How can we tell whether dark energy is composed by multiple fields?

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Dark energy is often assumed to be composed by a single scalar field. The background cosmic expansion is not sufficient to determine whether this is true or not. We study multi-field scalar-tensor models with a general dark matter source and write the observable modified gravity parameters (effective gravitational constant and anisotropic stress) in the form of a ratio of polynomials in the Fourier wavenumber k of order $2N$, where N is the number of scalar fields. By comparing these observables to real data it is in principle possible to determine the number of dark energy scalar fields coupled to gravity. We also show that there are no realistic non-trivial cases in which the order of the polynomials is reduced.

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I. INTRODUCTION

Testing the nature of dark energy and its possible modification of gravity at very large scales is currently one of the most interesting research activities in cosmology. Modifications of standard gravity are often modeled by introducing additional mediating fields in the gravitational Lagrangian. One of the most well studied example is the so-called Horndeski theory, which adds to the Einstein-Hilbert Lagrangian a single scalar field that obeys the most general second order equation of motion [1].

It is however in principle possible that dark energy is actually a composite state of two or more scalar fields interacting through a common potential and coupled to gravity with different strengths (see e.g. [2–5]). Since the only observable at background level is the Hubble function $H(z)$ as a function of redshift, and since to any $H(z)$ one can always associate a particular single-field potential (see appendix), it is impossible to tell whether dark energy is driven by multiple fields instead of just one, as almost universally assumed. In this paper we show how linear cosmological perturbations produce instead distinguishable effects that depend on the number of fields.

As shown in several papers (e.g. [6–10]), a generic modification of gravity introduces at linear perturbation level two new functions that depend only on background time-dependent quantities and, in Fourier space, on the wavenumber k. One function, that we denote here with $Y(t,k)$ (sometimes also called $G_{\rm eff}$), modifies the standard Poisson equation, while the second one, $\eta(t,k)$, the anisotropic stress or tilt, provides the relation between the two gravity potentials Ψ,Φ. In standard gravity, one has $Y=\eta=1$.

In the so-called quasi-static regime (i.e. for linear scales that are below the sound horizon) the two functions Y,η take a particularly simple form in the Horndeski models and can be directly constrained through observations [11, 12]. This also holds true in some cases [13] of bimetric models [14].

Since the equation of motion of the Horndeski Lagrangian is second order, the equations contain at most second order space derivatives and therefore k^2 terms in Fourier space. It is then no surprise that the resulting forms of the functions Y,η include indeed polynomials of second order in k

$$\eta\equiv-\frac{\Phi}{\Psi}=h_2\left(\frac{1+k^2h_4}{1+k^2h_5}\right),\, Y\equiv-\frac{k^2\Psi}{4\pi Ga^2\rho\delta}=h_1\left(\frac{1+k^2h_5}{1+k^2h_3}\right). \tag{1}$$

where the h_i functions are time dependent functions that depend on the specific Horndeski model, a is the scale factor, ρ is the average matter density in the Universe, δ is the matter density contrast and Φ,Ψ are the two perturbation potentials, to be defined below . Also the combination $Y(1+\eta)$ that appears in the lensing potential equation has the same structure. As shown in e.g. [15], the fact that the k structure is fixed regardless of the specific model (within the Horndeski or the bimetric gravity class), allows one to combine observations of weak lensing, redshift distortions and galaxy clustering to constrain or detect modifications of gravity at cosmological scales in a relatively model-independent way. It is therefore interesting to inquire about how general the form of Eq.(1) is.

In [8] (see also [10]) it has been shown that models that obey n-th order equations of motion generalize the k-structure of Y,η so that

$$\eta=h_2\frac{P_n^{(1)}(k)}{P_n^{(2)}(k)},\, Y=h_1\frac{P_n^{(2)}(k)}{P_n^{(3)}(k)}. \tag{2}$$

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where $P_n^{(1)}, P_n^{(2)}, P_n^{(3)}$ are even polynomials of $n-$th order in k with time-dependent coefficients, normalized for convenience so that $P_n^{(1)}(0) = P_n^{(2)}(0) = P_n^{(3)}(0) = 1$. This case includes Lagrangians which depend on several scalar fields, because they also obey a system of second-order differential equations that is dynamically equivalent to a single higher-order equation. A Lagrangian built out of several scalar fields is in general stable provided the interaction potential is bounded from below.

In order to compare the models to observations however one needs the explicit form of the polynomials: this is the main goal of this paper. Here we consider the simplest non-trivial scalar tensor theory, namely a multi-field Brans-Dicke model, parametrized by several coupling constants ω_i and by a general potential $V(\phi_1, \phi_2, ...)$. Moreover, we generalize the Brans-Dicke model also in a different way: we include a generic fluid matter source with equation of state w and sound speed c_s, both of which generally time dependent. This might turn out useful to compare real data to models in which dark matter includes hot or warm components.

Finally, we investigate whether there are cases in which the higher-order k terms in the polynomials in Eq.(2) cancel out, thereby effectively reducing the order of the system and, consequently, the number of observable scalar fields. We find that this is not possible unless the fields are decoupled from gravity or the Universe is filled only with radiation. We argue therefore that the k-structure of Y, η is a unique signature of the number of dynamically active dark energy fields coupled to gravity.

Multi-scalar-field scenarios are being considered in gravitational physics and cosmology since a while. In the context of modified theories of gravity such models are studied in e.g. [16–19] while for the inflationary context see e.g. [20, 21].

