Abstract—This paper investigates the problem of fault detection filter design for continuous-time switched systems. The designed fault detection filter is assumed to be asynchronous with the original systems. Attention is focused on designing a fault detection filter such that the estimation error between the residual and the fault is minimized in the sense of $H_\infty$ norm. By employing piecewise Lyapunov function and average dwell time techniques, a sufficient condition for the existence of such a filter is exploited in terms of certain linear matrix inequalities. Finally, an example is provided to illustrate the effectiveness of the proposed approach.

I. INTRODUCTION

In recent years, the process of fault detection and isolation (FDI) for dynamic systems has been of considerable interest, and fruitful model-based fault detection results have been obtained in several excellent papers [1]-[10] and the books [11]-[12]. Among these model-based approaches of FDI, the basic idea is to use state observer or filter to construct a residual signal and, based on this, to determine a residual evaluation function to compare with a predefined threshold. When the residual evaluation function has a value larger than the threshold, an alarm of faults is generated. On another hand, it is well known that control inputs, unavoidable unknown inputs, and faults are coupled in many industrial systems, which are potential sources of false alarm. This means that FDI systems have to be robust to control inputs and unknown inputs, and at the same time enhance the sensitivity to the faults. Therefore, it is of great significance to design a model-based fault detection system. In reviewing the development of the theories and techniques for different FDI system designs, a number of results have been obtained in designing FDI system. For examples, in [13], an $H_\infty$ filter formulation of robust FDI has been considered for uncertain LTI systems. The issue of $H_\infty$ fault detection filter design for linear discrete-time systems with multiple time delays is investigated in [9].

On another research direction, switched systems have attracted increasing attention in the literature of control problems due to their great significance in theory and practical applications. Switched systems belong to hybrid systems, which consist of a class of subsystems and a switching signal. The switching signal specifies which subsystem will be activated along the trajectory at each instant of time. Presently, a lot of achievements have been achieved on the control of switched system [14]-[19]. Especially for recent years, it is a hot topic to study the problem of fault diagnosis and fault tolerant control for switched systems. For examples, fault detection problem is separately considered in [20]-[21] for discrete case and in [22] for continuous case. Fault diagnosis for continuous-time switched system was investigated in [23], while [24] studied fault tolerant control problem for continuous time switched systems. However, all the above literatures assume that the observer or filter is synchronous with the original systems. Actually, it often takes time to identify the system. Therefore, the phenomenon of asynchronous switching generally exists. Moreover, the developed methods of fault detection for switched systems under asynchronous switching are no longer suitable to solve synchronous switching case. New technique should be contrived to solve this issue. Until now, to the best of our knowledge, the problem of fault detection for continuous-time switched systems under asynchronous switching has not been considered yet. Research is still under way into the development of an effective solutions for this issue, which motivates us to study this interesting and practical issue.

In this paper, the problem of fault detection for continuous-time switched systems under asynchronous switching is investigated. Firstly, by using piecewise Lyapunov function and average dwell time technique, a sufficient condition for the $H_\infty$ fault detection filter is exploited in the formation of LMI. Then, based-on the obtained condition, a desired fault detection filter is constructed. Finally, to demonstrate the feasibility and effectiveness of the proposed method, a simulation example is included.

The rest of this paper is organized as follows. In Section 2, system descriptions and problem formulation are presented. A sufficient condition on the existence of a fault detection filter for continuous-time switched systems is derived in terms of LMIs, and the parameters of the desired filter are constructed by solving the corresponding LMIs in Section 3. To demonstrate the validity of the proposed approach,
an example is given in Section 4 which is followed by a conclusion in Section 5.

