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Non-wave mechanism of transverse ion heating in magnetic flux tubes

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Abstract
We introduce a new mechanism for ion heating in the plane perpendicular to the magnetic field lines of a magnetic flux tube. The transverse energization is shown to be contributed by the non-wave solitons developed from nonlinear waves in magnetized plasmas. We use a cylindrical geometry in the study. Solitons carry space charges, which produce an electrostatic electric field by means of Gauss's law. For electron solitons, positive ions are accelerated by and gain energy from the radially inward space-charge electric field. Both the bulk kinetic energy and temperature of ions are enhanced from the initial states. For a single train of solitons, the heating effect is insignificant. By contrast, for a dense cluster of soliton sets, the space charges form an electric field, which is proportional to the radius; if the electric field is stochastic with time, ions can be heated efficiently to a level at which the enhanced parameters are observable by the high-resolution satellites, e.g. FAST.

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(Some figures in this article are in colour only in the electronic version.)

1. Introduction

Since the 1970s, transversely heated ions have been detected by observations in upward ion flows along geomagnetic flux tubes from the ionosphere into the magnetosphere in very thin filaments of 50–100 m across and a few hundred kilometers long (e.g. Chang *et al* 1989, Chaston *et al* 1999, Eriksson *et al* 2006, Karlsson *et al* 2005, Sharp *et al* 1979, Shelley *et al* 1976). These flows contain H^+, He^+, O^+ and O^{2+}, in which H^+ and O^+ ions dominate the population with energies ranging from eV to keV.

Most early arguments connected ion heating to wave energization. The waves include broadband extremely low-frequency (BBELF), electromagnetic ion cyclotron (EMIC) and lower-hybrid (LH) waves (see, for instance, André *et al* 1998, Daglis and Axford 1996, Kintner 1992, Kintner *et al* 1986, Kiwamoto and Tanaca 1976, Knudsen *et al* 1998, Lund *et al* 2000, Norqvist *et al* 1998, Rothwell *et al* 1992, 1995). Specifically, below 2000 km, LH waves were more probably related to electric fields (**E**) with large amplitudes 150–500 mV m^{-1}, and were responsible for most of the transverse ion heating in the auroral geomagnetic field (**B**) (Kintner *et al* 1986). By contrast, above 2000 km, LH waves seemed almost irrelevant to transverse ion heating (Lund *et al* 2000), but other weaker wave fields (tens of mV m^{-1}) should behave as the driver to energize ions (Kintner 1992). At ∼1700 km, Freja data showed that almost all of the ions were energized by BBELF, EMIC and LH waves (André *et al* 1998). Ions were considered as being heated in three modes (Norqvist *et al* 1998): (i) BBELF wavefields—producing resonant heating events around the ion gyrofrequency (LH waves exist, but do not take part in the process); (ii) BBELF wavefields plus precipitating keV protons and ∼0.1 keV electrons—responsible for the heating events where the $\mathbf{E}\times\mathbf{B}$ drifts are smaller than those in the first mode; (iii) LH and EMIC wavefields plus keV downward auroral electrons—contributing the heating events resonant at about the EMIC oscillation (around half the proton gyrofrequency). In the last mode, heavier ions (e.g. O^+) were found to be more significantly energized.

However, when taking a closer look at the measurements of ion energy distributions, there were some problems for the wave mechanism (Klumpar 1986, Moore *et al* 1986, Sckopke *et al* 1983): first of all, a one-to-one correspondence was not found between heated ions and any particular type of waves; secondly, energy was not always transferred from the wave(s) to ions or, rather, it can flow from ions to the wave(s); lastly, the interaction of ions with small-scale potential structures (a special form of wave–particle interactions) seemed closest to being responsible for the transverse ion heating. These potential structures were described as spikelets or non-wave cavitons (Chang 1993, McFadden *et al* 1999a, 1999b), although the correlation between ion energization and pulsative field strengths was still open (Schuck and Bonnell 2003). Consequently, the detected ion-heating scenarios should not be brought about by waves; or, if there were waves locally, they should not be the heater for ions but only a by-product in the process of ion energization (Moore *et al* 1986).

Observationally, the non-wave structures have attracted considerable attention since the 1970s. They were first recorded by IMP-7 in the neutral sheet (Scarf *et al* 1974), and then measured by IMP-8 in the plasma sheet boundary layer (Gurnett *et al* 1976). They were named as intense broadband electrostatic noises (BENs). Along with them were also measured plasma waves, such as magnetic noise bursts, electrostatic electron cyclotron waves and upper hybrid (UH) waves (Cattell *et al* 1986, Gurnett *et al* 1976). BENs were described as bursts extending from the lowest frequency to as high as the electron cyclotron frequency. However, the intensity decreases in frequency, with a peak at or below the LH frequency. Interestingly, by using high temporal resolution data of Geotail, Matsumoto *et al* (1994) confirmed that BENs were nothing but propagating non-wave solitons. The data showed localized transverse electric field structures of bipolar pulses with a time span of 2–5 ms and peak-to-peak amplitudes of a few tenths of $\mathrm{mV\,m^{-1}}$. The field structures implied respective small space-charge elements as being either cylindrically symmetric or pancake-like in shape, each sustaining a parallel electric field while producing a spiky perpendicular one (e.g. Ergun *et al* 1998a, 1998b, McFadden *et al* 2003, Pickett *et al* 2005). In observations, the non-wave solitary structures had been described as ‘phase-space (electron or ion) holes’, ‘Bernstein, Green and Kruskal (BGK) waves’, ‘solitary (electrostatic) waves’, ‘solitons’, ‘kinks’, ‘electrostatic shocks’, ‘(weak or strong) double layers’, ‘space-charge clumps’ or ‘bipolar/tripolar electric-field structures’. Numerous satellites have measured these non-wave solitons and confirmed the simultaneity between spikelet electric field structures and energetic ion populations (see e.g. Carlqvist and Boström 1970, Cattell *et al* 1991, Dusenbery *et al* 1988, Mottez 2001, and references in Ma and Hirose 2009, Temerin *et al* 1982, Vago *et al* 1992).

