NATURAL PROOFS FOR STRUCTURE, DATA, AND SEPARATION

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MOTIVATION

Automatic Software Verification

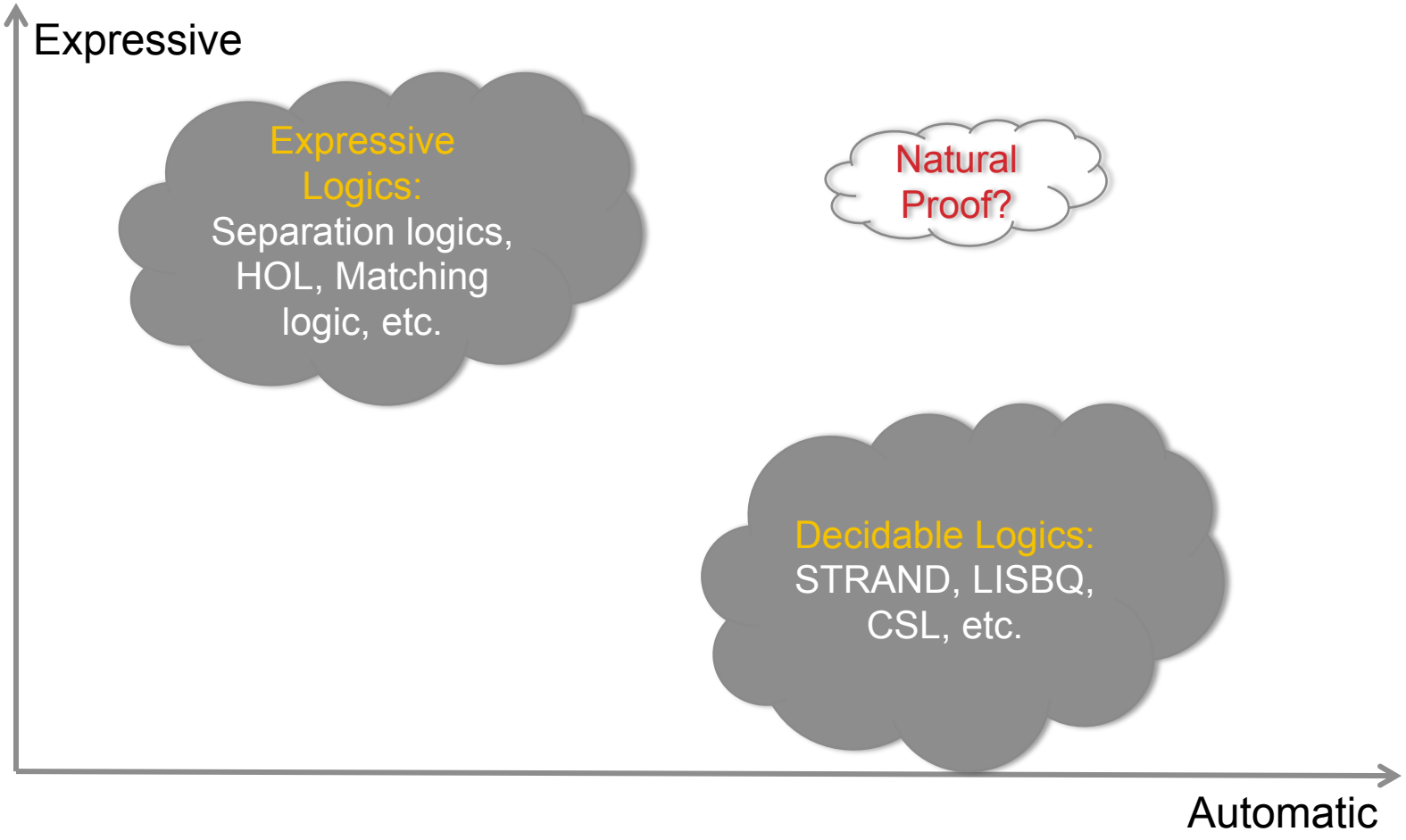
- Modular annotations + VC-Generation + SMT solvers
- Success stories (Boogie, VCC, Verifast, etc.)

How about dynamically modified heap?

- Structure (e.g., "p points to the root of a tree")
- Data (e.g., "the integers stored in a list is sorted")
- Separation (e.g., "the procedure modifies only elements reachable using the $next$ field")