II. PROBLEM FORMULATION

Consider the following class of continuous-time switched systems:
\[
\begin{align*}
\dot{x}(t) &= A_{\sigma(t)} x(t) + B_{1\sigma(t)} d(t) + D_{1\sigma(t)} f(t) \\
y(t) &= C_{\sigma(t)} x(t) + B_{2\sigma(t)} d(t) + D_{2\sigma(t)} f(t)
\end{align*}
\]  
where \( x(t) \in \mathbb{R}^n \) is the state vector, \( y(t) \in \mathbb{R}^m \) is the state vector, \( d(t) \in \mathbb{R}^p \) is the unknown input vector (including disturbance, noise or structured model uncertainty), and \( f(t) \in \mathbb{R}^q \) is the fault. \( \sigma(t) : [0, +\infty) \rightarrow \psi = \{1, \cdots, N\} \) is the switching signal which specifies which subsystem will be activated, and \( N \) denotes the number of subsystems. In this paper, we assume that the switching signal \( \sigma(t) \) is time-dependent, i.e. \( \sigma(t) : \{t_0, \sigma(t_0)\}, \{t_1, \sigma(t_1)\}, \cdots \), where \( t_0 \) denotes the initial time, and \( t_k \) denotes the \( k \)-th switching instant. \( A_{\sigma(t)}, B_{1\sigma(t)}, B_{2\sigma(t)}, C_{\sigma(t)}, D_{1\sigma(t)}, \) and \( D_{2\sigma(t)} \) are constant matrices with appropriate dimensions for all \( \sigma(t) \in \psi \). We denote the matrices associated with \( \sigma(t) = i \) by \( A_i, B_{1i}, B_{2i}, C_i, D_{1i}, \) and \( D_{2i} \). Actually, there inevitably exists asynchronous switching between the filter and the original system. Therefore, we suppose the \( i \)-th subsystem is activated at the switching instant \( t_{k-1} \), the \( j \)-th subsystem is activated at the switching instant \( t_{k} \), and the corresponding switching filter is activated at the switching instants \( t_{k-1} + \Delta_k-1 \) and \( t_k + \Delta_k \), respectively. Owing to asynchronous switching, the switching instant of the fault detection filter corresponding to \( j \)-th subsystem is \( t_{k} + \Delta_k \), then there exists a matched period at time interval \([t_{k-1} + \Delta_k-1, t_k]\) and a mismatched period at time interval \([t_k, t_{k} + \Delta_k]\).

For the purpose of residual generation, the following fault detection filter is constructed as a residual generator. For convenience, we use \( \sigma'(t) \) to denote the switching signal of the fault detection filter:
\[
\begin{align*}
\dot{x}_f(t) &= A_{f\sigma'(t)} \dot{x}_f(t) + B_{f\sigma'(t)} y(t) \\
y_f(t) &= C_{f\sigma'(t)} \dot{x}_f(t) + D_{f\sigma'(t)} y(t)
\end{align*}
\]  
where \( \dot{x}_f(t) \in \mathbb{R}^n \) is the filter’s state, and \( y_f(t) \in \mathbb{R}^m \) is the residual signal. \( A_{f\sigma'(t)}, B_{f\sigma'(t)}, C_{f\sigma'(t)}, \) and \( D_{f\sigma'(t)} \) are filter parameters to be determined.

Denoting \( e(t) = r(t) - f(t) \) and augmenting state vector \( \tilde{x}(t) = [x^T(t) \quad \dot{x}^T(t)]^T \), \( \omega(t) = [\dot{x}^T(t) \quad f^T(t)]^T \). When \( t \in \{t_0, t_1\} \cup \{t_{k-1} + \Delta_k-1, t_k\} \), \( k = 2, 3, 4, \cdots \), we obtain the augmented system as follows:
\[
\Sigma: \begin{cases} 
\dot{x}(t) = \tilde{A}_i \tilde{x}(t) + \tilde{B}_i \omega(t) \\
e(t) = \tilde{C}_i \tilde{x}(t) + \tilde{D}_i \omega(t)
\end{cases}
\]  
When \( t \in [t_k, t_k + \Delta_k) \), \( k = 1, 2, 3, \cdots \), we obtain the augmented system as follows:
\[
\Sigma: \begin{cases} 
\dot{x}(t) = \tilde{A}_{ij} \tilde{x}(t) + \tilde{B}_{ij} \omega(t) \\
e(t) = \tilde{C}_{ij} \tilde{x}(t) + \tilde{D}_{ij} \omega(t)
\end{cases}
\]  
where
\[
\begin{align*}
\tilde{A}_i &= \left[ \begin{array}{cc} A_i & 0 \\
B_{fi} C_i & A_{fi} \end{array} \right], \quad \tilde{B}_i &= \left[ \begin{array}{c} B_{fi} \\
B_{fi} B_{2i} \\
B_{fi} D_{1i} \end{array} \right], \\
\tilde{C}_i &= \left[ \begin{array}{ccc} D_{fi} C_i & C_{fi} \\
D_{fi} B_{2i} & D_{fi} D_{2i} - I \end{array} \right], \\
\tilde{A}_{ij} &= \left[ \begin{array}{cc} A_j & 0 \\
B_{fj} C_j & A_{fj} \end{array} \right], \quad \tilde{B}_{ij} &= \left[ \begin{array}{c} B_{fj} \\
B_{fj} B_{2j} \\
B_{fj} D_{1j} \end{array} \right], \\
\tilde{C}_{ij} &= \left[ \begin{array}{cc} D_{fj} C_j & C_{fj} \\
D_{fj} B_{2j} & D_{fj} D_{2j} - I \end{array} \right].
\end{align*}
\]

