Minimal blocking sets in $PG(2,9)$

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Abstract

We classify the minimal blocking sets of size 15 in $PG(2,9)$. We show that the only examples are the projective triangle and the sporadic example arising from the secants to the unique complete 6-arc in $PG(2,9)$. This classification was used to solve the open problem of the existence of maximal partial spreads of size 76 in $PG(3,9)$. No such maximal partial spreads exist [13]. In [14], also the non-existence of maximal partial spreads of size 75 in $PG(3,9)$ has been proven. So, the result presented here contributes to the proof that the largest maximal partial spreads in $PG(3, q = 9)$ have size $q^2 - q + 2 = 74$.

1 Introduction

A spread of $PG(3, q)$ is a set of $q^2 + 1$ lines partitioning the point set of $PG(3, q)$. A partial spread of $PG(3, q)$ is a set of pairwise disjoint lines of $PG(3, q)$ not forming a spread. A partial spread is called maximal when it is not contained in a larger partial spread. Let $S$ be a maximal partial spread of size $q^2 + 1 - \delta$, then $\delta$ is called the deficiency of $S$.

A lot of attention has been paid to the construction of maximal partial spreads. Until recently, the largest known maximal partial spreads in $PG(3, q)$, $q > 3$, were constructed by Bruen [6], Bruen and Thas [7], Freeman [9] and Jungnickel [19], and were maximal partial spreads of size $q^2 - q + 2$.

This led to the conjecture that $q^2 - q + 2$ is the largest size for a maximal partial spread.

However, Heden recently found a maximal partial spread in $PG(3,7)$ of size $(q^2 - q + 3 =)45$ [12].

The validity of this conjecture for $q = 8$ was recently proved by Barát, Del Fra, Innamorati and Storme [1].

Concentrating on $q = 9$, presently, it is known that the deficiency of a maximal partial spread in $PG(3,9)$ satisfies $\delta \geq 6$.

So the first open case is whether there exists a maximal partial spread with deficiency $\delta = 6$.

The standard technique to study this problem is to rely on the link between maximal partial spreads of $PG(3, q)$ and blocking sets of $PG(2,q)$.

A plane of $PG(3, q)$ containing one line of a maximal partial spread $S$ is called a rich plane of $S$. In the other case, this plane is called poor. A point not
lying on a line of $S$ is called a hole of $S$.

Let $S$ be a maximal partial spread of deficiency $\delta$. Then a rich plane contains $\delta$ holes and a poor plane contains $q + \delta$ holes. Moreover, the holes in a poor plane $P$ form a blocking set in $P$. This means that every line of $P$ contains at least one hole. For proofs, we refer to [21, Lemma 2.1]. A trivial blocking set is a blocking set containing a line.

When $S$ is maximal, no line consists entirely of holes. This means that the holes in $P$ form a non-trivial blocking set in $P$.

Hence, lower bounds on the cardinality of non-trivial blocking sets in $PG(2, q)$, and information on the structure of minimal blocking sets in $PG(2, q)$, yield information on maximal partial spreads in $PG(3, q)$.

Presently, the following results are known on non-trivial blocking sets in $PG(2, q)$, which have led to the following results on maximal partial spreads in $PG(3, q)$.

**Theorem 1.1** (1) (Bruen [5]) The smallest non-trivial blocking sets in $PG(2, q)$, $q$ square, have cardinality $q + \sqrt{q} + 1$ and are equal to Baer subplanes $PG(2, \sqrt{q})$.

(2) (Blokhuis, Storme, Szönyi [4]) In $PG(2, q)$, $q$ non-square, $q = p^h, h > 2, p \geq 5, p$ prime, $|B| \geq q + q^{2/3} + 1$ for every non-trivial blocking set $B$.

(3) (Blokhuis [2]) In $PG(2, q)$, $q$ prime, $q > 2, |B| \geq 3(q + 1)/2$ for every non-trivial blocking set $B$.

(4) (Blokhuis, Storme, Szönyi [4]) In $PG(2, q)$, $q$ square, $q = p^h, h > 2, p \geq 5, p$ prime, every non-trivial blocking set $B$ of cardinality $|B| < q + q^{2/3} + 1$ contains a Baer subplane.

