arXiv:1308.1339v1 [quant-ph] 6 Aug 2013

Single-photon transport in a one dimentional waveguide coupling to a hybrid atom-optomechanical system

W. Z. Jia[1] and Z. D. Wang[1, *]

[1]*Department of Physics and Center of Theoretical and Computational Physics, The University of Hong Kong, Pokfulam Road, Hong Kong, China*

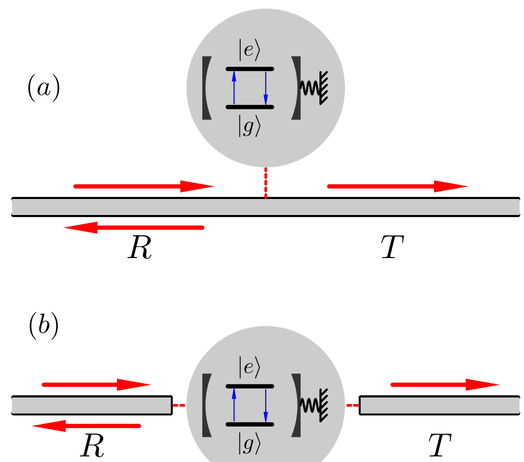
We explore theoretically the single-photon transport in a single-mode waveguide that is coupled to a hybrid atom-optomechanical system in a strong optomechanical coupling regime. Using a full quantum real-space approach, transmission and reflection coefficients of the propagating single-photon in the waveguide are obtained. The influences of atom-cavity detuning and the dissipation of atom on the transport are also studied. Intriguingly, the obtained spectral features can reveal the strong light-matter interaction in this hybrid system.

PACS numbers: 42.50.Wk, 42.50.Pq, 42.79.Gn, 07.10.Cm

I. INTRODUCTION

Recently, research of controllable single photon transport in low dimensional systems has attracted a growing interest for its significant importance in quantum control including quantum information processing. Usually, this kind of manipulation is achieved by strongly coupling a propagating single-photon in the waveguide to a local quantum system [1–3]. A desired single-photon control process is resulted from an interference between the directly transmitted photon and the photon re-emitted by the emitter. Specifically, such a waveguide-emitter system can be realized by a photonic nanowire with an embedded quantum dot [4], surface plasmons coupled to a single two-level emitter [5], a superconducting transmission line coupled to a superconducting artificial atom [6], or a single-mode waveguide coupled to a cavity interacting with a two-level atom [7–10].

As is known, a new type of optomechanical cavity was also developed to couple photons and phonons via radiation pressure. Significant research interest in this frontier of optomechanics is motivated by its potential applications in ultrasensitive measurements, quantum information processing, and implementation of novel quantum phenomena at macroscopic scales [11–13]. Important experimental progress on optomechanical systems has recently been made to reach the so-called single-photon strong coupling regime [14–20], where the single-photon coupling strength of the radiation pressure is comparable to (or even larger than) the cavity decay rate. This progress also inspired a series of theoretical investigations, including photon blockade [21–23] and photon-induced tunneling [24], single-photon cooling [25], optomechanically induced transparency in the single-photon strong coupling regime [26], and optomechanical instability [27]. In most quantum optomechanical devices, the cavity is side or direct coupled to a waveguide [13]. Thus in the single-photon regime, optomechanical systems, rather than traditional quantum emitters, may enable us to control the propagating single-photon in the waveguide. Also, the single-photon transmission spectra can be used to probe and characterize the strong-



directcouple.00

FIG. 1: (Color online) Schematic plot of a coupling system considered here. An optomechanical cavity interacting with a two-level atom is coupled to a single-mode waveguide, in which single photons propagate along the arrow direction. (a) Side-coupled cases. (b) Direct coupled cases.

coupling regime [28, 29].

In this paper, we explore theoretically the single photon transport in a waveguide coupled to a hybrid atom-optomechanical system in the single-photon strong coupling regime. This kind of hybrid atom-optomechanical system consists of an optomechanical cavity interacting with a single two-level atom, and is suggested to achieve a strong coupling between a single trapped atom and the motion of a membrane [30]. Notably, a weak continuous-wave laser scattering problem in this hybrid atom-optomechanical system was perturbatively treated in a recent study by assuming the weak coupling, [31]. Here, we employ a full quantum-mechanical approach [1, 2, 29] to study the transmission and reflection properties of the propagating photon in the waveguide in the strong optomechanical coupling regime. Our results also show that the single-photon transmission and reflection spectra can be used to probe and characterize the strong light-matter interaction in this kind of hybrid systems.

The paper is organized as follows. In Sec. II, we introduce

*Electronic address: zwang@hku.hk

our model for describing the single-photon transport. Then, in Sec. III, we look into the single-photon transport properties in detail. The influences of detuning and dissipation are also addressed. Finally, further discussions and conclusions are given in Sec. IV.

