Compressed Sensing in Astronomy

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- **Introduction:**
  - ➡**Compressed Sensing (CS)**
  - ➡**Sparse representation: Morphological Diversity**
- **Compressed Sensing in Astronomy**
  - ➡**CS Data**
  - ➡**CS and Inpainting**
  - ➡**CS and the Herschel Satellite**

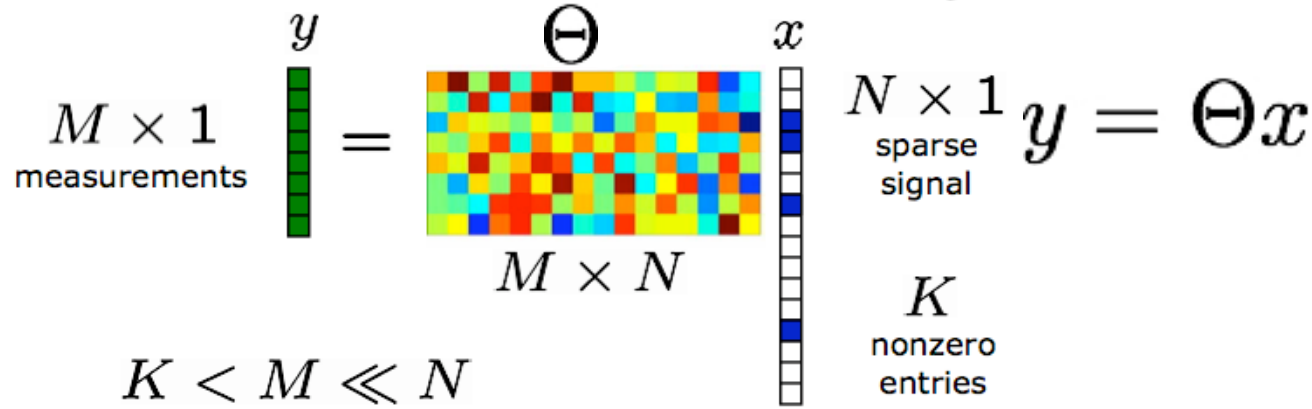
Compressed Sensing

* E. Candès and T. Tao, "Near Optimal Signal Recovery From Random Projections: Universal Encoding Strategies? ", IEEE Trans. on Information Theory, 52, pp 5406-5425, 2006.
* D. Donoho, "Compressed Sensing", IEEE Trans. on Information Theory, 52(4), pp. 1289-1306, April 2006.
* E. Candès, J. Romberg and T. Tao, "Robust Uncertainty Principles: Exact Signal Reconstruction from Highly Incomplete Frequency Information", IEEE Trans. on Information Theory, 52(2) pp. 489 - 509, Feb. 2006.

A non linear sampling theorem

"Signals with exactly K components different from zero can be recovered perfectly from ~ K log N incoherent measurements"

Replace samples with *few linear projections* $y = \Theta x$



Reconstruction via non linear processing: $\min_x \|x\|_1 \quad \text{s.t.} \quad y = \Theta x$

⇒Application: Compression, tomography, ill posed inverse problem.

Compressed Sensing Reconstruction

Measurements: $y_k = \left\langle x, \theta_k \right\rangle$

Reconstruction via non linear processing: $\min_x \|x\|_1 \quad \text{s.t.} \quad y = \Theta_\Lambda x$

In practice, x is sparse in a given dictionary: $x = \Phi\alpha$

and we need to solve: $\min_\alpha \|\alpha\|_1 \quad \text{s.t.} \quad y = \Theta_\Lambda \Phi\alpha$

The mutual incoherence is defined as $\mu_{\Theta,\Phi} = \max_{i,j} \left|\left\langle \theta_i, \phi_j \right\rangle\right|$

the number of required measurements is : $m \geq C.\mu^2_{\Theta,\Phi}.S.\log n$

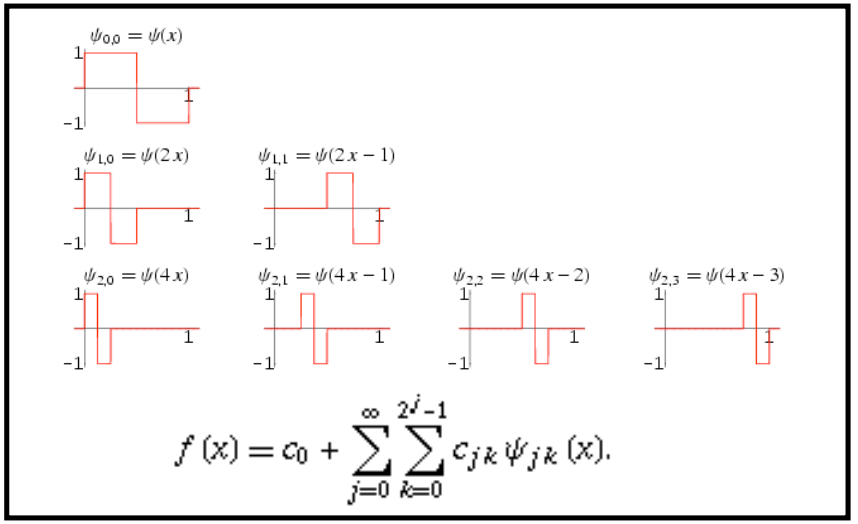
What is Sparsity ?

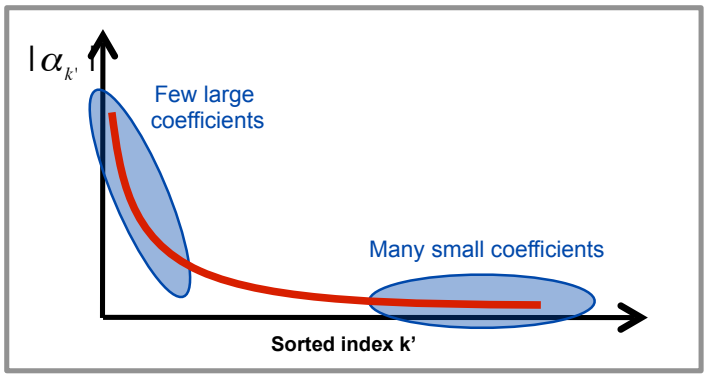
A signal *s* (*n* samples) can be represented as sum of weighted elements of a given dictionary

$$\Phi = \{\phi_1, \ldots, \phi_K\}$$

$$s = \sum_{k=1}^{K} \alpha_k \phi_k = \Phi\alpha$$

Ex: Haar wavelet





Multiscale Transforms

Critical Sampling

(bi-) Orthogonal WT
Lifting scheme construction
Wavelet Packets
Mirror Basis

Redundant Transforms

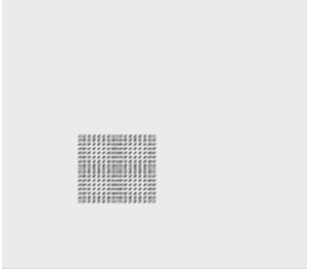
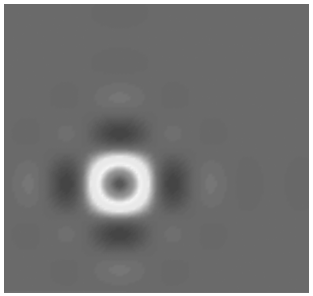
Pyramidal decomposition (Burt and Adelson)
Undecimated Wavelet Transform
Isotropic Undecimated Wavelet Transform
Complex Wavelet Transform
Steerable Wavelet Transform
Dyadic Wavelet Transform
Nonlinear Pyramidal decomposition (Median)

New Sparse Representations

Contourlet
Bandelet
Finite Ridgelet Transform
Platelet
(W-)Edgelet
Adaptive Wavelet
KSVD
Grouplet

Ridgelet
Curvelet (Several implementations)
Wave Atom

Looking for adapted representations

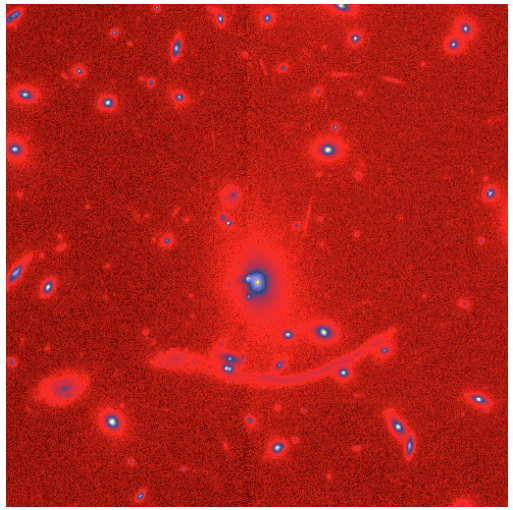
Local DCT	Stationary textures Locally oscillatory	
Wavelet transform	Piecewise smooth Isotropic structures	
Curvelet transform	Piecewise smooth, edge	

