Fault Detection using PI Observers*

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Abstract

The fault detection and isolation (FDI) problem in connection with Proportional Integral (PI) Observers is considered in this paper. A compact formulation of the FDI design problem using PI observers is given. An analysis of the FDI design problem is derived with respect to the time domain properties. A method for design of PI observers applied to FDI is given.

Keywords: Fault detection and isolation, Proportional Integral observers, Loop transfer recovery, LQG/LTR design.

1 Introduction

A major part of the methods in FDI are based on different types of observers/filters as e.g. full order and minimal order observers, Luenberger observers etc., just to mention a few. For a more detailed description of these types, see [6] and the references herein. The applied design methods for the FDI filters are spread out over a large number of methods, see e.g. again [6] and the references herein. Some of the applied methods are standard methods, as e.g. $\mathcal{H}_\infty$ filter design methods, [1], [9]. Other methods are based on modifications of existing methods as e.g. the eigenstructure assignment method of Patton et.al. [7], [8].

The key issue of this paper is to introduce a new observer type in connection with FDI, give a standard formulation of the design problem, and at last apply a standard method for the design. The new observer type is a PI observer, known from Loop Transfer Recovery (LTR) design, [5], [10]. The motivation for the introduction of a PI observer in connection with FDI is a combination of both the time domain properties for the PI observer [5], and the result in [2] where the FDI and LTR designs are connected. The results in [2] show that it is possible to design observers for FDI by using the LTR concept.

The rest of this paper is organized as follows. In Section 2, the FDI filter design problem is formulated followed by a short introduction to the PI observer in Section 3. An analysis of the FDI problem with PI observers is given in Section 4. The design of PI observers for FDI is considered in Section 5. An example is given in Section 6, where the PI observer is applied, followed by a conclusion in Section 7.

2 FDI Design Setup

The FDI design setup will be given in the following. Consider the following system G given by:

$$\begin{array}{rcccccc} \dot{x} & = & Ax & + & B_w w & + & B_f f \\ y & = & C_y x & + & D_{yw} w & + & D_{yf} f \end{array} \tag{1}$$

or as transfer functions:

$$\begin{array}{rcl} y & = & (C_y(sI-A)^{-1}B_w + D_{yw})w \\ & & +(C_y(sI-A)^{-1}B_f + D_{yf})f \\ & = & G_{yw}(s)w + G_{yf}(s)f \end{array}$$

(We shall throughout the paper assume 'compatible' dimensions of vectors and matrices to avoid tedious listing of dimensions.)

w is a disturbance signal vector and f is a fault signal vector. The general system formulation given in (1) will be used throughout this paper. By the selection of B_f and D_{yf}, actuator, sensor and internal faults can be handled in this setup, see e.g. [4].

For obtaining a good estimation of the individual faults, fault models need to be included in the system as frequency weightings on the fault signals:

$$f = V_f(s)v \tag{2}$$

where v is a signal that is anticipated to have a flat power spectrum.

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A filter is now applied to estimate the fault signal vector f out from the measurement signal vector y. Let the filter be given by $F(s)$, i.e. the estimate of the fault signal vector is given by:

$$\hat{f} = F(s)y$$

The estimation error is then given by:

$$e = f - \hat{f} = V_f(s)v - F(s)\left(G_{yw}(s)w + G_{yf}(s)V_f(s)v\right) \quad (3)$$

The system (1) together with the equation for the estimation error (3) makes it possible to setup the filter design problem in the standard formulation by using the external output z as the estimation error $f - \hat{f}$. The generalized system is then given by:

$$\begin{pmatrix} z \\ y \end{pmatrix} = \bar{G}(s) \begin{pmatrix} w \\ v \\ u \end{pmatrix} \quad (4)$$

with

$$\bar{G}(s) = \left(\begin{array}{cc|c} 0 & V_f & -I \\ \hline G_{yw} & G_{yf}V_f & 0 \end{array}\right)$$

or in a state space realization with $V_f = (A_v, B_v, C_v)$:

$$\begin{array}{rclllllll} \dot{\bar{x}} & = & \bar{A}\bar{x} & + & \bar{B}_w w & + & \bar{B}_v v & + & 0u \\ z & = & \bar{C}_z \bar{x} & & & & & - & Iu \\ y & = & \bar{C}_y \bar{x} & + & D_{yw} w & & & & \end{array} \quad (5)$$