II. HAMILTONIAN AND THE SOLUTIONS

We consider a hybrid atom-optomechanical system (i.e., a single two-level atom coupled to an optomechanical cavity) to be coupled to an open one dimensional waveguide. With well-developed techniques for confining a single atom in a usual optical cavity [32], it seems achievable in the near future to couple atoms with optomechanical cavities. Usually, the atom-optomechanical system can either side-coupled or directly coupled to a waveguide, which is schematically illustrated in Fig. 1(a) and (b). In this paper, we focus on the single-photon transport problem of the side-coupling case, for one can straightforwardly map the reflection amplitude of the side-coupled case into the transmission amplitude of the direct-coupled cases [2]. A model Hamiltonian of this system may be written as ($\hbar = 1$)

$$\begin{aligned}\hat{H} &= \int dx a_R^\dagger(x)\left(-iv_g\frac{\partial}{\partial x}\right)a_R(x) + \int dx a_L^\dagger(x)\left(iv_g\frac{\partial}{\partial x}\right)a_L(x)\\ &+\frac{\omega_a}{2}\sigma_z - i\gamma_a|e\rangle_a\langle e|_a + \omega_c c^\dagger c + \Omega b^\dagger b - g_0 c^\dagger c\left(b+b^\dagger\right) + \lambda\left(c\sigma^+ + c^\dagger\sigma^-\right)\\ &+V\int dx\delta(x)\left(a_R^\dagger(x)c + a_R(x)c^\dagger + a_L^\dagger(x)c + a_L(x)c^\dagger\right). \end{aligned} \quad (1)$$

The first line denotes the waveguide optical mode, where v_g is the group velocity of the photons, and $a_R^\dagger(x)$ ($a_L^\dagger(x)$) is a bosonic operator creating a right-going (left-going) photon at x. The second line describes the isolated atom-optomechanical system, where $c^\dagger$ ($b^\dagger$) is the photon (phonon) creation operator, $\sigma^+(\sigma^-)$ is the atomic raising (lowering) operator generating transition between ground state and exited state: $\sigma^+|g\rangle_a = |e\rangle_a$, $\sigma^-|e\rangle_a = |g\rangle_a$. ω_a is the atomic transition frequency, ω_c is the cavity resonance frequency, Ω is the mechanical frequency, g_0 is the single-photon coupling strength of the radiation pressure between the cavity and the mirror, λ is the coupling strength between the cavity and the atom, γ_a is the dissipation rate of the the atom, due to coupling to the reservoir. The third line represents the coupling between the waveguide and the atom-optomechanical system, where V is the coupling strength between the cavity and the waveguide. And the according cavity-waveguide's decay rate can be defined as $\Gamma = V^2/v_g$ [2]. Note that in our treatment, it is assumed that the majority of the decayed light from the cavity is guided into waveguide modes, i.e., the "strong coupling" exists between the cavity and the waveguide [33]. Thus the decay rate κ of the cavity into channels other than the 1D continuum is negligible. We also assume that the decay rate γ_M of the mirror motion is much smaller than the cavity-waveguide's decay rate. As a result, $\Gamma \gg \kappa, \gamma_M$, the optomechanical decoherence processes can safely be ignored.

For an input one-photon Fock state, the stationary state of the system satisfies the eigen equation

$$H|\epsilon\rangle = \epsilon|\epsilon\rangle. \quad (2)$$

We assume that, initially, the mirror is in state $|n_0\rangle_b$, the atom is in the ground state and the cavity is empty, and a single-photon comes from the left with energy $v_g k$ with k as the wave vector of the photon. In this case, the total energy of the coupled system is $\epsilon = -\omega_a/2 + v_g k + n_0\Omega$. In the single-photon subspace, $|\epsilon\rangle$ can be expanded as

$$\begin{aligned}|\epsilon\rangle &= \sum_n \int dx\varphi_R(x,n)a_R^\dagger(x)|\varnothing\rangle|n\rangle_b\\ &+\sum_n\int dx\varphi_L(x,n)a_L^\dagger(x)|\varnothing\rangle|n\rangle_b\\ &+\sum_n e_n c^\dagger|\varnothing\rangle|\tilde{n}\rangle_b + \sum_n f_n\sigma^+|\varnothing\rangle|n\rangle_b, \end{aligned} \quad (3)$$

where $|\varnothing\rangle = |0\rangle_k|0\rangle_c|g\rangle_a$ is the vacuum state, with zero photon in both the waveguide and the cavity, and with the atom in the ground state. $|n\rangle_b$ represents the number state of the mechanical mode. $\varphi_{R,L}(x,n)$ is the single-photon wave function in the R/L mode. e_n and f_n are excitation amplitudes of the cavity and the atom, respectively. $|\tilde{n}\rangle_b = \exp\left[\frac{g_0}{\Omega}\left(b^\dagger - b\right)\right]|n\rangle_b$ is the single-photon displaced number state of the mechanical oscillator satisfying the eigen equation

$$\begin{aligned}&\left[\omega_c c^\dagger c + \Omega b^\dagger b - g_0 c^\dagger c\left(b+b^\dagger\right)\right]|1\rangle_c|\tilde{n}\rangle_b\\ &= (\omega_c + n\Omega - \delta)|1\rangle_c|\tilde{n}\rangle_b, \end{aligned} \quad (4)$$

where $\delta = g_0^2/\Omega$ is the photon-state frequency shift caused by a single-photon radiation pressure.

By substituting Eq. (3) into Eq. (2), we obtain the following equations of motion

$$-iv_g\frac{\partial\varphi_R(x,n)}{\partial x}+\delta(x)V\sum_m e_m U_{nm}=\left(\epsilon+\frac{\omega_a}{2}-n\Omega\right)\varphi_R(x,n),\qquad(5a)$$

$$iv_g\frac{\partial\varphi_L(x,n)}{\partial x}+\delta(x)V\sum_m e_m U_{nm}=\left(\epsilon+\frac{\omega_a}{2}-n\Omega\right)\varphi_L(x,n),\qquad(5b)$$

$$V\int dx\delta(x)\left[\varphi_R(x,n)+\varphi_L(x,n)\right]+\lambda f_n=\sum_m\left(\epsilon+\frac{\omega_a}{2}-\omega_c-m\Omega+\delta\right)e_m U_{nm},\qquad(5c)$$

$$\lambda\sum_m e_m U_{nm}=\left(\epsilon-\frac{\omega_a}{2}-n\Omega+i\gamma_a\right)f_n,\qquad(5d)$$

with $U_{nm}=\langle n\mid\tilde{m}\rangle_b$.

