An Argument-Dependent Approach to Determining OWA Operator Weights Based on the Rule of Maximum Entropy

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The methods for determining OWA operator weights have aroused wide attention. We first review the main existing methods for determining OWA operator weights. We next introduce the principle of maximum entropy for setting up probability distributions on the basis of partial knowledge and prove that Xu’s normal distribution-based method obeys the principle of maximum entropy. Finally, we propose an argument-dependent approach based on normal distribution, which assigns very low weights to these “false” or “biased” opinions and can relieve the influence of the unfair arguments. A numerical example is provided to illustrate the application of the proposed approach. © 2007 Wiley Periodicals, Inc.

1. INTRODUCTION

The process of information aggregation appears in many applications, which will have a great affect on the development of intelligent systems. Yager1 introduced a new information aggregation technique based on the ordered weighted averaging operators (OWA), which has already been applied in neural networks,2,3 fuzzy logic controllers and fuzzy systems,4,5 information fusion,6 expert systems,7 and decision-making aids.8–11

One key issue in the theory of the OWA operator is to determine its associated weights. O’Hagan7 proposed a maximum entropy approach, which involved a constrained nonlinear optimization problem with a predefined degree of orness as its constraint and the entropy as the objective function. Fullér and Majlender12 transformed the maximum entropy model into a polynomial equation that can be solved analytically. Liu and Chen13 suggested a parametric geometric approach that could

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be used to obtain maximum entropy weights. Filev and Yager\textsuperscript{14} developed a procedure that learns the weights from a collection of samples with their aggregated value. Nettleton and Torra\textsuperscript{15} suggested a genetic algorithm (GA). Xu and Da\textsuperscript{16} established a linear objective-programming model for obtaining the OWA weights from observational data by using partial weight information.\textsuperscript{17} Yager\textsuperscript{18} expressed another kind of measure of entropy. Füllér and Majlender\textsuperscript{19} suggested a minimum variance approach to obtain the minimal variability OWA operator weights. Wang and Parkan\textsuperscript{20} proposed a minimax disparity approach for obtaining OWA operator weights.

The above approaches are generally quite complex because these approaches require the solution of a constrained nonlinear or linear optimization problem. There are some simple approaches for determining the OWA weights without optimization problems. Yager suggested an interesting way to compute the weights of the OWA operator using linguistic quantifiers\textsuperscript{1} and an exponential smoothing method.\textsuperscript{14} Xu\textsuperscript{21} developed an argument-independent method based on normal distribution for determining the OWA weights. The weights derived by the argument-independent approaches are associated with particular ordered positions of the aggregated arguments and have no connection with the aggregated arguments, whereas the argument-dependent approaches determine the weights based on the input arguments. In this article, we propose an argument-dependent approach based on normal distribution, which obeys the principle of maximum entropy.

2. AN OVERVIEW OF THE EXISTING MAIN METHODS

An OWA operator of dimension $n$ is a mapping, $OWA : R^n \rightarrow R$, that has an associated $n$ vector $= (w_1, w_2, \ldots, w_n)^T$ such that $w_j \in [0,1]$ and $\sum_{j=1}^n w_j = 1$. Furthermore,

$$OWA(a_1, a_2, \ldots, a_n) = \sum_{j=1}^n w_j b_j$$

where $b_j$ is the $j$th largest element of the collection of the aggregated objects $a_1, a_2, \ldots, a_n$.

Clearly, the key point of the OWA operator is to determine its associated weights. Yager\textsuperscript{1} introduced the ideal of aggregate dependent weights, which allows the weights to be a function of the aggregated arguments, in this case

$$OWA(a_1, a_2, \ldots, a_n) = \sum_{j=1}^n f_j(b_1, b_2, \ldots, b_j) b_j$$

The first family of the aggregate dependent weights that Yager\textsuperscript{1} studied is as follows:

$$w_j = \frac{b_j^a}{\sum_{j=1}^n b_j^a}, \quad j = 1, 2, \ldots, n$$

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where $\alpha \in (-\infty, +\infty)$. In this case, it leads to a neat OWA operator:

$$OWA(a_1, a_2, \ldots, a_n) = \frac{\sum_{j=1}^{n} b_j^{\alpha+1}}{\sum_{j=1}^{n} b_j^{\alpha}} = \frac{\sum_{j=1}^{n} a_j^{\alpha+1}}{\sum_{j=1}^{n} a_j^{\alpha}}$$  \hspace{1cm} (4)