II. STANDARD BRANS-DICKE THEORY

Let us first derive the cosmological perturbation equations for the standard Brans-Dicke theory with a potential. The corresponding action is:

$$S_1 = \int d^4x\sqrt{-g}(\phi R - \frac{\omega_{BD}}{\phi}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)) + \int d^4x\sqrt{-g}\mathcal{L}_{matter}, \tag{3}$$

where as matter fluid we consider a general component characterized by an equation of state w and a sound speed c_s, both arbitrarily time dependent. The equation of motion (e.o.m.) emerging from the variation with respect to the metric is

$$\phi G_{\mu\nu} + [\Box\phi + \frac{1}{2}\frac{\omega_{BD}}{\phi}(\nabla\phi)^2]g_{\mu\nu} - \nabla_\mu\nabla_\nu\phi - \frac{\omega_{BD}}{\phi}\nabla_\mu\phi\nabla_\nu\phi + \frac{V(\phi)}{2}g_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{4}$$

where

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu} \tag{5}$$

is the stress-energy tensor of the perfect fluid, with ρ being the energy density, p being the pressure and u_μ the 4-velocity. We will need the trace of Eq.(4) which reads as

$$-\phi R + 3\Box\phi + 2\frac{\omega_{BD}}{\phi}(\nabla\phi)^2 - \frac{\omega_{BD}}{\phi}\nabla^\alpha\phi\nabla_\alpha\phi + 2V(\phi) = 8\pi G T. \tag{6}$$

The scalar field e.o.m. is

$$R + 2\frac{\omega_{BD}}{\phi}\Box\phi - \frac{\omega_{BD}}{\phi^2}\partial^\alpha\phi\partial_\alpha\phi - V_{,\phi} = 0, \tag{7}$$

where the derivative with respect to the field ϕ is denoted as $V_{,\phi}$. Using Eq.(6) we obtain

$$(3 + 2\omega_{BD})\Box\phi + 2V(\phi) - V_{,\phi} = 8\pi G T. \tag{8}$$

From this equation it follows that in the limit $\omega_{BD} \to \infty$ the scalar field is constant (see e.g. [22]) and therefore in this large-ω_{BD} limit the gravitational equation of motion in Eq.(4) coincides with the Einstein-Hilbert equation of motion. It is interesting to note that the Brans-Dicke Lagrangian reproduces also the so-called $f(R)$ models in the limit in which ω_{BD} vanishes and $\phi = df(R)/dR$ (see e.g. [23]).

We start deriving now the cosmological perturbation equations for the Brans-Dicke theory in a spatially flat Friedmann-Lemaitre-Robertson-Walker (FLRW) metric. We assume a single fluid matter source with general equation of state and general sound speed. As it is well know, at first order in the perturbation parameter one can bring the scalar line element into the form

$$ds^2 = a^2(\tau)[-(1+2\Psi)d\tau^2 + (1+2\Phi)dx^i dx_i], \tag{9}$$

where τ is the conformal time, a is the scale factor and Ψ, Φ are the scalar potentials. We also decompose the scalar field into the background sector and the perturbed sector as $\phi(t,\vec{x}) = \phi(t) + \varphi(t,\vec{x})$. After switching to Fourier space we obtain from the $(0,0)$ element of Eq.(4)

$$-3\frac{\mathcal{H}^2}{a^2}\varphi + \phi\frac{2}{a^2}[3\mathcal{H}(\mathcal{H}\Psi - \Phi') - k^2\Phi] - \frac{k^2}{a^2}\varphi - \frac{1}{2a^2}\frac{\omega_{BD}}{\phi^2}\varphi\phi'^2 + \frac{1}{a^2}\frac{\omega_{BD}}{\phi}\varphi'\phi' + \frac{V_{,\phi}}{2}\varphi = -8\pi G\delta\rho, \tag{10}$$

where $\mathcal{H} = a'/a$ is the conformal Hubble function, the prime denotes differentiation with respect to the conformal time τ and $\delta\rho$ denotes the matter density perturbation.

Now we assume the validity of the so called quasistatic approximation (see e.g. [11, 24]), which means we consider only subhorizon scales $k^2 \gg \mathcal{H}^2$ and also that the perturbation fields vary slowly enough with time so that $\Phi, \Phi', \Psi, \Psi', \varphi, \varphi'$ are all negligible with respect to $\delta\rho$ unless multiplied by k^2 or by the effective scalar field mass $M^2 \equiv V_{,\phi\phi}$. In this approximation the previous expression becomes

$$2\phi\frac{k^2}{a^2}\Phi + \frac{k^2}{a^2}\varphi - 8\pi G\delta\rho = 0. \tag{11}$$

Perturbing the scalar field e.o.m. we obtain

$$\frac{1}{a^2}[2k^2\Psi + 4k^2\Phi - 12(\mathcal{H}' + \mathcal{H}^2)\Psi] + 6\frac{1}{a^2}\Phi'' - 6\frac{\mathcal{H}}{a^2}(\Psi' - 3\Phi') - 2\frac{\omega_{BD}}{\phi^2}\varphi\nabla^0\nabla_0\phi + 2\frac{\omega_{BD}}{\phi}\nabla^i\nabla_i\varphi + 2\frac{\omega_{BD}}{\phi}\nabla^0\nabla_0\varphi - \frac{2}{a^2}\frac{\omega_{BD}}{\phi^3}\varphi\phi'^2 + \frac{2}{a^2}\frac{\omega_{BD}}{\phi^2}\varphi'\phi' - V_{,\phi\phi}\varphi = 0. \tag{12}$$

In the quasistatic limit this becomes:

$$4\frac{k^2}{a^2}\Phi + 2\frac{k^2}{a^2}\Psi - (2\frac{\omega_{BD}}{\phi}\frac{k^2}{a^2} + M^2)\varphi = 0. \tag{13}$$

Finally we use the traceless part of the gravitational e.o.m. to derive one more perturbation equation:

$$\phi G^0_0 - 3\nabla^0\nabla_0\phi - 2\nabla^i\nabla_i\phi + \phi R - \frac{1}{2}\frac{\omega_{BD}}{\phi}\nabla^i\phi\nabla_i\phi - \frac{3}{2}\frac{\omega_{BD}}{\varphi}\nabla^0\phi\nabla_0\phi - \frac{3}{2}V(\phi) = 8\pi G(T^0_0 - T). \tag{14}$$