Assuming that the mirror is initially prepared in state $|n_0\rangle_b$ and a single-photon comes from the left with energy v_gk, $\varphi_R(x,n)$ and $\varphi_L(x,n)$ should take the form

$$\varphi_R(x,n)=\theta(-x)\delta_{nn_0}e^{i\left(k+(n_0-n)\frac{\Omega}{v_g}\right)x}+\theta(x)t_n e^{i\left(k+(n_0-n)\frac{\Omega}{v_g}\right)x},\qquad(6a)$$

$$\varphi_L(x,n)=\theta(-x)r_n e^{-i\left(k+(n_0-n)\frac{\Omega}{v_g}\right)x},\qquad(6b)$$

where t_n and r_n are the transmission and reflection amplitude, respectively. Substituting Eqs. (6a) and (6b) into Eqs. (5a)-(5d), the equations for t_n, r_n, e_n and f_n are given by

$$-iv_g(-\delta_{nn_0}+t_n)+V\sum_m e_m U_{nm}=0,\qquad(7a)$$

$$-iv_g r_n+V\sum_m e_m U_{nm}=0,\qquad(7b)$$

$$\frac{1}{2}V\left[\delta_{nn_0}+t_n+r_n\right]+\lambda f_n=\sum_m\left(\Delta_c+(n_0-m)\Omega+\delta\right)e_m U_{nm},\qquad(7c)$$

$$\lambda\sum_m e_m U_{nm}=\left(\Delta_c-\Delta_{ac}+(n_0-n)\Omega+i\gamma_a\right)f_n,\qquad(7d)$$

with $\Delta_c=v_gk-\omega_c$, $\Delta_{ac}=\omega_a-\omega_c$. If $\lambda\ll\Gamma,\gamma_a$, we can have the series solutions of r_n and t_n:

$$r_n=-i\Gamma\left(\sum_{n'}\frac{U_{nn'}U^*_{n_0n'}}{\tilde{\Delta}_c(n')}+\sum_{n'mn''}\frac{\lambda^2U_{nn'}U^*_{mn'}U_{mn''}U^*_{n_0n''}}{\tilde{\Delta}_c(n')\tilde{\Delta}_a(m)\tilde{\Delta}_c(n'')}+\sum_{n'mn''m'n'''}\frac{\lambda^4U_{nn'}U^*_{mn'}U_{mn''}U^*_{m'n''}U_{m'n'''}U^*_{n_0n'''}}{\tilde{\Delta}_c(n')\tilde{\Delta}_a(m)\tilde{\Delta}_c(n'')\tilde{\Delta}_a(m')\tilde{\Delta}_c(n''')}+\cdots\right),\qquad(8a)$$

$$t_n=\delta_{nn_0}+r_n\qquad(8b)$$

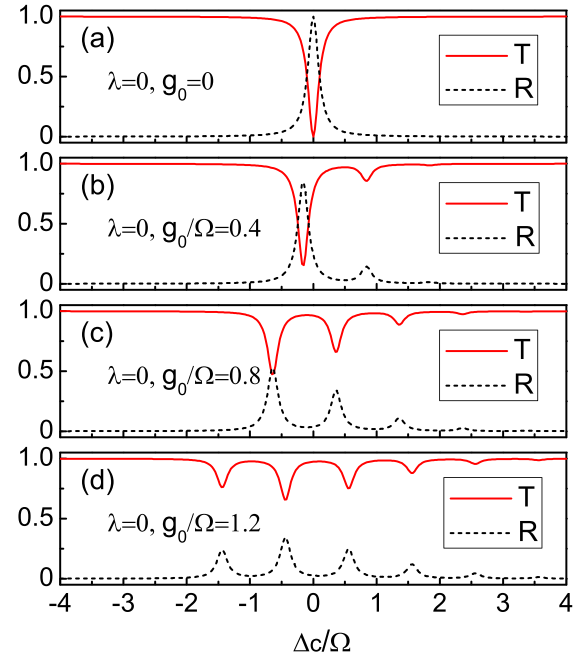


FIG. 2: (Color online) Single-photon transmission (reflection) spectra of a standard optomechanical cavity (i.e., the cavity-atom coupling strength $\lambda=0$) for various g. The cavity-waveguide decay rate $\Gamma=0.1\Omega$ is chosen for plotting the spectra.

with

$$\tilde{\Delta}_c(m)=\Delta_c+(n_0-m)\Omega+\delta+i\Gamma,$$
$$\tilde{\Delta}_a(n)=\Delta_c-\Delta_{ac}+(n_0-n)\Omega+i\gamma_a.$$

For an arbitrary λ, Eqs. (7a)-(7d) can be solved numerically by choosing the upper limit of n large enough, namely, $n_{max}\gg n_0$, and solving the attained $4(n_{max}+1)$ equations. Note that r_n (t_n) represents the amplitude of reflecting (transmitting) a single-photon with frequency $v_gk-(n-n_0)\Omega$. Thus the total single-photon transmission and reflection coefficients should be given by

$$T=\sum_n|t_n|^2,\ R=\sum_n|r_n|^2.\qquad(9)$$

III. SINGLE-PHOTON SCATTERING SPECTRA

A. Single-photon transmission(reflection) coefficient: atom cavity in tune and nondissipative case

We first investigate the single-photon transmission and reflection spectra of an optomechanical cavity containing no atom. Only the sideband resolved regime $\Gamma\ll\Omega$ is considered in this work. We plot the transmission and reflection coefficient as the functions of the photon-cavity detuning Δ_c