Now, the problem of fault detection filter design can be formulated as an \( H_\infty \) filter problem; to develop filter (2) for system (1) such that the augmented system \( \Sigma \) is stable when \( \omega(t) = 0 \) and, under zero-initial condition, the minimum of \( \gamma \) is made small in the feasibility of
\[
\sup_{\omega(t) \neq 0} \{ ||e(t)||_2 \} < \gamma, \quad \gamma > 0
\]  
After designing the residual generator, the last step to a successful fault detection is the residual evaluation stage including an evaluation function and a threshold. In this paper, the threshold \( J_{th} \) and residual evaluation function \( J_r(L) \) are selected as
\[
J_r(L) = ||r(t)||_2 = \left( \int_{0}^{L} r^T(\zeta) r(\zeta) d\zeta \right)^{\frac{1}{2}}
\]
\[
J_{th} = \sup_{d \in \mathbb{I}_2, f_2=0} \| r(t) \|_2
\]  
where \( L \) is the evaluation time steps. Based on this, the occurrence of faults can be detected by comparing \( J_r(L) \) and \( J_{th} \) according the following test:
\[
J_r(L) > J_{th} \Rightarrow \text{ with faults} \Rightarrow \text{alarm}
\]  
\[
J_r(L) \leq J_{th} \Rightarrow \text{ no faults}
\]

Remark 1: The fault detection filter design for continuous-time switched systems with time-varying delay has been exploited in [22]. However, the designed fault detection filter must be matched with the original systems. In this note, the mismatched case is considered, and the obtained results can be directly extended to delay case and parameter uncertain case.

Definition 1: [25] For any switching signal \( \sigma(k) \) and any \( t_2 > t_1 > 0 \), let \( N_{\omega}(\tau) \) denote the number of switchings of \( \sigma(t) \) on an interval \( (t_1, t_2) \).

\[
N_{\omega}(\tau) \leq N_0 + \frac{t_2 - t_1}{\tau_a}
\]  
holds for a given \( N_0 \geq 0 \) and \( \tau_a > 0 \), then the constant \( \tau_a \) is called the average dwell-time and \( N_0 \) the chattering bound.

Lemma 1: [26] If there exist functions \( \phi(t) \) and \( v(t) \) satisfying
\[
\dot{\phi}(t) \leq -\zeta \phi(t) + \kappa v(t)
\]  
then
\[
\phi(t) \leq e^{-\zeta(t-t_0)} \phi(t_0) + \kappa \int_{t_0}^{t} e^{-\zeta(t-t)} v(t) d\tau
\]
III. FAULT DETECTION FILTER DESIGN
A. H∞ performance analysis

Theorem 1: Given constants $\alpha > 0$, $\beta > 0$, $\mu_1 \geq 1$, and $\mu_2 \geq 1$, if there exist matrices $P_i > 0$, $P_{ij} > 0$, for $i \neq j$, $i, j \in N$, such that