Recalling that $\delta T^0_0 - \delta T = -\delta\rho - (3c_s^2 - 1)\delta\rho = -3c_s^2\delta\rho$, where $c_s^2 = \delta p/\delta\rho$ is the sound speed, we get:

$$-3\varphi\frac{\mathcal{H}^2}{a^2} + \phi[-6\frac{\mathcal{H}}{a^2}\Phi' + 6\frac{\mathcal{H}^2}{a^2}\Psi - \frac{2}{a^2}k^2\Phi] + \frac{3}{a^2}\varphi'' + 2\frac{k^2}{a^2}\varphi + 6\varphi[\frac{\mathcal{H}^2}{a^2} + \frac{\mathcal{H}'}{a^2}] - \frac{1}{a^2}\phi[-2k^2\Psi - 4k^2\Phi + 12(\mathcal{H}' + \mathcal{H}^2)\Psi] + 6\frac{1}{a^2}\phi\Phi'' - 6\frac{\mathcal{H}}{a^2}\phi(\Psi' - 3\Phi') + \frac{3}{2}\frac{\omega_{BD}}{\phi^2}\varphi\nabla^0\phi\nabla_0\phi - 3\frac{\omega_{BD}}{\phi}\nabla^0\varphi\nabla_0\phi - \frac{3}{2}V_{,\phi}\varphi = -24\pi Gc_s^2\delta\rho. \tag{15}$$

In the quasi-static limit this reduces to

$$2\phi\frac{k^2}{a^2}\Phi + 2\frac{k^2}{a^2}\varphi + 2\phi\frac{k^2}{a^2}\Psi = -24\pi Gc_s^2\delta\rho. \tag{16}$$

Collecting all the above-derived perturbation equations together we have the following system of equations:

$$2\phi\frac{k^2}{a^2}\Phi + 2\frac{k^2}{a^2}\varphi + 2\phi\frac{k^2}{a^2}\Psi = -24\pi Gc_s^2\delta\rho, \tag{17}$$

$$4\frac{k^2}{a^2}\Phi + 2\frac{k^2}{a^2}\Psi - (2\frac{\omega_{BD}}{\phi}\frac{k^2}{a^2} + M^2)\varphi = 0, \tag{18}$$

$$2\phi\frac{k^2}{a^2}\Phi + \frac{k^2}{a^2}\delta\varphi = 8\pi G\delta\rho. \tag{19}$$

In case of $c_s^2 = 0$, these expressions agree with [7]. Solving the system and replacing $\delta\rho$ with $\delta \equiv \delta\rho/\rho = 8\pi G(\delta\rho)/3\Omega_m H^2$ we obtain:

$$k^2\Psi = -\frac{3\Omega_m \mathcal{H}^2\delta}{2}Y, \tag{20}$$

where the modified gravity parameter Y is defined to be

$$Y \equiv \frac{1}{\phi}\frac{(1+3c_s^2)M^2\phi + \frac{k^2}{a^2}(4+2\omega_{BD}+6c_s^2(1+\omega_{BD}))}{M^2\phi + \frac{k^2}{a^2}(3+2\omega_{BD})}. \tag{21}$$

Since local gravity measurement constrain the Brans-Dicke coupling parameter to be very large [25], we can expand Y in powers of $1/\omega_{BD}$, which at the first non-trivial order gives

$$Y \backsimeq \frac{(1+3c_s^2)}{\phi} + \frac{1-3c_s^2}{2\omega_{BD}\phi}. \tag{22}$$

We define the anisotropic stress as (see e.g. [12])

$$\eta \equiv -\frac{\Phi}{\Psi} = \frac{M^2\phi + \frac{k^2}{a^2}(2+3c_s^2+2\omega_{BD})}{(1+3c_s^2)M^2\phi + \frac{k^2}{a^2}(4+2\omega_{BD}+6c_s^2(1+\omega_{BD}))}. \tag{23}$$

If we set $M^2 = \omega_{BD} = c_s^2 = 0$, we get $Y = 4/3\phi$, $\eta = 1/2$, which, as expected, are the same values as in a standard $f(R)$ model in the limit of small mass M provided that $\phi = df(R)/dR$. The first order expansion of η in $1/\omega_{BD}$ is

$$\eta \simeq \frac{1}{1+3c_s^2} + \frac{(3c_s^2-1)(3c_s^2+2)}{2\left(3c_s^2+1\right)^2\omega_{BD}}. \tag{24}$$

Notice that for $c_s^2 = 1/3$, i.e. for a relativistic fluid, both Y and η are independent of ω_{BD}. This reflects the fact that under a conformal rescaling the Brans-Dicke theory can be recast into a ordinary gravity model with field-matter coupling proportional to the trace of the energy momentum; when this vanishes, as for a relativistic fluid, the coupling vanishes as well.

III. TWO-FIELD BRANS-DICKE THEORY

We move now to the case in which the Lagrangian includes two fields, both coupled to gravity. We adopt then the following two-field Brans-Dicke Action

$$S_2 = \int d^4x\sqrt{-g}[\phi_1 R - \frac{\omega_1}{\phi_1}g^{\mu\nu}\partial_\mu\phi_1\partial_\nu\phi_1 + \phi_2 R - \frac{\omega_2}{\phi_2}g^{\mu\nu}\partial_\mu\phi_2\partial_\nu\phi_2 - V(\phi_1,\phi_2)] + \int d^4x\sqrt{-g}\mathcal{L}_{matter}. \tag{25}$$