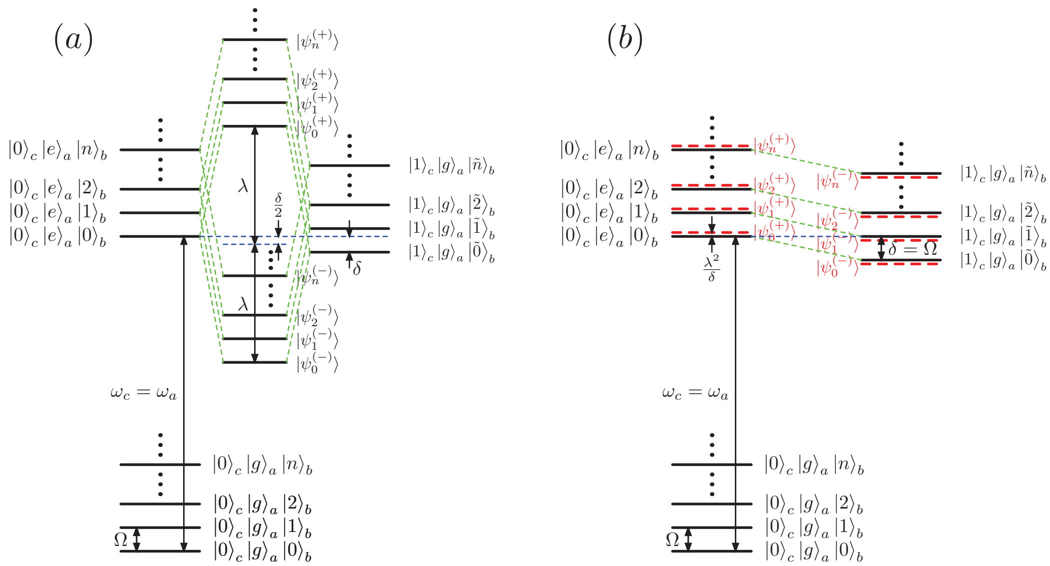


FIG. 3: (Color online) The energy-level structure of a hybrid atom-optomechanical cavity (limited to the zero- and one-photon subspaces) in the single-photon strong coupling regime, where the two-level atom is in resonance with the cavity (i.e., $\Delta_{ac} = 0$). The atom-cavity coupling strength $\lambda \gg \Omega \gg \Gamma$ for (a); and $\lambda < \Gamma$ with $g_0 = \Omega$ (i.e., $\delta = \Omega$) for (b).

for various values of g_0 when the mirror is initially prepared in the ground state $|0\rangle_b$, as shown in Fig. 2. When $g_0 = 0$, the coherently interference of the leaked waves out of the cavity and the propagating modes in the one-dimensional continuum results in a complete suppression of the transmission for a resonantly incident photon with $\Delta_c = 0$. When entering the single-photon strong regime $g_0 > \Gamma$, the transmission dips (reflection peaks) appear at $\Delta_c = -\delta + n\Omega$ $(n = 0, 1, 2\cdots)$, exhibiting a global red shift δ and more sidebands. This means that an incident single-photon with frequency $\omega_c - \delta + n\Omega$ can be strongly reflected by the optomechanical system because of the strong optomechanical coupling.

To investigate the single-photon transmission and reflection property of the hybrid atom-optomechanical system, we first give the eigen energies and eigen states of an isolate atom-optomechanical system. The Hamiltonian of an isolate atom-optomechanical system can be written as

$$\hat{H}_{AO} = \hat{H}_0 + \hat{H}_I \tag{10}$$

with

$$\hat{H}_0 = \frac{\omega_a}{2}\sigma_z + \omega_c c^\dagger c + \Omega b^\dagger b - g_0 c^\dagger c \left(b + b^\dagger\right), \tag{11a}$$

$$\hat{H}_I = \lambda \left(c\sigma^+ + c^\dagger \sigma^-\right). \tag{11b}$$

In the single-photon subspace, the eigen states of $\hat{H}_0$ are $|0\rangle_c |e\rangle_a |n\rangle_b$ with the eigen energy $\epsilon_{\uparrow n} = \omega_a/2 + n\Omega$, and $|1\rangle_c |g\rangle_a |\tilde{n}\rangle_b$ with the eigen energy $\epsilon_{\downarrow n} = -\omega_a/2 + \omega_c + n\Omega - \delta$. In this subspace, exact diagonalization of the Hamiltonian $\hat{H}_{AO}$ yields the eigen states

$$\left|\psi_n^{(+)}\right\rangle = \sin\theta\, |1\rangle_c |g\rangle_a |\tilde{n}\rangle_b + \cos\theta\, |0\rangle_c |e\rangle_a |n\rangle_b\,, \tag{12a}$$

$$\left|\psi_n^{(-)}\right\rangle = -\cos\theta\, |1\rangle_c |g\rangle_a |\tilde{n}\rangle_b + \sin\theta\, |0\rangle_c |e\rangle_a |n\rangle_b \tag{12b}$$

with the corresponding eigen energies

$$E_n^{(+)} = \frac{\omega_c}{2} + n\Omega - \frac{\delta}{2} + \frac{1}{2}\sqrt{\left(\Delta_{ac} + \delta\right)^2 + 4\lambda^2}, \tag{13a}$$

$$E_n^{(-)} = \frac{\omega_c}{2} + n\Omega - \frac{\delta}{2} - \frac{1}{2}\sqrt{\left(\Delta_{ac} + \delta\right)^2 + 4\lambda^2}, \tag{13b}$$