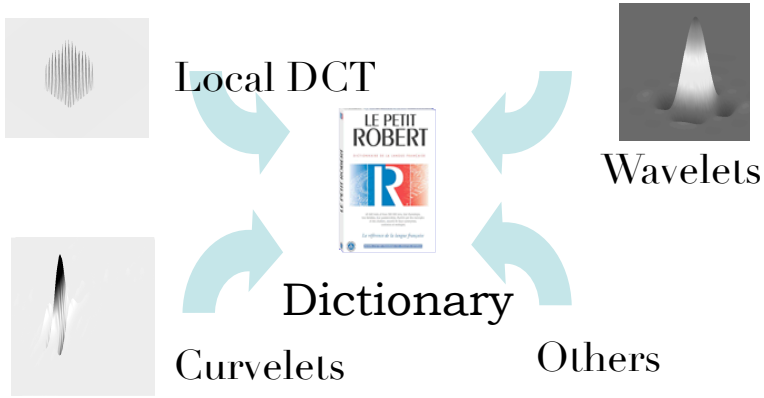
Morphological Diversity

•J.-L. Starck, M. Elad, and D.L. Donoho, ***Redundant Multiscale Transforms and their Application for Morphological Component Analysis***, *Advances in Imaging and Electron Physics, 132, 2004.*

•J.-L. Starck, M. Elad, and D.L. Donoho, ***Image Decomposition Via the Combination of Sparse Representation and a Variational Approach***, *IEEE Trans. on Image Proces., 14, 10, pp 1570--1582, 2005.*

•J.Bobin et al, ***Morphological Component Analysis: an adaptive thresholding strategy***, IEEE Trans. on Image Processing, Vol 16, No 11, pp 2675--2681, 2007.





$$\phi = [\phi_1, \ldots, \phi_L], \quad \alpha = \{\alpha_1, \ldots, \alpha_L\}, \quad s = \phi\alpha = \sum_{k=1}^{L} \phi_k \alpha_k$$

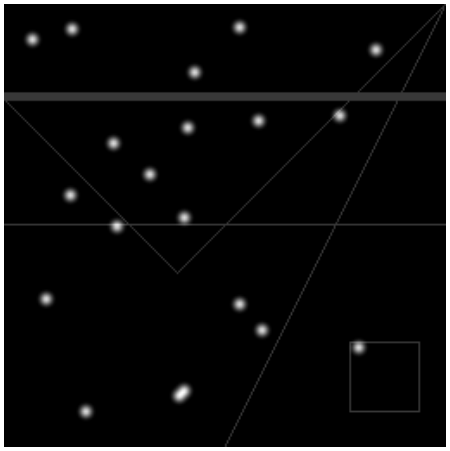
Model:

$$s = \sum_{k=1}^{L} s_k + n$$

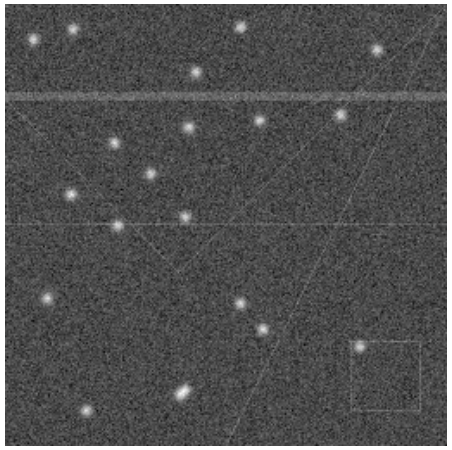
and s_k ($s_k = \phi_k \alpha_k$) is sparse in ϕ_k .

→ Algorithm MCA (Morphological Component Analysis)

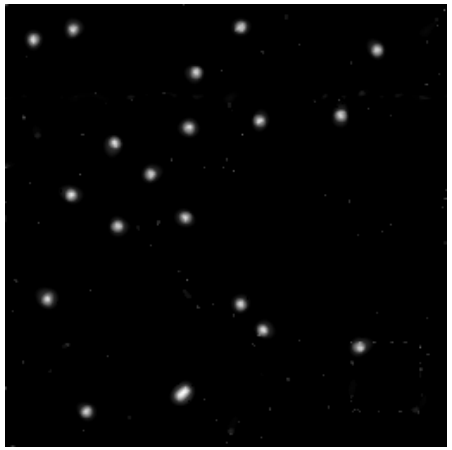
$$MIN_{s_1,s_2}\left(\left\|Ws_1\right\|_p + \left\|Cs_2\right\|_p\right) \text{ subject to } \left\|s-(s_1+s_2)\right\|_2^2 < \varepsilon$$



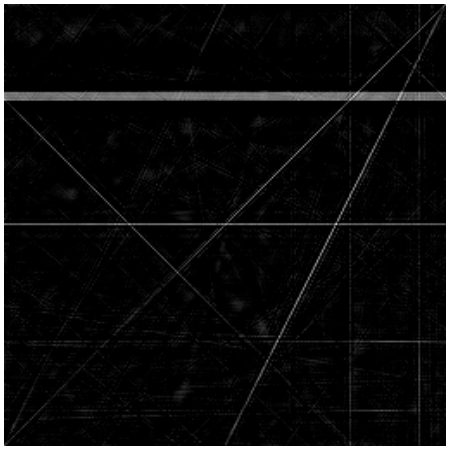
a) Simulated image (gaussians+lines)



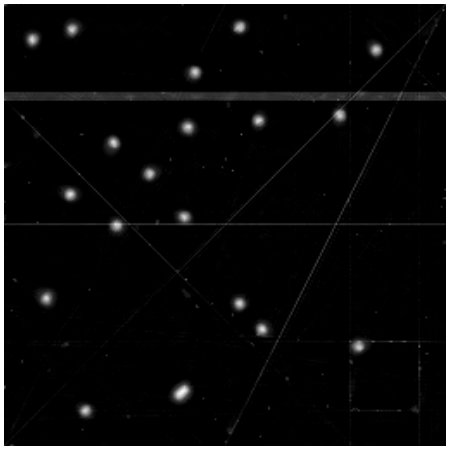
b) Simulated image + noise



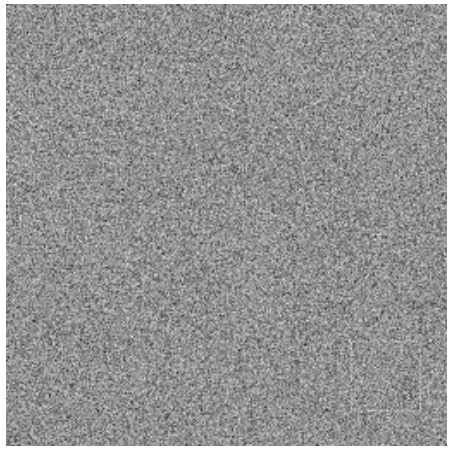
c) A trous algorithm



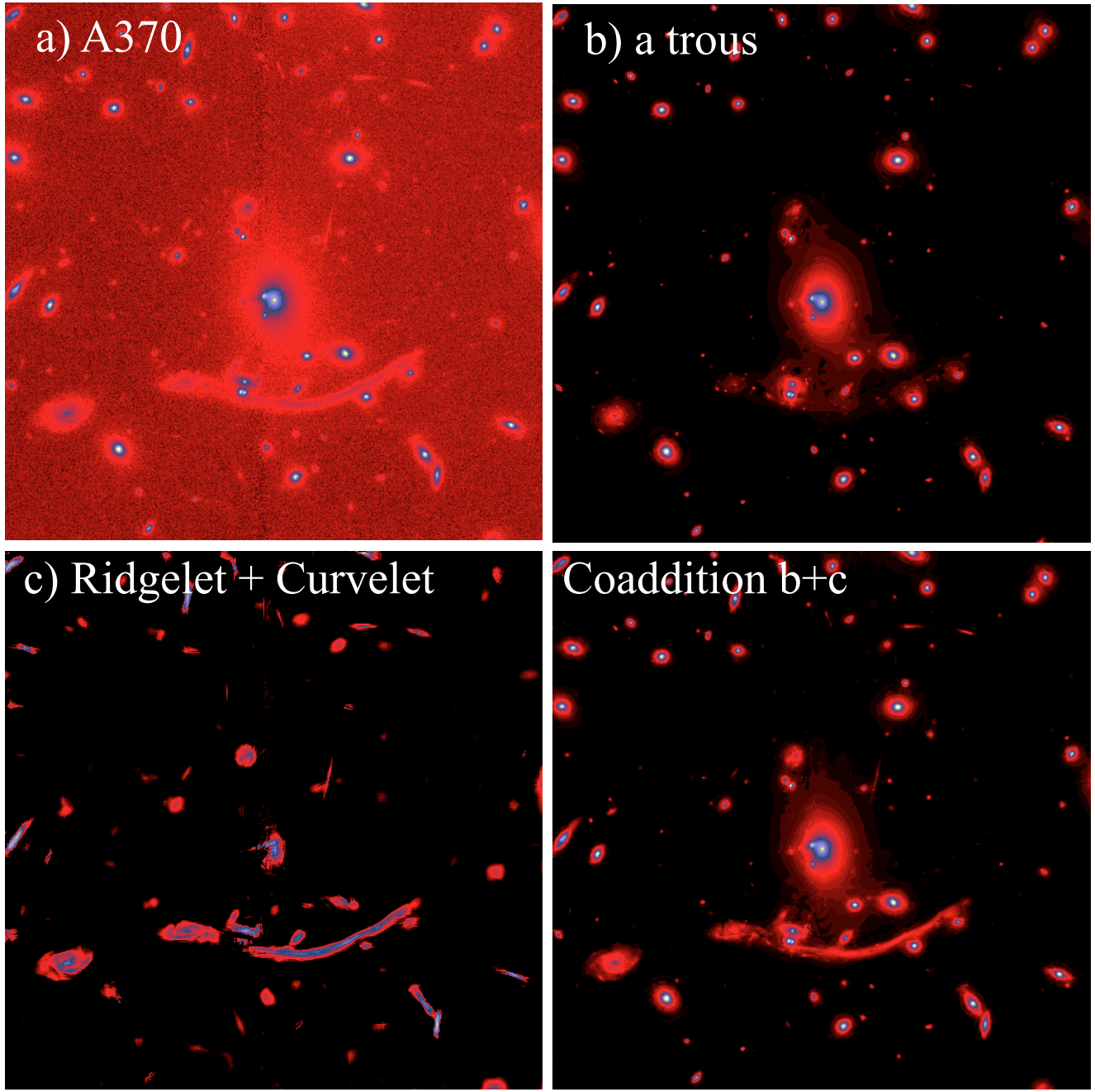
d) Curvelet transform



e) coaddition c+d



f) residual = e-b


a) A370
b) a trous
c) Ridgelet + Curvelet
Coaddition b+c

New Perspectives Based on Union of Overcomplete Dictionaries: *The Morphological Diversity*

Denoising and Deconvolution

Starck, Candes, Donoho, ***Very High Quality Image Restoration***, SPIE, Vol 4478, 2001.