The gravitational e.o.m. of this theory is

$$(\phi_1+\phi_2)G_{\mu\nu} + [\Box(\phi_1+\phi_2) + \frac{1}{2}\frac{\omega_1}{\phi_1}(\nabla\phi_1)^2 + \frac{1}{2}\frac{\omega_2}{\phi_2}(\nabla\phi_2)^2]g_{\mu\nu} - \nabla_\mu\nabla_\nu(\phi_1+\phi_2) - \frac{\omega_1}{\phi_1}\nabla_\mu\phi_1\nabla_\nu\phi_1 - \frac{\omega_2}{\phi_2}\nabla_\mu\phi_2\nabla_\nu\phi_2 + \frac{V(\phi_1,\phi_2)}{2}g_{\mu\nu} = 8\pi G T_{\mu\nu}. \tag{26}$$

Taking the trace we obtain

$$-(\phi_1+\phi_2)R + 3\Box(\phi_1+\phi_2) + \frac{\omega_1}{\phi_1}(\nabla\phi_1)^2 + \frac{\omega_2}{\phi_2}(\nabla\phi_2)^2 + 2V(\phi_1,\phi_2) = 8\pi G T. \tag{27}$$

The ϕ_1 and ϕ_2 e.o.m. are

$$R + 2\frac{\omega_i}{\phi_i}\Box\phi_i - \frac{\omega_i}{\phi_i^2}\partial^\alpha\phi_i\partial_\alpha\phi_i - V_{,\phi_i} = 0,\ i = 1, 2. \tag{28}$$

Summing Eqs.(27),(28) together we obtain:

$$(3 + 2\omega_1)\Box\phi_1 + (3 + 2\omega_2)\Box\phi_2 + 2V(\phi_1, \phi_2) - V_{,\phi_1} - V_{,\phi_2} = 8\pi GT. \tag{29}$$

Repeating the arguments after Eq.(8) we conclude that this theory reduces to standard general relativity in the large-ω_1, ω_2 limit.

Proceeding as in the previous sectiom, the perturbed (0,0) component of Eq.(26) in the quasi-static approximation leads to the corresponding perturbation equation:

$$2(\phi_1 + \phi_2)\frac{k^2}{a^2}\Phi + \frac{k^2}{a^2}(\varphi_1 + \varphi_2) - 8\pi G\delta\rho = 0. \tag{30}$$

Similarly, from the scalar field e.o.m. Eqs.(28) we derive the corresponding perturbation equations:

$$4\frac{k^2}{a^2}\Phi + 2\frac{k^2}{a^2}\Psi - [2\frac{\omega_i}{\phi_i}\frac{k^2}{a^2} + M_i^2]\varphi_i = 0,\ i = 1, 2 \tag{31}$$

where $M_i^2 \equiv V_{,\phi_i\phi_i}$, $i = 1, 2$. Perturbing also the (0,0) component of the traceless part of the gravitational e.o.m. in the quasistatic limit, we obtain the following system of perturbation equations:

$$2(\phi_1 + \phi_2)\frac{k^2}{a^2}\Phi + 2\frac{k^2}{a^2}(\varphi_1 + \varphi_2) + 2(\phi_1 + \phi_2)\frac{k^2}{a^2}\Psi = -24\pi G c_s^2\delta\rho, \tag{32}$$

$$4\frac{k^2}{a^2}\Phi + 2\frac{k^2}{a^2}\Psi - [2\frac{\omega_i}{\phi_i}\frac{k^2}{a^2} + M_i^2]\varphi_i = 0,\ i = 1, 2 \tag{33}$$

$$2(\phi_1 + \phi_2)\frac{k^2}{a^2}\Phi + \frac{k^2}{a^2}(\varphi_1 + \varphi_2) = 8\pi G\delta\rho. \tag{34}$$

Solving this system and replacing $\delta\rho$ with $\delta \equiv \delta\rho/\rho = 8\pi G(\delta\rho)/3\Omega_m H^2$ as before, we obtain finally

$$k^2\Psi = -\frac{3\Omega_m\mathcal{H}^2\delta}{2}Y, \tag{35}$$

where

$$Y \equiv A_1\frac{1 + A_2k^2 + A_3k^4}{1 + A_4k^2 + A_5k^4}, \tag{36}$$

with the coefficients being

$$A_1 = \frac{1 + 3c_s^2}{\phi_1 + \phi_2}, \tag{37}$$

$$A_2 = \frac{2}{(3c_s^2 + 1)(\phi_1 + \phi_2)a^2}[\frac{(1 + 3c_s^2)(\phi_1 + \phi_2)\omega_1 + \left(2 + 3c_s^2\right)\phi_1}{M_1^2\phi_1} + \frac{\left(1 + 3c_s^2\right)(\phi_1 + \phi_2)\omega_2 + \left(2 + 3c_s^2\right)\phi_2}{M_2^2\phi_2}], \tag{38}$$

$$A_3 = \frac{4(2 + 3c_s^2)(\omega_2\phi_1 + \omega_1\phi_2) + 4\omega_1\omega_2(\phi_1 + \phi_2)(1 + 3c_s^2)}{(1 + 3c_s^2)M_1^2M_2^2\phi_1\phi_2(\phi_1 + \phi_2)a^4}, \tag{39}$$

$$A_4 = \frac{1}{(\phi_1 + \phi_2)a^2}\left[\frac{2\omega_1(\phi_1 + \phi_2) + 3\phi_1}{M_1^2\phi_1} + \frac{2\omega_2(\phi_1 + \phi_2) + 3\phi_2}{M_2^2\phi_2}\right], \tag{40}$$

$$A_5 = \frac{4\omega_1\omega_2(\phi_1 + \phi_2) + 6(\phi_1\omega_2 + \phi_2\omega_1)}{M_1^2M_2^2\phi_1\phi_2(\phi_1 + \phi_2)a^4}. \tag{41}$$

Considering the limit of ω_1 and ω_2 both much larger than unity, we find for the effective gravitational constant the result

$$Y \backsimeq \frac{3c_s^2+1}{\phi_1+\phi_2} + \frac{(-3c_s^2+1)(\omega_1\phi_2+\omega_2\phi_1)}{2\omega_1\omega_2(\phi_1+\phi_2)^2}. \tag{42}$$