where

$$\theta = \frac{1}{2}\tan^{-1}\frac{2\lambda}{\Delta_{ac} + \delta}.$$

We now consider the case that the two-level atom is in resonance with the cavity, i.e. $\Delta_{ac} = 0$. When the atom-cavity coupling strength $\lambda \gg \Omega \gg \Gamma$ and the optmechanical coupling strength $g_0 = 0$, the transmission(reflection) spectrum shows vacuum Rabi splitting [2, 7] with the splitting width 2λ, as shown in Fig. 4(a). Figs. 4(b)-(d) show that how the moving mirror modify vacuum Rabi spectrum in the single-photon strong coupling regime. When the coupling strength g_0 increases to enter into the single-photon strong coupling regime, the spectra will undergo a red shift $\delta/2$. Additionally, on the right side of each main peak, more sidebands will appear with interval Ω, corresponding to the energy levels (under the condition $\lambda \gg \delta$)

$$E_n^{(+)} \approx \frac{\omega_c}{2} + n\Omega - \frac{\delta}{2} + \lambda, \tag{14a}$$

$$E_n^{(-)} \approx \frac{\omega_c}{2} + n\Omega - \frac{\delta}{2} - \lambda. \tag{14b}$$

The corresponding eigen states take the form

$$\left|\psi_n^{(+)}\right\rangle \sim \frac{1}{\sqrt{2}}\left(|1\rangle_c |g\rangle_a |\tilde{n}\rangle_b + |0\rangle_c |e\rangle_a |n\rangle_b\right), \tag{15a}$$

$$\left|\psi_n^{(-)}\right\rangle \sim \frac{1}{\sqrt{2}}\left(|1\rangle_c |g\rangle_a |\tilde{n}\rangle_b - |0\rangle_c |e\rangle_a |n\rangle_b\right). \tag{15b}$$

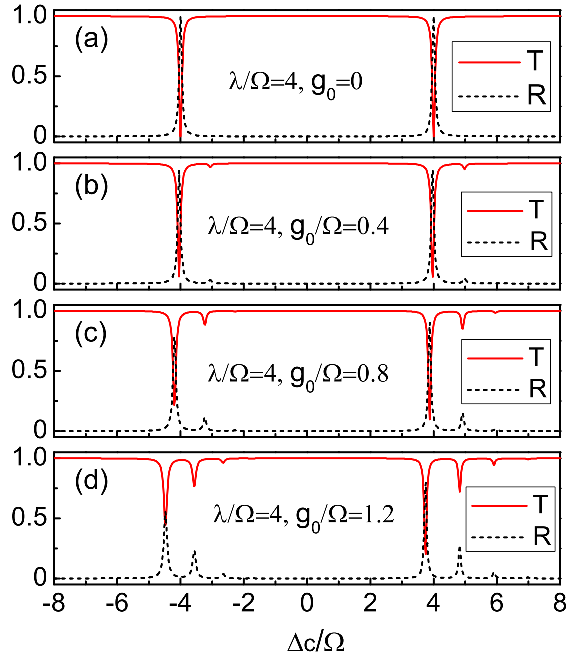


FIG. 4: (Color online) Single-photon transmission(reflection) spectra of the atom-optomechanical system for $\lambda \gg \Gamma$. The parameters are $\lambda = 4\Omega$, $\Delta_{ac} = 0$, $\gamma_a = 0$, and $\Gamma = 0.1\Omega$.

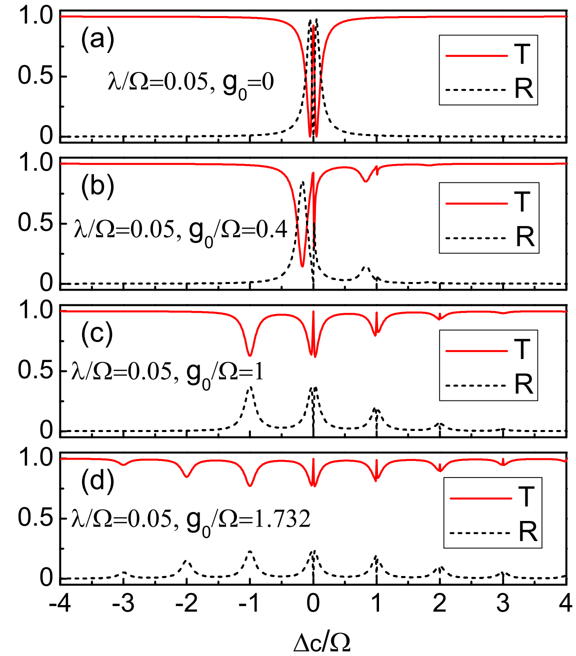


FIG. 5: (Color online) Single-photon transmission(reflection) spectra of the atom-optomechanical system for $\lambda < \Gamma$. The parameters are $\lambda = 0.05\Omega$, $\Delta_{ac} = 0$, $\gamma_a = 0$, and $\Gamma = 0.1\Omega$.

The energy-level structure in this case is potted in Fig. 3(a).