Key Challenge: Expressive Power vs. Automated Reasoning

EXPRESSIVENESS VS. AUTOMATICITY



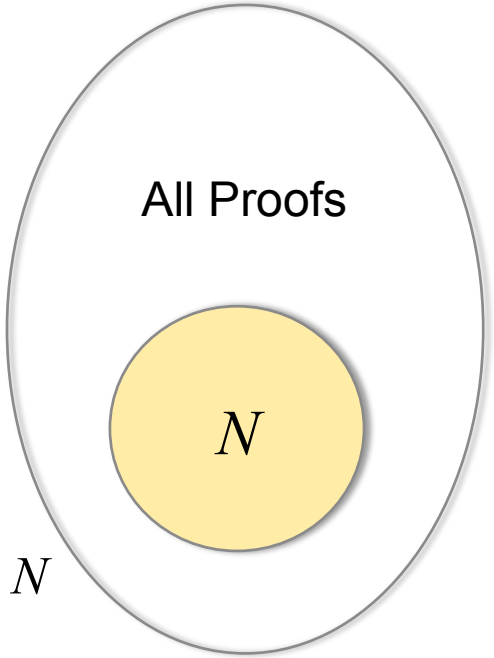
NATURAL PROOFS: IN A NUTSHELL

- **Handle a logic that is very expressive**

  (inevitably undecidable)

- **Retain automaticity at the same level as decidable logics**

- **Identify a class of simple and natural proofs N such that**
  - Many correct programs can be proved using a proof in class N
  - The class N is effectively searchable (searching thoroughly for a proof in N is efficiently decidable)
  - "Unfold recursive defs + formula abstraction"



CONTRIBUTION

- **DRYAD$_{tree}$ [POPL'12]: Natural proofs for trees**
  - only for trees (can't say "x points to a tree and y points into the tree")
  - only functional recursion (no while-loops, too weak for loop invariants)
  - only classical logic (no frame reasoning)
- **Aim**
  - To provide a single logical framework that supports natural proofs for general properties of structure, data, and separation
- **Contribution**
  - DRYAD: A dialect of separation logic
    - no explicit quantification, but supports recursive definitions
    - admits a deterministic translation to classical logic
  - Develop natural proofs for this logic using decision procedures (powered by SMT solvers)
  - A much wider variety of programs verified automatically and efficiently

BINARY SEARCH TREE: AN EXAMPLE OF DRYAD

$$bst^*(x) \overset{def}{=} (x = \text{nil} \wedge \text{emp}) \vee (x \overset{l,r,key}{\mapsto} xl, xr, xk) * \left(bst^*(xl) \wedge keys^*(xl) < \{xk\}\right) * \left(bst^*(xr) \wedge \{xk\} < keys^*(xr)\right)$$

$$keys^*(x) \overset{def}{=} (x = \text{nil} : \emptyset ;$$

$$(x \overset{l,r,key}{\mapsto} xl, xr, xk) * \text{true} : \{xk\} \cup keys^*(xl) \cup keys^*(xr) ;$$

$$\text{default} : \emptyset$$

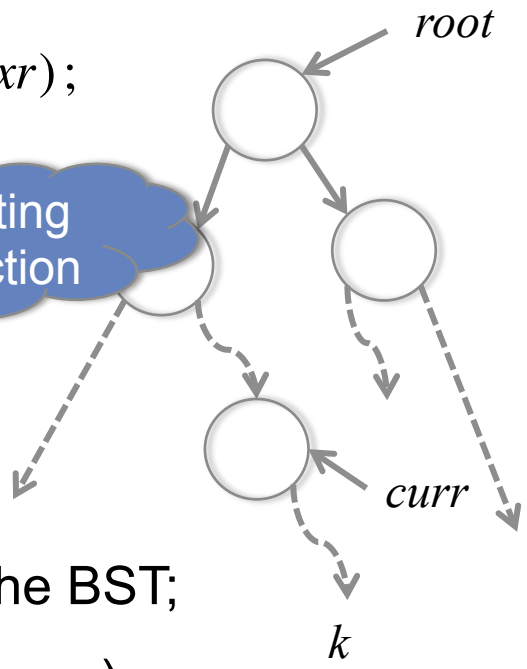
Recursive predicate

Recursive function

Separating Conjunction

$$\psi \equiv bst^*(root) \wedge (bst^*(curr) * \text{true})$$

$$\wedge (k \in keys^*(root) \leftrightarrow k \in keys^*(curr) * \text{true})$$



(loop invariant for "*bst-search*":

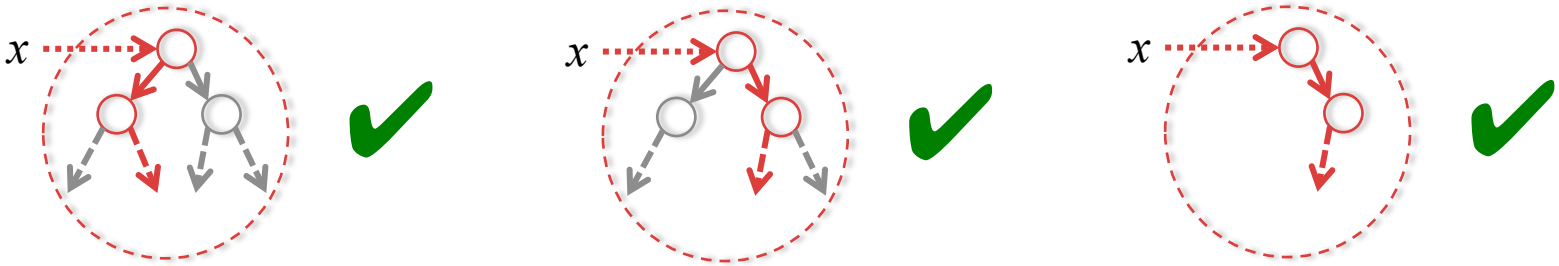
root points to a BST and *curr* points into the BST;

k is stored in the BST iff *k* is stored under *curr*)