Similarly, the anisotropic stress is

$$\eta \equiv -\frac{\Phi}{\Psi} = B_1\frac{1+B_2k^2+B_3k^4}{1+B_4k^2+B_5k^4}, \tag{43}$$

with the coefficients defined as

$$B_1 = \frac{1}{(3c_s^2+1)}, \tag{44}$$

$$B_2 = \frac{1}{(\phi_1+\phi_2)a^2}\left[\frac{\left(\left(3c_s^2+2\right)\phi_2+2\omega_2(\phi_1+\phi_2)\right)}{M_2^2\phi_2} + \frac{\left(\phi_1\left(3c_s^2+2\right)+2\omega_1(\phi_1+\phi_2)\right)}{M_1^2\phi_1}\right], \tag{45}$$

$$B_3 = \frac{2\left(2+3c_s^2\right)(\omega_1\phi_2+\omega_2\phi_1)+4\omega_1\omega_2(\phi_1+\phi_2)}{M_1^2M_2^2\phi_1\phi_2(\phi_1+\phi_2)a^4}, \tag{46}$$

$$B_4 = A_2 = \frac{2}{(3c_s^2+1)(\phi_1+\phi_2)a^2}\left[\frac{(1+3c_s^2)(\phi_1+\phi_2)\omega_1+\left(2+3c_s^2\right)\phi_1}{M_1^2\phi_1} + \frac{\left(1+3c_s^2\right)(\phi_1+\phi_2)\omega_2+\left(2+3c_s^2\right)\phi_2}{M_2^2\phi_2}\right], \tag{47}$$

$$B_5 = A_3 = \frac{4(2+3c_s^2)(\omega_2\phi_1+\omega_1\phi_2)+4\omega_1\omega_2(\phi_1+\phi_2)(1+3c_s^2)}{(1+3c_s^2)\,M_1^2M_2^2\phi_1\phi_2(\phi_1+\phi_2)a^4}. \tag{48}$$

As expected, we find again that Y and η are independent of $\omega_{1,2}$ if $c_s^2 = 1/3$. Notice also that since $A_2 = B_4$ and $A_3 = B_5$, the lensing potential $\Psi - \Phi$ obeys the equation

$$k^2(\Psi-\Phi) = -\frac{3}{2}Y(1+\eta)\Omega_m\mathcal{H}^2\delta, \tag{49}$$

in which the combination $Y(1+\eta)$ has in general the same k-structure as Y and η. However, for the particular case here studied, i.e. the simple Brans-Dicke model, the null geodesics must remain the same as in General Relativity and therefore $Y(1+\eta)$ should be independent of k. In fact we find $Y(1+\eta) = (2+3c_s^2)/(\phi_1+\phi_2)$. This shows that from weak lensing alone it would be impossible to detect the presence of several Brans-Dicke fields.

In the large ω_1 and ω_2 limit we obtain for η

$$\eta \backsimeq \frac{1}{3c_s^2+1} + \frac{(3c_s^2-1)(3c_s^2+2)(\omega_1\phi_2+\omega_2\phi_1)}{2\left(3c_s^2+1\right)^2\omega_1\omega_2(\phi_1+\phi_2)}. \tag{50}$$

It is also interesting to compare our two-field results with the standard single-field ones in the limit when one of the Brans-Dicke parameters is very large, i.e. when the corresponding field effectively decouples from gravity. Keeping only the zeroth order term in such an expansion and setting ϕ_2 to zero we have:

$$Y \backsimeq \frac{\left(1+3c_s^2\right)M_1^2\phi_1+2\frac{k^2}{a^2}\left(3c_s^2(1+\omega_1)+\omega_1+2\right)}{\phi_1\left(M_1^2\phi_1+\frac{k^2}{a^2}(2\omega_1+3)\right)}, \tag{51}$$

$$\eta \backsimeq \frac{M_1^2\phi_1+\frac{k^2}{a^2}\left(2+3c_s^2+2\omega_1\right)}{(1+3c_s^2)\,M_1^2\phi_1+2\frac{k^2}{a^2}\left(3c_s^2(\omega_1+1)+\omega_1+2\right)}. \tag{52}$$

As we see they identically coincide with Eqs.(21) and (23). In this limit, therefore, the two-field Lagrangian effectively reduces to a single-field one.

IV. MULTI-FIELD BRANS-DICKE THEORY

Finally, we can generalize the previous steps to the N-field Brans-Dicke. By analyzing the previous perturbation equations we can immediately write down the N-field quasi-static perturbation equations as follows:

$$2\sum_{i=1}^{N}\phi_i\frac{k^2}{a^2}\Phi+2\frac{k^2}{a^2}\sum_{i=1}^{N}\varphi_i+2\sum_{i=1}^{N}\phi_i\frac{k^2}{a^2}\Psi=-24\pi Gc_s^2\delta\rho, \tag{53}$$

$$4\frac{k^2}{a^2}\Phi+2\frac{k^2}{a^2}\Psi-[2\frac{\omega_i}{\phi_i}\frac{k^2}{a^2}+M_i^2]\varphi_i=0,\ i=1,2,...,N, \tag{54}$$

$$2\sum_{i=1}^{N}\phi_i\frac{k^2}{a^2}\Phi+\frac{k^2}{a^2}\sum_{i=1}^{N}\varphi_i=8\pi G\delta\rho. \tag{55}$$

From Eqs.(54) we have

$$\sum_{i=1}^{N}\varphi_i=2\frac{k^2}{a^2}(2\Phi+\Psi)\sum_{i=1}^{N}\frac{1}{2\frac{\omega_i}{\phi_i}\frac{k^2}{a^2}+M_i^2}. \tag{56}$$