(5) (Szönyi [26]) In $PG(2, q)$, $q = p^2, p$ prime, every non-trivial blocking set $B$ of cardinality $|B| < 3(q + 1)/2$ contains a Baer subplane.

**Theorem 1.2** (Polverino, Polverino and Storme [22, 23, 24]) The smallest minimal blocking sets in $PG(2, p^3)$, $p = p_0^h, p_0 \geq 7$, with exponent $e \geq h$, are:

(1) a line,

(2) a Baer subplane of cardinality $p^3 + p^{3/2} + 1$, when $p$ is a square,

(3) a set of cardinality $p^3 + p^2 + 1$, equivalent to

$$\{(x, T(x), 1) | x \in GF(p^3)\} \cup \{(x, T(x), 0) | x \in GF(p^3) \setminus \{0\}\},$$

with $T$ the trace function from $GF(p^3)$ to $GF(p)$,

(4) a set of cardinality $p^3 + p^2 + p + 1$, equivalent to

$$\{(x, x^p, 1) | x \in GF(p^3)\} \cup \{(x, x^p, 0) | x \in GF(p^3) \setminus \{0\}\}.$$

**Corollary 1.3** Let $S$ be a maximal partial spread of $PG(3, q)$ of deficiency $\delta$.

Then

(1) $\delta \geq \sqrt{q} + 1$ when $q$ is square,

(2) $\delta \geq q^{2/3} + 1$ when $q$ is non-square, $q = p^h, h > 2, p \geq 5, p$ prime,

(3) $\delta \geq (q + 3)/2$ when $q$ is an odd prime.
Corollary 1.4 (Metsch and Storme [21]) (a) Suppose that $\delta$ is an integer and $q$ square, $q = p^h, h > 2, p \geq 5, p$ prime, such that $0 < \delta < q^{2/3} + 1$.

If $\mathcal{S}$ is a maximal partial spread of $PG(3, q)$ with $q^2 + 1 - \delta$ lines, then $\delta = s(\sqrt{q} + 1)$ for an integer $s \geq 2$ and the set of holes is the disjoint union of $s$ Baer subgeometries $PG(3, \sqrt{q})$.

(b) Suppose that $\delta$ is an integer and $q = p^2, p$ prime, $q > 4$, such that $0 < 2\delta \leq q + 1$.

If $\mathcal{S}$ is a maximal partial spread of $PG(3, q)$ with $q^2 + 1 - \delta$ lines, then $\delta = s(\sqrt{q} + 1)$ for an integer $s \geq 2$ and the set of holes is the disjoint union of $s$ Baer subgeometries.

Theorem 1.5 (Metsch and Storme [21]) Let $\mathcal{S}$ be a maximal partial spread of $PG(3, q^3), q$ non-square, $q = p^h, h \geq 2, p$ prime, $p \geq 7$, of deficiency $\delta \leq q^2 + q + 1$. Then $\delta = q^2 + q + 1$ and the set of holes forms a projected subgeometry $PG(3, q^3)$.

Theorem 1.6 (Metsch and Storme [21]) Let $\mathcal{S}$ be a maximal partial spread of $PG(3, q^3), q = p^h, h \geq 2, p$ prime, $p \geq 7$, of deficiency $\delta \leq q^2 + q + 1$.

Then, (1) $\delta \equiv 0 \pmod{q^{3/2} + 1}$, $\delta \geq 2(q^{3/2} + 1)$, and the set of holes is the union of disjoint subgeometries $PG(3, q^{3/2})$, or (2) $\delta = q^2 + q + 1$ and the set of holes forms a projected subgeometry $PG(5, q)$ in $PG(3, q^3)$.

In the following theorems, for $q = p^3, p$ prime, $p \geq 17, \delta_0$ is the largest integer smaller than $(3p^3 + 27p^2 - 5p + 25)/25$. For $p = 7, 11, 13$, $\delta_0 = 90, \delta_0 = 285$ and $\delta_0 = 441$ respectively. For $q = p^3, p = p_0^h, p_0$ prime, $p_0 \geq 7, h > 1, \delta_0$ is defined as the largest integer smaller than $(3p^3 + 27p^2 - 5p + 25)/25$ and smaller than the value $\delta'$ for which $p^3 + \delta'$ is the cardinality of the smallest non-trivial minimal blocking set in $PG(2, p^3)$ of cardinality larger than $p^3 + p^2 + p + 1$.

