Image Segmentation with Fuzzy Clustering Based on Generalized Entropy

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Abstract—Aimed at fuzzy clustering based on the generalized entropy, an image segmentation algorithm by joining space information of image is presented in this paper. For solving the optimization problem with generalized entropy’s fuzzy clustering, both Hopfield neural network and multi-synapse neural network are used in order to obtain cluster centers and fuzzy membership degrees. In addition, to improve anti-noise characteristic of algorithm, a window is introduced. In experiments, some commonly used images are selected to verify performance of algorithm presented. Experimental results show that the image segmentation of fuzzy clustering based on generalized entropy using neural network performs better compared to FCM and BCFCM_S1.

Index Terms—image segmentation, spatial information, generalized entropy, neural network

I. INTRODUCTION

Image segmentation is considered as an important part of image processing and pattern recognition system, and it directly impacts on the quality of the image analysis and the final discrimination result. For this reason, some scholars have done a lot of researches about it. After that, they presented some image segmentation algorithms, where image segmentation based on clustering is commonly used method. Since Zadeh proposed fuzzy concept, Ruspini firstly proposed fuzzy c-partitions in 1969 and some researchers started to focus on fuzzy clustering algorithm. In 1974, Dunn proposed fuzzy C-means with a weighting exponent m equal to 2, later in 1981 Bezdek popularized it with m>1, that is fuzzy C-means(FCM). Since then, fuzzy clustering algorithm attracted a lot of attentions and successfully was applied in many fields, such as image processing, pattern recognition, medical, artificial intelligence and data mining, etc. In 2002, Ahmed et al. [1] considered a bias-corrected fuzzy c-means (shortening as BCFCM) with introducing spatial neighborhood information into the objective function to overcome the drawback of FCM that is sensitive to salt and pepper noise and image artifacts. In 2004, Chen et al. [2] pointed out a shortcoming of a computational complexity for BCFCM and then proposed BCFCM_S1. In 2012, Wang et al. [3] presented a pavement image segmentation algorithm based on FCM using neighborhood information to reduce the effect of noise. In 2013, Gong et al. [4] introduced a tradeoff weighted fuzzy factor to improve FCM. Later, Despotovic et al. [5] presented a new method based on FCM for spatially coherent and noise-robust. Besides, Karayiannis et al. [6], Li et al. [7] and Tran et al. [8] also studied entropy based fuzzy clustering method. What attracts us most is that scholars have combined entropy with fuzzy clustering method, and proposed fuzzy clustering based on entropy. The general case of fuzzy weight m in generalized FCM has come out by Zhu et al. [9]. Sun et al. [10] redistricted segmented regions and further classified the segmented image pixels with the method of the minimum fuzzy entropy to improve segment result of entropy in 2012. In this paper, we study the problem of image segmentation with fuzzy clustering based on the generalized entropy, where Hopfield neural network and multi-synapses neural network[11] are used to solve optimization problem with fuzzy clustering based on the generalized entropy. In addition, spatial information of image is also considered.

The rest of this paper is organized as follows. In section 2, we briefly introduce the fuzzy clustering and other related algorithms. In section 3, we present the method of image segmentation with fuzzy clustering based on the generalized entropy. In section 4, some experimental results are given. In section 5, we have our conclusion and prospects for future work.

II. FUZZY CLUSTERING BASED ON GENERALIZED ENTROPY

A. Fuzzy C-Means Clustering and Its Variants

Clustering method attempts to organize unlabeled data
into clusters or groups, such that the data within a group are more similar to each other than the ones belonging to different groups. One of commonly used methods is fuzzy c-means clustering (FCM). The fuzzy clustering problem is described as follows: Given that \( X=\{x_1, x_2, \ldots, x_n\} \) \((n>1)\) is a finite data set, \( c \) is the number of cluster, \( m \) is fuzzy weight with \( 1<m<\infty \), \( \mathcal{V}=\{v_1, v_2, \ldots, v_c\} \) represents the cluster center, and \( \mathcal{U}=\{\mu_{ij}, 1\leq i \leq c, 1\leq j \leq n\} \) represents membership degree matrix, where \( \mu_{ij} \) is the fuzzy membership degree from the data point \( x_j \) to center \( v_i \). The objective function of FCM is written as

\[
J(U,V) = \sum_{j=1}^{n} \sum_{i=1}^{c} \mu_{ij}^m \| x_j - v_i \|^2 .
\] (1)

Fuzzy clustering is viewed as solving the following optimization problem:

\[
\min_{U,V} J(U,V)
\]

such that \( \sum_{i=1}^{c} \mu_{ij} = 1, 1 \leq j \leq n \).

