

## Inverse Technique: MIMO-OFDM Channel Estimation and PAPR Reduction

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### ABSTRACT

*In general, for wireless communication systems, Multiple-Input-Multiple-Output and orthogonal frequency division multiplexing (MIMO-OFDM) systems provide high spectral efficiency. However, they have a major drawback of high PAPR which results in inefficient use of a power amplifier and also improper detections. Now a days, in wireless communication systems, channel estimation is mandatory for higher data rates with low bit error rates. The use of OFDM causes a high peak-to-average power ratio (PAPR), which necessitates expensive and power-inefficient radio-frequency (RF) components at the base station. The proposed precoder focused around the summed up converse is made out of two sections. One is for minimizing PAPR, and the other is for getting the multiplexing addition. Additionally, the proposed precoder contains a scalar parameter  $\beta$  that measures the got signal-to-noise power ratio (SNR) misfortune at the expense of PAPR diminishment. Indeed in instances of little SNR misfortune, the proposed plan drastically lessens PAPR. Moreover, reenactment results demonstrate that we can acquire a PAPR near to 0.5 by utilizing many transmission radio wires with little SNR misfortune.*

**Keywords:** OFDM, MIMO, pre-coding, PAPR, PAR, convex optimization, SU-MIMO.

### INTRODUCTION

Multiple Input Multiple Outputs (MIMO)-OFDM is widely recognized as a key technology for future wireless communications due to its high spectral efficiency

and superior robustness to Multipath fading channels [1]. For MIMO-OFDM systems, accurate channel estimation is essential to guarantee the system performance [2]. In this letter, a more practical sparse MIMO-OFDM channel estimation scheme based on spatial and temporal correlations of sparse wireless MIMO channels is proposed to deal with arbitrary path delays and also we exploit a generalized inverse of the right singular matrix of the MIMO channel to use redundant spatial dimensions at the transmitter. The generalized inverse of a matrix inherently includes an arbitrarily controllable matrix which is our key design parameter to minimize PAPR, and has a fixed part that we use for obtaining the spatial multiplexing gain.

The main contributions of this letter are summarized as follows. First, the proposed scheme can achieve super-resolution estimates of arbitrary path delays, which is more suitable for wireless channels in practice.

Second, due to the small scale of the transmit and receive antenna arrays compared to the long signal transmission distance in typical MIMO antenna geometry, channel impulse responses (CIRs) of different transmit-receive antenna pairs share common path delays [5], which can be translated as a common sparse pattern of CIRs due to the spatial correlation of MIMO channels. Meanwhile, such common sparse pattern is nearly unchanged along several adjacent OFDM symbols due to the temporal correlation of wireless channels [6], [7]. Compared with previous work which just simply extends the sparse channel estimation scheme in single antenna systems to that in MIMO by exploiting the spatial correlation of MIMO

channels [5] or only considers the temporal correlation for single antenna systems [6], [7], the proposed scheme exploits both spatial and temporal correlations to improve the channel estimation accuracy.

Third, we reduce the pilot overhead by using the finite rate of innovation (FRI) theory [8], which can recover the analog sparse signal with very low sampling rate, as a result, the average pilot overhead per antenna only depends on the channel scarcity level instead of the channel length. Finally, PAPR performance for large-scale MIMO systems, has effectively improved.

### NOTATIONS

In this paper indicating the following notations are,  $(\bullet)^T$  – transpose,  $(\bullet)^H$  – conjugate transpose,  $(\bullet)^{-1}$  – inverse,  $(\bullet)^{\dagger}$  – pseudo inverse,  $(\bullet)^{-}$  – generalized inverse,  $(\bullet)^{\perp}$  – orthogonal projection,  $\|\bullet\|_{\infty}$  – infinite norm,  $\|\bullet\|_F$  – frobenius norm,  $\text{blkdiag}(\bullet)$  – block digitalization,  $\text{tr}(\bullet)$  – trace and  $(\bullet)^{i, k}$  – trace of  $k^{\text{th}}$  subcarrier  $i^{\text{th}}$  row vector of a matrix.

### MIMO-OFDM SYSTEM DESCRIPTION

The block diagram of MIMO-OFDM (down link) system is shown in the following figure.1, this system contains number of transmission antennas  $M_T$  and number of receiver antennas  $M_R$ , in this system transmission antennas are greater than the receiver antennas i.e.  $M_T > M_R > d_k$ . Where  $k$  representing as  $k^{\text{th}}$  subcarrier  $\forall k \in \{1 \dots NC\}$ .

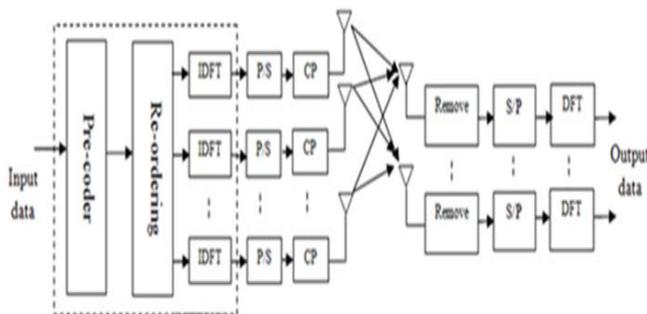


Fig: Block diagram of MIMO OFDM system

The transmitter transfer  $d_k \times 1$  symbol vectors  $s_k = [s_{k,1} \dots s_{k,d_k}]^T$  satisfy the following equation  $E_{s_k}[S_k S_k^H] = (p_s/d_k I)$  the received signal  $y_k$  is defined as