**Note.** An OWA operator is called neat if the aggregated value is independent of the ordering.\(^{18}\)

Another interesting case of the aggregate dependent weights is

$$w_j = \frac{(1 - b_j)^{\alpha}}{\sum_{j=1}^{n} (1 - b_j)^{\alpha}}, \quad j = 1, 2, \ldots, n$$  \hspace{1cm} (5)

In this case, it follows that

$$OWA(a_1, a_2, \ldots, a_n) = \frac{\sum_{j=1}^{n} (1 - b_j)^{\alpha} b_j}{\sum_{j=1}^{n} (1 - b_j)^{\alpha}} = \frac{\sum_{j=1}^{n} (1 - a_j)^{\alpha} a_j}{\sum_{j=1}^{n} (1 - a_j)^{\alpha}}$$  \hspace{1cm} (6)

which is also a neat aggregation.

Yager\(^{1}\) further introduced two characterizing measures called *orness measure* and *dispersion measure*, respectively, associated with the weighting vector $w$ of an OWA operator, where the *orness measure* of the aggregation is defined as

$$orness(w) = \frac{1}{n - 1} \sum_{j=1}^{n} (n - j) w_j$$  \hspace{1cm} (7)

which lies in the unit interval $[0, 1]$ and characterizes the degree to which the aggregation is like an *or* operation. The second one, the *dispersion measure* of the aggregation, is defined as

$$disp(w) = -\sum_{j=1}^{n} w_j \ln w_j$$  \hspace{1cm} (8)

which measures the degree to which $w$ takes into account the information in the arguments during the aggregation.

O’Hagan\(^{7}\) suggested a maximum entropy approach, which requires the solution of the following constrained nonlinear optimization problem:
Maximize $D_{sp}(W) = -\sum_{i=1}^{n} w_i \ln w_i$ (9)

\[ \text{s.t. orness}(W) = \frac{1}{n-1} \sum_{i=1}^{n} (n - i) w_i = \alpha \]

\[ \sum_{i=1}^{n} w_i = 1 \]

$0 \leq \alpha \leq 1; \quad w_i \in [0,1], \quad i = (1, \ldots, n)$

Yager\textsuperscript{18} expressed a measure of entropy as $1 - \text{Max}_i[w_i]$, and then extended Model 9 to the following:

Minimize : $\text{Max}_i(w_i)$ (10)

\[ \text{s.t. orness}(W) = \frac{1}{n-1} \sum_{i=1}^{n} (n - i) w_i = \alpha \]

\[ \sum_{i=1}^{n} w_i = 1 \]

$0 \leq \alpha \leq 1; \quad w_i \in [0,1], \quad i = (1, \ldots, n)$

Fullér and Majlender\textsuperscript{19} suggested a minimum variance approach, which minimizes the variance of OWA operator weights under a given level of orness. A set of OWA operator weights with minimal variability could then be generated. Their approach requires the solution of the following mathematical programming problem:

Minimize $D^2(W) = \frac{1}{n} \sum_{i=1}^{n} \left( w_i - \frac{1}{n} \right)^2$ (11)

\[ \text{s.t. orness}(W) = \frac{1}{n-1} \sum_{i=1}^{n} (n - i) w_i = \alpha \]

\[ \sum_{i=1}^{n} w_i = 1 \]

$0 \leq \alpha \leq 1; \quad w_i \in [0,1], \quad i = (1, \ldots, n)$

Wang and Parkan\textsuperscript{20} proposed a minimax disparity approach, which generates the OWA operator weights by minimizing the maximum difference between any two adjacent weights. Their approach requires the solution of the following mathematical programming problem:

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Minimize \[
\left\{ \text{Max}_{i \in \{1, \ldots, n-1\}} |w_i - w_{i+1}| \right\}
\]

s.t. orness \(W\) = \[
\frac{1}{n-1} \sum_{i=1}^{n} (n - i)w_i = \alpha
\]

\[
\sum_{i=1}^{n} w_i = 1
\]

\[
0 \leq \alpha \leq 1; \quad w_i \in [0,1], \quad i = (1, \ldots, n)
\]