By using Lagrange method, membership degree \( \mu_{ij} (1 \leq i \leq c, 1 \leq j \leq n) \) and cluster center \( v_j (1 \leq i \leq c) \) are obtained in the following.

\[
\mu_{ij} = \frac{1}{\sum_{j=1}^{n} \left( \frac{\| x_j - v_i \|^2}{\| x_j - v_j \|^2} \right)^{\frac{2}{m-1}}} \] (2)

\[
v_j = \frac{\sum_{j=1}^{n} \mu_{ij}^m x_j}{\sum_{j=1}^{n} \mu_{ij}^m} .
\] (3)

It can be seen from (2) and (3) that membership degree \( \mu_{ij} \) and cluster center \( v_j \) are dependent to each other. So, FCM algorithm uses iterative method to find the optimal fuzzy clustering partition.

Since FCM did not consider the spatial information of image, it is sensitive to salt and pepper noise and image artifacts. To overcome this drawback, Ahmed et al. proposed BCFCM with the following objective function:

\[
J_{BCFCM}(\mu, \alpha) = \sum_{j=1}^{n} \sum_{i=1}^{c} \mu_{ij}^m \| x_j - v_i \|^2 + \alpha \sum_{i=1}^{c} \sum_{j=1}^{N_j} \| x_j - \overline{x}_j \|^2 .
\] (4)

where \( N_j \) represents the set of pixels that exist in a window around \( x_j \) and \( N_R \) is the cardinality of \( N_j \). Later, for reducing computation, Chen et al.[2] modified objective function (4) which is given as follows:

\[
J_{m}(\mu, \alpha) = \sum_{j=1}^{n} \sum_{i=1}^{c} \mu_{ij}^m \| x_j - v_i \|^2 + \alpha \sum_{j=1}^{n} \sum_{i=1}^{c} \mu_{ij}^m \| x_j - \overline{x}_j \|^2 ,
\] (5)

where \( \overline{x}_j \) is the mean of pixel within the window around \( x_j \), \( J_{m}^{\alpha} \) becomes objective function of BCFCM_S. The effect of neighboring pixels is controlled by the parameter \( \alpha \).

B. Fuzzy Clustering of the Generalized Entropy

The concept of entropy is proposed by Rudolf Clausius, which is used to represent the uniformity of spatial distribution for energy. Specifically, more uniform energy distributes, the greater the entropy is. Later, Shannon firstly introduced the concept of entropy into information theory as a measure of the uncertainty. After that, Karayiannis et al. [6], Li et al. [7] and Tran et al. [8] employed entropy in fuzzy clustering problem and proposed maximum entropy clustering algorithm. In the process of fuzzy clustering, by introducing the entropy of membership degree and distance from the sample points to center, clustering process is gradually transformed from the maximum uncertainty into determination. In 2012, Li et al. [12] generalized fuzzy clustering based on entropy and gave the following objective function with generalized entropy

\[
J_{G}(\mu, \alpha) = \sum_{j=1}^{n} \sum_{i=1}^{c} \mu_{ij}^m \| x_j - v_i \|^2 + \delta H(U, \alpha) \alpha > 0, \alpha \neq 1
\] (6)

where \( H(U, \alpha) = \sum_{j=1}^{n} (2^\alpha - 1)^{-1} (\sum_{i=1}^{c} \mu_{ij}^\alpha - 1) \) is the generalized entropy and \( \alpha \) is called as index of generalized entropy.

III. IMAGE SEGMENTATION WITH FUZZY CLUSTERING BASED ON GENERALIZED ENTROPY

A. Objective Function with Generalized Entropy for Image Segmentation

Motivated by BCFCM algorithm, we introduce spatial information into (6) to obtain the following objective function:

\[
J_{G}(\mu, \alpha) = \sum_{j=1}^{n} \sum_{i=1}^{c} \mu_{ij}^m \| x_j - v_i \|^2 + \delta \sum_{j=1}^{n} (2^\alpha - 1)^{-1} (\sum_{i=1}^{c} \mu_{ij}^\alpha - 1) + \sum_{j=1}^{n} (1-p_j)\overline{x}_j^2 \]

where \( p_j = N_j / N_R \) represents the distribution of pixels around \( x_j \), \( N_j \) is the number of neighboring pixels that belong to the same cluster as \( x_j \), \( N_R \) is the cardinality of set \( N_j \), and \( \overline{x}_j \) is the mean of pixels within the window around \( x_j \).