Therefore Eqs.(53) and (55) read respectively

$$2\sum_{i=1}^{N}\phi_i\frac{k^2}{a^2}\Phi+4\frac{k^4}{a^4}(2\Phi+\Psi)\sum_{i=1}^{N}\frac{1}{2\frac{\omega_i}{\phi_i}\frac{k^2}{a^2}+M_i^2}+2\sum_{i=1}^{N}\phi_i\frac{k^2}{a^2}\Psi=-24\pi Gc_s^2\delta\rho, \tag{57}$$

$$2\sum_{i=1}^{N}\phi_i\frac{k^2}{a^2}\Phi+2\frac{k^4}{a^4}(2\Phi+\Psi)\sum_{i=1}^{N}\frac{1}{2\frac{\omega_i}{\phi_i}\frac{k^2}{a^2}+M_i^2}=8\pi G\delta\rho. \tag{58}$$

Solving this system one finds

$$\eta=\frac{\sum_{i=1}^{N}\phi_i+\left(2+3c_s^2\right)\frac{k^2}{a^2}\sum_{i=1}^{N}\frac{1}{2\frac{\omega_i}{\phi_i}\frac{k^2}{a^2}+M_i^2}}{\left(1+3c_s^2\right)\sum_{i=1}^{N}\phi_i+2\left(2+3c_s^2\right)\frac{k^2}{a^2}\sum_{i=1}^{N}\frac{1}{2\frac{\omega_i}{\phi_i}\frac{k^2}{a^2}+M_i^2}}, \tag{59}$$

$$Y=\frac{\left(1+3c_s^2\right)\sum_{i=1}^{N}\phi_i+2\left(2+3c_s^2\right)\frac{k^2}{a^2}\sum_{i=1}^{N}\frac{1}{2\frac{\omega_i}{\phi_i}\frac{k^2}{a^2}+M_i^2}}{(\sum_{i=1}^{N}\phi_i)^2+3(\sum_{i=1}^{N}\phi_i)\sum_{i=1}^{N}\frac{1}{2\frac{\omega_i}{\phi_i}\frac{k^2}{a^2}+M_i^2}\frac{k^2}{a^2}}. \tag{60}$$

The expression for η can now be written as

$$\eta=C_0\frac{1+\sum_{i=1}^{N}C_ik^{2i}}{1+\sum_{i=1}^{N}D_ik^{2i}}, \tag{61}$$

by explicitly specifying the coefficients

$$C_0=\frac{1}{1+3c_s^2}, \tag{62}$$

and

$$C_d = \frac{1}{a^{2d}C_\star}[(\sum_{i=1}^{N}\phi_i)2^d \sum_{i_1>i_2>...>i_d} \frac{\omega_{i_1}}{\phi_{i_1}}\frac{\omega_{i_2}}{\phi_{i_2}}...\frac{\omega_{i_d}}{\phi_{i_d}} \prod_{j\neq i_1,i_2,...,i_d} M_j^2 +$$

$$\left(2+3c_s^2\right)2^{d-1}\sum_{i=1}^{N} \sum_{\substack{i_j\neq i,\\ j=1,...,d-1\\ i_1>i_2>...>i_{d-1}}} \frac{\omega_{i_1}}{\phi_{i_1}}\frac{\omega_{i_2}}{\phi_{i_2}}...\frac{\omega_{i_{d-1}}}{\phi_{i_{d-1}}} \prod_{j\neq i,i_1,i_2,...,i_{d-1}} M_j^2], \tag{63}$$

$$D_d = \frac{1}{a^{2d}D_\star}[\left(1+3c_s^2\right)(\sum_{i=1}^{N}\phi_i)2^d \sum_{i_1>i_2>...>i_d} \frac{\omega_{i_1}}{\phi_{i_1}}\frac{\omega_{i_2}}{\phi_{i_2}}...\frac{\omega_{i_d}}{\phi_{i_d}} \prod_{j\neq i_1,i_2,...,i_d} M_j^2 +$$

$$2\left(2+3c_s^2\right)2^{d-1}\sum_{i=1}^{N} \sum_{\substack{i_j\neq i,\\ j=1,...,d-1\\ i_1>i_2>...>i_{d-1}}} \frac{\omega_{i_1}}{\phi_{i_1}}\frac{\omega_{i_2}}{\phi_{i_2}}...\frac{\omega_{i_{d-1}}}{\phi_{i_{d-1}}} \prod_{j\neq i,i_1,i_2,...,i_{d-1}} M_j^2], \tag{64}$$

with $d = 1, 2, ..., N$. Here and in the following expressions by the second summation sign in the second line of Eq.(63) we mean multiple sum over the indices $i_j,\ j = 1, ..., d-1$, where each of i_j runs from 1 to N omitting the value i. In the case of $d = 1$ it is meant that there is no summation at all. The products of M_j^2 are over index j running from 1 to N omitting the corresponding indices mentioned under the product signs. $C_\star$ and $D_\star$ are given by:

$$C_\star = (\sum_{i=1}^{N}\phi_i)\prod_{j=1}^{N} M_j^2, \tag{65}$$

$$D_\star = \left(1+3c_s^2\right)(\sum_{i=1}^{N}\phi_i)\prod_{j=1}^{N} M_j^2. \tag{66}$$