Theoretically, non-wave structures were first predicted by Bernstein, Green and Kruskal (1957) at an equilibrium state of trapped plasma particles. They were nominated as ‘clumps’ in turbulent plasmas (Dupree 1972). In general, the ‘clumps’ were considered to originate from some kind of plasma instability: the instability grows to break down the linearization procedure; nonlinear effects tend to limit the growth, and solitons propagate along a specific direction without appreciable deformation in amplitudes (see fundamental theories in e.g. Dauxois and Peyrard 2006, Davidson 1972, Mamun *et al* 2002, Shivamoggi 1988, Shukla and Yu 1978, Reddy *et al* 2002). Specifically, the ‘clumps’ were described as a result of the incomplete blending of a Vlasov plasma (Chiueh and Diamond 1986, Pécseli 1985): wave–particle interactions make stochastic orbits of particles by turbulent electric fields; the phase-space density tends to decrease to smaller scales in a finite time, and thus generate cylindrically symmetric density granulations (or ‘space-charge’ elements that are either over-dense or under-dense). In the development, the turbulent forces produced by the turbulent electromagnetic field tend to tear the chunk of particles apart and cause the decay of clumps. However, the size of clumps is so small that they keep every particle feeling the same force. Thus, space charges retain their structural integrity for a relatively longer time than the average correlation time of the system. When these clumps ballistically propagate along a specific direction at a resonant speed, a ‘train’ of solitons develops from any initial perturbation. If the propagation is in a magnetic field, the electrostatic structures can be striating along magnetic field lines, called field-aligned ‘filamentation’ (Mottez *et al* 1992).

Both observational and theoretical studies have shown that besides waves there exist other kinds of ingredients in turbulent plasmas: non-wave solitary structures. These structures are cylindrically symmetric, carrying space charges while sustaining localized, electrostatic electric fields. An investigation of previous studies in the last 30 years on non-thermal plasmas under various electric field structures has led us to a new understanding of the transverse ion-heating mechanism: the perpendicular electrostatic electric field of propagating solitons is the prime mover of ion energization. It provides energy to ions just as electric power provides energy to an electric bulb; a single soliton appears in space periodically just as the power is turned on and off regularly. For a negatively charged soliton, when the switch is on, local ions face a process of acceleration and gain energy from the field; if the switch is off, ions will return to an initial state but with an enhanced perpendicular drift velocity and/or temperature gained in the previous electric field. When the electric field continues to be on and off repeatedly during the propagation of solitons, ions can be heated or accelerated continuously to extraordinary levels. This field-excitation (rather than wave-excitation) mechanism was suggested by one of the authors (J-PSM) in the research of auroral energetic outflows affected by either large-scale (up to thousands of km) or small-scale (tens of meters) transverse dc electric field structures. Although irrelevant of soliton physics when proposed, the brilliant idea sheds light on the new perspective for the mechanism of transverse ion heating: ions are energized by the electrostatic space-charge electric field of the non-wave solitons that propagate along magnetic flux tubes in turbulent plasmas.

The behavior of charged particles in crossed **E** and **B** is a classical topic in plasma physics (see e.g. a newly published textbook (Smirnov 2007) and references therein).

For ions under different electric field structures, $\mathbf{E} \times \mathbf{B}$ drifts are different. Usually, the drift is considerable (of the order of $\mathrm{km\,s^{-1}}$) compared with the normal ion thermal speeds, but several orders smaller than the electron thermal speeds. As a result, there can be strong distortions for ions (St Maurice and Schunk 1979): they depart from a thermal (Maxwellian) distribution to a non-Maxwellian one, and the related bulk parameters (e.g. drift and temperature) deviate from their respective thermal equilibrium values. But for electrons, they still maintain thermal equilibrium.

In spatially homogeneous electric fields that are constant with time, various non-Maxwellian distribution functions have been studied in great detail since Cole (1971). In this pioneering work, Cole determined how far ion distribution can digress from the equilibrium Maxwellian in a uniform electric field in the collisionless case. He found that the distribution function is determined completely by the characteristics of ion motion or, precisely, by the energy conservation of ion motion in the electromagnetic field. By considering an initial Maxwellian ion distribution in the absence of the electric field, he obtained an ion velocity distribution that pulsated about the $\mathbf{E} \times \mathbf{B}$ drift. For $E \geqslant 10\,\mathrm{mV\,m^{-1}}$, the average ion distribution started to deviate significantly from a Maxwellian. Schunk and Walker (1972) generalized Cole's work by including the collisional term. Formally, the collision term is described by the Boltzmann collision integral. By expanding the distribution function using a complete orthonomal polynomial series of the products of Sonine polynomials and spherical tensors, the authors obtained a Maxwellian-weighting, infinite-component solution of the Boltzmann equation. The results were applicable to small distortions from the initial Maxwellian, such as can be found with small electric fields, and/or high ion–neutral collision frequencies. They found that the departure from a Maxwellian was small below 120 km (where the collision frequency is much greater than the ion-cyclotron frequency). Above that altitude, the departure was basically the same at all altitudes below 300 km. As with Cole's results, a $10\,\mathrm{mV\,m^{-1}}$ electric field was deemed to be strong enough to cause appreciable non-Maxwellian effects.

St Maurice and Schunk (1973, 1974) expanded Schunk and Walker's work to strong electric fields. They used a simple relaxation model in place of the Boltzmann collision integral. Physically, this model assumes that the target neutral has a velocity $\mathbf{v}_{\mathrm{n}}$, satisfying a Maxwellian distribution. When an ion with velocity $\mathbf{v}_{\mathrm{i}}$ collides with the neutral, the particles exchange velocities ($\mathbf{v}_{\mathrm{n}}$ is replaced by $\mathbf{v}_{\mathrm{i}}$ and vice versa). The ion takes the neutral's Maxwellian velocity distribution in phase space before regaining momentum and energy from the electromagnetic fields. The neutral subsequently moves with the traded velocity until colliding with other neutrals. The timescales over which the entire ion velocity distribution will change is, not surprisingly, ν_{in}^{-1}, a time long enough for all ions to experience roughly one collision. They obtained analytical solutions to the Boltzmann equation for all electric field strengths and collision frequencies. In the small collision regime ($\nu_{\mathrm{in}} \ll \Omega_{\mathrm{i}}$), namely, above 150 km, the ion distribution formed a torus for electric fields higher than $40\,\mathrm{mV\,m^{-1}}$. By contrast, in a region close to 120 km ($\nu_{\mathrm{in}} \sim \Omega_{\mathrm{i}}$), the distribution was bean-shaped for electric fields of $100\,\mathrm{mV\,m^{-1}}$ or more. The departure of ion distribution from Maxwellian distribution was confirmed by radar and satellite measurements (Lockwood *et al* 1987, St Maurice *et al* 1976, Swift 1975).