Here, we use the augmented Lagrange method to solve the constrained optimization problem for objective function (7), where the constrained condition is

\[
\sum_{i=1}^{c} \mu_{ij} = 1, 1 \leq j \leq n .
\]

So the augmented Lagrange function is
Here, the number of neurons in the network is \( n \). Some details are seen in ref. [12]. By using Hopfield neural network, we obtain the equation (9). In multi-synapse neural network, whose structure is shown in Fig. 1, and multi-synapse neural network is used to solve cluster center and multi-synapse neural network is used to solve fuzzy membership degree. After solution of cluster centers are obtained, these values are seen as constants to transfer to multi-synapse neural network and vice versa.

**B. Solving Cluster Center Using Hopfield Neural Network**

It is seen from (8) that this function is a quadratic function about cluster center \( v_i \). For solving cluster centers, we discard the fixed parts in (8). So, function above is expressed as follows:

\[
\begin{align*}
Z_s(U,v) & = \sum_{j=1}^{c} \mu_j \| v_j - v_i \|^2 + \sum_{j=1}^{c} \| v_j - v_i \|^2 , \\
& \quad + \sum_{j=1}^{c} ( \sum_{i=1}^{c} \mu_i | \mu_i | ) ( v_i - v_j ) ( v_j - v_i ) ,
\end{align*}
\]

where \( \lambda_i \) (\( j=1,2,\ldots,n \)) is Lagrange multipliers and \( \gamma \) is a large value. In the following, we combine Hopfield neural network, where the structure is shown in Fig. 1, and multi-synapse neural network to solve (8), where Hopfield neural network is used to solve cluster center and multi-synapse neural network is used to solve fuzzy membership degree. After solution of cluster centers are obtained, these values are seen as constants to transfer to multi-synapse neural network and vice versa.

In (12), superscript \( g \) represents the \( g \)-th loop and parameter \( \delta_i \) is a small initial value for adjusting \( v_i \).

**C. Solving Membership Degree Using Multi-synapse Neural Network**

In the following, we mainly optimize membership degree \( \mu \) in objective function (8) which is high-order about membership degree. We expand (8) to obtain following expression

\[
\begin{align*}
\sum_{j=1}^{c} \sum_{i=1}^{c} \left[ ( p_j d_{ij}^{(1)} + (1 - p_j) d_{ij}^{(2)}) | \mu_j^{m} \right] \\
& + \mu_j^{m} \\
& + \delta (2^{1-a} - 1)^{-1} \mu_j^{a} \\
& + (\lambda_j - 2 \gamma) \mu_j^{a}
\end{align*}
\]

where \( d_{ij}^{(1)} = || v_j - v_i ||^2 \) and \( d_{ij}^{(2)} = || v_j - v_i ||^2 \). Now, we use multi-synapse neural network to optimize membership degree. Here, the number of neurons \( s \) is \( c \times n \). The structure of multi-synapse neural network is seen in Fig. 2, where two dimensional subscripts for membership degree are converted into one dimensional subscript. Note that there are more than two weights between every two neurons.

In multi-synapse neural network, the conversions are as follows:

\[
\begin{align*}
\mu \text{ convert to } \mu_{(j-1)oc+i} & \text{, } d_{ij} \text{ convert to } d_{(j-1)oc+i}
\end{align*}
\]

The matrix form used to express the total input of multi-synapse neural network is

\[
\begin{align*}
NET = W \cdot U + Z \cdot U + Y \cdot U + I
\end{align*}
\]

In addition, we define the following matrices:

\[
U_{(m-1)} = \begin{bmatrix} \mu_{e}^{m-1} \\ \vdots \\ \mu_{n}^{m-1} \end{bmatrix}, \quad m > 1, \text{ where } U_{(1)} = U
\]
$$
abla E_{\alpha}(i)=−\lambda(i−c)(1−c)\leq\frac{i}{c}c\leq\frac{j}{c}c \quad \text{&} \quad i,j=1,2,\cdots,s
$$

The transposition of $U_{\alpha,i\cdot} \in \mathbb{R}^{1 \times s}$ can be written as $U_{\alpha,i\cdot}^T$ and $Y_{\alpha,i\cdot} \in \mathbb{R}^{s \times 1}$ as $Y_{\alpha,i}^T$. For (14), the energy function is

$$
E(\mu_{\alpha\beta},\mu_{\alpha\beta})=\frac{1}{2}\mu_{\alpha\beta}^{T}\cdot Z\cdot U−\frac{1}{\alpha}\mu_{\alpha\beta}^{T}\cdot Y\cdot U_{\alpha\beta}^{T}\cdot I
$$