Theorem 1.7 (Ferret and Storme [8]) Let $p = p_0^h, p_0 \geq 7$ a prime, $h \geq 1$ odd. The set of holes of a maximal partial spread in $PG(3, p^3)$ of deficiency $\delta \leq \delta_0$ is the disjoint union of projected $PG(5, p')$'s of cardinality $p^3 + p^3 + p^3 + p^2 + p + 1$, and so $\delta = s(p^2 + p + 1)$ for some integer $s$.

Theorem 1.8 (Ferret and Storme [8]) Let $p = p_0^h, p_0 \geq 7$ a prime, $h > 1$ even. The set of holes of a maximal partial spread in $PG(3, p^3)$ of deficiency $\delta \leq \delta_0$ is the disjoint union of $PG(3, p^{3/2})$'s and of projected $PG(5, p')$'s of cardinality $p^3 + p^3 + p^3 + p^2 + p + 1$ and so the deficiency $\delta$ of a maximal partial spread in $PG(3, p^3)$ can be written as $\delta = r(p^{3/2} + 1) + s(p^2 + p + 1)$ for some integers $r$ and $s$.

In $PG(2, 8)$, the following results on the smallest non-trivial blocking sets are known.

Theorem 1.9 (Innamorati and Zuanni [17]) Let $\mathcal{B}$ be a non-trivial minimal blocking set of size 13 in $PG(2, 8)$, then $\mathcal{B}$ is projectively equivalent to the set

$$\{(t, t + t^2 + t^4, 1)|t \in GF(8)\} \cup \{(t, t + t^2 + t^4, 0)|t \in GF(8) \setminus \{0\}\}.$$
Theorem 1.10 (Barát, Del Fra, Innamorati and Storme [1]) There do not exist minimal blocking sets of size 14 in $\text{PG}(2, 8)$.

The two preceding results led to the following sharp result on the size of the largest maximal partial spreads in $\text{PG}(3, 8)$.

Theorem 1.11 (Barát, Del Fra, Innamorati and Storme [1]) The largest maximal partial spreads in $\text{PG}(3, 8)$ have size $q^2 - q + 2$.

In all of the preceding results on maximal partial spreads in $\text{PG}(3, q)$ of deficiency $\delta$, information on minimal blocking sets of size $q + \delta$ in $\text{PG}(2, q)$ was of crucial importance.

To prove the non-existence of maximal partial spreads of deficiency $\delta = 6$ in $\text{PG}(2, 9)$ in [13], we will classify the non-trivial blocking sets of size $15 = q + \delta$ in $\text{PG}(2, q = 9)$. We will show that next to the classical example of the projective triangle, there is a unique second example.

The minimal blocking sets of size 15 in $\text{PG}(2, q = 9)$ are minimal blocking sets of size $3(q + 1)/2$.

Regarding their classification in other planes $\text{PG}(2, q)$, for small odd values of $q$, we note that also in $\text{PG}(2, 7)$ and in $\text{PG}(2, 13)$, there is a unique example different from the projective triangle. But in $\text{PG}(2, q)$, $q = 11$, or $q$ an odd prime number satisfying $17 \leq q \leq 37$, the projective triangles are the only examples of minimal blocking sets of size $3(q + 1)/2$ (see Blokhuis, Brouwer and Wilbrink [3]).

Regarding the classification of the largest maximal partial spreads in $\text{PG}(3, 9)$, we note that also the non-existence of maximal partial spreads of size 75 in $\text{PG}(3, 9)$ has been proven [14]. This altogether proves that the largest maximal partial spreads in $\text{PG}(3, q = 9)$ have size $q^2 - q + 2 = 74$.

2 The known minimal blocking sets of size 15

Presently, there are two known examples of minimal blocking sets of size 15 in $\text{PG}(2, 9)$.

2.1 The projective triangle

The first example is the projective triangle [15, Lemma 13.6]. This is the set of points projectively equivalent to the set

$$\{(0, 1, a_0), (1, 0, a_1), (-a_2, 1, 0) | a_0, a_1, a_2 \text{ squares of } GF(9)\}.$$  

There are exactly three non-concurrent 6-secants to the projective triangle. The intersection points of two of these 6-secants are called the vertices of the projective triangle.