If the electric field is inhomogeneous, that is, there are spatial divergences in $\mathbf{E}$ (and thus shears in the $\mathbf{E} \times \mathbf{B}$ drift), the physical mechanism and mathematical formulations become much more complicated than what was discussed in the last paragraphs. Even the simplest parameter, the ion gyrofrequency Ω_{i}, changes from $\Omega_{\mathrm{i}} = qB/m_{\mathrm{i}}$ in the homogeneous field to a new definition in the present inhomogeneous field. For example, if the first derivative of the field ($\nabla \cdot \mathbf{E} = \mathrm{d}E_x/\mathrm{d}x$) is nonzero while the second derivative ($\mathrm{d}^2E_x/\mathrm{d}x^2$) is zero, Cole (1976) showed that the effective gyrofrequency ω satisfies $\omega^2 = \Omega_{\mathrm{i}}^2 + \Omega_{\mathrm{i}}\mathrm{d}(E_x/B)/\mathrm{d}x$. In addition, he found that it is possible to accelerate ions when the gyroradius is larger than the scale length of the potential well. Rothwell *et al* (1995) stated that for $\mathrm{d}^2E_x/\mathrm{d}x^2 \neq 0$, there are two criteria about the gradients: $\mathrm{d}(E_x/B)/\mathrm{d}x \geqslant \Omega_{\mathrm{i}}$ and $\mathrm{d}^2(E_x/B)/\mathrm{d}x^2 \geqslant 3\omega\Omega_{\mathrm{i}}/(8v_0)$ (where v_0 is the initial velocity). For the first condition, the kinetic theory has to be used instead of either ideal or non-ideal magneto-hydro-dynamic (MHD) theory; if the second one is satisfied, unbounded ions come into being. For a more complicated field, $E_x \sim x\mathrm{e}^{-x^2}$, Rothwell *et al* (1992) and Anastasiadis *et al* (2004) showed that particles are either trapped or untrapped in the potential well; also, if the characteristic length of the well is comparable to the gyroradius, stochastic heating can occur depending on initial conditions of the phase angle $\theta = \arctan(v_{y0}/v_{x0})$ (where v_{x0} and v_{y0} are initial velocity components) and kinetic energy.

In the collision-free regime, Ganguli *et al* (1988) obtained collision-free ion velocity distributions following the onset of a perpendicular electric field with a linear divergence. The authors had in mind instability calculations in the presence of shears at high enough altitudes, so that collisions would not affect the velocity distributions for a sufficiently long time. After finding the invariants for the problem, the authors used an arbitrary distribution of this invariant without attempting to relate this distribution to a particular initial or boundary condition. This choice of solution may have been valid for a slowly changing electric field in time. By contrast, in the collisional regime, St Maurice *et al* (1994) solved the complicated Boltzmann equation in an inhomogeneous, transverse electric field that varies linearly in space along a particular direction: $\mathbf{E} = E_0(1 + y/L)\hat{\mathbf{y}}$ in Cartesian geometry (where E_0 is the field at $y = 0$, L the characteristic length and y the ion position). The calculations were carried out for a simple relaxation collision model, focusing on the collision-dominated case. The authors found that a phase-average model involving ion characteristics could be used to solve the ion velocity distribution, and associated transport properties. The results showed 'horseshoe-shaped' ion distributions. The stronger the electric field, the more evident the horse-shoe shape. Stimulated by the above two studies, Ma and St. Maurice *et al* (2008) assumed a cylindrically symmetric electric field that is 'constant' in time, but proportional to the radial position, i.e. a 'radially linear' electric field in space. They solved the collision-free Boltzmann equation by tracking the ions back in time, using the temporal link between the initial position and velocity of an ion and the arbitrary position and velocity at any time. They obtained completely analytical solutions for the ion trajectories and the ion distribution function, in addition to the transport properties everywhere in space and at any time. The results showed that individual ions gyrate in phase at an effective gyrofrequency (ω) that is different from the conventional magnetic gyrofrequency (Ω_i), while the associated velocity distribution pulsated at a non-steady rate with time. Nevertheless, for an initially uniform Maxwellian velocity distribution, the distribution remained Maxwellian for all times, although the drift, density and temperature of that distribution kept changing with time but stayed independent of position.

Based on observational and theoretical achievements reported so far by previous authors in the study of solitary waves and ions' kinetics in crossed electric and magnetic fields, this paper develops a theoretical framework to illuminate the new mechanism for transverse ion heating: the heating is energized efficiently by propagating solitons, the space charges of which contribute a localized transverse electric field. As a first step, we tackle a collision-free situation in a cylindrical geometry for which the electric field is produced by electron solitons, that is, the space charge is negative. We assume that the field changes in time sinusoidally due to the periodic occurrence of solitons at a specific position. We also take a simple inhomogeneous field structure in space to simulate the measured localized electric field of solitary waves in space. We focus on ions that are well inside the space-charge cylinder to calculate the heating effect. The layout of this paper is as follows. Section 2 introduces an algorithm to solve the collision-free Boltzmann equation, and shows how the ion distribution function and temperature components are obtained for the simplest case where only a single train of solitons exists in the flux tube. Section 3 considers a dense cluster of soliton sets. For simplicity, we assume a continuum space-charge model for soliton sets in any cross section of the flux tube. Under the condition that the charge density is constant in time, we present results of a numerical study for the enhancements in both the bulk kinetic energy and the temperature of ions. The last section summarizes the results and discusses the heating process by means of a cartoon. SI units are used throughout the paper.