where

$$\begin{array}{rcl} \bar{A} & = & \begin{pmatrix} A & B_f C_v \\ 0 & A_v \end{pmatrix}, \quad \bar{B}_w = \begin{pmatrix} B_w \\ 0 \end{pmatrix} \\ \bar{B}_v & = & \begin{pmatrix} 0 \\ B_v \end{pmatrix} \\ \bar{C}_z & = & \begin{pmatrix} 0 & C_v \end{pmatrix} \\ \bar{C}_y & = & \begin{pmatrix} C_y & D_{yf} C_v \end{pmatrix} \end{array}$$

Note that there is no direct term in the weight matrix $V_f(s)$. A direct term can be included without problems, but from a practical point of view, this will in general not be necessary.

3 PI Observer

The PI observer has been described in e.g. [5] for the continuous time case and in [10] for the discrete time case. In both papers, the PI observers has been applied in connection with Loop Transfer Recovery (LTR) to obtain time recovery, i.e. good recovery at low frequencies. The PI observer will shortly be introduced in the following.

Let a dynamic system be given by:

$$\begin{array}{rclll} \dot{x} & = & Ax & + & Bu \\ y & = & Cx & & \end{array} \quad (6)$$

A PI observer for the system in (6) is given by, [5]:

$$\begin{array}{rl} \dot{\hat{x}} = & A\hat{x} + K_P(C\hat{x} - y) + Bu + B\zeta \\ \dot{\zeta} = & K_I(C\hat{x} - y) \\ u = & -F\hat{x} \end{array} \quad (7)$$

where K_P is the proportional observer gain, K_I is the integral observer gain and F is the state feedback gain. Note that when $K_I = 0$, we have a conventional full order (P) observer.

The stability condition for the PI observer requires that the eigenvalues of R given by:

$$R = \begin{bmatrix} A + K_P C & B \\ K_I C & 0 \end{bmatrix}$$

have negative real parts.

For derivation of systematic design methods, the PI observer based controller can be represented in the following compact form:

$$\begin{array}{rl} \dot{x}_{PI} = & A_{PI}x_{PI} + K_{PI}(C_{PI}x_{PI} - y) + B_{PI}u \\ u = & -F_{PI}x_{PI} \end{array} \quad (8)$$

where

$$\begin{array}{rclrcl} A_{PI} & = & \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}, & B_{PI} & = & \begin{bmatrix} B \\ 0 \end{bmatrix} \\ C_{PI} & = & \begin{bmatrix} C & 0 \end{bmatrix}, & K_{PI} & = & \begin{bmatrix} K_P \\ K_I \end{bmatrix} \\ F_{PI} & = & \begin{bmatrix} F & 0 \end{bmatrix} & & & \end{array}$$

By using this compact form of the PI observer, the PI observer has exactly the same form as a standard full order observer based controller. Now methods such as LQG, eigenstructure assignment etc. can be applied as ordinary observer design methods to determine the observer gain K_{PI}.

4 PI Observers for FDI

To derive the design conditions for the FDI problem given in Section 2 when a PI observer is applied, we need to calculate the estimation error in (3). The transfer function for the PI observer is given by:

$$L_{PI}(s) = -F_{PI}(sI - A_{PI} - K_{PI}C_{PI})^{-1}K_{PI} \quad (9)$$

The estimation error e is given by:

$$
\begin{aligned}
e &= \bar{C}_z(sI-\bar{A})^{-1}\begin{pmatrix} \bar{B}_w & \bar{B}_v \end{pmatrix}\begin{pmatrix} w \\ v \end{pmatrix} \\
&\quad -L_{PI}(s)[\bar{C}_y(sI-\bar{A})^{-1}\begin{pmatrix} \bar{B}_w & \bar{B}_v \end{pmatrix} \\
&\quad +\begin{pmatrix} D_{yw} & 0 \end{pmatrix}]\begin{pmatrix} w \\ v \end{pmatrix} \\
&= T_e(s)\begin{pmatrix} w \\ v \end{pmatrix}
\end{aligned}
$$