If $\lambda < \Gamma$, we can get a spectrum that is analogous to that for electromagnetically induced transparency (EIT) phenomena [2, 35]. Typically, when $g_0 = 0$, the spectrum exhibits a standard EIT one with a very narrow transmission window, as shown in Fig. 5 (a). When entering the single-photon strong coupling regime $g_0 > \Gamma$, more EIT structures appear in the sideband regime (Figs. 5(b)-(d)). The transmission maxima are located at $\Delta_c = n\Omega$ $(n = 0, 1, 2\cdots)$. Typically, when $g_0 = \sqrt{m}\Omega$ $(m = 1, 2, \cdots)$, i.e., $\delta = m\Omega$, we have $\epsilon_{\uparrow n} = \epsilon_{\downarrow n+m} = \omega_c/2 + n\Omega$. Namely, the eigen states of H_0, $|0\rangle_c |e\rangle_a |n\rangle_b$ and $|1\rangle_c |g\rangle_a \left|n \tilde{+} m\right\rangle_b$ are degenerate. This degeneracy is perturbed by the relatively weak atom-cavity interaction H_I, resulting in a pair of near degenerate states

$$\left|\psi_n^{(+)}\right\rangle \sim |0\rangle_c |e\rangle_a |n\rangle_b + \frac{\lambda}{\delta} |1\rangle_c |g\rangle_a |\tilde{n}\rangle_b, \tag{16a}$$

$$\left|\psi_{n+m}^{(-)}\right\rangle \sim |1\rangle_c |g\rangle_a \left|n \tilde{+} m\right\rangle_b - \frac{\lambda}{\delta} |0\rangle_c |e\rangle_a |n+m\rangle_b, \tag{16b}$$

with the eigen energies

$$E_n^{(+)} \approx \frac{\omega_c}{2} + n\Omega + \frac{\lambda^2}{\delta}, \tag{17a}$$

$$E_{n+m}^{(-)} \approx \frac{\omega_c}{2} + n\Omega - \frac{\lambda^2}{\delta}. \tag{17b}$$

The energy-level structure of this case is depicted in Fig. 3(b) by choosing $m = 1$ as an example. Thus, when a single-photon with detuning $\Delta_c = n\Omega$ $(n = 0, 1, 2\cdots)$ injected, destructive quantum interference occurs between the two possible transition channel $|0\rangle_c |g\rangle_a |0\rangle_b \rightarrow \left|\psi_n^{(+)}\right\rangle$ and $|0\rangle_c |g\rangle_a |0\rangle_b \rightarrow \left|\psi_{n+m}^{(-)}\right\rangle$, resulting in a complete transmission of the single-photon. This generate a EIT-like structure at $\Delta_c = n\Omega$ in the transmission(reflection) spectrum, as seen in Figs. 5(c) and (d). In addition, there are single transmission dips (reflection peaks) located at $\Delta_c = -l\Omega$ $(l = 1, \cdots, m)$, corresponding to the $|0\rangle_c |g\rangle_a |0\rangle_b \rightarrow \left|\psi_{m-l}^{(-)}\right\rangle$ $(l = 1, \cdots, m)$ transition.

B. Single-photon transmission(reflection) coefficient: effects of atom-cavity detunings and dissipations

We next consider the case of atom cavity to be detuned. When the optomechanical coupling strength $g_0 = 0$, for a photon on resonance with the atom $\Delta_c = \Delta_{ac}$, the transmission amplitude is always 1, as seen from Figs. 6 (a) and (b), which was indicated in Ref. [2]. When entering the single-photon strong coupling regime $g > \Gamma$, we can see that these maxima will appear at $\Delta_c = \Delta_{ac} + n\Omega$, corresponding to the $|0\rangle_c |g\rangle_a |0\rangle_b \rightarrow |0\rangle_c |e\rangle_a |n\rangle_b$ transitions, as shown in Figs. 6 (c)-(f).

There exist unavoidable intrinsic dissipative processes in the system, resulting in the leakage of photons into non-

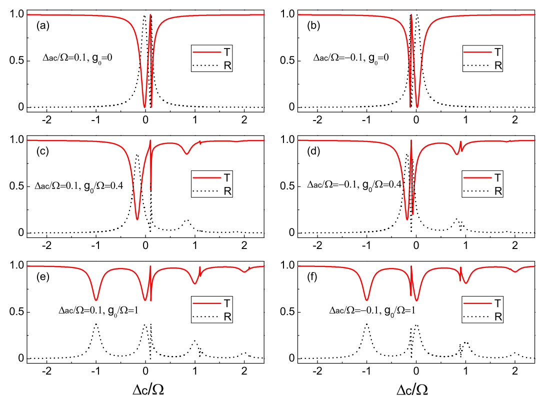


FIG. 6: (Color online) Single-photon transmission(reflection) spectra for detuned atom-cavity cases. In (a), (c), and (e), the atom-cavity detuning is $\Delta_{ac} = -0.1\Omega$; in (b), (d), and (f), $\Delta_{ac} = 0.1\Omega$. The other parameters are $\lambda = 0.05\Omega$, $\gamma_a = 0$, and $\Gamma = 0.1\Omega$.

waveguided degrees of freedom. Here we assume that the cavity strongly coupled to the waveguide, namely, the majority of the decayed light from the cavity is guided into waveguide modes [33]. Thus the decay rate κ of the cavity into channels other than the 1D continuum is negligible. The main dissipative processes are originated from the decay of atom. Experimentally, in both typical cavity QED and solid-state circuit QED systems, the ratio between the atom (artificial atom) decay rate and the cavity decay rate is about $\gamma_a/\Gamma \sim 0.1$ [34]. Figs. 7 give the transmission(reflection) spectrum of the dissipative atom case. The leakage of photons into non-waveguided degrees of freedom can be measured in terms of $T + R$ (the grey thin lines). In the Rabi-splitting like cases with strong atom-cavity coupling ($\lambda \gg \Omega \gg \Gamma$),

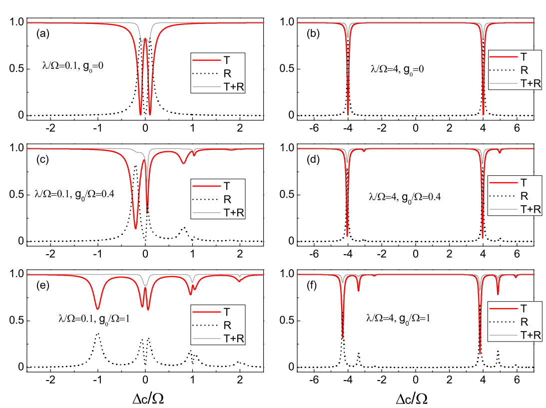


FIG. 7: (Color online) Single-photon transmission (reflection) spectra for dissipative atom cases ($\gamma_a = 0, 01\Omega$). In (a), (c), and (e), the atom-cavity coupling strength is $\lambda = 0.1\Omega$; in (b), (d), and (f), $\lambda = 4\Omega$. The other parameters are $\Delta_{ac} = 0$ and $\Gamma = 0.1\Omega$.