2. Heating effect by a single train of solitons

In order to provide the most basic picture for the new mechanism, and thus to gain important insights into more complicated situations, while still being able to illustrate the process clearly, we start by calculating the enhancement in bulk kinetic energy and temperature components of ions contributed by a single train of over-dense electron solitons in a magnetic flux tube. When a soliton appears, the space charges it carries are assumed to be cylindrically symmetric in space and homogeneous with a characteristic radius R_c. It has a density $n_{sc} = n_0 + \delta n_0$ (where n_0 is the background plasma density in the flux tube and δn_0 is the difference between n_0 and n_{sc}). We neglect the end effects of solitons by assuming that they are long charge cylinders extending to infinity at both ends along magnetic field lines. We do not consider the parallel electric field sustained by solitons in the present study.

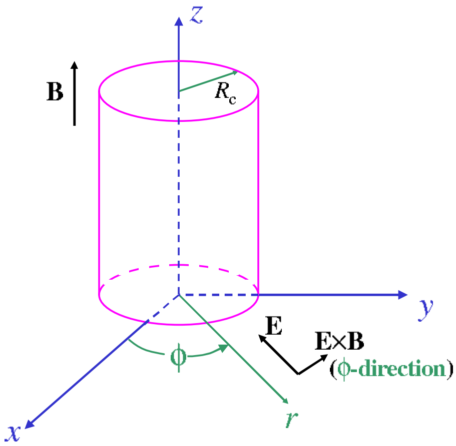

Figure 1. Cylindrical coordinates with reference to the Cartesian frame for a single train of solitons in a magnetic flux tube, the symmetric line of which lies on the z-axis. R_c is the characteristic radius of the flux tube.

We choose a cylindrical frame (r, ϕ, z) with the axis along $\mathbf{B}$ that is assumed to be homogeneous in space: $\mathbf{B} = B\hat{\mathbf{e}}_z$ (where $\hat{\mathbf{e}}_z$ is the unit axial vector), as shown in figure 1.

Soliton space charges produce a radial electric field: $\mathbf{E} = \mathbf{E}_r = \mathbf{E}_r\hat{\mathbf{e}}_r$ (where $\hat{\mathbf{e}}_r$ is the unit radial vector). Strength E_r is defined by

$$E_r = -E_c \cdot g_1(r) g_2(t), \tag{1}$$

where E_c is a characteristic strength of E_r, and $g_1(r)$ and $g_2(t)$ denote the radial structure and temporal variation of E_r, respectively. In the crossed $\mathbf{E}$ and $\mathbf{B}$ fields, ions have an $\mathbf{E}\times\mathbf{B}$-drift in the ϕ-direction. For typical values $B = 5\times 10^{-5}$ T (0.5 G) and $E_c = 0.1$–$1\ \mathrm{V\,m^{-1}}$, $\mathbf{E}\times\mathbf{B}$ drift speed is $E_c/B = 2$–$20\ \mathrm{km\,s^{-1}}$ if $g = 1$. Take ions and electrons in the auroral plasma for a discussion. For the main ion gradient, O^+, its equilibrium thermal speed v_{th} is $1\ \mathrm{km\,s^{-1}}$ at a temperature $T_0 = 1000$ K, while for electrons, the thermal speed exceeds $100\ \mathrm{km\,s^{-1}}$. This means that ions are strongly modulated by the $\mathbf{E}\times\mathbf{B}$ drift. In other words, the ion velocity distribution will depart from the thermal equilibrium greatly, resulting in a different ion temperature $T = \{T_r, T_\phi\}$ in the perpendicular plane. If $T > T_0$, we say ions are heated. Another parameter to indicate the heating effect is the bulk kinetic energy K_i of ions. It is defined as $K_i = m_i v_D^2/2$, where m_i is the ion mass and $\mathbf{v}_D = \{v_{dr}, v_{d\phi}\}$ is the drift velocity (also called 'average velocity', 'guiding-center velocity') of ions. Initially, $K_i = 0$ because the average velocity of ions is zero at the Maxwellain thermal equilibrium. For electrons, by contrast, any departure from a Maxwellian velocity distribution is so small as to be neglected due to the large gap between their thermal speed and the $\mathbf{E}\times\mathbf{B}$ drift. Thus, electrons are still Maxwellian and experience $\mathbf{E}\times\mathbf{B}$ drifting as a fluid for the situation at hand.

We calculate T and $\mathbf{v}_D$ by using the velocity moments of the ion distribution function, f_i, via (Schunk and Nagy 2000)

$$\left.\begin{aligned}
&T_r = 2\left(\langle v_r^2\rangle - \langle v_r\rangle^2\right), \quad T_\phi = 2\left(\langle v_\phi^2\rangle - \langle v_\phi\rangle^2\right),\\
&v_{dr} = \langle v_r\rangle = \frac{1}{n_i}\int v_r f_i \mathrm{d}\mathbf{v}, \quad v_{d\phi} = \langle v_\phi\rangle = \frac{1}{n_i}\int v_\phi f_i \mathrm{d}\mathbf{v},\\
&\langle v_r^2\rangle = \frac{1}{n_i}\int v_r^2 f_i \mathrm{d}\mathbf{v}, \quad \langle v_\phi^2\rangle = \frac{1}{n_i}\int v_\phi^2 f_i \mathrm{d}\mathbf{v}, \quad n_i = \int f_i \mathrm{d}\mathbf{v},
\end{aligned}\right\} \tag{2}$$

in which T_r and T_ϕ are normalized by T_0; v_r and v_ϕ are the ion speed components in r and ϕ directions, respectively, normalized by v_{th}; $\langle\cdot\rangle$ denotes the average in velocity space; n_{i} is the ion density normalized by the initial $n_{\text{i}0}$; and f_{i} is obtained numerically by solving the collision-free Boltzmann equation:

$$\frac{\mathrm{D}f_{\text{i}}}{\mathrm{D}t}=\frac{\partial f_{\text{i}}}{\partial t}+\mathbf{v}\cdot\nabla f_{\text{i}}+\frac{e}{m_{\text{i}}}(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\nabla_{\mathbf{v}}f_{\text{i}}=0. \qquad (3)$$