The transfer function for the estimation error $T_e(s)$ can now be rewritten by using simple matrix manipulations. Using $F_{PI} = \begin{pmatrix} \bar{C}_z & 0 \end{pmatrix}$ and the equations for the estimation error from [2] when a Luenberger observer has been applied, we get the following equation for $T_e(s)$:

$$
T_e(s) = \tilde{C}_z(sI - A_{PI} - K_{PI}C_{PI})^{-1}(B_{PI} - K_{PI}\tilde{D}_{yw}) \tag{10}
$$

where

$$
\begin{aligned}
A_{PI} &= \begin{pmatrix} \bar{A} & \bar{B} \\ 0 & 0 \end{pmatrix}, \quad B_{PI} = \begin{pmatrix} \bar{B} \\ 0 \end{pmatrix} \\
C_{PI} &= \begin{pmatrix} \bar{C}_y & 0 \end{pmatrix} \\
\tilde{C}_z &= \begin{pmatrix} \bar{C}_z & 0 \end{pmatrix}, \quad \tilde{D}_{yw} = \begin{pmatrix} D_{yw} & 0 \end{pmatrix} \\
\bar{B} &= \begin{pmatrix} \bar{B}_w & \bar{B}_v \end{pmatrix}
\end{aligned}
$$

The design condition is to minimize a suitable norm of the transfer function from $\begin{pmatrix} w^T & v^T \end{pmatrix}^T$ to e, $T_e(s)$, to obtain a good estimate of the fault signal v.

It will in the rest of this paper be assumed that there is no direct term, i.e. $D_{yw} = 0$. It is impossible to obtain the time domain properties (time recovery in LTR design) when the transfer function T_e include $\tilde{D}_{yw}$. Other methods need to be applied which will not be considered here.

Based on T_e in (10) with $\tilde{D}_{yw} = 0$, T_e can be rewritten into:

$$
\begin{aligned}
T_e(s) &= \\
&s\bar{C}_z(s^2I - s(\bar{A}+K_P) - \bar{B}K_I\bar{C}_y))^{-1}\bar{B}
\end{aligned} \tag{11}
$$

Further, let us define time fault detection and isolation, TFDI, in the following way:

Definition 1 *Let $T_e(s)$ be the error transfer function. TFDI is obtained if and only if*

$$
T_e(0) = 0
$$

TFDI means that we obtain exact detection in the steady state ($t \to \infty$) even if there are faults with nonzero DC components. It is in general difficult to obtain TFDI with an arbitrary observer type. However, the PI observer architecture facilitates TFDI under mild conditions. These conditions are given in the following theorem.

Theorem 2 *TFDI is obtained if and only if the largest invariant subspace of the matrix $\bar{A}_K^{-1}\bar{B}K_I\bar{C}_y$ where $\bar{A}_K = \bar{A} + K_P\bar{C}_y$, contained in the controllable subspace of the pair $(\bar{A}_K^{-1}\bar{B}K_I\bar{C}_y, \bar{A}_K^{-1}\bar{B})$ corresponding to the eigenvalue $s = 0$ is itself contained in the unobservable subspace of the pair $(\bar{C}_z, \bar{A}_K^{-1}\bar{B}K_I\bar{C}_y)$.*

Proof: A proof can be found in [5].

A further discussion of the above conditions are given in [5].

5 Design of PI Observers for FDI

The applied design method is based on LQG design of a full order observer.

First, consider standard LQG/LTR design of a full order observer. Consider a system given by $S(A,B,C)$. The LQG/LTR observer gain K is obtained by solving the following Riccati equation:

$$
AP + PA^T + \Gamma - PC^T\Sigma^{-1}CP = 0 \tag{12}
$$

where Γ and Σ is selected as

$$
\begin{aligned}
\Gamma &= \Gamma_0 + q^2BB^T, \Gamma_0 \geq 0, 0 \leq q < \infty \\
\Sigma &= \Sigma_0, \Sigma_0 > 0
\end{aligned} \tag{13}
$$

The gain is given by

$$
K = -PC^T\Sigma^{-1} \tag{14}
$$

As q approaches ∞ we will get

$$
F(sI - A - BF - KC)^{-1}KG(s) \to F(sI - A)^{-1}B
$$

(pointwise convergence) or

$$
F(sI - A - KC)^{-1}B \to 0 \tag{15}
$$

when $S(A,B,C)$ is minimum phase, see [3] for further details.

