Condition Monitoring of Rotating Machinery using Active Magnetic Bearings

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Abstract
The concept that changes in the dynamic behaviour of a rotor could be used for general fault detection and monitoring is well established. Current methods rely on the response of the machine to unbalance excitation and are mainly based on pattern recognition approaches. However these methods are relatively insensitive and the crack must be large before it can be robustly detected. Active magnetic bearings (AMB) have been used in high speed applications or where oil contamination must be prevented, although their low load capacity restricts the scope of applications. Recently AMBs have been proposed as an actuator to apply force to the shaft of a machine. The presence of the crack generates responses containing frequencies at combinations of the rotor spin speed and applied force. This paper discusses some of the issues to be addressed to enable this approach to become a robust condition monitoring technique for cracked shafts. The approach is illustrated with a number of simulated examples.

1 Introduction

The idea that changes in a rotor’s dynamic behaviour could be used for general fault detection and monitoring was first proposed in the 1970s. Of all machine faults, probably cracks in the rotor pose the greatest danger and research in crack detection has been ongoing for the past 30 years. Current methods examine the response of the machine to unbalance excitation during run-up, run down or during normal operation. In principle, the presence of a crack in the rotor will change the dynamic behaviour of the system but in practice it has been found that small or medium size cracks make such a small change to the dynamics of machine system that they are virtually undetectable by this means. Only if the crack grows to a potentially dangerous size can they be readily detected. It is a race between detection and destruction!

If the vibration due to any out-of-balance forces acting on a rotor is greater than the static deflection of the rotor due to gravity, then the crack will remain either opened or closed depending on the size and location of the unbalance masses. In the case of the permanently opened crack, the rotor is then asymmetric and this condition can lead to stability problems. Of more importance in large machines is the case where the vibration due any out-of-balance forces acting on a rotor is less than the deflection of the rotor due to gravity. In this case the crack will open and close (or breathe) due to the turning of the rotor. The problem was initially studied by Gasch [1,2] who modelled the breathing crack by a “hinge”. In this model the crack is opened for one half and closed for the other half a revolution of the rotor, and the transition from opened to closed (and visa-versa) occurs abruptly as the rotor turns. Mayes and Davies [3,4] developed a similar model except that the transition from fully opened to fully closed is described by a cosine function (see Section 2). Relating crack size to shaft stiffness is not easy and Papadopoulos and Dimarogonas [5] and Jun et al. [6] have proposed crack models based on fracture mechanics. Papadopoulos and

Gasch [2] carried out a survey of the stability of a simple rotor with a crack and also considered the response of the rotor due to unbalance. Gasch also showed that as long as the resulting vibrations remain small the essentially non-linear equations of motion become linear with periodically time varying coefficients. Pu et al. [10] also considered a self-weight dominated rotor and solved the resulting equations using the harmonic balance method.

Active magnetic bearings (AMB) have been used in high speed applications or where oil contamination must be prevented, although their low load capacity restricts the scope of applications. Recently a number of authors considered the use of AMBs as an actuator that is able to apply force to the shaft of a machine. Bash [11] studied the use of AMBs as an actuator for rotor health monitoring in conjunction with conventional support bearings, by applying a variety of known force inputs to a spinning rotor system. If the applied force is periodic, then the presence of the crack generates responses containing frequencies at combinations of the rotor spin speed and applied forcing frequency. The character and frequency of the force, and the dynamics of the AMB are important for the crack detection. Mani et al. [12,13] and Quinn et al. [14] studied the effect of the force from an AMB on the response of rotating machinery, including the external forcing frequency and amplitude. The excitation by unbalance and AMB forces produces combination resonances between critical speed of the shaft, the rotor spin speed and the frequency of the AMB excitation. The key is to determine the correct excitation frequency to induce a combination resonance that can be used to identify the magnitude of the time-dependent stiffness arising from the breathing mode of the shaft crack.

2 The Mayes model for an opening and closing crack

Consider a Jeffcott rotor that is modelled using two degrees of freedom. The stiffness matrix of the machine, with a crack and in rotating coordinates, is

$$\mathbf{K}(\theta) = \begin{bmatrix} k_x(\theta) & 0 \\ 0 & k_y(\theta) \end{bmatrix}$$

(1)

where $\hat{x}$, $\hat{y}$ are the directions of the rotating coordinates, $\theta$ is the angle between the crack and the rotor response and

$$k_x(\theta) = k_{xM} + k_{xD} \cos(\theta), \quad k_y(\theta) = k_{yM} + k_{yD} \cos(\theta)$$

(2)

In these equations,

$$k_{xM} = \frac{1}{2} (k_u + k_{o\bar{x}}) \quad \text{and} \quad k_{xD} = \frac{1}{2} (k_u - k_{o\bar{x}}),$$