$$P_j \leq \mu_1 P_i, \quad P_{ij} \leq \mu_2 P_i;$$

then by (9), we get

$$\dot{V}_{ij}(t) \leq -\alpha V_{ij}(t) \leq \beta V_{ij}(t) \leq 0$$

it follows that

$$\dot{V}_{ij}(t) \leq \beta V_i(t)$$

then during the matched period, $V_{ij}(t)$ satisfy

$$V_{ij}(t) \leq V_{ij}(t_k) e^{\beta(t-t_k)}$$

Let $t_1, t_2, \cdots, t_k, \cdots$ denote the switching instant of $\sigma(t)$ over the interval $[t_0, t]$. Consider the following piecewise Lyapunov functional candidate for system $\Sigma$ in (3) and (4):

$$V(t) = \begin{cases} V_i(t) = \hat{x}^T(t)P_i\hat{x}(t), & t \in [t_0, t_1) \cup [t_k-1 + \Delta_{k-1}, t_k), k = 2, 3, 4, \cdots \\ V_i(t) = \hat{x}^T(t)P_i\hat{x}(t), & t \in [t_k, t_k + \Delta_k), k = 1, 2, 3, \cdots \end{cases}$$

When $t \in [t_k, t_k + \Delta_k), k = 1, 2, 3, \cdots$, and with the condition in (8) (16) and (21), we have

$$V(t) \leq \mu_1 V_{i(t_k)} e^{\beta(t-t_k)}$$

By (11), we have

$$\beta T^+ (t_0, t) - \alpha T^- (t_0, t) \leq -\zeta^*(t-t_0)$$

then (22) and (23) imply that

$$V(t) \leq \mu_1 V_{i(t_k)} e^{\beta(t-t_k)}$$

Therefore, if the average dwell time satisfies (11), we conclude $V(t)$ converges to zero as $t \rightarrow \infty$. Then the stability of system $\Sigma$ can be deduced.

For any nonzero $\omega(t) \in L_2[0, \infty]$ and zero initial condition $\hat{x}(0) = 0$. When $t \in [t_0, t_1) \cup [t_{k-1} + \Delta_{k-1}, t_k)$, $k = 2, 3, 4, \cdots$, the augmented system can be written as
\[ V(t) \leq \int_{t_0}^t e^{\alpha(t-s)} \Gamma(s) \Gamma(s) ds \]

Multiplying both sides of (31) by \( e^{-[N_{\sigma_1}(t_0,t) + N_{\sigma_2}(t_0,s)] \ln \mu_1} \) yields
\[
\int_{t_0}^t e^{\alpha(t-s)} \Gamma(s) \Gamma(s) ds \leq \int_{t_0}^t e^{\alpha(t-s)} \Gamma(s) \Gamma(s) ds
\]

When \( t \in [t_k, t_k + \Delta_k] \), \( k = 1, 2, 3, \ldots \), by the condition in (11), we have
\[
-\frac{t - t_0}{\tau_a} \ln (\mu_1 \mu_2) - \ln \mu_1
\]

By (22) and (33), we have
\[
e^{-[N_{\sigma_1}(t_0,t) + N_{\sigma_2}(t_0,s)] \ln \mu_1} V(t)
\]

which means
\[
\int_{t_0}^t e^{\alpha(t-s)} \Gamma(s) \Gamma(s) ds \leq \gamma^2 \int_{t_0}^t \omega^2(s) \omega(s) ds
\]

when \( t \to \infty \), then the proof is completed.

B. Fault detection filter design

Theorem 2: Given constants \( \alpha > 0, \beta > 0, \mu_1 \geq 1, \) and \( \mu_2 \geq 1 \), if there exist matrices positive-definite matrices \( X_i, P_{r_{i1}}, P_{r_{i2}}, P_{r_{i3}}, \) and any matrices \( P_{12i}, P_{13i}, A_{fi}, B_{fi}, C_{fi}, \) and \( D_{fi} \), for \( i \neq j, i, j \in N \), such that
\[
\begin{bmatrix}
X_i & P_{12i} & P_{13i}
\end{bmatrix} > 0
\]

Under the initial condition \( x(t_0) = 0 \), we get
\[
V(t) \leq \int_{t_0}^t e^{\alpha(t-s)} \Gamma(s) \Gamma(s) ds
\]

where
\[
\Theta_i = \begin{bmatrix}
\tilde{A}_i^T P_i + P_i \tilde{A}_i - \beta P_i + \tilde{C}_i^T \tilde{C}_i
\end{bmatrix}
\]