A vertex lies on two 6-secants, four 2-secants and four tangents to the projective triangle. A non-vertex point of the projective triangle lies on one 6-secant, four 3-secants, one 2-secant and four tangents.
2.2 The sporadic blocking set

In $PG(2,9)$, there is a unique complete 6-arc [15, p. 386]. The 15 bisecants to this complete 6-arc form a minimal blocking set in the dual projective plane.

So, dualizing this situation, a sporadic example of a minimal blocking set of size 15 arises.

The characteristic properties of this sporadic example are:

1. There are exactly six 5-secants to this blocking set which form a complete 6-arc of lines.
2. There are ten 3-secants to the blocking set. These ten 3-secants form a dual conic.
3. And furthermore, there are fifteen 2-secants to the blocking set. These fifteen 2-secants are the secants to a complete 6-arc in $PG(2,9)$.

3 The classification of the minimal blocking sets of size 15

From now on, let $B$ be a minimal blocking set of size 15 in $PG(2,9)$. Since $B$ is non-trivial, a line $L$ intersects $B$ in at most 6 points. Namely, for a fixed point $p \in L \setminus B$, the nine lines through $p$ which are different from $L$ all contain at least one point of $B$, so $L$ contains at most 6 points of $B$. Blocking sets of size 15 in $PG(2,9)$ having at least one 6-secant are called blocking sets of Rédei-type [25].

3.1 Introductory results

**Lemma 3.1** Every point of $B$ lies on at least four tangents to $B$.

**Proof:** Let $p \in B$ and let $L$ be a tangent line to $B$ at $p$. Consider $PG(2,9) \setminus L$ and call this $AG(2,9)$. Then a set $B \setminus L$ of size 14 remains.

A minimal blocking set in $AG(2,9)$ contains at least 17 points [18]. This means that we need to add at least three points to $B \setminus L$ to get a blocking set in $AG(2,9)$.

The only external lines to $B \setminus L$ in $AG(2,9)$ are the tangents to $B$ at $p$ (different from $L$). Since at least three points need to be added to $B \setminus L$ to obtain a blocking set in $AG(2,9)$, there are at least three external lines to $B \setminus L$ in $AG(2,9)$; so $p$ lies already on at least three tangents to $B$, different from $L$. Also $L$ is a tangent line to $B$. Hence $p$ lies on at least four tangents to $B$. 

**Lemma 3.2** $B$ has at least one secant with at least four points.

**Proof:** Suppose there are only 1-, 2- and 3-secants. Let the number of them be denoted by $a$, $b$ and $c$ respectively. Then the following equations must hold by standard counting arguments.
\[
a + b + c = 91 \\
a + 2b + 3c = 150 \\
2b + 6c = 210
\]

From these equations, \( b = -33 \), which is a contradiction. \( \square \)

3.2 There are at least 5- and/or 6-secants

Suppose that there are only 1-, 2-, 3- and 4-secants. Let the respective numbers be \( a, b, c, d \). Then the standard counting arguments imply that

\[
b = -3a + 201 \\
c = 3a - 188 \\
d = -a + 78
\]

So \( a \geq 63 \).

It is impossible that there is a point lying on at least 9 tangents. Namely, if a point \( p \) of \( B \) lies on at least 9 tangents, then the 14 other points of \( B \) lie on the tenth line through \( p \), which is impossible. If a point \( p \) not belonging to \( B \) lies on 9 tangents, then the tenth line contains the 6 remaining points of \( B \), but this contradicts the fact that there are at most 4-secants to \( B \). So, the tangents form a \((k,8)\)-arc in the dual plane of \( PG(2,9) \). Table 5.4 of [16] shows us that a \((k,8)\)-arc in \( PG(2,9) \) contains at most 65 elements, so there are at most 65 tangents to \( B \).

So, there are only the following three possibilities:

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**Lemma 3.3** Only the case \((a, b, c, d) = (65, 6, 7, 13)\) occurs.

**Proof:** Otherwise, the number of 4-secants is at least 14. Two 4-secants always intersect in a point of \( B \). For assume they intersect in a point \( p \) not in \( B \). Then since the eight other lines through \( p \) all contain at least one point of \( B \), \(|B| \geq 2 \times 4 + 8 = 16\), which is false.