When following ion trajectories in velocity space, we obtain the solution of the above equation:

$$\left.\begin{aligned} &f_{\text{i}}(r,\mathbf{v},t)=f_{\text{i}}(r_0,\mathbf{v}_0,0)=f_0=\frac{n_0}{\pi}\mathrm{e}^{-(v_{r0}^2+v_{\phi0}^2)}\\ &=\frac{n_0}{\pi}\mathrm{e}^{-[v_r^2+v_\phi^2+(P_r-P_{r_0})]}=\frac{n_0}{\pi}\mathrm{e}^{-\left[v_r^2+v_\phi^2-2g_2(t)\frac{E_{\text{c}}}{B}R_{\text{c}}\Omega_{\text{i}}\int_r^{r_0}g_1(r)\mathrm{d}r\right]}\end{aligned}\right\} \qquad (4)$$

in which r is normalized by R_{c}; E_{c}/B and $R_{\text{c}}\Omega_{\text{i}}$ are normalized by v_{th}; t is in units of $T_\Omega=2\pi/\Omega_{\text{i}}$; P is the dimensionless potential energy in units of $m_{\text{i}}v_{\text{th}}^2/2$. The relation between r_0 and r is numerically obtained by employing a back-tracking approach from the equations of motion:

$$\left.\begin{aligned} &\frac{\mathrm{d}r_0}{\mathrm{d}t}=-\frac{2\pi}{R_{\text{c}}\Omega_{\text{i}}}v_{r0},\\ &\frac{\mathrm{d}v_{r_0}}{\mathrm{d}t}=\frac{2\pi}{R_{\text{c}}\Omega_{\text{i}}}\left[\left(\frac{R_{\text{c}}\Omega_{\text{i}}}{2}\right)^2r_0+\frac{E_{\text{c}}}{B}R_{\text{c}}\Omega_{\text{i}}\cdot g_1(r)g_2(t)-\frac{c_k^2}{r_0^3}\right],\\ &c_k=r^2\left(\frac{v_\phi}{r}+\frac{1}{2}R_{\text{c}}\Omega_{\text{i}}\right).\end{aligned}\right\} \qquad (5)$$

The unknown input functions $g_1(r)$ and $g_2(t)$ are specified from an observation-oriented perspective. Ergun *et al* (1998c) observed that the solitary structures produce localized electric fields diminishing to zero for $r\to\infty$, and are evenly spaced close to the proton cyclotron frequency Ω_{i} in time series. We thus choose

$$\begin{aligned} g_1(r)&=r\mathrm{e}^{-0.1r^3},\\ g_2(t)&=|\sin(\pi t)|, \end{aligned} \qquad (6)$$

which shows that in space E_r increases linearly with r for $r<1$ and then decreases quickly after $r>1$; with time, it changes sinusoidally with Ω_{i} but the state of polarization is always positive. Numerical calculations by using equations (2)–(6) then produce evolutions of the bulk kinetic energy K_{i} (represented by the dimensionless v_{D}^2) and the temperature components T_r and T_ϕ with time, as shown in figure 2, under two different electric field strengths: a weak one $E_{\text{c}}/B=0.5$ and a strong one $E_{\text{c}}/B=2$ at $r=0.5$, and $R_{\text{c}}\Omega_{\text{i}}=3$. Both v_{D}^2 and T_{i} evolve in repeated peaks due to the temporal change of $g_2(t)$. The peaks have different amplitudes. They have a fundamental frequency that is determined by the input cyclotron frequency Ω_{i} of $g_2(t)$ in time series, as well as a second-order harmonic which has a frequency of $2\Omega_{\text{i}}$. For v_{D}^2, it increases from zero at the initial state to peak values smaller than 0.35 at $E_{\text{c}}/B=0.5$. After the electric field goes up to $E_{\text{c}}/B=2$, the v_{D}^2 peak can fly up to 14. For T_{i}, we easily see that the two components, T_r and T_ϕ, are different at any time, but both of them are always higher than 1. This means that ions are heated radially and azimuthally with different heating effect. Besides, at the weak electric field, the temperature enhancement is obviously lower than that at the strong one: the maximum peak is ~1.9 in the former case, while it is about 12 in the latter one.

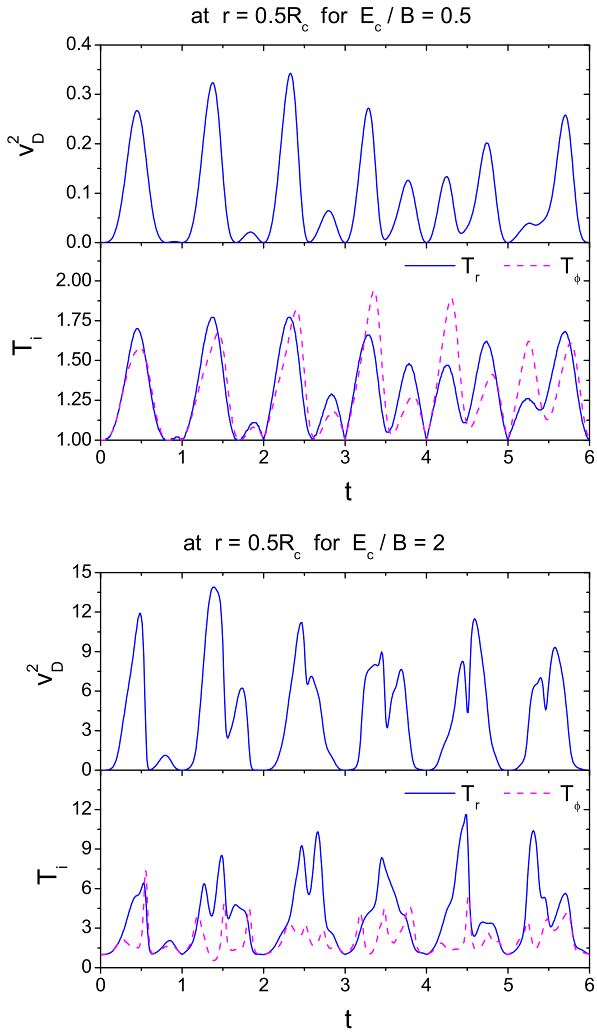

Figure 2. Ion heating by a single train of solitons with an electric profile expressed by equation (6) with $E_{\text{c}}/B=0.5$, 2 and $R_{\text{c}}\Omega_{\text{i}}=3$ (in units of v_{th}) at $r=0.5$ (in units of R_{c}).