The properties of (15) is exactly what we need to apply in connection with FDI.

Now consider the design of the PI observer for the FDI problem. First, note from (11) that TFDI is almost always obtained for $K_I \neq 0$. This can be seen by rewritten (11) into:

$$T_e(s) = s\bar{C}_z(sI - \bar{A} - K_P\bar{C}_y)^{-1}\bar{B} (sI - K_I\bar{C}_y(sI - \bar{A} - K_P\bar{C}_y)^{-1}\bar{B}) \tag{16}$$

When the conditions in Theorem 2 is satisfied, we get directly:

$$T_e(s) \to 0 \; as \; s \to 0$$

This mean that we get an estimation error equal to zero in the steady state under very weak conditions.

However, we do not only want FDI at steady state, we will also have FDI in a low frequency range. For making such a design, we can apply the LQG/LTR design method from the full order observer on the PI observer. By using the LQG/LTR design method directly, the integral effect vanished, i.e. $K_I \to 0$ as $q \to \infty$, we will just obtain a standard full observer, [5]. Therefore, the LQG/LTR design method must be modified before it is applied for the design of PI observers. This can be done by including an additional parameter in the weight matrix Γ. Now let Γ in (13) instead be given as:

$$\Gamma = \Gamma_0 + q^2 B_\alpha B_\alpha^T, \Gamma_0 \geq 0, 0 \leq q < \infty \tag{17}$$

where

$$B_\alpha = \begin{pmatrix} \bar{B} \\ \alpha I \end{pmatrix}, \alpha \geq 0$$

The α parameter is related to the time domain properties whereas the q parameter is related to the frequency domain properties.

By proper selection of the two parameters (α, q), it is possible to obtain quite reasonable FDI filters with good fault detections properties. It is here important to mention that it is possible to obtain TFDI also if the design problem is non minimum phase. We can derive an explicit equation for $T_e(s)$ as $q \to \infty$. For doing this, we need to consider the full order case first. Consider the system $S(A, B, C)$ and the minimum phase system $S(A, Z, C)$. Further, the estimation error transfer function is given by:

$$T_e(s) = \bar{C}_z(sI - \bar{A} - K\bar{C}_y)^{-1}\bar{B} \tag{18}$$

Now, let the observer gain satisfies:

$$\frac{K}{q} \to ZW, \; det(W) \neq 0 \; as \; q \to \infty \tag{19}$$

(obtained by using $\Gamma = \Gamma_0 + q^2 ZZ^T$)

The limit value of T_e is then given by:

$$T_e(s) = \bar{C}_z(sI - \bar{A})^{-1} (\bar{B} - Z(\bar{C}_y(sI - \bar{A})^{-1}Z)^{-1} \bar{C}_y(sI - \bar{A})^{-1}\bar{B}) \tag{20}$$

Based on (20), we can calculate the equivalent equation for the PI observer. This can be done by using $\bar{A} = A_{PI}$, $\bar{B} = B_{PI}$, $\bar{C}_y = C_{PI}$, $\bar{C}_z = \tilde{C}_z$ and $Z = \begin{pmatrix} \bar{B}_m \\ \alpha I \end{pmatrix}$ satisfies:

$$C_{PI}(sI - A_{PI})^{-1}B_\alpha = C_{PI}(sI - A_{PI})^{-1}ZB_z(s) \tag{21}$$

where $S(A_{PI}, Z, C_{PI})$ is the minimum phase image of $S(A_{PI}, B_\alpha, C_{PI})$ and $B_z(s)$ is an all-pass factor, in (20). After some simple manipulations, we get the following equation for the estimation error for PI observers:

$$T_e(s) = \bar{C}_z\Phi(s) [\bar{B} - (\tfrac{s}{\alpha}\bar{B}_m + \bar{B})(\tfrac{s}{\alpha}\bar{C}_y\Phi(s)\bar{B}_m + \bar{C}_y\Phi(s)\bar{B})^{-1}\bar{C}_y\Phi(s)\bar{B}] \tag{22}$$

where $\Phi(s) = (sI - \bar{A})^{-1}$.