Xu\(^{21}\) developed a simple method based on the normal distribution for determining the OWA weights, which can be defined as follows:

\[
w_i = \frac{1}{\sqrt{2\pi \sigma_n}} e^{-[(i-u_n)^2/2\sigma_n^2]}, \quad i = 1, 2, \ldots, n
\]

where \(w = (w_1, w_2, \ldots, w_n)^T\) is the weight vector of the OWA operator; \(u_n\) is the mean of the collection of \(1, 2, \ldots, n\), and \(\sigma_n (\sigma_n > 0)\) is the standard deviation of the collection of \(1, 2, \ldots, n\). \(u_n\) and \(\sigma_n\) are obtained by the following formulas, respectively:

\[
u_n = \frac{1 + n}{2}
\]

\[
\sigma_n = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (i - u_n)^2}
\]

Consider that \(w_i \in [0,1]\) and \(\sum_{i=1}^{n} w_i = 1; w_i\) can be defined as follows:

\[
w_i = \frac{e^{-[(i-(1+n)/2)^2/2\sigma_n^2]}}{\sum_{j=1}^{n} e^{-[(j-(1+n)/2)^2/2\sigma_n^2]}}, \quad i = 1, 2, \ldots, n
\]

3. THE PRINCIPLE OF MAXIMUM ENTROPY

Jaynes\(^{22}\) proposed the principle of maximum entropy for setting up probability distributions on the basis of partial knowledge. It is the least biased estimate possible on the given information. The maximum entropy distribution may be asserted for the positive reason that it is uniquely determined as the one that is maximally noncommittal with regard to missing information, instead of the negative one that there was no reason to think otherwise. Mathematically, the maximum entropy distribution has the important property that no possibility is ignored; it assigns positive weight to every situation that is not absolutely excluded by the given information.

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3.1. Discrete Distribution

The quantity \( x \) is capable of assuming the discrete values \( x_i (i = 1, 2, \ldots, n) \). We are not given the corresponding probabilities \( p_i \); all we know is \( \langle f_r(x) \rangle \), which is the expectation value of the function \( f_r(x) \). To determine the probabilities \( p_i \) fairly, Jaynes proposed the optimization problem:

\[
\text{Max } H(p_1, \ldots, p_n) = - \sum_{i=1}^{n} p_i \ln p_i
\]

s.t. \( \langle f_r(x) \rangle = \sum_{i=1}^{n} p_i f_r(x_i) \)

\[
\sum_{i=1}^{n} p_i = 1
\]

\( p_i \geq 0; \quad r = 1, \ldots, m, \quad i = 1, \ldots, n \)

To resolve Model 17, we introduce Lagrangian Multipliers \( \lambda_0, \lambda_1, \ldots, \lambda_r, \ldots, \lambda_m \), in the usual way, and obtain the result

\[
p_i = \exp\{-[\lambda_0 + \lambda_1 f_1(x_i) + \cdots + \lambda_m f_m(x_i)]\}
\]

in which the constants \( \lambda_0, \lambda_1, \ldots, \lambda_r, \ldots, \lambda_m \) are determined from

\[
Z(\lambda_1, \ldots, \lambda_m) = \sum_{i=1}^{n} \exp\{-[\lambda_1 f_1(x_i) + \cdots + \lambda_m f_m(x_i)]\}
\]

\[
\langle f_r(x) \rangle = - \frac{\partial}{\partial \lambda_r} \ln Z(\lambda_1, \ldots, \lambda_r, \ldots, \lambda_m)
\]

\[
\lambda_0 = \ln Z(\lambda_1, \ldots, \lambda_r, \ldots, \lambda_m)
\]

Especially when \( f_r(x_i) = 0, \ r = 2, \ldots, m, \ i = 1, \ldots, n \), we can get

\[
\text{Max } H(p_1, \ldots, p_n) = - \sum_{i=1}^{n} p_i \ln p_i
\]

s.t. \( \langle f_1(x) \rangle = \sum_{i=1}^{n} p_i f_1(x_i) \)

\[
\sum_{i=1}^{n} p_i = 1
\]