(3)

with similar expressions for the $\hat{y}$ direction. When $\cos(\theta) = 1$ the crack is fully closed and $k_x(\theta) = k_y(\theta) = k_u$, which is the uncracked machine stiffness. Thus we are assuming that the rotor is symmetric when the crack is closed. When $\cos(\theta) = -1$ the crack is fully open and $k_x(\theta) = k_{o\bar{x}}$ and $k_y(\theta) = k_{o\bar{y}}$, where $k_{o\bar{x}}$ and $k_{o\bar{y}}$ represents the stiffness of the machine in the two directions when the crack is fully open. Note that when the crack opens the rotor is asymmetric.

The stiffness matrix in fixed coordinates can be determined by transforming from rotating coordinates so that
\[
K = T^T \tilde{K} T, \quad \text{where} \quad T = \begin{bmatrix}
\cos(\phi) & \sin(\phi) \\
-sin(\phi) & \cos(\phi)
\end{bmatrix}.
\] (4)

For a rotor running at constant spin speed, \(\Omega, \phi = \Omega t\). Thus, in fixed coordinates we have
\[
K(\theta, t) = K_0 - K_c(\theta, t)
\] (5)
where \(K_0\) is the diagonal stiffness matrix for the undamaged rotor and \(K_c(\theta, t)\) is the stiffness change due to the crack, obtained from Equations (1) to (4).

3 The equations of motion

The analysis may be performed in fixed or rotating coordinates. If the bearings and foundations are axi-symmetric then the stator dynamic stiffness will appear constant in the rotating frame, and there is some benefit in analysing the machine in rotating coordinates. Typically foundations will be stiffer vertically than horizontally, and in this case the advantage in using rotating coordinates is significantly reduced. In this paper and axi-symmetric stator is assumed and the machine is analysed in fixed coordinates.

In fixed coordinates we have \(K(\theta, t) = K_0 - K_c(\theta, t)\) where \(\theta\) is the angle between the crack axis and the rotor response at the crack location and determines the extent to which the crack is open. Let the deflection of the system be \(q = q_{st} + q_{dy}\) where \(q_{st}\) is the static deflection and \(q_{dy}\) is the dynamic deflection due to the rotating out of balance and/or the dynamic forces applied to the rotor by the AMB. Thus, \(\dot{q} = \dot{q}_{dy}\) and \(\ddot{q} = \ddot{q}_{dy}\), and the equation of motion in fixed coordinates is
\[
M\ddot{q}_{dy} + (D + G)q_{dy} + (K_0 - K_c(\theta, t))(q_{st} + q_{dy}) = Q_u + Q_{AMB} + W
\] (6)
where \(Q_u\), \(Q_{AMB}\) and \(W\) are the out of balance forces, the external forces applied to the rotor by the active magnetic bearing and the gravitational force respectively. Damping and gyroscopic effects have been included as a symmetric positive semi-definite matrix \(D\) and a skew-symmetric matrix \(G\), although they have little direct bearing on the analysis. If there is axi-symmetric damping in the rotor then there will also be a skew-symmetric contribution to the undamaged stiffness matrix, \(K_0\). We refer to Equation (6) as the “full equations”.

There are two approximations that are commonly used in the analysis of cracked rotors, namely weight dominance and neglecting the parametric excitation terms. Only weight dominance will be considered here.

3.1 Weight dominance

One common assumption is that the static deflection is much greater that the response due to the unbalance or rotating asymmetry, that is \(|q_{st}| \gg |q_{dy}|\). For example, for a large turbine rotor the static deflection might be of the order of 1 mm whereas at running speed the amplitude of vibration is typically 50 \(\mu\)m. Even at a critical speed the allowable level of vibration will only be 250 \(\mu\)m. In this situation that the crack opening and closing is dependent only on the static deflection and thus \(\theta = \Omega t + \theta_0\), where \(\Omega\) is the rotor speed and \(\theta_0\) is the initial angle.