The evolving features of both v_{D}^2 and T_{i} expose that the heating effect is directly related to the energy gain of ions in the solitons' electric field. Positive ions are accelerated by the soliton's negative electric field, and thus acquire energy from the field which increases the bulk kinetic energy and temperature. Stronger strength means that ions can obtain more energy and reach higher heated levels. Undoubtedly, the ion heating effect does exist for ions in the solitons' space-charge electric field. However, the single train of solitons is unable to provide sufficient electric power to drive the bulk kinetic energy or the temperature to such a level that can be used to illustrate the observations: v_{D}^2 (or T_{i}) is increased by only one order of magnitude from the initial state, whereas FAST data suggest that ions dominating

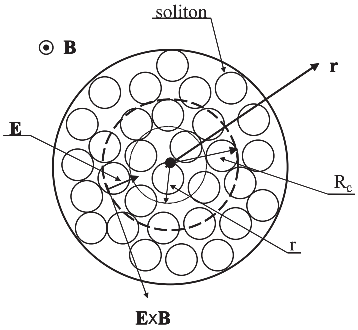

Figure 3. Cross section of figure 1 viewed from top. The magnetic flux tube is teeming with soliton sets (see e.g. Ergun *et al* 1998c, Ergun 1999). The diagram is not to scale in reality, but used to dramatically illustrate the point.

the upward current region are often energized to 1 keV, three orders of increase in magnitude in the up-going fluxes (figure 2(b) in Ergun *et al* 1998a).

3. Heating effect by soliton sets

Rather than a single train of solitons in a magnetic flux tube as assumed in the last section, FAST observations (see e.g. Ergun 1999, Ergun *et al* 1998c) suggested that the solitary structures in auroral current regions should be in sets, the spatial extent of which is perpendicular to the magnetic field scales with ion Larmor radii ρ_i, namely, $2\lambda_D < \rho_i < 20\lambda_D$ (where $\lambda_D = 82$ m is the Debye length); more importantly, the strength of the transverse electric field can be as high as $2\,\mathrm{V\,m^{-1}}$ (or, equivalently, $E/B = 40$). Figure 3 uses a cartoon to show the soliton sets (a dense cluster of soliton trains) in a flux tube.

In this more complicated case, we simplify the system with an emphasis on tackling the mechanism of ion heating: Firstly, we still focus on a collision-free problem, neglecting the ion–neutral interactions. Secondly, we assume that all soliton sets have the same uniform space-charge density δn_0 for each set. Thirdly, we choose a circle (heavy dashed line in figure 3), the characteristic radius of which is R_c. The circle is well inside the flux tube cylinder (heavy solid boundary line); thereby the cylinder-edge effects and features associated with the decaying field outside the cylinder are neglected. Lastly, spaces among soliton sets are negligible.

Under these assumptions, the space-charge density in the flux tube is δn_c if the tube is completely filled with soliton sets. However, all soliton trains are not in phase to appear in space, but come into being randomly in both magnitude and time due to the plasma turbulence. We use a new parameter $\delta\tilde{n}_c$ to denote the stochastic δn_c (the sign '$\tilde{\ }$' means 'stochastic' hereafter). In the area within the radius r (thin solid line), $\delta\tilde{n}_c$ produces a radial electric field $\tilde{E}_r$ by means of Gauss's law:

$$\tilde{E}_r = -\tilde{E}_c r, \tag{7}$$

where $\tilde{E}_c = [eR_c/(2\epsilon_0)]\delta\tilde{n}_c$ is positive. Clearly, the electric field within the cylinder is proportional to r, and points radially inward. We can check that the plasma is quasi-neutral even if the plasma has a density perturbation of $\delta\tilde{n}_c$: if we were to let $\tilde{E}_c = 2\,\mathrm{V\,m^{-1}}$ at $R_c = 1$ km, the corresponding space-charge number density would need to be $\delta\tilde{n}_c \approx 2\times 10^5\,\mathrm{m^{-3}}$. This is 10^4–10^6 times smaller than the ambient plasma density n_0. In such a radial electric field proportional to r, at any time before $\tilde{E}_c$ switches to a new amplitude next, ions reside in a dc electric field with gyrations in a cyclotron frequency ω, which is different from the magnetic gyrofrequency Ω_i (Mikhailovskii 1974):

$$\omega = \Omega_i\sqrt{1 + 4\frac{\tilde{E}_c/B}{R_c\Omega_i}} \tag{8}$$

with a shifted Maxwellian that pulsates with time (see the complete analytical solutions under a dc electric field structure proportional to r in Ma and St Maurice (2008)):

$$f_i(\mathbf{r},\mathbf{v},t) = \frac{n_0}{\pi}\mathrm{e}^{-a_0[(v_r-v_{dr})^2+(v_\phi-v_{d\phi})^2]}, \tag{9}$$

in which

$$\left.\begin{aligned} a_0 &= 1-\frac{1}{2}\left[1-\left(\frac{\Omega_i}{\omega}\right)^2\right](1-\cos\omega t)\,,\\ v_{dr} &= \frac{E_r/B}{a_0}\frac{\Omega_i}{\omega}\sin\omega t,\\ v_{d\phi} &= -\frac{E_r/B}{a_0}\left(\frac{\Omega_i}{\omega}\right)^2(1-\cos\omega t)\,,\\ \left(\mathbf{v}_D-\frac{\mathbf{E}\times\mathbf{B}}{B^2}\right)^2 &= \left(\frac{E_r}{B}\right)^2. \end{aligned}\right\} \tag{10}$$

Note again that the parameters v_{dr}, $v_{d\phi}$ and $\tilde{E}_c/B$ are normalized by v_{th}. Clearly, the bulk velocity $\mathbf{v}_D$ of ions rotates around the $\mathbf{E}\times\mathbf{B}$-drift velocity $\mathbf{v}_\mathbf{E} = \{0, -E_r/B\}$ with a radius $|E_r/B|$, and the ion temperature T_i is

$$T_i = \frac{T_r + T_\phi + T_\parallel}{3} = \frac{1}{3}\left(1+\frac{2}{a_0}\right), \tag{11}$$

in which $T_r = T_\phi = 1/a_0$ and $T_\parallel = 1$.