\( p_i \geq 0, \quad i = 1, \ldots, n \)
Then the maximum entropy probability distribution is given by

$$p_i = \exp\{ -[\lambda_0 + \lambda_1 f_i(x_i)] \}$$

(23)

in which the constants $\lambda_0, \lambda_1$ are determined from

$$Z(\lambda_1) = \sum_{i=1}^{n} \exp\{ -\lambda_1 f_i(x_i) \}$$

(24)

$$\langle f_i(x) \rangle = -\frac{\partial}{\partial \lambda_1} \ln Z(\lambda_1)$$

(25)

$$\lambda_0 = \ln Z(\lambda_1)$$

(26)

Obviously Model 22 is the same as Model 9 in O’Hagan. So we can find that Model 9 obeys the principle of maximum entropy, which assigns positive weight to every possibility that is not absolutely excluded by the given information. Model 9 is complex for requiring the solution of a nonlinear optimization problem. So Models 10, 11, and 12 relax the entropy function, respectively. But all of them obey the principle of maximum entropy approximately.

Xu developed a simple and practical method based on the normal distribution for determining the OWA weights where $u_n$ and $\sigma_n$ are known ($u_n$ is the mean of the collection of $1, 2, \ldots, n$, and $\sigma_n$ is the standard deviation of the collection of $1, 2, \ldots, n$). We can provide that the normal distribution is a kind of maximum entropy distribution with the information of $u_n$ and $\sigma_n$.

### 3.2. Continuous Distribution

Let $x$ ($x \in \Theta$, $\Theta$ is the universe of discourse) be the continuous random variable. We are not given the corresponding probability density function $f(x)$; all we know are $g_i(x)$ and $E_i$, which represent the known information. To determine the probability density function $f(x)$ fairly, Kullback set up the following optimization problem:

$$\text{Max } H(x) = -\int_{x \in \Theta} f(x) \ln f(x) \, dx$$

(27)

s.t. $\int_{x \in \Theta} f(x) \, dx = 1$

$$\int_{x \in \Theta} f(x) g_i(x) \, dx = E_i, \quad i = 1, 2, \ldots, n$$

Let $L(H, \alpha, \beta_i)$ denote the Lagrange function of Equation 27, where $\alpha$ and $\beta_i$ are real numbers.
Then the partial derivatives of $L$, $\alpha$, $\beta_i$ are computed as
\[
\frac{\partial L}{\partial x} = f(x) \ln \left[ f(x)^{-1} e^{-\alpha - \sum_{i=1}^{n} \beta_i g_i(x)} \right] = 0
\]
Obviously $f(x) \neq 0$, so we get
\[
f(x)^{-1} e^{-\alpha - \sum_{i=1}^{n} \beta_i g_i(x)} = 1
\]
That is,
\[
f(x) = e^{-\alpha - \sum_{i=1}^{n} \beta_i g_i(x)} \tag{28}
\]
The constants $\alpha$ and $\beta_i$ are determined by substituting Equation 28 into Model 27. The result may be written in the form
\[
\int_{x \in \Theta} e^{-\alpha - \sum_{i=1}^{n} \beta_i g_i(x)} \, dx = 1 \tag{29}
\]
\[
\int_{x \in \Theta} e^{-\alpha - \sum_{i=1}^{n} \beta_i g_i(x)} g_i(x) \, dx = E_i, \quad i = 1, 2, \ldots, n \tag{30}
\]

**Theorem 1.** Let $f(x)$ be the probability density function of $X$. When the expectation $E(X) = u$ and the variance $D(X) = \int_{-\infty}^{+\infty} x^2 f(x) \, dx - u^2 = \sigma^2$ are known, the normal distribution $f(x) = \left(1/\sqrt{2\pi\sigma}\right) e^{(x-u)^2/2\sigma^2}$ is the maximum entropy distribution.