It is now easy to see that

$$T_e(0) = 0$$

which indicate that we will obtain an exact FDI and disturbance rejection in the steady state.

6 Design Example

An example of design of FDI filters for the FDI problem from [7] will be considered in this section. The LQG/LTR design methodology will be applied for designing a PI observer as well as a standard full order observer for the FDI problem.

The system is a reduced order model of a jet engine. A reduced order model of order 5 has been given in [7]. The model has 1 disturbance input, 2 control inputs and all 5 states are measured.

The fault detection case that will be considered here is where faults can appear in the two actuators. This correspond to $B_f = B_u$ in this case.

For the FDI design, first order weights are applied to model the two fault signals. The two weights are given by:

$$f = \begin{pmatrix} \frac{1}{1+\tau_1 s} & 0 \\ 0 & \frac{1}{1+\tau_2 s} \end{pmatrix} v$$

where $\tau_1 = \frac{1}{50}$ and $\tau_2 = \frac{1}{55}$.

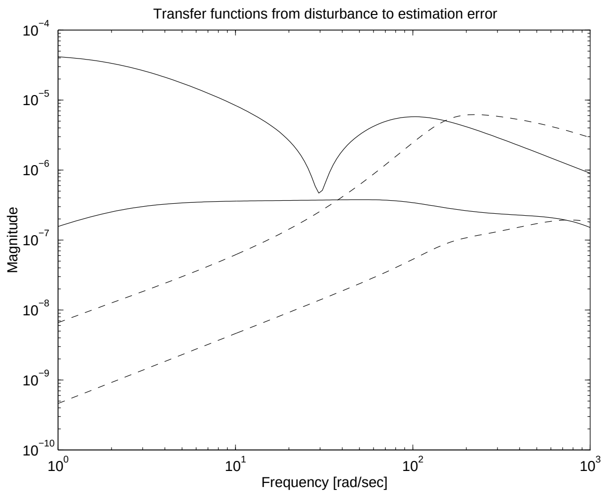

Figure 1: The transfer functions from the disturbance input signal w to the 2 fault estimation error signals e for the full order observer (solid lines) and for the PI observer (dashed lines)

The design condition for the two FDI observers are as follows:

Design objectives: *All six transfer functions from the three external input (disturbance and two fault signals) to the two fault estimation error signals must be smaller than* 0.1 *for* $\omega \leq 25$ *rad/sec.*

The results of the design of is shown in Figure 1 - 3 for $q = 10^4$ and $\alpha = 500$. The matrix norm for the observer gains is 1.65×10^7 and 1.72×10^7 for the full-order observer and for the PI observer resp.

As it can be seen from the 3 figures, the PI observer satisfies the design conditions, i.e. the six transfer functions, from the external inputs to the two estimation error signals, have a gain less than 0.1 for $\omega \leq 25$ rad/sec. From Figure 3, we can see that the full order observer does not satisfy the design conditions for the selected q. However, it is possible to design a full order observer such that the design conditions are satisfied. By increasing q to 7.0×10^5, the design condition will be satisfied. The matrix norm for this observer gain is 1.17×10^9.

In general, it turns out from Figure 3 that the estimation of fault on actuator 2 is the most difficult part in the design of both observers. The design conditions for the disturbance reduction (Figure 1) and the estimation of fault on actuator 1 (Figure 2) are more than satisfied for both observers.

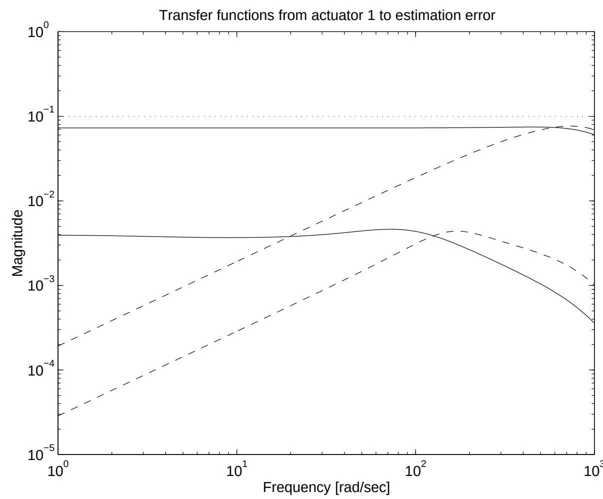

Figure 2: The transfer functions from the fault input signal f on actuator 1 to the two fault estimation error signals e for the full order observer (solid lines) and for the PI observer (dashed lines)