By contrasting (13) and (17), we can express $W, Z, Y$, and $I$, respectively as follows:

$$
w_j=\begin{cases}
-\mu_j \mu^{(0)}(i−p_j) & i=j \quad i,j=1,2,\cdots,s \\
0 & \text{otherwise}
\end{cases}
$$

$$
z_j=\begin{cases}
-2\gamma \left[\frac{i}{c}−1\right] & c<j \leq\frac{i}{c} \quad c,i,j=1,2,\cdots,s \\
0 & \text{otherwise}
\end{cases}
$$

$$
y_j=\begin{cases}
-\alpha \delta (2^{\alpha−c}−1)^{i} & i=j \quad i,j=1,2,\cdots,s \\
0 & \text{otherwise}
\end{cases}
$$

$$
i_j=2\gamma−\lambda_i \left[\frac{i}{c}−1\right] & c<k \leq\frac{i}{c} \quad c \quad j=1,2,\cdots,s
$$

Next, we need a relation between the energy function and the input of neural network. What we hope is that the value of energy function decreases while the iteration increases, just like the values of objective function is. To discover the relation between the energy function and the input of neural network, we find that matrix $W$ and $Y$ is symmetrical, so $L\cdot W^T=R\cdot W\cdot L^T$ and $L\cdot Y^T=R\cdot Y\cdot L^T$, where matrix $L$ and $R$ are nonzero matrixes with size of $1 \times s$. Therefore, we obtain a variant of (17) as follows:

$$
E=\frac{1}{m}\mu_{\alpha\beta}^{T}\cdot W\cdot U_{\alpha\beta}−\frac{1}{2}\mu_{\alpha\beta}^{T}\cdot Z\cdot U−\frac{1}{\alpha}\mu_{\alpha\beta}^{T}\cdot Y\cdot U_{\alpha\beta}^{T}\cdot I
$$

With the same way of transforming (14) to (17), we can get a new meaning between (14) and (22). Specifically, the net turns $U_{\alpha\beta,i\cdot}$ to weight $W$, a new $U$ to $Z$ and $U_{\alpha\beta,i\cdot}$ to $Y$. So far, we get a new multi-synapse neural network, whose input matrix of the new net can be written as:

$$
Y_{\alpha,i}\left[\begin{array}{c}
y_{i}^{(a-1)} \\
y_{i}^{(a-1)} \\
\vdots \\
y_{i}^{(a-1)}
\end{array}\right], \quad a>1, \quad \text{where} \quad Y_{\alpha}\left[\begin{array}{c}
y_{i} \\
y_{i} \\
\vdots \\
y_{i}
\end{array}\right]
$$

The total input of new neural network is given as:

$$
\text{net}_j=\sum_{i=1}^{s}(w_{ji}\mu_{ji}^{(e)}+y_{ji}\mu_{ji}^{(e)}+i_j) \quad j=1,2,\cdots,s
$$

What we can get by comparing (23) and (24) is shown as:

$$
\nabla E=−\text{net}_j \quad j=1,2,\cdots,s
$$

Equation (25) is exact what we need, and it means that the energy function decreases with the increase of iteration. The relevance for $\mu$ is the same with the cluster center $\nu$, so we can use a same activation function with solution of $\nu$ to adjust $\mu$.

$$
\mu_j^{(\nu)}=f(\text{net}_j^{(\nu)})=\begin{cases}
\mu_j^{(e)}+\delta_j & \text{if} \quad \text{net}_j^{(\nu)}\geq 0 \\
\mu_j^{(e)}−\delta_j & \text{if} \quad \text{net}_j^{(\nu)}< 0 \\
1,2,\cdots,s
\end{cases}
$$

In (26), $g$ represents the $g$th loop and $\delta_j$ is a small initial value for adjusting $\mu_j$.

On the other hand, we may need to adjust the membership degree $\mu$ by (27) when necessary.

$$
\mu_j=\begin{cases}
1 & \text{if} \quad \mu_j>1 \\
0 & \text{if} \quad \mu_j<0 \\
\mu_j & \text{otherwise}
\end{cases} \quad j=1,2,\cdots,s
$$

The termination condition of iteration loop is that the difference of membership degree value between this cycle and the last cycle is less than a given value $\varepsilon$. The proposed algorithm is named as ISGEFCM (Image Segmentation of Generalized Entropy Fuzzy C-Means) which is given in the following.

Step 1 Initialize the value of cluster number $c$, fuzzy coefficient $m$, augmented Lagrange coefficient $\gamma$, adjustment $\delta_i$ and $\delta_i$ for $\nu$ and $\mu$ respectively, termination condition $\Delta \nu$ for Hopfield neural network, $\Delta \mu$ for multi-synapse neural network and $\varepsilon$ for the algorithm.