**Proof.** Theory 1 can be transformed to the following optimization problem:
\[
\max H(x) = -\int_{-\infty}^{+\infty} f(x) \ln f(x) \, dx \tag{31}
\]
s.t. $\int_{x \in \Theta} f(x) \, dx = 1$
\[
\int_{x \in \Theta} x f(x) \, dx = u
\]
\[
\int_{x \in \Theta} x^2 f(x) \, dx = \sigma^2 + u^2, \quad \Theta = (-\infty, +\infty)
By Equations 28, 29, and 30, we can get

\[ \int_{x \in \Theta} e^{-\alpha - x \times \beta_1 - x^2 \times \beta_2} \, dx = 1 \]  
\[ \int_{x \in \Theta} x 	imes e^{-\alpha - x \times \beta_1 - x^2 \times \beta_2} \, dx = u \]  
\[ \int_{x \in \Theta} x^2 \times e^{-\alpha - x \times \beta_1 - x^2 \times \beta_2} \, dx = \sigma^2 + u^2 \]  

By resolving Equations 32, 33, and 34, we have

\[ \alpha = \frac{u^2}{2\sigma^2} + \ln(\sqrt{2\pi}\sigma) \]

\[ \beta_1 = -\frac{u}{\sigma^2} \]

\[ \beta_2 = \frac{1}{2\sigma^2} \]

So

\[ f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-u)^2/2\sigma^2} \]  

Then, it is reasonable to infer that \( X \) is normally distributed only with the information of \( u \) and \( \sigma \). Xu’s method\(^{21}\) based on the normal distribution also obeys the principle of maximum entropy.

4. ARGUMENT-DEPENDENT APPROACH BASED ON NORMAL DISTRIBUTION

Yager\(^1\) introduced an argument-dependent approach for determining the OWA operator weights. Xu\(^{21}\) develop an argument-independent method based on the normal distribution for determining the OWA weights, which assigns very low weights to these “false” or “biased” opinions. The weights derived by the argument-independent approaches are associated with particular ordered positions of the aggregated arguments and have no connection with the aggregated arguments, whereas the argument-dependent approaches determine the weights based on the input arguments. In light of work by Yager\(^1\) and Xu,\(^{21}\) this article proposes an argument-dependent approach for determining the OWA operator weights based on the normal distribution.
Definition 1. Suppose \( a_1, a_2, \ldots, a_n \) is a collection of arguments. Let \( u \) be the average value and \( \sigma \) be the variance of these arguments, where \( b_j \) is the jth largest element of the collection of the aggregated objects \( a_1, a_2, \ldots, a_n \):

\[
\begin{align*}
  u &= \frac{1}{n} \sum_{j=1}^{n} b_j \\
  \sigma &= \sqrt{\frac{1}{n} \sum_{j=1}^{n} (b_j - u)^2}
\end{align*}
\]  

Let \( w = (w_1, w_2, \ldots, w_n)^T \) be the weight vector of the OWA operator; then we define the following:

\[
w_j = \frac{1}{\sqrt{2\pi \sigma}} e^{-\frac{(b_j-u)^2}{2\sigma^2}}, \quad j = 1, 2, \ldots, n
\]  

Consider that \( \sum_{j=1}^{n} w_j = 1 \) and \( w_j \in [0, 1] \); then by Equation 38, we can obtain

\[
w_j = \frac{1}{\sum_{j=1}^{n} \frac{1}{\sqrt{2\pi \sigma}} e^{-\frac{(b_j-u)^2}{2\sigma^2}}} \frac{\sum_{j=1}^{n} e^{-\frac{(b_j-u)^2}{2\sigma^2}}}{\sum_{j=1}^{n} e^{-\frac{(b_j-u)^2}{2\sigma^2}}}, \quad j = 1, 2, \ldots, n
\]  

In this case, Equations 1, 7, and 8 can be written as Equations 40, 41, and 42, respectively:

\[
\begin{align*}
  OWA(a_1, a_2, \ldots, a_n) &= \frac{\sum_{j=1}^{n} e^{-\frac{(b_j-u)^2}{2\sigma^2}} \times b_j}{\sum_{j=1}^{n} e^{-\frac{(b_j-u)^2}{2\sigma^2}}} = \frac{\sum_{j=1}^{n} e^{-\frac{(a_j-u)^2}{2\sigma^2}} \times a_j}{\sum_{j=1}^{n} e^{-\frac{(a_j-u)^2}{2\sigma^2}}} \\
  disp(w) &= -\sum_{j=1}^{n} w_j \ln w_j = \frac{\sum_{j=1}^{n} e^{-\frac{(b_j-u)^2}{2\sigma^2}} \left( \frac{(b_j-u)^2}{2\sigma^2} + \ln \left( \sum_{j=1}^{n} e^{-\frac{(b_j-u)^2}{2\sigma^2}} \right) \right)}{\sum_{j=1}^{n} e^{-\frac{(b_j-u)^2}{2\sigma^2}}} \\
  orness &= \frac{\sum_{j=1}^{n} (n-j) \times e^{-\frac{(b_j-u)^2}{2\sigma^2}}}{(n-1) \sum_{j=1}^{n} e^{-\frac{(b_j-u)^2}{2\sigma^2}}}
\end{align*}
\]
From Equation 40, it is easy to see this is a neat and dependent OWA operator. We use the following numerical example in Xu\textsuperscript{21} to illustrate the application of the proposed approach.

**Example 1.** Suppose that there are five experts \(e_j (j = 1,2,\ldots,5)\); these experts provide their individual preferences for a university faculty with respect to the criterion research. Assume that the given preference arguments are as follows:

\[
\begin{align*}
    a_1 &= 80, & a_2 &= 75, & a_3 &= 100, & a_4 &= 50, & a_5 &= 85
\end{align*}
\]

Therefore, the reordered arguments \(a_j (j = 1,2,\ldots,5)\) in descending order are

\[
\begin{align*}
    b_1 &= 100, & b_2 &= 85, & b_3 &= 80, & b_4 &= 75, & b_5 &= 50
\end{align*}
\]

By Equations 36, 37, and 39–42, we have

\[
\begin{align*}
    u &= 78; & \sigma &= 16.3095 \\
    w_1 &= 0.1144, & w_2 &= 0.2591, & w_3 &= 0.2820, & w_4 &= 0.2794, & w_5 &= 0.0651
\end{align*}
\]

\[
\text{OWA}(a_1,a_2,\ldots,a_n) = \sum_{j=1}^{5} w_j b_j = 80.2335
\]

\[
\text{orness} = 0.5196
\]

\[
\text{disp}(w) = 1.4890
\]

**Example 2.** Let us change the argument \(a_4\) from 50 to 60, and get the new reordered arguments in descending order as follows:

\[
\begin{align*}
    b_1 &= 100, & b_2 &= 85, & b_3 &= 80, & b_4 &= 75, & b_5 &= 60
\end{align*}
\]

By Equation 36, 37, and 39–42, we have

\[
\begin{align*}
    u &= 80; & \sigma &= 13.0384 \\
    w_1 &= 0.0887, & w_2 &= 0.2674, & w_3 &= 0.2878, & w_4 &= 0.2674, & w_5 &= 0.0887
\end{align*}
\]

\[
\text{OWA}(a_1,a_2,\ldots,a_n) = \sum_{j=1}^{5} w_j b_j = 80
\]

\[
\text{orness} = 0.5
\]

\[
\text{disp}(w) = 1.4936
\]

In the above Example 1 and Example 2, we assign low weights to those “unduly high” and “unduly low.” In Example 1, we assign the lowest weight \(w_5 = 0.0651\) to the lowest preference value \(b_5 = 50\), which has the biggest departure from the average value, and assign the most weight \(w_3 = 0.2820\) to the preference value \(b_3 = 80\), which is closest to the average value.
Example 2 is a situation similar to Example 1. When the argument $a_4$ changes from 50 to 60, the weights by Xu’s method\textsuperscript{21} still remains the following:
\[
\begin{align*}
    w_1 &= 0.1117, \\
    w_2 &= 0.2365, \\
    w_3 &= 0.3036, \\
    w_4 &= 0.2365, \\
    w_5 &= 0.1117
\end{align*}
\]
But the weight $w_5$ by our argument-dependent approach turns from 0.0651 to 0.0887, which seems more reasonable in some circumstances because our argument-dependent approach makes use of the information of the arguments themselves.

5. CONCLUSIONS

In this article, we introduce the principle of maximum entropy for determining the OWA operators weights and prove that O’Hagan’s approach and Xu’s approach also obey the principle of maximum entropy. Based on the normal distribution, we propose an argument-dependent approach, which assigns very low weights to these “false” or “biased” opinions and can relieve the influence of the unfair arguments.

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