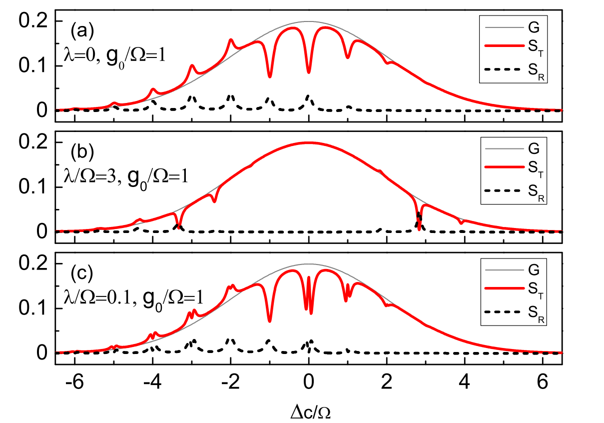


FIG. 8: (Color online) Transmitted (reflected) photon spectra $S_T(\Delta_c)$ ($S_R(\Delta_c)$). The parameters are $\Delta_0 = 0$, $d = 4\Omega$, $\Delta_{ac} = 0$, $\gamma_a = 0, 01\Omega$, and $\Gamma = 0.1\Omega$. The grey thin curve is the spectral density G of incident photons.

the atom dissipation has a stronger effect on the transmission of an photon at the frequencies of resonant absorption. On the contrary, in the EIT-like cases with relatively small atom-cavity coupling strength ($\lambda \sim \Gamma$), the atom dissipation has a stronger effect on the transmission of a photon with detuning around $\Delta_c = n\Omega$ ($n = 0, 1, 2, \cdots$).

C. The final reservoir occupation spectrum

We have above discussed the transmission (reflection) coefficients of a monochromatic incident photon. Note that for an optomechanical system in single-photon strong coupling regime, the inelastic scattering should have an influential effect, resulting in a re-emitted photon with red/blue sideband frequency. This is different from the case of photon transmitting (reflecting) from a usual cavity, where the frequency of the photon remains unchanged after scattering. To see this point more clearly, we calculate the final reservoir occupation spectrum [23, 28, 29], which describes probability density for finding the single photon with a specific frequency of the transmission (reflection) fields. Let us consider an incident photon with a Gaussian-type spectral amplitude $\alpha(\Delta_c) = \left(2/\pi d^2\right)^{1/4} \exp[-\left(\Delta_c - \Delta_0\right)^2/d^2]$, where Δ_0 and d is the detuning and spectrum width of the photon, respectively. The according spectral density can be represented as $G = |\alpha|^2$. We plot in Fig. 8 the spectra $S_T(\Delta_c)$ and $S_R(\Delta_c)$ of resonantly incident single-photon (i.e., $\Delta_0 = 0$) scattering when the mirror is initially prepared in the ground state $|0\rangle_b$, with strong optomechanical coupling strength $g_0 = \Omega$ and different atom-cavity coupling strength λ. It can be seen from Fig. 8 that phonon sidebands appear in the spectrum in the single-photon strong coupling regime. The dips in the spectrum $S_T(\Delta_c)$ appear at the same position as in the spectrum T, correspond to the resonant transition from $|0\rangle_c |0\rangle_a |0\rangle_b$ to the excited states. Thus after scattering, the probability density for finding the single photon at these frequencies decreases. And the peaks in the red sideband indicate that the re-emitted photon can lose its energy by $n\Omega$, leading to the final state $|0\rangle_c |0\rangle_a |n\rangle_b$ of system and increasing the value of $S_T(\Delta_c)$ at these transition frequencies.

IV. CONCLUSIONS

In summary, we have for the first time explored the single-photon transport in a waveguide coupled to a hybrid atom-optomechanical system in the single-photon strong-coupling regime. These spectra can characterize the mirror-cavity and atom-cavity couplings. On one hand, an optomechanical coupling dependent frequency shift and more sidebands appear in the transmission (reflection) spectra when the optical coupling strength increases. For the existence of atomic degrees of freedom, we can get a Rabi-splitting like or an EIT-like spectrum, depending on the atom-cavity coupling strength. Here we wish to make some further remarks on the possible experimental realizations of hybrid atom-optomechanical systems. Such a hybrid system can be possibly realized by directly combining the well developed technology of optomechanical cavities with moving mirror[32] and trapping a single atom in cavity QED [11–13]. Also, this set up may be more easily achieved using an on-chip circuit cavity electromechanics with a spiral inductor shunted by a parallel-plate capacitor, an analog of optomechanical cavity in microwave domain [17], which can be easily coupled to a superconducting artificial atom using currently availabe circuit QED technology [7]. These systems may provide quantum interface allowing the coherent transfer of quantum states between the mechanical oscillator and atoms, opening a door for coherent preparation and manipulation of micromechanical resonators [30].