Starck et al, ***Wavelets and Curvelets for Image Deconvolution: a Combined Approach***, *Signal Processing, 83, 10, pp 2279-2283, 2003.*

Morphological Component Analysis (MCA)

•Starck, Elad, Donoho, ***Redundant Multiscale Transforms and their Application for Morphological Component Analysis***, *Advances in Imaging and Electron Physics, 132, 2004.*

•Starck, Elad, Donoho, ***Image Decomposition Via the Combination of Sparse Representation and a Variational Approach***, *IEEE Trans. on Image Proces., 14, 10, pp 1570--1582, 2005.*

•*Bobin et al*, ***Morphological Component Analysis: an adaptive thresholding strategy***, *IEEE Trans. on Image Proces.*, Vol 16, No 11, pp 2675--2681, 2007.

MCA Inpainting

•Elad, Starck, Donoho, ***Simultaneous Cartoon and Texture Image Inpainting using Morphological Component Analysis (MCA),*** *ACHA, Vol. 19, pp. 340-358, 2005.*

•Fadili, Starck, Murtagh, ***Inpainting and Zooming using Sparse Representations***, The Computer Journal , *in press.*

MCAlab available at: http://www.greyc.ensicaen.fr/~jfadili

Extension to Multichannel/Hyperspectral Data

• Bobin et al, ***Morphological Diversity and Source Separation***", IEEE Trans. on Signal Processing , Vol 13, 7, pp 409--412, 2006.

• Bobin et al, ***Sparsity, Morphological Diversity and Blind Source Separation***" , IEEE Trans. on Image Processing , Vol 16, pp 2662 - 2674, 2007.

• Bobin et al, **Morphological Diversity and Sparsity for Multichannel Data Restoration**, J. of Mathematical Imaging and Vision, in press.

GMCAlab available at: http://perso.orange.fr/jbobin/gmcalab.html

Compressive Sensing Resources

http://www.dsp.ece.rice.edu/cs/

More than 200 related papers already!

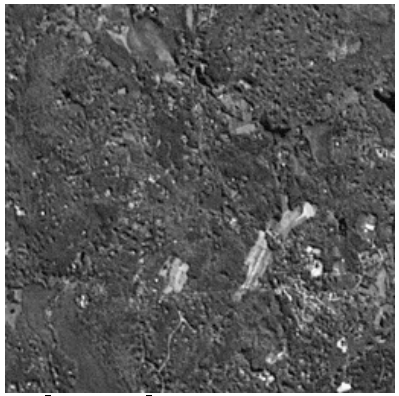
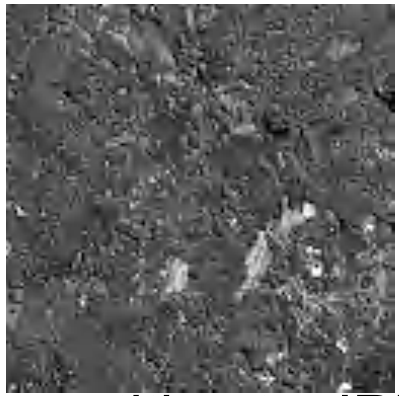
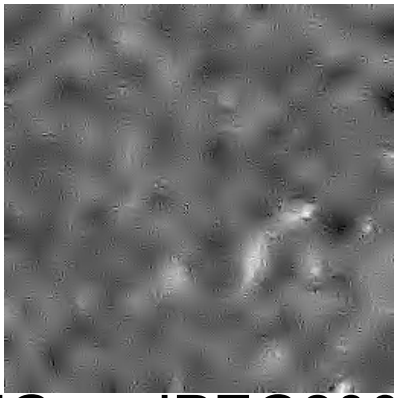
- Compressive Sensing
- Extensions of Compressive Sensing
- Multi-Sensor and Distributed Compressive Sensing
- Compressive Sensing Recovery Algorithms
- Foundations and Connections
- High-Dimensional Geometry
- Ell-1 Norm Minimization
- Statistical Signal Processing
- Machine Learning
- Bayesian Methods
- Finite Rate of Innovation
- Multi-band Signals
- Data Stream Algorithms
- Compressive Imaging
- Medical Imaging
- Analog-to-Information Conversion
- Biosensing
- Geophysical Data Analysis
- Hyperspectral Imaging
- Compressive Radar
- Astronomy
- Communications

+ software available

Compressed Sensing For Data Compression

Compressed Sensing presents several interesting properties for data compress:

- Compression is **very fast** ==> good for on-board applications.
- Very **robust** to bit loss during the transfer.
- **Decoupling** between compression/decompression.
- **Data protection**.
- **Linear Compression**.

But clearly not as competitive to JPEG or JPEG2000 to compress an image.

Compressed Sensing Impact in astronomy

Typical Astronomical Data related to CS

- (radio-) Interferometry: Θ = Fourier transform
  Φ = Id (or Wavelet transform)
- Period detection in temporal series
  Θ = Id
  Φ = Fourier transform
- Gamma Ray Instruments (Integral) - Acquisition with coded masks

CS gives another point of view on some existing methods

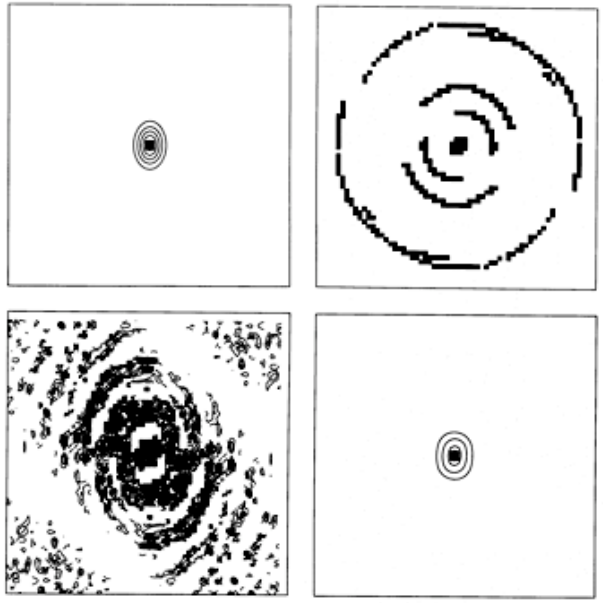
- Inpainting: Θ = Id $\quad M = \Theta_\Lambda$

$$\min_\alpha \|\alpha\|_1 \quad \text{s.t.} \quad y = M\Phi\alpha$$

New problems that can be addressed by CS

==> Data compression: the case of Herschel satellite

(Radio-) Interferometry

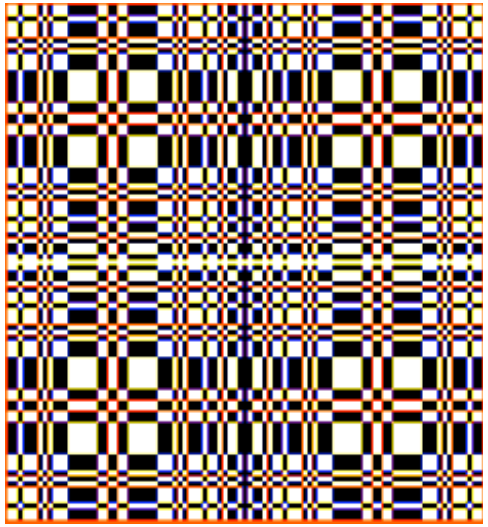


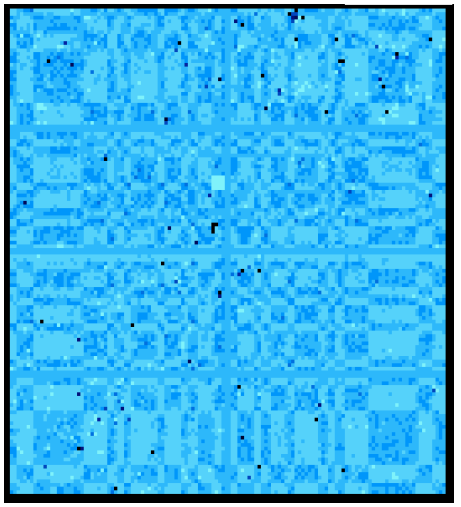
J.L. Starck, A. Bijaoui, B. Lopez, and C. Perrier, "Image Reconstruction by the Wavelet Transform Applied to Aperture Synthesis", Astronomy and Astrophysics, 283, 349--360, 1994.

