cell. This occurs because mean path loss increases exponentially with distance. So proper cell size must be chosen in order to guarantee that receivers can get signals from the three transmitter groups.

V. CONCLUSION

In this paper, we introduced virtual MIMO-based SFN which can exploit the full spatial diversity available in SFN and greatly improve reliability of multicast/broadcast by using orthogonal space-time block codes. Then, we studied the BER performance of virtual MIMO-based SFN and derived the average BER formula for M-QAM. Moreover, the formula was verified through the simulations. The effect of distance on the BER performance was taken into account in our analysis. We believe that the proposed method for performance analysis in this study is universal and can be applied to analyzing the performance of the other orthogonal space-time block codes in virtual MIMO-based SFN.

REFERENCES