Step 2 Initialize the center $\nu_j$ among $x_j$, membership degree $u_j$ within $[0,1]$, and $\lambda$ within $[1,10]$. 

![Figure 2. The structure of multi-synapse neural network for solving membership degrees](image-url)
Step 3 Set the initial value of input net(0)\(j=0\) and the iteration counter \(g=1\) in Hopfield neural network.
Step 4 Calculate \(NET\) by (9), \(I\) by (10) and \(W\) by (11) in Hopfield neural network.
Step 5 for \(j=1\) to \(s\)
   if net\(^{(g)}\)_\(j\)*net\(^{(g-1)}\)_\(j\) <= 0
      then \(\delta^g_j = \delta^g_j / 2\).
Step 6 for \(j=1\) to \(s\)
   if net\(^{(g)}\)_\(j\) > 0
      then \(v^*_j = v^*_j + \delta_j\)
   else \(v^*_j = v^*_j - \delta_j\).
Step 7 If ((\(\delta^0_1 <= \Deltaivent\))&((\(\delta^0_2 <= \Deltaivent\))&...&(\(\delta^0_s <= \Deltaivent\)))
   then go to step (8)
   else \(g = g + 1\), go to step (4).
Step 8 Set iteration counter \(g=1\) in multi-synapse neural network.
Step 9 Calculate \(W\) by (18), \(Z\) by (19), \(Y\) by (20), \(I\) by (21) and \(NET\) by (23) in multi-synapse neural network.
Step 10 for \(j=1\) to \(s\)
   if net\(^{(g)}\)_\(j\)*net\(^{(g-1)}\)_\(j\) <= 0
      then \(\delta^g_\mu = \delta^g_\mu / 2\).
Step 11 for \(j=1\) to \(s\)
   if \(\mu^*_j > 1\) then \(\mu^*_j = 1\)
   else if \(\mu^*_j < 0\)
      then \(\mu^*_j = 0\).
Step 12 If ((\(\delta^0_1 <= \Deltaueq\))&((\(\delta^0_2 <= \Deltaueq\))&...&(\(\delta^0_\mu <= \Deltaueq\)))
   then go to step 14
   else \(g = g + 1\), go to step 9.
Step 13 If \(||U^{(g)}-U^{(g-1)}|| <= \epsilon\)
   then exiting the algorithm else go to step 3.

IV. EXPERIMENT

For image segmentation, it is emphasized that \(x_j (1 \leq j \leq n)\) is 1-dimensional data which represents the gray value of pixels in image. Thus, in Hopfield neural network, the number of cluster center is \(c\) and the number of neurons \(s\) is equal to \(c\). In multi-synapse neural network, the membership degree \(\mu_i (1 \leq i \leq c \times n)\) is 1-dimensional matrix after conversion subscript, thus the number of neurons \(s\) is equal to \(c \times n\).

In this paper, all pictures are set with size 72×72, all algorithms are implemented under the same initial value and all algorithms are set with the same stopping thread \(\epsilon=0.0001\). In experiments, the window to calculate the mean of pixels is set as 3×3.

First, we use four images without noise to perform the algorithm we proposed. The picture Lena is downloaded from Internet and the others are built-in-matlab. On the other hand, we select FCM and BCFCM_S1 as comparison algorithms. Experimental results are given in Fig. 3 to Fig. 6.

Next, salt and pepper noise is added to the images to test the robust of these algorithms. The picture binimage is artificially synthesized. The results are shown in Fig. 7 to Fig. 11.
Experimental results of the first group show that the proposed algorithm can maintain more details and segment more accurately, for example the bordering of third coin in Fig. 2 and the edge of hat in Fig. 6. Results of the second group show that FCM is invalid to noise, BCFCM S 1 algorithm removes most noise, and our method almost removes all noise. The comparisons suggest that our algorithm achieves better segmentation to noise and multi-level image like Fig. 11. What can be concluded is that our method is robust to noise and effective for image segmentation with some certain selection of parameters. Of course, the selections of parameters need lots of experiments.

V. CONCLUSION

In this paper, it is mainly to realize the image segmentation based on generalized entropy by using multi-synapse neural network, and introduce spatial information of image to the algorithm. The equation \( p_j \) represents the distribution of pixels around \( x_j \), by which we can automatically adjust the effect of neighbors around \( x_j \). We get good segmentation results by comparing with control experiments. In the future, we will focus on the segmentation efficiency of algorithm and further study the entropy method combined with fuzzy clustering.

ACKNOWLEDGMENT

This work is support by Natural Science Foundation of China (No. 61375075) and Nature Science Foundation of Hebei Province (No. F2012201014).

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