Suppose the rotor is initialize so that \(\theta_0 = 0\) and thus \(\theta = \phi\). Carrying out the matrix multiplications of Equation (4) and expanding the trigonometric expressions in multiple angles gives
\[ k_{12}(\theta) = \frac{1}{2}(k_{3M} - k_{3M})\sin(2\theta) + \frac{1}{4}(k_{3D} - k_{3D})(\sin(\theta) + \sin(3\theta)) \]
\[ = \frac{1}{4}(k_{a} - k_{b})(\sin(2\theta) - \frac{1}{2}(\sin(\theta) + \sin(3\theta))) . \]

Also
\[ k_{11} = k_{u} - k_{c11}(\theta) , \quad k_{22} = k_{u} - k_{c22}(\theta) \]

and
\[ k_{c11}(\theta) = \frac{1}{2}(k_{3D} + k_{3D}) + \frac{1}{2}(k_{3D} - k_{3D})\cos(2\theta) - \frac{1}{4}\left(3k_{3D} + k_{3D}\right)\cos(\theta) + \left(k_{3D} - k_{3D}\right)\cos(3\theta) \]
\[ k_{c22}(\theta) = \frac{1}{2}(k_{3D} + k_{3D}) + \frac{1}{2}(k_{3D} - k_{3D})\cos(2\theta) - \frac{1}{4}\left(3k_{3D} + k_{3D}\right)\cos(\theta) + \left(k_{3D} - k_{3D}\right)\cos(3\theta) \]

Thus \( K = K_0 - K_c(\theta, t) \) where
\[ K_0 = \begin{bmatrix} k_0 & 0 \\ 0 & k_0 \end{bmatrix} \quad \text{and} \quad K_c(\theta) = \begin{bmatrix} k_{c11} & k_{c12} \\ k_{c21} & k_{c22} \end{bmatrix} . \] (10)

Thus in fixed coordinates the Mayes model generates a constant term plus 1X, 2X and 3X rotor angular velocity components in the diagonal stiffness terms and 1X, 2X and 3X rotor angular velocity components in the off-diagonal stiffness terms.

The above analysis shows that \( K_c(0, t) = K_c(t) \) is a periodic function of time and the full non-linear Equation (6) becomes a linear parametrically excited equation. We refer to this as the “weight dominance assumption” and the equation of motion becomes
\[ M\ddot{q}_{dy} + (C + G)q_{dy} + (K_0 - K_c(t))(q_{st} + q_{dy}) = Q_u + Q_{AMB} + W \] (11)

4 Condition Monitoring using Active Magnetic Bearings

The force applied from the AMBs is already included in the equations of motion, although there was no discussion of the character of this force. The key aspect of the analysis is that the system has three different frequencies, namely the natural frequency (or critical speed), the rotor spin speed and the forcing frequency from the AMB. The parametric terms in the equations of motion (or non-linear terms in the full equations) cause combinational resonances in the response of the machine. Mani et al. [12,13] and Quinn et al [14] used a multiple scales analysis to determine the conditions required for a combinational resonance, which occurs when
\[ \Omega_2 = |n\Omega - \omega_n| \]
for \( n = \pm 1, \pm 2, \pm 3 \) (12)

where \( \Omega \) is the rotor spin speed, \( \Omega_2 \) is the frequency of the AMB force, and \( \omega_n \) is the natural frequency of the system. This analysis was based on a two degree of freedom Jeffcott rotor model with weight dominance, equivalent to that described in this paper. Mani et al. also considered the effect of detuning, that is excitation close to this exact excitation frequency for resonance, and investigated the effect on the magnitude of the primary resonance close to the natural frequency of the machine. In the examples the running speed of the machine was five times higher than the natural frequency. This ratio is not practical since there is likely to be a second unmodelled resonance below the running speed. Indeed the fact that higher resonances are not modelled is a serious omission, particularly as the combinational resonances are likely to excite any higher frequency resonances.

The AMB force is usually sinusoidal. Mani et al. [13] introduced the possibility of a periodic step force profile. This profile has two major disadvantages. First a step change in force will excite the higher modes of the structure making the identification of the key features that determine a cracked shaft difficult.
Secondly AMBs are unable to provide such a force profile and their maximum rate of change of force is limited.

5 Simulated Example

The approach described in the preceding sections will be tested on a simulated example of a two degree of freedom Jeffcott rotor. The mass of the rotor is 1 kg, the natural frequency is 40 Hz and the damping ratio is 1%. Unless otherwise specified the unbalance force is given by $Q_u = 10^{-4} \Omega^2 \begin{bmatrix} \cos \Omega t \\ \sin \Omega t \end{bmatrix}$ N, and the AMB force is vertical with magnitude 1 N. The equations of motion are integrated using ode45 in MATLAB, and once the steady state has been established the FFT is calculated, taking care to avoid leakage problems.

Figure 1 shows the vertical response due to the unbalance force, with no damage or AMB, and the rotor spin speed at 90% of the machine natural frequency. As expected the major response is at the rotor spin speed, although the response does still contain a small transient at 40 Hz. Figure 2 shows the vertical response where a crack of depth equal to 40% of the radius is modelled, with a stiffness reduction is 9% and 4% in the directions parallel and orthogonal to the crack face. The full equations of motion are used. The major difference is that the response now contains harmonics of the rotor spin speed. Figure 3 shows the effect of exciting the rotor with the AMB when there is no crack, and the vertical response mainly contains the frequencies corresponding to the rotor spin speed and the AMB excitation.