Similarly, we find

$$Y = \bar{C}_0 \frac{1+\sum_{i=1}^{N}\bar{C}_i k^{2i}}{1+\sum_{i=1}^{N}\bar{D}_i k^{2i}}, \tag{67}$$

where

$$\bar{C}_0 = \frac{1+3c_s^2}{\sum_{i=1}^{N}\phi_i}, \tag{68}$$

and

$$\bar{C}_d = \frac{1}{a^{2d}\bar{C}_\star}[\left(1+3c_s^2\right)(\sum_{i=1}^{N}\phi_i)2^d \sum_{i_1>i_2>...>i_d} \frac{\omega_{i_1}}{\phi_{i_1}}\frac{\omega_{i_2}}{\phi_{i_2}}...\frac{\omega_{i_d}}{\phi_{i_d}} \prod_{j\neq i_1,i_2,...,i_d} M_j^2 +$$

$$2\left(2+3c_s^2\right)2^{d-1}\sum_{i=1}^{N} \sum_{\substack{i_j\neq i,\\ j=1,...,d-1\\ i_1>i_2>...>i_{d-1}}} \frac{\omega_{i_1}}{\phi_{i_1}}\frac{\omega_{i_2}}{\phi_{i_2}}...\frac{\omega_{i_{d-1}}}{\phi_{i_{d-1}}} \prod_{j\neq i,i_1,i_2,...,i_{d-1}} M_j^2], \tag{69}$$

$$\bar{D}_d = \frac{1}{a^{2d}\bar{D}_\star}[(\sum_{i=1}^{N}\phi_i)^2 2^d \sum_{i_1>i_2>...>i_d} \frac{\omega_{i_1}}{\phi_{i_1}}\frac{\omega_{i_2}}{\phi_{i_2}}...\frac{\omega_{i_d}}{\phi_{i_d}} \prod_{j\neq i_1,i_2,...,i_d} M_j^2 +$$

$$3(\sum_{i=1}^{N}\phi_i)2^{d-1}\sum_{i=1}^{N} \sum_{\substack{i_j\neq i,\\ j=1,...,d-1\\ i_1>i_2>...>i_{d-1}}} \frac{\omega_{i_1}}{\phi_{i_1}}\frac{\omega_{i_2}}{\phi_{i_2}}...\frac{\omega_{i_{d-1}}}{\phi_{i_{d-1}}} \prod_{j\neq i,i_1,i_2,...,i_{d-1}} M_j^2], \tag{70}$$

with $d = 1, 2, ..., N$. $\bar{C}_\star$ and $\bar{D}_\star$ are given by:

$$\bar{C}_\star = \left(1 + 3c_s^2\right) \left(\sum_{i=1}^{N} \phi_i\right) \prod_{j=1}^{N} M_j^2, \tag{71}$$

$$\bar{D}_\star = \left(\sum_{i=1}^{N} \phi_i\right)^2 \prod_{j=1}^{N} M_j^2. \tag{72}$$

Note that $D_d = \bar{C}_d$ identically for $\forall d$ (see also [8]).

V. REDUCING THE POLYNOMIALS ORDER?

In this section we discuss under which circumstances the polynomials in η and Y effectively reduce to lower order ones at all times. If this were possible then the observations (at least at first order in the perturbations) would not be able to establish the number of active coupled dark energy fields, not even in principle.

From Eqs.(59) and (60) one can see that the order of the polynomials can be effectively reduced only in one of the following cases: $\underline{a}$) $c_s^2 = 1/3$, $\underline{b}$) at least one of the coupling constants is infinitely large, $\underline{c}$) one or more $M_i^2 \to \infty$, $\underline{d}$) one or more $\phi_i \to 0$, $\underline{e}$) there are i and j such that $\omega_{i,j} \to 0$, $\underline{f}$)) there are i and j such that $\frac{\omega_i}{\phi_i} = \alpha \frac{\omega_j}{\phi_j}$ and $M_i^2 = \alpha M_j^2$, where α is a non-zero real number. Note that $\alpha = -1$ is conceptually different from all the other values of α; in the former case two summands for each pair i, j disappear from the sum, whereas in the latter case the number of summands reduces only by one for each pair i, j. We will comment more on this difference below.

As we have already discussed before, in case of $c_s^2 = 1/3$, i.e. radiation, our theory is conformally equivalent to the standard general relativity and the fields (regardless of their number) do not have an observable impact except for a time-dependence of the effective gravitational constant.

The case $\underline{b}$) is the limit when the field with infinite (or very large) ω_i is completely decoupled from the Brans-Dicke theory (as an example of this reduction see the end of Section III). The case $\underline{c}$) implies that the field with infinite (very large) mass is static and just renormalizes the gravitational constant. Case $\underline{d}$) is unphysical because it leads to a singular Lagrangian. In the case $\underline{e}$) the kinetic terms of the i and j fields are absent from the Lagrangian and the $\phi_i R + \phi_j R$ term can be interpreted as a non-minimal coupling of a single field: $\psi_{ij} R = (\phi_i + \phi_j) R$.

Case $\underline{f}$) is non trivial and we need to check the consistency of this case with the field equations of motion. To do so we substitute the condition $\underline{f}$) in the ϕ_i equation of motion (28):

$$R^{(0)} + 2\alpha \frac{\omega_j}{\phi_j} \nabla^0 \nabla_0 \phi_i + 2\alpha \frac{\omega_j}{\phi_i \phi_j a^2} \phi_i'^2 - V_{,\phi_i}|_{\phi_i = \omega_i \phi_j / \alpha \omega_j} = 0, \tag{73}$$

where $R^{(0)}$ is the background Ricci scalar. In order for the above equation to be consistent with the ϕ_j e.o.m:

$$R^{(0)} + 2 \frac{\omega_j}{\phi_j} \nabla^0 \nabla_0 \phi_j + 2 \frac{\omega_j}{\phi_j^2 a^2} \phi_j'^2 - V_{,\phi_j}|_{\phi_i = \omega_i \phi_j / \alpha \omega_j} = 0, \tag{74}$$

we need that $\phi_j = \alpha \phi_i$ (leading to $\omega_i = \omega_j$) and $V_{,\phi_i}|_{\phi_i = \phi_j/\alpha} = V_{,\phi_j}|_{\phi_i = \phi_j/\alpha}$. This means that depending on whether $\alpha = -1$ or $\alpha \neq -1$, either these two fields decouple completely from the Lagrangian or we can collect the terms corresponding to ϕ_i and ϕ_j in a way that they effectively act as one field.