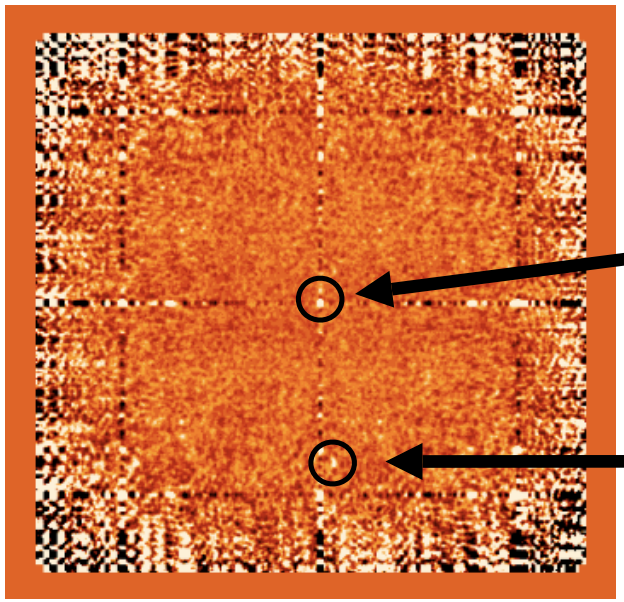
Wavelet - CLEAN minimizes well the l_0 norm

But recent l_0-l_1 minimization algorithms would be clearly much faster.

INTEGRAL/IBIS Coded Mask







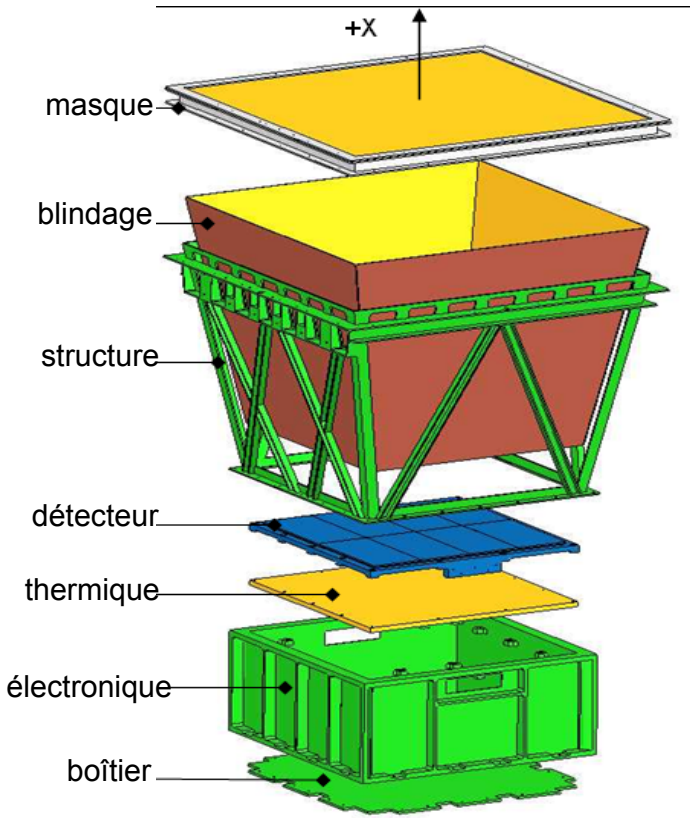
Excess 1
Position (SPSF fit)
Identification
→ Modelling

Excess 2
Position (SPSF fit)
Identification
→ Modelling

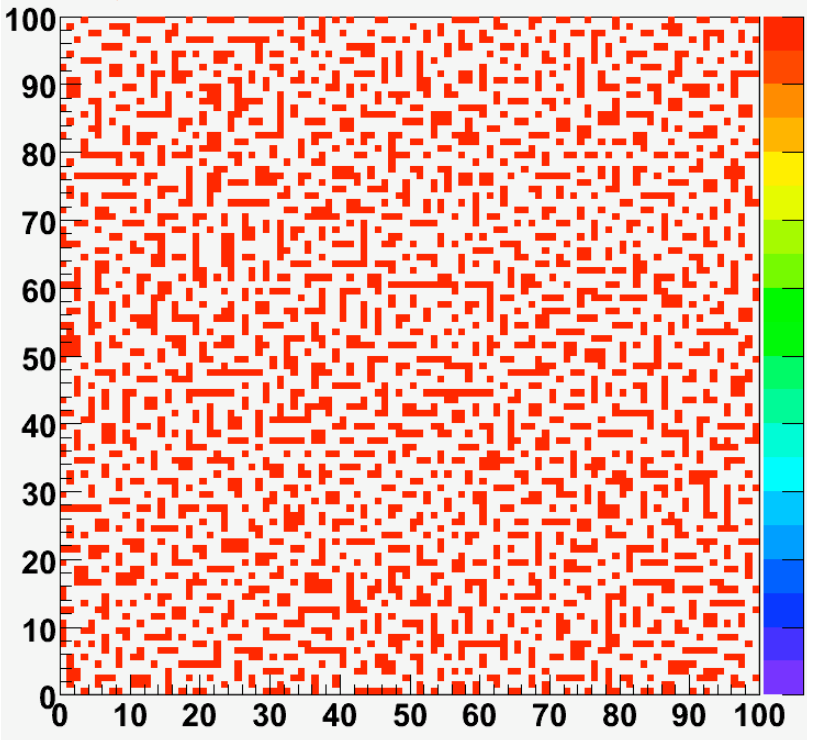
ECLAIRs

dapnia
saclay

- ***ECLAIRs france-chinese satellite 'SVOM' (launch in 2013)***
  Gamma-ray detection in energy range 4 - 120 keV
  Coded mask imaging (at 460 mm of the detector plane)
- *detector 1024 cm^2 of Cd Te (80 x 80 pixels)*
- *mask (100 x 100 pixels)*



blanc = opaque, rouge = transparent



Stéphane Schanne, CEA

Interpolation of Missing Data: Inpainting

•*M. Elad, J.-L. Starck, D.L. Donoho, P. Querre, "Simultaneous Cartoon and Texture Image Inpainting using Morphological Component Analysis (MCA)", ACHA, Vol. 19, pp. 340-358, 2005.*
•*M.J. Fadili, J.-L. Starck and F. Murtagh, "Inpainting and Zooming using Sparse Representations", in press.*

$$\Theta_\Lambda = \mathbf{Id}_\Lambda \qquad \min_\alpha \|\alpha\|_1 \quad \text{s.t.} \quad y = M\Phi\alpha$$

Where M is the mask: M(i,j) = 0 ==> missing data
M(i,j) = 1 ==> good data

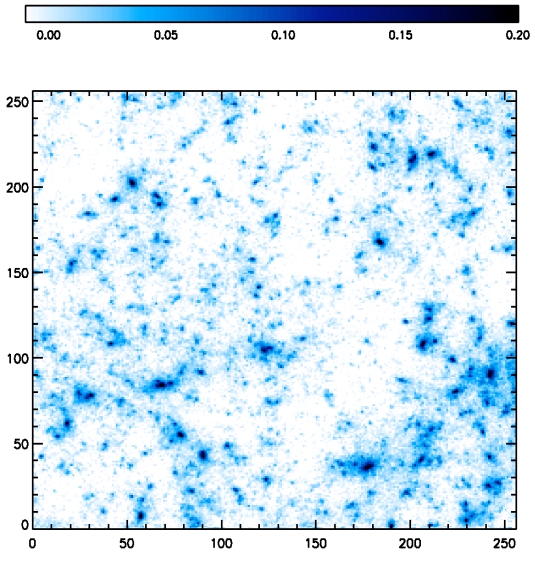
$$x^{(n+1)} = \mathcal{S}_{\Phi,\lambda^{(n)}} \left\{ x^{(n)} + M\left(y - x^{(n)}\right) \right\}$$