![Figure 1: The vertical response with no damage, $\Omega=2160$ rev/min (36 Hz), no AMB.](image-url)
Figure 2: The vertical response with 40% crack, $\Omega=2160$ rev/min (36 Hz), no AMB.

Figure 3: The vertical response with no damage, $\Omega=2160$ rev/min (36 Hz), AMB frequency 1920 rev/min (32 Hz).

Figure 4 shows the vertical response when the cracked rotor is excited by the AMB at 1920 rev/min (corresponding to $n = 2$ in Equation (12)). There is a response at the frequencies corresponding to the machine natural frequency (40 Hz), the rotor spin speed (36 Hz) and the AMB excitation frequency (32 Hz). The response also contains significant components at many combinations of these frequencies. Figure 5 shows a similar response when the AMB excitation frequency is changed to 4080 rev/min (68 Hz), corresponding to $n = 3$ in Equation (12). Although similar frequency components appear, they are fewer in number than in Figure 4. In Figure 6 the rotor spin speed is increased to 2640 rev/min (44 Hz) and the AMB excitation frequency changed to 5520 rev/min (92 Hz, $n = 3$). The number of frequency components is now significantly reduced. Figure 7 shows the vertical response when the AMB force is applied horizontal rather than vertically as in Figure 6 (the other parameters remain unaltered). The only real difference is the response at the AMB excitation frequency (92 Hz).
Figure 4: The vertical response with 40% crack, $\Omega = 2160$ rev/min (36 Hz), AMB frequency 1920 rev/min (32 Hz).

Figure 5: The vertical response with 40% crack, $\Omega = 2160$ rev/min (36 Hz), AMB frequency 4080 rev/min (68 Hz).

Figure 6: The vertical response with 40% crack, $\Omega = 2640$ rev/min (44 Hz), AMB frequency 5520 rev/min (92 Hz).
Figure 7: The vertical response with 40% crack, $\Omega=2640$ rev/min (44 Hz), AMB frequency 5520 rev/min (92 Hz), AMB force applied horizontally.

Figure 8 shows the effect of increasing the unbalance force to $10^{-2}\Omega^2$N rather than $10^{-4}\Omega^2$N. The AMB is now applied vertically, and other parameters remain the same as the system that produced Figure 6. Now the unbalance force dominates which means that the number of frequency components in the response is reduced. Figure 9 retains this increased level of unbalance but also increases the AMB force to 20 N (note the weight of the rotor is only 10 N). Figure 10 is the same model but with the assumption of weight dominance. Clearly in this case the assumption of weight dominance increases the number of frequency components in the response because the model assumes the crack opens and closes, when in fact it does not.

Figure 8: The vertical response with 40% crack, $\Omega=2640$ rev/min (44 Hz), AMB frequency 5520 rev/min (92 Hz), unbalance force increased by a factor of 100.
Figure 9: The vertical response with 40% crack, $\Omega = 2640$ rev/min (44 Hz), AMB frequency $5520$ rev/min (92 Hz), unbalance force increased by a factor of 100, AMB force increased to 20 N, full equations of motion.

Figure 10: The vertical response with 40% crack, $\Omega = 2640$ rev/min (44 Hz), AMB frequency $5520$ rev/min (92 Hz), unbalance force increased by a factor of 100, AMB force increased to 20 N, weight dominance assumed.

6 Conclusions

This simulated study of a cracked Jeffcott rotor shows that using an active magnetic bearing (AMB) to excite the rotor with an harmonic force at an appropriate frequency causes components in the system response at many frequencies which are combinations of the rotor speed, the AMB excitation frequency and the system natural frequency. Such frequencies are not in the system response when the rotor is undamaged. These combination frequencies could be used to detect cracks in the rotor.

A possible scenario might be that a reference response is determined for the new, undamaged rotor with the harmonic force generated by the AMB acting so that Figure 3 is obtained. Then at prescribed intervals the AMB would be activated and the response compared with the reference response. When a crack occurs in the rotor a response similar to Figure 4 would be obtained and the changes between it and the reference response could be indicative of the presence of a crack in the rotor. This change in response might be most clearly seen by displaying the difference between Figure 4 and Figure 3.

In this preliminary study only a 2 DoF model of Jeffcott rotor has been considered and the crack, equal to 40% of rotor radius, is large: one would want to detect the crack before it grew to this level. Thus it is
planned to study more complex FE models of rotors with cracks, to examine the problem of detecting much smaller cracks and to assess the robustness of the method in the presence of noise.

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References