In simulations, we use $R_c = 50$ m (well inside a flux tube of scales 1 km) and $\Omega_i = 300\,\mathrm{rad\,s^{-1}}$ (for O^+). Thus, $R_c\Omega_i = 15$ (in units of v_{th}). In addition, we consider both a weak and a strong field with $\tilde{E}_c$-peaks to be 0.05 and $0.75\,\mathrm{V\,m^{-1}}$, respectively. The corresponding $\tilde{E}_c/B$-peaks are 1 and 15 (in units of v_{th}), respectively. Thus, the maximum ω turns out to be 1.125 and 2.236 (in units of Ω_i), respectively. Moreover, the random changes of $\tilde{E}_c$ in magnitude and time are simulated by two random number generators, respectively, in the code. We still use v_D^2 and T_i to expose the ion-heating effect.

Figure 4 shows the evolutions of v_D^2 and T_i at two $\tilde{E}_c/B$-peaks (1 and 15), respectively. The electric field switches on at $t = 0$ and off after 4000 cyclotron cycles. These two parameters are greatly influenced by the $\tilde{E}_c/B$-peak: for the peak-1 case, the kinetic energy of ions increases to several hundreds of the initial energy, and the temperature increases to tens of the initial temperature; by contrast, for the peak-15 case, the former goes up to tens of thousands of the initial energy, while the latter becomes more than 1000 times the

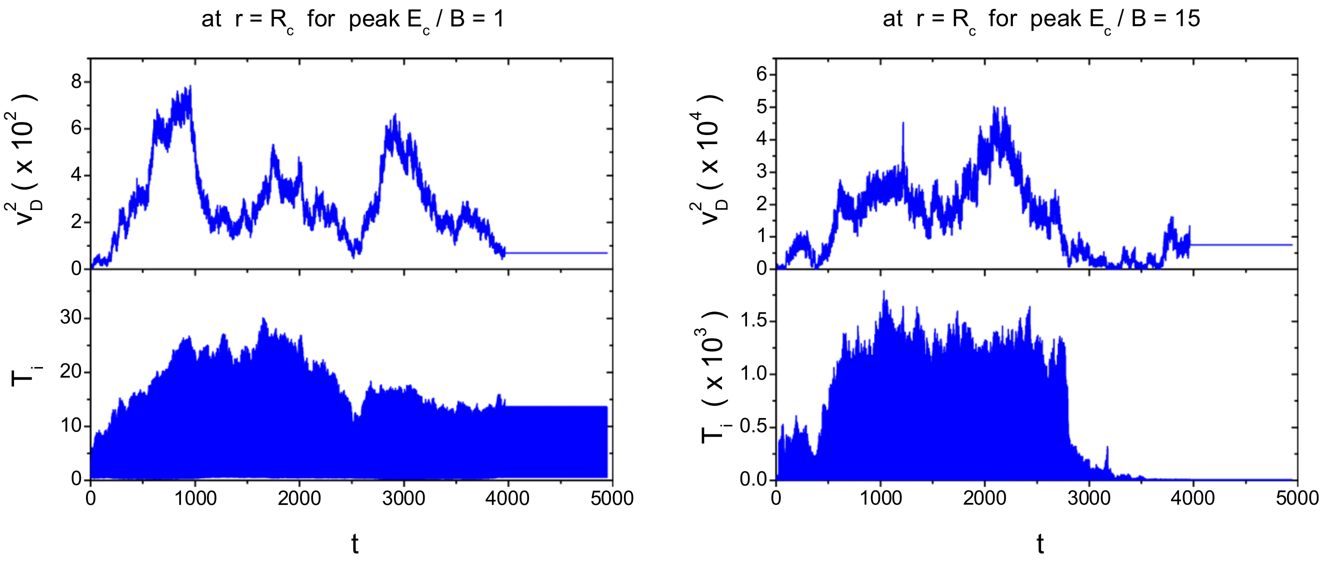

Figure 4. Evolutions of the ion bulk kinetic energy v_{d}^2 (in units of v_{th}^2) and temperature T_{i} (in units of T_0) with time at $\tilde{E}_{\mathrm{c}}/B$-peak taking 1 and 15, respectively.

initial temperature. In addition, after the electric field is turned off, ions continue to maintain v_{D}^2 and T_{i} values obtained in the previous electric field, but do not return them to their respective initial values. This interesting result tells us that even if there is no electric field anymore, the heated ions will remain heated till a new electric field is applied.