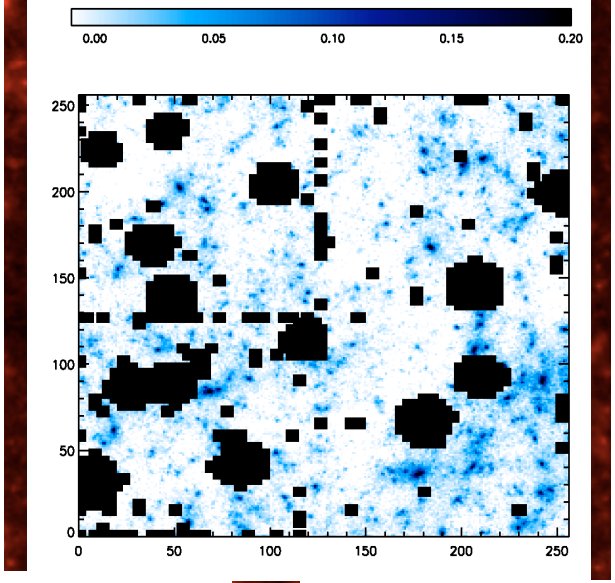
20%

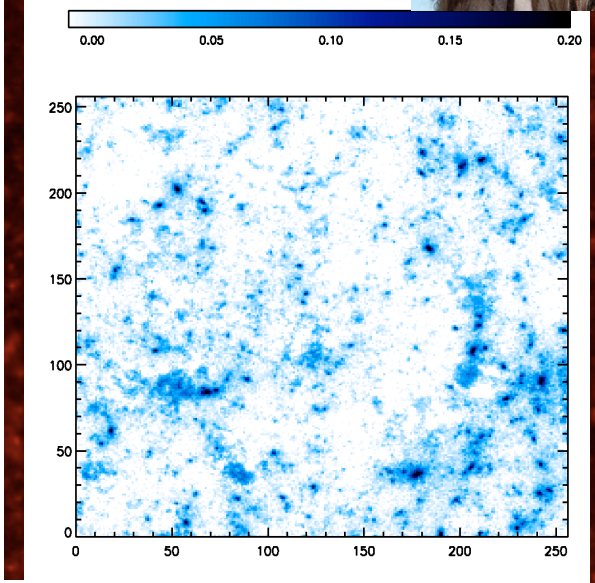
50%

80%

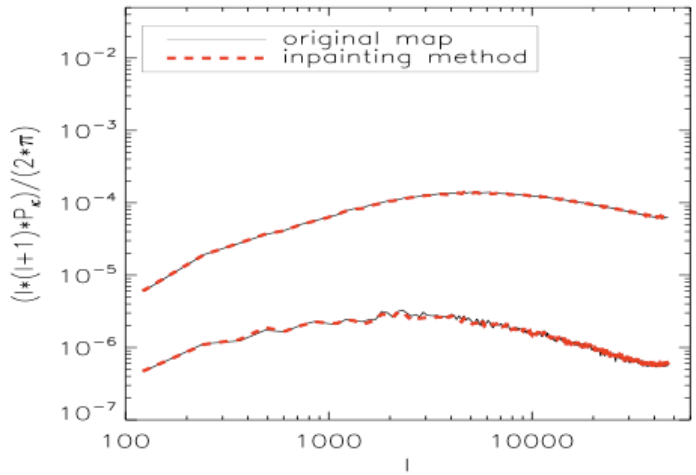
Inpainting :



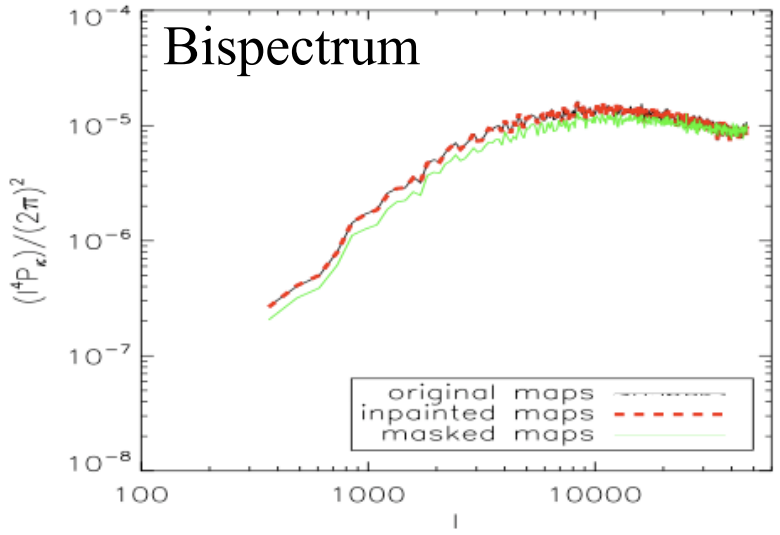




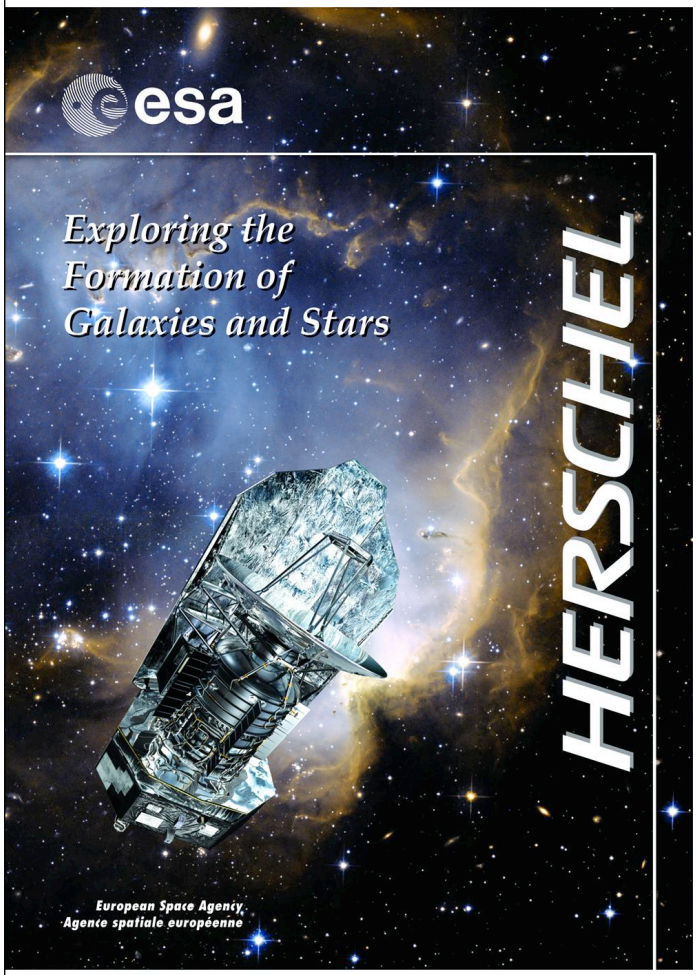
Power spectrum



Bispectrum



HERSCHEL



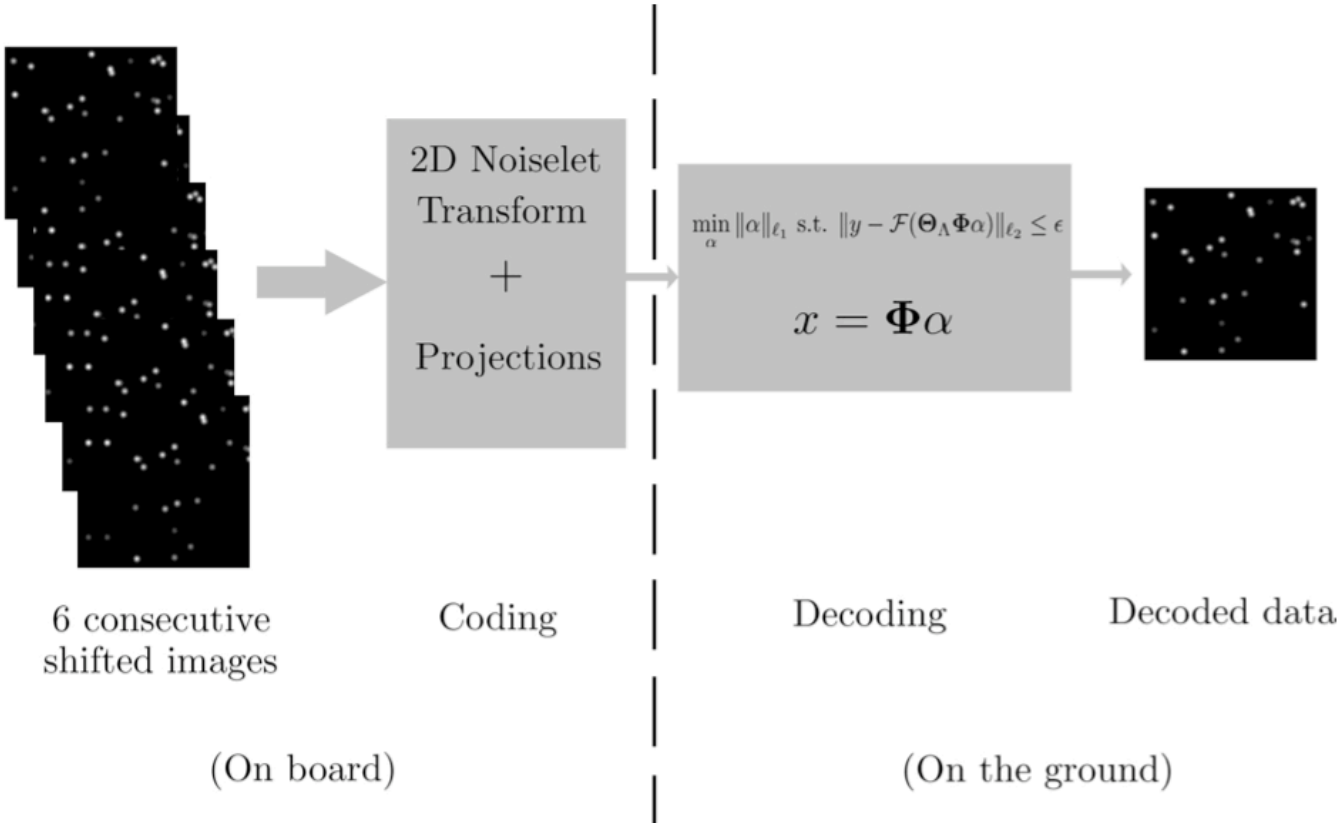
This space telescope has been designed to observe in the far-infrared and sub-millimeter wavelength range.
Its launch is scheduled for the beginning of 2009.
The shortest wavelength band, 57-210 microns, is covered by PACS (Photodetector Array Camera and Spectrometer).