Consider a 4-secant \( L \). The (at least) 13 other 4-secants intersect \( L \) in a point of \( B \), so some point \( p \) of \( L \cap B \) lies on at least five 4-secants, the line \( L \) included. But then \(|B| \geq 1 + 5 \times 3 = 16\) when counting the number of points of \( B \) on the lines through \( p \), which is false. \( \square \)

Let \( L \) be a 4-secant. Let \( L : Z = 0 \) where the coordinates of a point are \((x, y, z)\). Let \( r_1 = (0, 1, 0), r_2 = (1, 0, 0) \) be points of \( L \) not belonging to \( B \). Let
Let $r_3, r_4, r_5, r_6$ be the other points of $S = L \setminus B$. We identify $r_1$ with $(\infty)$, $r_2$ with $(0)$, and the points $(1, y, 0)$ with $(y)$. We also identify the affine points $(x, y, 1)$ with $(x, y)$. Let $(a_i, b_i), i = 1, \ldots, 11$, be the points of $B \setminus L$.

Then the following result is valid.

**Lemma 3.4** At most two of the points $r_i, i = 1, \ldots, 6$, lie on a 3-secant.

**Proof:** Suppose the points $r_1$ and $r_2$ lie on 3-secants to $B$. Let $p = (0, 0)$ be the intersection of these 3-secants. Since $r_1$ and $r_2$ are the points at infinity of respectively the vertical and horizontal line through the origin $p$, the vertical and horizontal line through the origin contain three affine points of $B$, and this implies $\{a_i|i = 1, \ldots, 11\} = \{b_i|i = 1, \ldots, 11\} = GF(9)$ where every non-zero element appears once and where zero appears three times in the sequence of elements $a_i$, respectively $b_i$. This shows that $\prod_{i=1}^{11} (X - a_i) = \prod_{i=1}^{11} (X - b_i) = X^{11} - X^3$.

Let 
\[
\sigma_{k,l}(a_1, \ldots, a_{11}; b_1, \ldots, b_{11}) = \sum a_{i_1} \cdots a_{i_k} \cdot b_{j_1} \cdots b_{j_l}
\]
where the sum is over all index sets $\{i_1, \ldots, i_k\}$ and $\{j_1, \ldots, j_l\}$ being disjoint subsets of $\{1, \ldots, 11\}$ of cardinality $k$ and $l$, respectively.

Then $\prod_{i}(X - a_i) = \prod_{i}(X - b_i) = X^{11} - X^3$ implies $\sigma_{1,0} = \sigma_{0,1} = \sigma_{2,0} = \sigma_{0,2} = 0$.

We now use the lacunary polynomial associated with the set $\{(a_i, b_i)|i = 1, \ldots, 11\}$. This is the polynomial
\[
H(X, Y) = \prod_{i=1}^{11} (X + a_iY - b_i) = X^{11} + a(Y)X^{10} + b(Y)X^9 + \cdots,
\]
where $a(Y) = \sigma_{1,0}Y - \sigma_{0,1}$ and where $b(Y) = \sigma_{2,0}Y^2 - \sigma_{1,1}Y + \sigma_{0,2}$.

Since $\sigma_{1,0} = \sigma_{0,1} = \sigma_{2,0} = \sigma_{0,2} = 0$, $a(Y)$ is identically zero and $b(Y) = -cY$, for some constant $c$.

So, $H(X, y) = (X^9 - X)(X^2 - cy)$ for all $(\infty) \neq (y) \in S = L \setminus B$ since all affine lines through such a point must contain a point of $B$.

If $c \neq 0$, then $X^2 - cy$ cannot have a double root for a fixed value $y \neq 0$, so these points $(y)$ lie on two 2-secants to the affine part. On the other hand, $c = 0$ would imply that all lines through $p$ and a point of $S$ are 3-secants. If $p \notin B$, then $|B| \geq 1 + 3 \times 6$, which is false. So $p \in B$, but then $B$ is not minimal. \(\square\)

**Lemma 3.5** It is impossible that $B$ has at most 4-secants.

**Proof:** The preceding lemma shows that there are at least eight 2-secants to $B$ since we know that there are at least four points $r_i$ lying on two 2-secants to $B$. But the number $b$ of 2-secants is $b = 6$ (Lemma 3.3). So we have a contradiction. \(\square\)
3.3 The computer search for a minimal blocking set of size 15 of Rédei-type

A minimal blocking set of size 15 of Rédei-type has at least one 6-secant \( L \).