However, the two parameters are unable to show the oscillating features of ions that are connected to the cyclotron frequency ω, which ranges from 1 to 1.125 and 2.236 (in units of Ω_{i}), respectively, in the two cases. This implies that the soliton sets devote two independent products to observations: one is the ion heating as introduced above, and the other is the observable stochastic cyclotron oscillations that can be expressed as follows under the present electric field structure (cf Ma and St Maurice *et al* 2008):

$$\left.\begin{aligned}
&\tilde{E}_{ri} = -\tilde{E}_{\mathrm{c}i} r,\\
&r^2 = A_{0i}[1 \pm \varepsilon_{0i} \sin(\omega_{\mathrm{i}} t + \phi_{0i})],\\
&A_{0i} = \frac{b_2}{2b_1}, \quad \varepsilon_{0i} = \sqrt{1 - 4\frac{b_1 b_3}{b_2^2}},\\
&\phi_{0i} = \sin^{-1}\frac{r_{0i}^2/A_{0i} - 1}{\varepsilon_{0i}},\\
&\omega_{\mathrm{i}} = \sqrt{1 + 4\frac{\tilde{E}_{\mathrm{c}i}/B}{R_{\mathrm{c}}\Omega_{\mathrm{i}}}},\\
&b_1 = (\Omega_{\mathrm{i}} R_{\mathrm{c}}/2)^2 + (\tilde{E}_{\mathrm{c}i}/B) R_{\mathrm{c}}\Omega_{\mathrm{i}},\\
&b_2 = b_1 r_{0i}^2 + v_{r0i}^2 + \left(v_{\phi 0i} + \Omega_{\mathrm{i}} r_{0i}/2\right)^2,\\
&b_3 = r_{0i}^2 \left(v_{\phi 0i} + \Omega_{\mathrm{i}} r_{0i}/2\right)^2,
\end{aligned}\right\} \qquad (12)$$

in which the sign '±' before ω takes '+' for $v_{r0i} > 0$ and '−' for $v_{r0i} < 0$; $\{\tilde{E}_{\mathrm{c}i}, \omega_{\mathrm{i}}, r_{0i}, v_{r0i}, v_{\phi 0i}\}$ are effective electric field strength, cyclotron frequency, and initial radius and velocity positions, respectively, at the ith step. This is equivalent to saying that cyclotron oscillations do not contribute to ion heating, but come into being as another product in the process of ion energization powered by the solitons' space-charge electric fields, as pointed out 20 years ago by Moore *et al* (1986).

4. Summary and discussion

Aiming at illustrating a new mechanism for transverse ion heating, we established a cylindrically symmetric model for solitons propagating parallel to a magnetic flux tube. To obtain easy access and comprehensive insight into the heating effect, we did not take into consideration the parallel electric field sustained by the solitons, and neglected the boundary effects on the heating process by choosing a radius that is well inside the flux tube cylinder. If only one train of solitons was considered, the transverse ion heating contributed by the solitons' space-charge electric field E_r perpendicular to the magnetic field did occur but the heating effect was not conspicuous, and thus could not be used to account for observations. By contrast, a dense cluster of soliton sets could provide sufficient heating energy to ions by using input parameters (e.g. the peak strength of the stochastic space-charge electric field) estimated from FAST observations. We also showed that the ion-cyclotron oscillations exist in the heating process as a by-product in the process of ion energization.

We address that the observed transverse ion energization is produced by the soliton sets in the nonlinear solitary waves propagating along magnetic flux tubes. The space-charge electric field of solitons is the prime mover for the ion heating. If the electric field is constant, ions oscillate about the $\mathbf{E} \times \mathbf{B}$ drift with a cyclotron frequency ω, which is larger than but in the same order of the ion gyrofrequency Ω_{i} in real space, and follow a shifted, pulsating Maxwellian distribution function in velocity space. This distribution exposes a temperature that changes with time periodically, described by $1/a_0$ (Ma and St Maurice 2008). However, every soliton train constituting the soliton sets appears stochastically in turbulent plasmas. Thus, the resultant electric field is not constant in strength and time but always switches from one state to the other randomly. Ions then experience a series of accelerations (decelerations

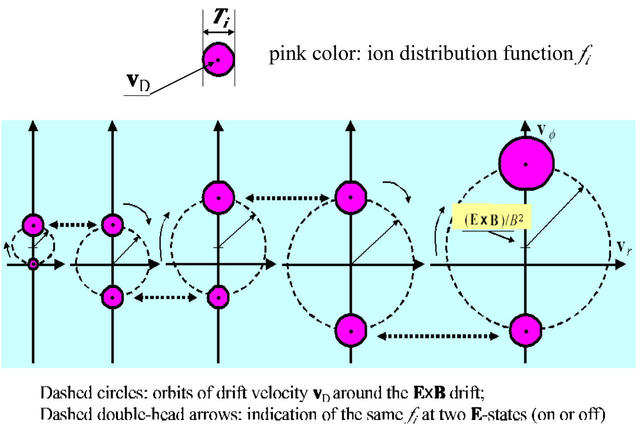

Figure 5. A cartoon showing transverse ion heating by random soliton electric fields at a frequency that matches the local cyclotron frequency and thus gives the maximum heating effect (not a requirement for the heating mechanism, but used to dramatically illustrate the point).

are also involved but with less possibility in a negative soliton electric field) in the electric fields, and eventually acquire high kinetic energy and temperature, which were observed in space projects.

For a better understanding of the mechanism, we consider a simple picture where the electric field has only two states: on and off, as shown by a cartoon in figure 5. When it is on, ions gyrate about $\mathbf{E}\times\mathbf{B}$-drift and gain energy from the field, and thus the drift velocity $\mathbf{v}_\mathrm{D}$ and T_i increase; if at some point E_r is switched off, ions will have the drift velocity and temperature reached just before E_r disappears, and start to pulsate around the origin in a cyclotron frequency but with a new amplitude and a zero mean-velocity. If the electric field continues to be on and off repeatedly, $\mathbf{v}_\mathrm{D}$ and T_i become larger and larger continuously to extraordinary levels.

The transverse ion heating energized by solitons has been benchmarked numerically and theoretically by using a model describing the space-charge electric field carried by solitons in magnetic flux tubes. The study provides a feasible mechanism to explain qualitatively the observed energetic ions observed by e.g. FAST. A quantitative data-fit modeling is thus needed to validate the model. This is going to be done in another paper. In addition, the effect of the parallel electric field sustained by solitons on the heating effect remains to be carried out. Furthermore, the model did not consider the ion–neutral collisions. If this term is considered, the ion distribution function will no longer be a shifted, pulsating Maxwellian but relaxed to a time-averaging shape. This will surely affect the heating effect. Last but not least, solitary waves may be propagating obliquely in the flux tube. We leave these issues for future investigations of a broad range of ion heating topics related to non-wave energization processes based on the present study.

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