Herschel data transfer problem:
-no time to do sophisticated data compression on board.
-a compression ratio of 6 must be achieved.

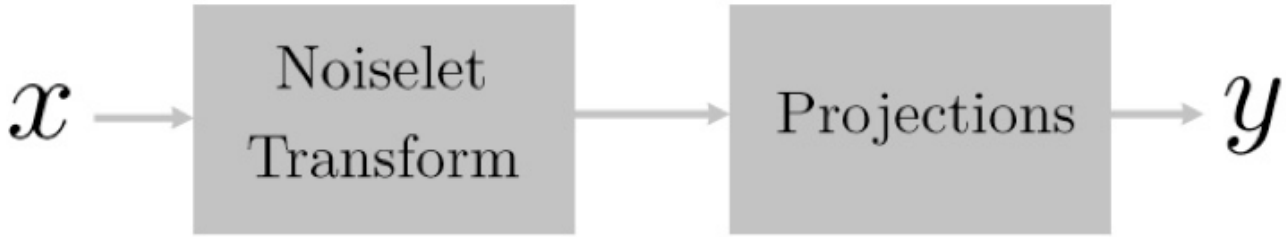
==> solution: averaging of six successive images on board

CS may offer another alternative.

The proposed Herschel compression scheme



The coding scheme



Good measurements must be incoherent with the basis in which the data are assumed to be sparse.

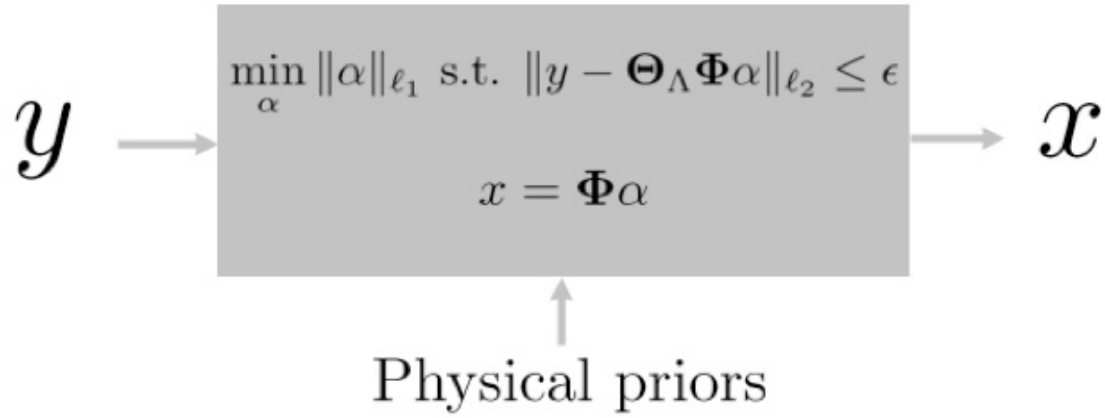
Noiselets (Coifman, Geshwind and Meyer, 2001) are an orthogonal basis that is shown to be highly incoherent with a wide range of practical sparse representations (wavelets, Fourier, etc).

Advantages:

Low computational cost (O(n))

Most astronomical data are sparsely represented in a wide range of wavelet bases

The decoding scheme



$$\alpha^{(h)} = \mathcal{S}_\gamma \left\{ \alpha^{(h-1)} + \mathbf{\Phi}^T \mathbf{\Theta}^T \left(y^\sharp - \mathbf{I}_\Lambda \mathbf{\Theta} \mathbf{\Phi} \alpha^{(h-1)} \right) \right\}$$

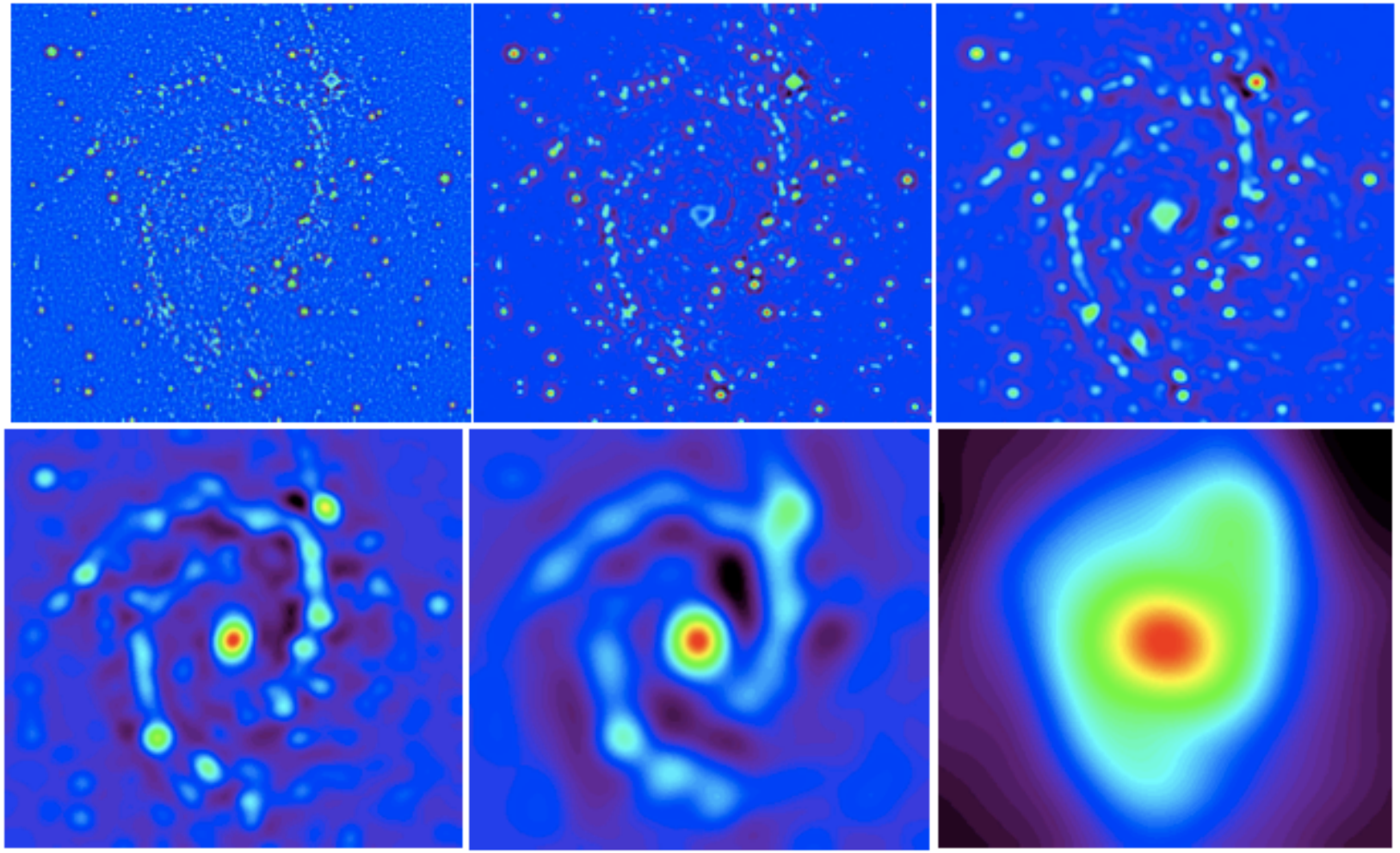
1. Set the number of iterations $I_{\max}$ and threshold $\gamma^{(0)} = \|\mathbf{\Phi^T \Theta^T} y^\sharp\|_\infty$. $x^{(0)}$ is set to zero.
2. While $\gamma^{(h)}$ is higher than a given lower bound $\gamma_{\min}$
   - Compute the measurement projection of $x^{(h-1)}$: $y^{(h)} = \mathbf{I}_\Lambda \mathbf{\Theta} x^{(h-1)}$.
   - Estimate the current coefficients $\alpha^{(h)}$: $\alpha^{(h)} = \mathcal{S}_{\gamma^{(h)}} \left\{ \alpha^{(h-1)} + \mathbf{\Phi^T \Theta^T} \left[y^\sharp - y^{(h)} \right] \right\}$.
   - Get the new estimate of x by reconstructing from the selected coefficients $\alpha^{(h)}$: $x^{(h)} = \mathbf{\Phi} \alpha^{(h)}$.
3. Decrease the threshold $\gamma^{(h)}$ following a given strategy.