For instance, if we consider a potential without ϕ_i, ϕ_j interaction,

$$V(\phi_1, \phi_2, ..., \phi_N) = \sum_{k=1}^{D} (\lambda_k^{(i)} (\phi_i)^k + \lambda_k^{(j)} (\phi_j)^k) + V(\phi_1, ..., \hat{\phi}_i, ..., \hat{\phi}_j, ..., \phi_N), \tag{75}$$

where the coefficients λ_k are some constant numbers and hat represents exclusion. For the derivatives of this potential we have:

$$V_{,\phi_i}|_{\phi_i = \phi_j/\alpha} = \sum_{k=1}^{N} k \lambda_k^{(i)} \frac{(\phi_j)^{k-1}}{\alpha^{k-1}},$$

$$V_{,\phi_j}|_{\phi_i = \phi_j/\alpha} = \sum_{k=1}^{D} k \lambda_k^{(j)} (\phi_j)^{k-1},$$

where D is a natural number. In order for the two derivatives to be equal one needs $\lambda_k^{(i)} = \alpha^{k-1}\lambda_k^{(j)}$. It is easy to see that this condition is consistent with the proportionality of the field masses: $V_{,\phi_i\phi_i}|_{\phi_i=\phi_j/\alpha} = \alpha V_{,\phi_j\phi_j}|_{\phi_i=\phi_j/\alpha}$.

To exemplify the $\alpha = -1$ case we study the conditions for A_3, A_5 and B_3 given in Eqs.(39, 41, 46) to be zero. It is easy to see that they are consistently zero in one of the following cases: $c_s^2 = \frac{1}{3}$, $\omega_1 = \omega_2 = 0$, at least one of M_1^2 and M_2^2 is infinitely large, at least one of ϕ_1 and ϕ_2 is zero, or, finally, $\phi_1 = -\phi_2$ and $\omega_1 = \omega_2$. In the latter case, which can be associated with the $\alpha = -1$ situation above, both fields completely decouple from the Lagrangian. This sheds more light on the case $\underline{f})$ above; the cancellation of higher order terms in this case is possible if a pair of fields decouples completely from the Lagrangian ($\alpha = -1$), or if they are still active but in Lagrangian one can group together the terms corresponding to the considered pair of fields in such a way that they lead to terms resembling a singe field ($\alpha \neq -1$). It is important to note that even though the cancellation of higher order terms was shown only at linear level, the fact that it leads to a lower number of active fields in the Lagrangian means that this in fact is a non-perturbative effect.

This means that apart from the case $\underline{a})$ all the other cases lead to an effectively smaller number of active fields and therefore are trivial, in the sense that there exists a field redefinition that brings the Lagrangian into a function of a smaller number of fields. This shows that if dark energy is composed by N fields that are dynamically active and coupled to gravity, their presence can in principle be detected in the observables Y and η. Needless to say, the effective detectability of multiple dark energy fields depends on many other factors and is beyond the scope of this paper.

VI. CONCLUSIONS

We investigated multi-field scalar-tensor models of the Brans-Dicke type, in presence of a generic source characterized by an equation of state and a sound speed. Our aim was to find the modified gravity parameters that affect the Poisson equation and the anisotropic stress in the quasi-static limit. The interest in this approach lies in the fact that future large scale observations will probe the k-structure of modified gravity [15] and possibly detect whether dark energy is composed by one or several distinct components.

We have shown that multi-field Brans-Dicke models with N scalar fields are described by the anisotropic stress η and the effective gravitational constant Y as

$$\eta = h_2 \frac{P_n^{(1)}(k)}{P_n^{(2)}(k)}\,,\ Y = h_1 \frac{P_n^{(2)}(k)}{P_n^{(3)}(k)}\,. \tag{76}$$

where $P_n^{(1)}, P_n^{(2)}, P_n^{(3)}$ are even polynomials of order $n = 2N$ in k, with coefficients that depend on the evolution of background quantities and on the matter sound speed c_s. We also find that there are no realistic non-trivial cases in which the higher-order k terms cancel out. Each term in the three polynomials introduces a new characteristic time-dependent scale k_n, with $n = 2, 4, ..., 2N$, for total of $3N$ scales, which can in principle be individually measured and employed to constrain the number of fields N.

An interesting question that naturally arises is how well can future surveys like e.g. the Euclid satellite [26, 27], probe the existence of more than one scalar field and constrain the values of the Brans-Dicke parameters ω_i. This is left to future work.

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APPENDIX

Here we show that there always exist a single field potential $V(\phi)$ that reproduces any observed Hubble parameter as a function of redshift, $H(z)$. We assume the Universe contains matter with a known generic equation of state w_m

and a single canonical minimally coupled scalar field. The two independent Einstein equations are (we set $8\pi G = 1$)

$$3H^2 = \rho_m + \frac{\dot{\phi}^2}{2} + V(\phi) \tag{77}$$

$$2\dot{H} = -\dot{\phi}^2 - \rho_m(1 + w_m) \tag{78}$$

Adopting the redshift z as time coordinate, we can write from (78)

$$H(z)^2(1+z)^2\left(\frac{d\phi}{dz}\right)^2 = 2(1+z)H(z)\frac{dH}{dz} - \rho_m(z)(1 + w_m(z)) \tag{79}$$

where $\rho_m(z)$ is the solution of

$$(1+z)\frac{d\rho_m}{dz} = 3\rho_m(z)(1 + w_m(z)) \tag{80}$$

From Eq.(79) one can obtain $\phi(z)$. Then by combining (79) and (77) one obtains

$$V = 3H(z)^2 - (1+z)H(z)\frac{dH(z)}{dz} + \frac{\rho_m(z)}{2}(w_m(z) - 1) \tag{81}$$

which gives $V(z)$. Finally, by inverting $\phi(z)$, it is possible to reconstruct $V(\phi)$ for any observed $H(z)$. There is no guarantee however that the formal solution so obtained is stable or unique or free of singularities.

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