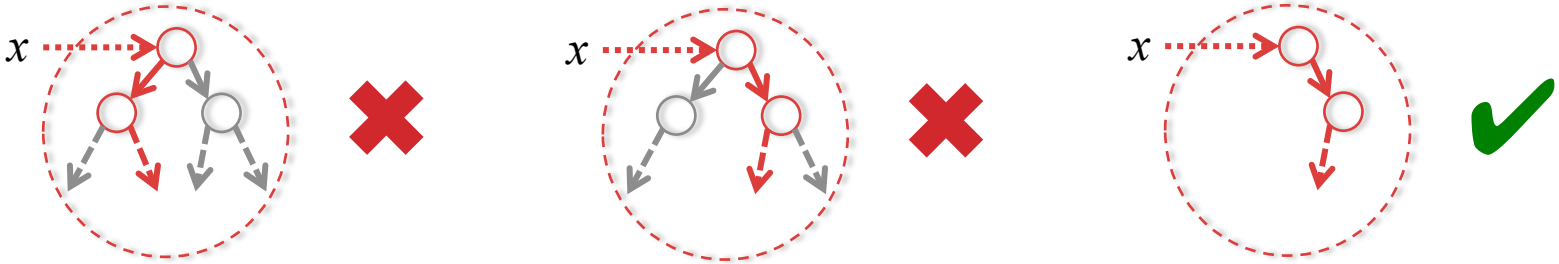
DRYAD VS. SEPARATION LOGIC

$$U^*(x) \overset{def}{=} (x = \text{nil} \wedge \text{emp}) \vee (x \overset{l,r}{\mapsto} xl, xr * U^*(xl)) \vee (x \overset{l,r}{\mapsto} xl, xr * U^*(xr))$$

Separation Logic: any tree and any path



DRYAD: the heaplet is exactly the reachable locations from x



TRANSLATE DRYAD TO A CLASSICAL LOGIC

The scope (heaplet required) of a formula can be syntactically determined

- *singleton heap* $x \mapsto y : \{x\}$
- *recursive definitions* $U^*(x) : \text{reach}(x)$
- *Connective* $t \sim t' : \text{scope}(t) \cup \text{scope}(t)$

the domain of a heaplet can be modeled as a set of locations, and the heaplet semantics can be expressed using free set variables

Example: $tree^*(x) * tree^*(y)$

can be translated to

$$tree(x) \wedge tree(y) \wedge$$
$$\text{reach}(x) \cap \text{reach}(y) = \emptyset \wedge$$
$$\text{reach}(x) \cup \text{reach}(y) = G$$

(still quantifier-free)

NATURAL PROOFS FOR DRYAD

We consider a toy programming language that explicitly manages the heap

We consider programs with modular annotations

(pre/post, loop invariant in DRYAD)

Natural Proofs in 4 steps

1. Translate DRYAD to classical logic
2. Generate the verification condition (compute strongest-post)
3. Unfold recursive definitions across the footprint (still precise)
4. Formula Abstraction and solve in SMT
   (recursive definitions uninterpreted, becomes sound but incomplete)

EXPERIMENTAL EVALUATION

A prototype verifier with Z3 as the backend solver

(more details at http://web.engr.illinois.edu/~qiu2/dryad/)

- 10+ data structures

  singly-linked list, sorted list, doubly-linked list, cyclic list, max-heap, BST, Treap, AVL tree, red-black tree, binomial heap…

- 100+ DRYAD-annotated programs

  textbook algorithms, Glib library, OpenBSD library, Linux kernel, an ongoing OS-Browser verification project…

- All these VCs were proved by Z3 !
- Few routines exceed 1 sec, even fewer exceed 100 secs

  "RBT-delete_iter" spent 225 secs, *"binomial-heap-merge_rec"* spent 153 secs

(To the best of our knowledge)

First terminating automatic mechanism that can prove such a wide variety of programs full-functionally correct

CONCLUSION

DRYAD is an expressive, tractable dialect of separation logic

A deterministic translation to classical logic

Extended the natural proof scheme to let the user

- specify structural properties of the heap
- reason with multiple structures
- reason with separation (frame reasoning)

A tool that handles Hoare-triples automatically using SMT solvers

(more details at http://web.engr.illinois.edu/~qiu2/dryad/)

Ongoing: Natural Proofs for C

(encode the natural proof scheme in ghost code for VCC)

THANK YOU!