Isotropic Undecimated Wavelet Transform

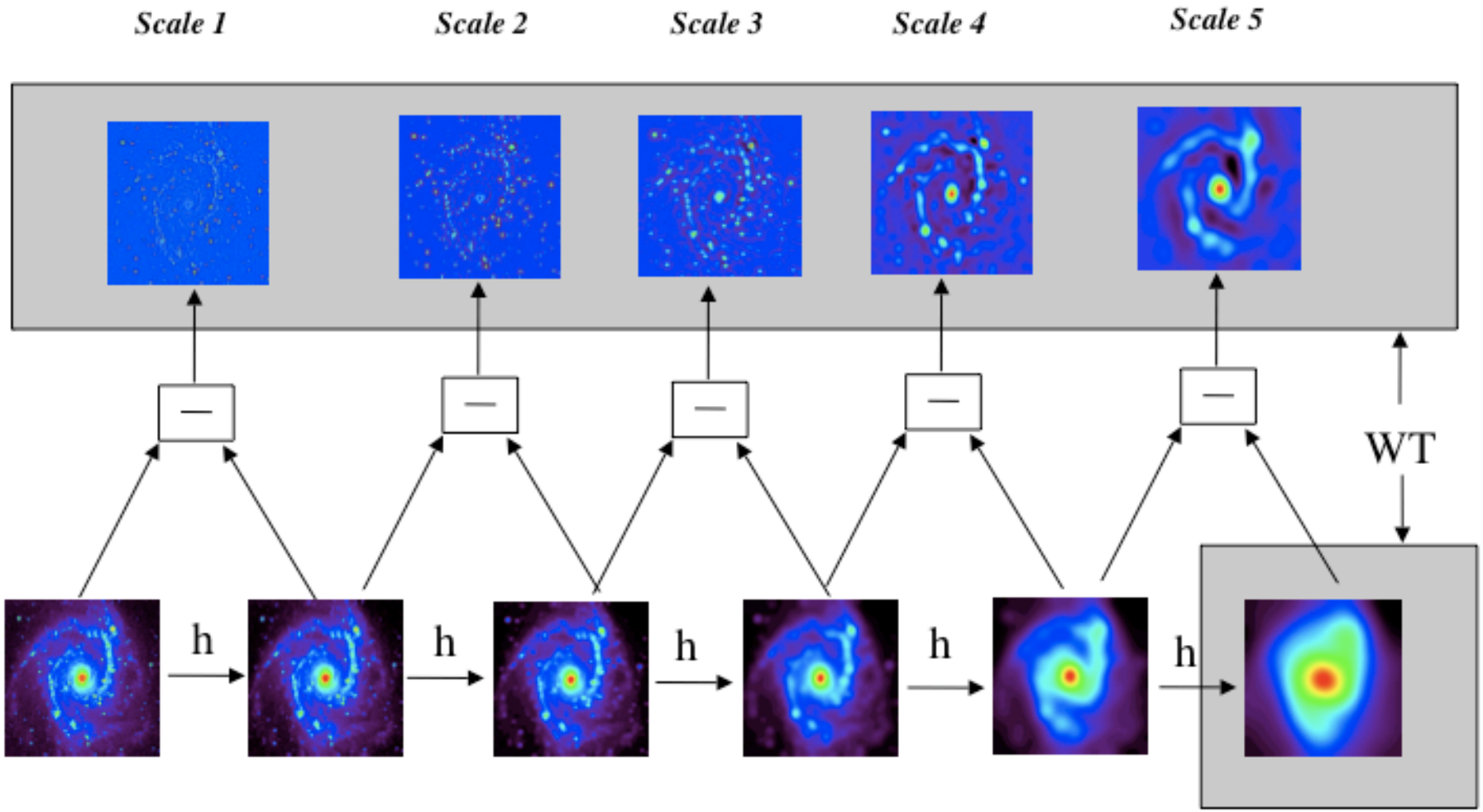
$$\varphi = B_3 - \text{spline}, \quad \frac{1}{2}\psi(\frac{x}{2}) = \frac{1}{2}\varphi(\frac{x}{2}) - \varphi(x)$$

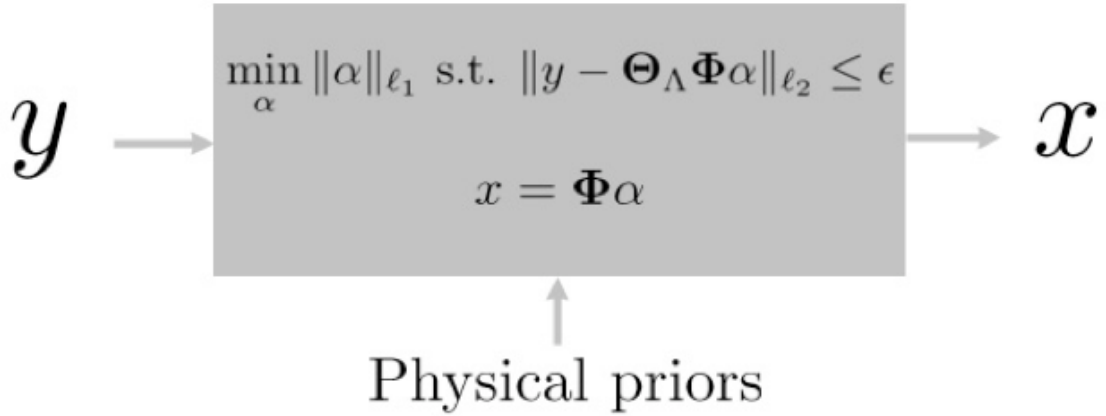
$$h = [1,4,6,4,1]/16, \quad g = \delta - h, \quad \tilde{h} = \tilde{g} = \delta$$

$$I(k,l) = c_{J,k,l} + \sum_{j=1}^{J} w_{j,k,l}$$



ISOTROPIC UNDECIMATED WAVELET TRANSFORM

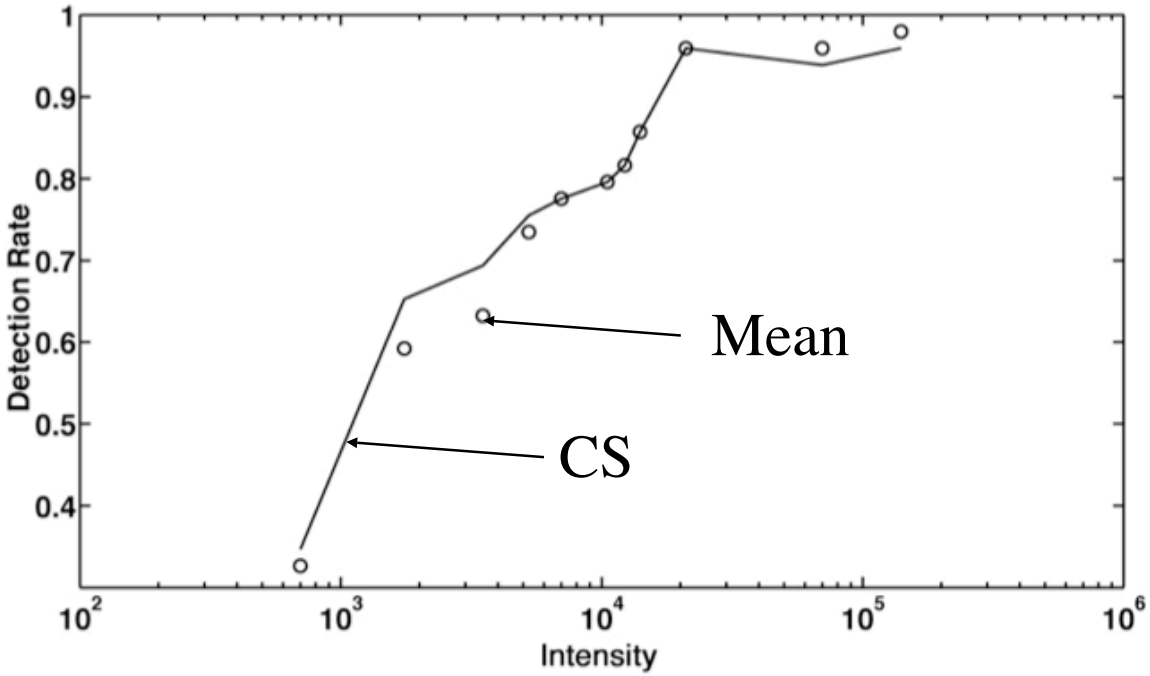




Six consecutive observations of the same field can be decompressed together:

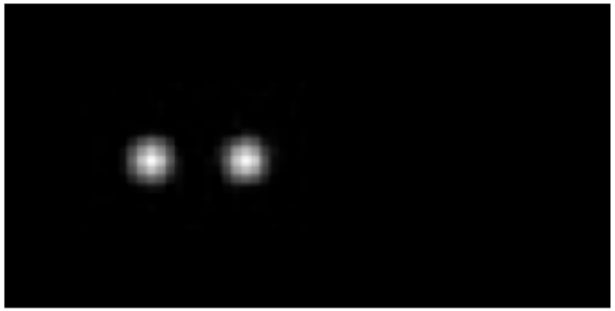
$$x^{(n+1)} = \frac{1}{6}\mathcal{S}_{\Phi,\lambda^{(n)}}\left\{x^{(n)} + \sum_{i=1,..6} \Theta^T\left(y - \Theta x^{(n)}\right)\right\}$$