By (9), it follows that
\[
\dot{V}_i(t) < -\alpha V_i(t) + \Gamma(t)
\]

Consider the piecewise Lyapunov function as in (22), when \( t \in [t_k, t_k + \Delta_k] \), \( k = 1, 2, 3, \ldots \), it follows from (26) and (29) that
\[
V(t) \leq \int_{t_0}^t e^{\alpha(t-s)} \Gamma(s) \Gamma(s) ds
\]

when \( t \to \infty \), then the proof is completed.
Define the following matrix $P_i$, let the positive-definite matrices $R_i, A_i, B_i, T_i$ be partitioned as

$$\begin{bmatrix}
R_i A_i & A_i T_i R_i + \alpha R_i; \\
R_i A_i & B_i C_i + C_i T_i B_i + \alpha R_i;
\end{bmatrix}$$

Remark 3: Noting that the conditions in (36)-(41) are mutually dependent. Therefore, we can first solve (39) and (41) to obtain matrices $A_j, B_j, C_j, and D_j$. Then, the feasible solutions of $P_{1ij}, P_{2ij}, and P_{22ij}$ can be found by solving (36), (38), and (40).

Remark 2: Noting that the conditions in (36)-(41) are mutually dependent. Therefore, we can first solve (39) and (41) to obtain matrices $A_j, B_j, C_j, and D_j$. Then, the feasible solutions of $P_{1ij}, P_{2ij}, and P_{22ij}$ can be found by solving (36), (38), and (40).

Remark 3: In practical operation, the condition in (11) is usually difficult to justify. We can assume that the maximum value of the lag $\tau_{max}$ between the fault detection filter and the original system is a known constant, then (11) can be reduced to the following condition

$$\tau_a > \tau_a^* = \max \left\{ \ln\left(\frac{\mu_1 \mu_2}{\mu_1^* \mu_2^*}\right), \left(\frac{\beta + \zeta^*}{\zeta^* - \alpha^*} + 1\right) \Delta_{max} \right\}$$

IV. Conclusion

In this paper, the problem of fault detection for continuous-time switched systems under asynchronous switching has been considered. The filter is assumed to be unmatched with the original system. An efficient condition has been given to construct the filter under the switching signal with average dwell time. Finally, an example has been provided to illustrate the proposed methods.

REFERENCES


detection filter for discrete-time switched systems with state delays,”
International Journal of Innovative Computing, Information and Con-
discriminant analysis theory in power systems,” ICIC Express Letters,
Markovian jump linear systems with polytopic uncertainties,” Inter-
national Journal of Innovative Computing, Information and Control,
vol.6, no.3(A), pp.995-1004, 2010.
of robust H control and RFD for LTI systems,” in Proc. IFAC World
in the theory of FDI,” in Proc. IFAC Safeprocess, Budapest, Hungary,
2000, pp. 16-27.
fault detection filter for uncertain LTI systems,” Automatica,
[14] J. Zhao, and D. J. Hill, “Passivity and stability of switched systems:
analysis for switched symmetric systems with time delay,” Proc. 2003
American Control conference, Denver, June, pp.4-6, 2003.
[16] C. Cui, F. Long, and C. Li, “Disturbance attenuation for switched sys-
tem with continuous-time and discrete-time subsystems: state feedback
for switched linear discrete-time systems with time-varying delays,”
[18] H. Yang; B. Jiang, and V. Cocquempot, “Fault tolerance analysis for
stochastic systems using switching diffusion processes” Int. J. Control,
switched systems with state delays via switched Lyapunov function
2007.
linear systems with state delays,” IEEE Trans. Systems, Man, and
switched systems with interval time-varying delays,” International
Journal of Control, Automation and Systems, vol. 9, no. 2, pp. 396-
401, 2011.
[22] D. Wang, P. Shi, and W. Wang, “Robust fault detection for continuous-
time switched delay systems: an linear matrix inequality approach,”
with time delay,” The 8th IEEE International Conference on Control
systems with time delay,” International Journal of Adaptive Control
average dwell time. In Proc. 38th conf. decision control, Phoenix,
dynamic systems with persistent bounded disturbance,” Proc. the 13th
IEEE International Conference on Fuzzy Systems, Budapest, Hungary,