Acknowledgments

We thank helpful discussions with Y. Li. This work was supported by the GRF (HKU7058/11P) and the CRF (HKU8/11G) of Hong Kong, and the URC fund of HKU.

[1] J. T. Shen and S. Fan, Phys. Rev. Lett., **95**, 213001 (2005).
[2] J. T. Shen and S. Fan, Phys. Rev. A **79**, 023837 (2009).
[3] L. Zhou, Z. R. Gong, Y. X. Liu, C. P. Sun, and F. Nori, Phys. Rev. Lett. **101**, 100501 (2008).
[4] J. Claudon, J. Bleuse, N. S. Malik, M. Bazin, P. Jaffrennou, N. Gregersen, C. Sauvan, P. Lalanne and Jean-Michel Grard, Nat. Photon. **4**, 174 (2010).
[5] D. E. Chang, A. S. Srensen, E. A. Demler, and M. D. Lukin, Nature Phys. **3**, 807 (2007).
[6] O. Astafiev, A. M. Zagoskin, A. A. Abdumalikov Jr., Y. A. Pashkin, T. Yamamoto, K. Inomata, Y. Nakamura and J. S. Tsai, Science **327**, 840 (2010).
[7] A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, R. S. Huang, J. Majer, S. Kumar, S. M. Girvin, and R. J. Schoelkopf, Nature (London) **431**, 162 (2004);
[8] T. Aoki, B. Dayan, E. Wilcut, W. P. Bowen, A. S. Parkins, T. J. Kippenberg, K. J. Vahala, and H. J. Kimble, Nature (London) **443**, 671 (2006).
[9] K. Srinivasan and O. Painter, Nature (London) **450**, 862 (2007).
[10] B. Dayan, A. S. Parkins, T. Aoki, E. P. Ostby, K. J. Vahala, and H. J. Kimble, Science **319**, 1062 (2008).
[11] T. J. Kippenberg and K. J. Vahala, Science **321**, 1172 (2008).
[12] F. Marquardt and S. M. Girvin, Physics **2**, 40 (2009).
[13] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, arXiv:1303.0733.
[14] K. W. Murch, K. L. Moore, S. Gupta, and D. M. Stamper-Kurn, Nat Phys **4**, 561 (2008).
[15] F. Brennecke, S. Ritter, T. Donner, and T. Esslinger, Science **322**, 235 (2008).
[16] D. W. C. Brooks, T. Botter, S. Schreppler, T. P. Purdy, N. Brahms, and D. M. Stamper-Kurn, Nature **488**, 476 (2012).
[17] J. D. Teufel, T. Donner, D. Li, J. W. Harlow, M. S. Allman, K. Cicak, A. J. Sirois, J. D. Whittaker, K. W. Lehnert, and R. W. Simmonds, Nature **475**, 359 (2011).
[18] J. Chan, T. P. M. Alegre, A. H. Safavi-Naeini, J. T. Hill, A. Krause, S. Groblacher, M. Aspelmeyer, and O. Painter, Nature **478**, 89 (2011).
[19] E. Verhagen, S. Deleglise, S. Weis, A. Schliesser, and T. J. Kippenberg, Nature **482**, 63 (2012).
[20] J. Chan, A. H. Safavi-Naeini, J. T. Hill, S. Meenehan, and O. Painter, Applied Physics Letters **101**, 081115 (2012).
[21] P. Rabl, Phys. Rev. Lett. **10**7, 063601 (2011).
[22] A. Nunnenkamp, K. Brkje, and S. M. Girvin, Phys. Rev. Lett. **10**7, 063602 (2011).
[23] Jie-Qiao Liao and C. K. Law, Phys. Rev. A **8**7, 043809 (2013).
[24] Xun-Wei Xu, Yuan-Jie Li, and Yu-xi Liu, Phys. Rev. A **87**, 025803 (2013).
[25] A. Nunnenkamp, K. Brkje, and S. M. Girvin, Phys. Rev. A **85**, 051803 (2012).
[26] A. Kronwald and F. Marquardt, arXiv:1304.5230.
[27] J. Qian, A. A. Clerk, K. Hammerer, and F. Marquardt, Phys. Rev. Lett. **109**, 253601 (2012).
[28] J. Q. Liao, H. K. Cheung, and C. K. Law, Phys. Rev. A **85**, 025803 (2012).
[29] Xue-Xin Ren, Hao-Kun Li, Meng-Yuan Yan, Yong-Chun Liu, Yun-Feng Xiao, and Qihuang Gong, Phys. Rev. A **87**, 033807 (2013).
[30] K. Hammerer, M. Wallquist, C. Genes, M. Ludwig, F. Marquardt, P. Treutlein, P. Zoller, J. Ye, and H. J. Kimble, Phys. Rev. Lett. **103**, 063005 (2009); M. Wallquist, K. Hammerer, P. Zoller, C. Genes, M. Ludwig, F. Marquardt, P. Treutlein, J. Ye, and H. J. Kimble, Phys. Rev. A **81**, 023816 (2010).
[31] Daniel Breyer and Marc Bienert, Phys. Rev. A **86**, 053819 (2012).
[32] R. Miller, T. E. Northup, K. M. Birnbaum, A. Boca, A. D. Boozer, and H. J. Kimble, J. Phys. B **38**, S551 (2005).
[33] Huaixiu Zheng, D. J. Gauthier, and H. U. Baranger, Phys. Rev. lett. **107**, 223601 (2011).
[34] A. Blais, R. S. Huang, A. Wallraff, S. M. Girvin, and R. J. Schoelkopf Phys. Rev. A **69**, 062320 (2004).
[35] S. E. Harris, Phys. Today **50**(7), 36 (1997).