Using MAGMA [20], it was determined that there are two orbits of the group \( \text{PGL}(2,9) \) on the subsets of size 4 of a line \( L \). This gives two possibilities for the orbits of sets of 6 points on such a line. So there are two possibilities for \( L \cap B \). The stabilizer group of the first 6-set acts transitively on the 6 points; the stabilizer group of the other 6-set has two orbits on the 6-set.

Consider the affine plane \( \text{PG}(2,9) \setminus L \). This shares 9 points with \( B \). Every secant \( M \) to \( B \setminus L \) intersects \( L \) in a point of \( B \). For let \( p \) be a point of \( L \setminus B \). Since \( L \) contains already 6 points of \( B \), there only remain 9 other points in \( B \), and since every one of the nine lines through \( p \) different from \( L \) must contain at least one point of \( B \), these nine points of \( B \setminus L \) must lie one by one on the nine lines through \( p \) different from \( L \). So a point of \( L \setminus B \) does not lie on a secant to \( B \setminus L \); secants to \( B \setminus L \) intersect \( L \) in a point of \( L \cap B \).

Suppose the 9 points of \( B \setminus L \) form a 9-arc, then the four points of \( L \setminus B \) extend this 9-arc to a 10-arc since they only lie on tangents to \( B \setminus L \). A 9-arc in \( \text{PG}(2,9) \) consists of 9 points of a conic [16, p. 386], so can only be extended by the tenth point of this conic to a 10-arc.

So there are at least three collinear points in \( B \setminus L \). The line containing these collinear points intersects \( L \) in a point of \( B \). Using the preceding results on the stabilizer groups of the two possibilities for the 6-sets \( B \cap L \), there are in total three possibilities for this intersection point.

So it is possible to determine 9 points of \( B \), without having too many possibilities.

The computer search showed that the projective triangles are the only examples.

**Theorem 3.6** The projective triangles are the only minimal blocking sets of size 15 in \( \text{PG}(2,9) \) that are of Rédei-type.

3.4 The computer search for a minimal blocking set of size 15 having no 6-secants, but at least one 5-secant

First of all, MAGMA showed that the group \( \text{PGL}(2,9) \) has two orbits on the 5-sets of a projective line. So, for the 5-secant \( L \) to \( B \), there are two possibilities for \( L \cap B \).

Consider now the affine part \( B \setminus L \) of size 10. Here, the following result of Gács gives crucial information on the structure of this affine part.

**Theorem 3.7** (Gács [10]) In \( \text{PG}(2,q) \), let \( B \) be a minimal blocking set of size \( q+k \), and suppose there is a line \( L \) intersecting \( B \) in exactly \( k-1 \) points. Then there is a point \( p \notin B \) such that every line joining \( p \) to a point of \( L \setminus B \) contains two points of \( B \). Hence \( k \geq (q+3)/2 \).
Using this result, we see that there is a point $p$ not in $B$ such that the five lines joining $p$ to the points of $L \setminus B$ each contain two points of $B$; so these lines contain the 10 points of $B \setminus L$.

This information was used to conduct a computer search. The computer search showed that the only example that satisfies this condition is the sporadic example coming from the complete 6-arc in $PG(2, 9)$.

**Theorem 3.8** Every minimal blocking set in $PG(2, 9)$ of size 15 having at least one 5-secant, but no 6-secant, is projectively equivalent to the minimal blocking set arising from the complete 6-arc in $PG(2, 9)$.

4 Application

As indicated in the introduction, this classification of the minimal blocking sets of size 15 in $PG(2, 9)$ was used in [13] to prove the non-existence of maximal partial spreads of size 76 (deficiency 6) in $PG(3, 9)$.

**Theorem 4.1** There do not exist maximal partial spreads of size 76 in $PG(3, 9)$.

In [14], the non-existence of maximal partial spreads of size 75 in $PG(3, 9)$ has been proven. There exist in $PG(3, q = 9)$ maximal partial spreads of size $q^2 - q + 2 = 74$. So the size of the largest maximal partial spreads is now also known in $PG(3, 9)$.

**Theorem 4.2** The largest maximal partial spreads in $PG(3, q = 9)$ have size $q^2 - q + 2 = 74$.

References


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