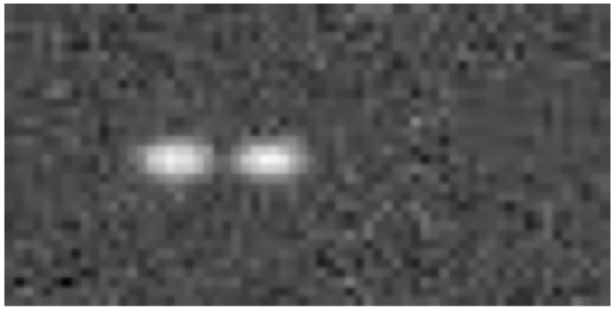
Sensitivity: CS versus mean of 6 images



Resolution: CS versus Mean

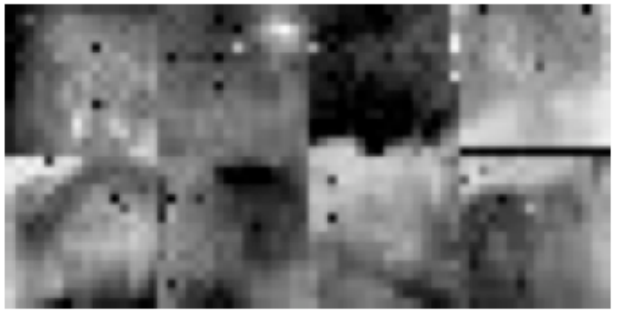
Simulated image

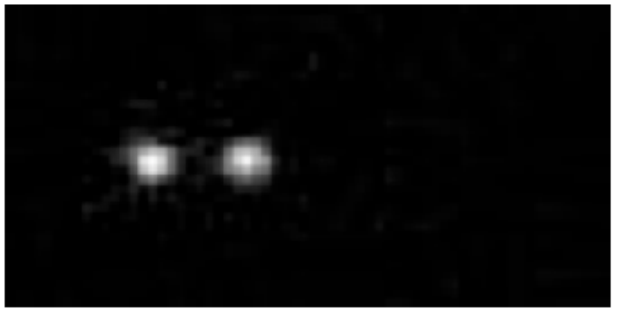




Mean of six images

Simulated noisy image with flat and dark





Compressed sensing reconstructed images

Resolution limit versus SNR

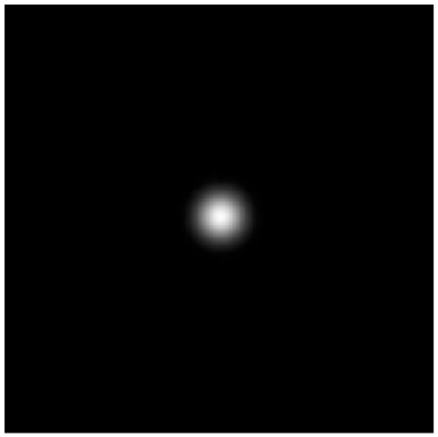
SNR	−17.3	−9.35	−3.3	0.21	2.7	4.7	6.2	7.6	8.7
Intensity	900	2250	4500	6750	9000	11250	13500	15750	18000
MO6	3	3	3	3	3	3	3	3	3
CS	2.33	2.33	2	2	2	2	2	2	2

THE CS-BASED COMPRESSION ENTAILS A RESOLUTION GAIN EQUAL TO A 30% OF THE SPATIAL RESOLUTION PROVIDED BY MO6.

CS and Herschel Status

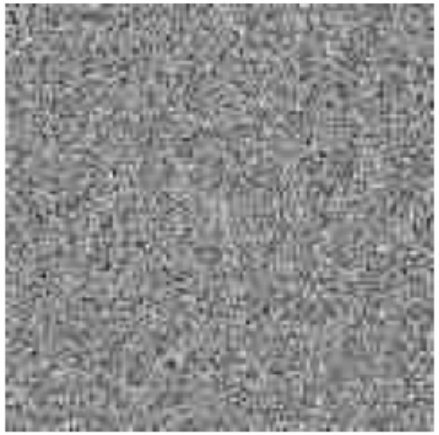
- CS compression **is** implemented in the Herschel on-board software (as an option).
- CS tests in flight will be done.
- Software developments are required for an efficient decompression (taking into account dark, flat-field, PSF, etc).
- The CS decompression needs to be fully integrated in the data processing pipeline (map making level).

Data Fusion: JPEG versus Compressed Sensing



Simulated source

One of the 10 observations



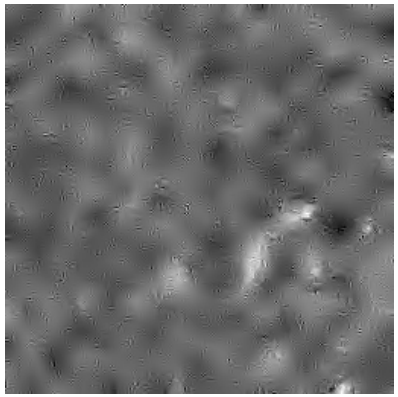
Averaged of the 10 JPEG compressed images (CR=4)

Reconstruction from the 10 compressed sensing images (CR=4)

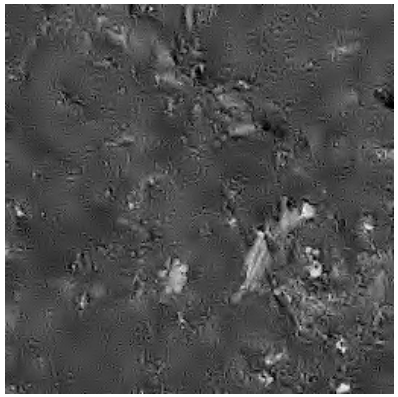
JPEG2000 Versus Compressed Sensing

Compression Rate: 25

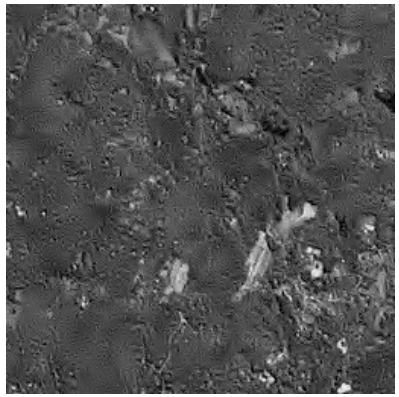
One observation



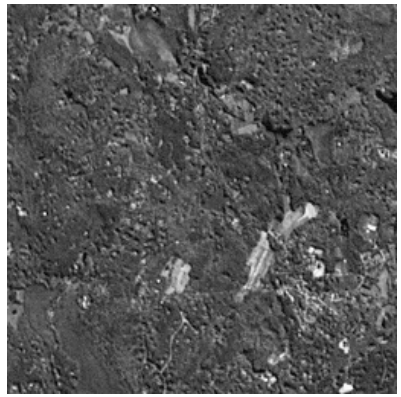
10 observations

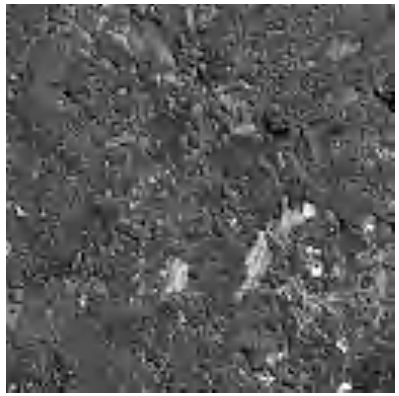


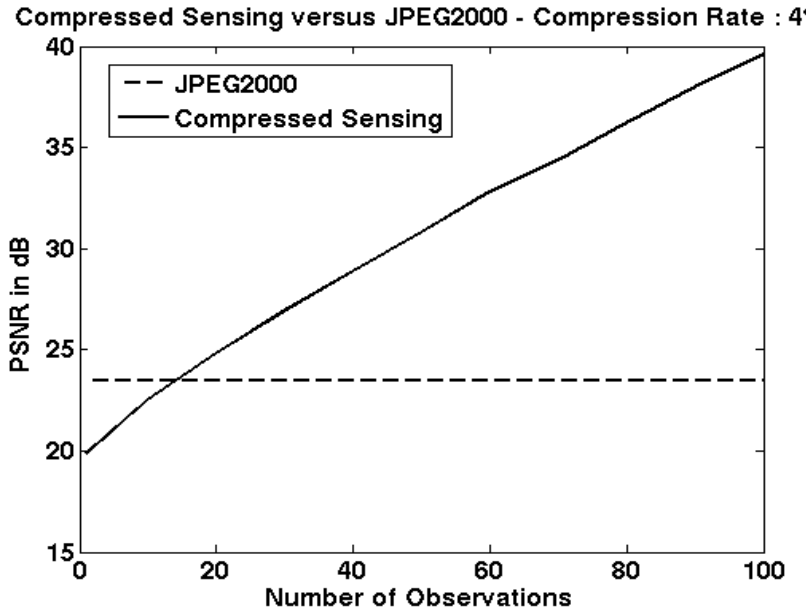
20 observations



100 observations







Conclusions

Compressed Sensing gives us a clear direction for:

- (radio-) interferometric data reconstruction
- periodic signals with sampled irregularly
- gamma-ray image reconstruction

CS provides an interesting framework and a good theoretical support for our inpainting work.

CS can be a good solution for on board data compression.

CS is a highly competitive solution for compressed data fusion.

Perspective

Design of CS astronomical instruments ?

PREPRINT: **http://fr.arxiv.org/abs/0802.0131**

Morphological Component Analysis (MCA)

$$\mathrm{MIN}_{s_1,\ldots,s_L} \sum_{k=1}^{L} \| T_k s_k \|_p \quad \text{subject to} \quad \| s - \sum_{k=1}^{L} s_k \|^2 < \epsilon$$

- Initialize all s_k to zero
- Iterate j=1,...,Niter
  - Iterate k=1,..,L

    Update the kth part of the current solution by fixing all other parts and minimizing:

$$J(s_k) = \left\| s - \sum_{i=1, i\neq k}^{L} s_i - s_k \right\|_2^2 + \lambda^{(j)} \left\| T_k s_k \right\|_p$$

Which is obtained by a simple **hard**/soft thresholding of : $s_r = s - \sum_{i=1, i\neq k}^{L} s_i$

- Decrease the threshold $\lambda^{(j)}$