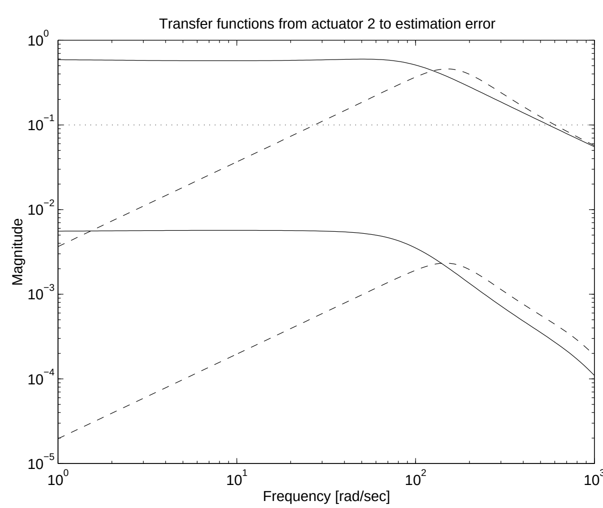

Figure 3: The transfer functions from the fault input signal f on actuator 2 to the two fault estimation error signals e for the full order observer (solid lines) and for the PI observer (dashed lines)

7 Conclusion

The FDI problem has been considered in this paper and a filter based on a PI observer has been introduced in connection with FDI. Using results from LTR design, it is possible to apply PI observers in connection with FDI and obtain time FDI, i.e. no estimation error in the steady state. Conditions for obtaining TFDI has been given in this paper.

Further, a modified LQG/LTR design method for FDI filters based on PI observers has been given. The TFDI has been shown both by analysis of the estimation error transfer function in Section 5 and in connection with an example in Section 6.

References

[1] A. Edelmayer, J. Bokor, and L. Keviczky. $\mathcal{H}_\infty$ detection filter design for linear systems: Comparison of two approaches. In *Proceedings of the 13th IFAC World Congress*, volume N, pages 37–42, San Francisco, CA, USA, 1996.

[2] H.H. Niemann. An application of LTR design in fault detection. *Optimal Control - Application and Methods*, 1998.

[3] H.H. Niemann, P. Søgaard-Andersen, and J. Stoustrup. Loop transfer recovery for general observer architectures. *International Journal of Control*, 53:1177–1203, 1991.

[4] H.H. Niemann and J. Stoustrup. Integration of control and fault detection: Nominal and robust design. In *SAFEPROCESS'97*, England, 1997.

[5] H.H. Niemann, J. Stoustrup, B. Shafai, and S. Beale. LTR design of proportional integral observers. *International Journal of Robust and Nonlinear Control*, pages 671–693, 1995.

[6] R.J. Patton. Robust model-based fault diagnosis: The state of art. In *Proceedings IFAC Symp. SAFEPROCESS '94*, pages 1–24, Espoo, Finland, 1994.

[7] R.J. Patton and J. Chen. Robust fault detection of jet engine sensor systems using eigenstructure assignment. In *Proceedings of the AIAA Guidance, Navigation and Control Conference*, pages 1666–1675, New Orleans, Louisiana, USA, 1991.

[8] R.J. Patton and J. Chen. Robust fault detection using eigenstructure assignment: A tutorial consideration and some new results. In *Proceedings of the 30th Conference on Decision and Control*, pages 2242–2247, Brighton, England, 1991.

[9] Z. Qiu and J. Gertler. Robust FDI systems and $\mathcal{H}_\infty$ optimization. In *Proceedings of the 32nd Conference on Decision and Control*, pages 1710–1715, San Antonio, Texas, USA, 1993.

[10] B. Shafai, S. Beale, H.H. Niemann, and J. Stoustrup. LTR design of discrete-time propertional-integral observers. *IEEE Transactions on Automatic Control*, 41:1056–1062, 1996.