$$y_k = R_k H_k F_k s_k + R_k n_k \quad \text{(i)}$$

Here  $F_k$  denotes the transmission pre-coder for  $k^{\text{th}}$  subcarrier and is satisfying  $E_{s_k}[F_k s_k s_k^H F_k^H] \leq p_s$ .  $R_k$  denotes the receiver filter of  $k^{\text{th}}$  subcarrier,  $n_k$  is Gaussian noise vector and it satisfy  $[n_k n_k^H] = \sigma_n^2 I$ .  $H_k$  is the  $M_T \times M_R$  MIMO fading channel. The entire frequency selective fading channel is subdivided into number of series narrowband fading channels. Rewrite the equation (i) for all  $N_C$  subcarriers is defined as

$$y = R H F s + R n \quad \text{(ii)}$$

Here  $n = [n_1^T \dots n_{N_C}^T]^T$ ,  $s = [s_1^T \dots s_{N_C}^T]^T$ ,  $F = \text{blkdiag}(F_1 \dots F_{N_C})$ ,  $R = (R_1 \dots R_{N_C})$  and  $H = \text{blkdiag}(H_1 \dots H_{N_C})$ .

### OFDM SIGNAL

In OFDM system, the message bits are grouped in blocks  $\{X_n, n = 0, 1 \dots N - 1\}$  and modulate in amplitude a set of  $N$  subcarriers  $\{f_n, n = 0, \dots N - 1\}$ . These sub-carriers are chosen to be orthogonal, that is  $f_n = n\Delta f$ , where  $\Delta f = 1/T$ , and  $T$  is the OFDM symbol period. The resulting signal can be written as

$$X(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi f_n t} \quad \text{(iii)}$$

### PAPR OF OFDM SIGNALS AND GENERALIZED INVERSE

The PAPR is a measure commonly used to quantify the envelope fluctuations of multicarrier signals. For a discrete time signal  $(n)$ , the PAPR is defined as the ratio of the maximum power to the average power. For MIMO OFDM system (for IDFT Consideration at the transmitter) PAPR is defined as

$$PAPR = \frac{\max_{j \in [1, N_C], i \in [1, M_T]} |\tilde{x}_j^{(i)}(n)|^2}{\frac{1}{N_C M_T} \sum_{i=1}^{M_T} \sum_{j=1}^{N_C} |\tilde{x}_j^{(i)}(n)|^2} \quad \text{(iv)}$$

Where denominator is the average power of the signal  $\tilde{x}^{(i)}(n)$ , numerator is the maximum power of the signal  $\tilde{x}^{(i)}(n)$  and  $l$  is the oversampling factor. The different PAPR reduction techniques are shown in below Table.1

Methods	Average Power	Computational Complexity	BandWidth Expansions	BER Degradation	Side Information
Clipping	No	Low	No	Yes	No
PTS/SLM	No	High	Yes	No	Yes
TR/TI	Yes	High	Yes	No	No
Precoding	No	Low	Yes	No	No

**Table.1: Precoding technique compared with different PAPR reduction techniques**

$H_k$  is divided into three parts based on singular value decomposition (SVD) i.e.  $H_k = U_k \sum_k \bar{V}_k^H$ ,

Where  $\bar{V}_k$ ,  $U_k$  are denotes the  $M_T \times M_T$ ,  $M_R \times M_R$  unitary matrix and  $\sum_k$  is denotes  $M_R \times M_T$  diagonal matrix with singular values on the diagonal. Vector Symbol Error Rate (VSER) is minimizing by using pre-coding matrix for MIMO OFDM data transmission. VSER is defined as  $F_k = \bar{V}_k$  [11], here  $\bar{V}_k$  is composed of first  $d_k$  columns of  $\bar{V}$ ,  $tr(\bar{V}_k^H \bar{V}_k) = d_k$ . The generalized inverse  $\bar{V}_k^H$  is denoted as  $\bar{V}_k^H$ - is satisfying  $\bar{V}_k^H \bar{V}_k^H \bar{V}_k^H = \bar{V}_k^H$  [12].

Then the matrix  $V_k^H$  is defined as

$$\bar{V}_k^H = \bar{V}_k^H + P_k^L T_k \tag{v}$$

Here  $P_k^L = \bar{V}_k^H \bar{V}_k^H$  is orthogonal projection matrix and  $\square_\square$  is  $\square_\square \times \square_\square$  random matrix.

**PRECODER DESIGN FOR MIMO**

The main objective is to design MIMO pre-coder, it minimize the PAPR while providing a subsequence data rate performance based on MIMO pre-coding gain. In this, first we discuss on PAPR minimization.

**PRECODING MATRIX SELECTION**

The preorder matrix will determine the performance of the entire system. Because we are interested in constructing a high-rate signaling scheme with low error rates, we will present bounds on the probability of vector symbol error (i.e., the robability that the receiver returns at least one symbol in error). Here the design factor is used the null space of the right singular matrix  $\square_\square$  and it exists in generalized inverse of  $\square_\square$ . From equation (i) the proposed pre-coding matrix is defined as

$$F_k = \sqrt{B} \cdot \bar{V}_k^H + P_k^L T_k \tag{viii}$$

Here  $\square$  is the received SNR loss factor and it satisfying the condition  $0 < \square < 10$ . The parameter  $\square$  is allowed to control the power consumption between the control signal  $\square \square^L \square \square$  and the effective data transmission based on the SVD generalized inverse. It is given in below equation.

$$tr(F_k^H F_k) = \sqrt{B} \cdot tr(T_k^H P_k^L \bar{V}_k + \bar{V}_k^H P_k^L T_k) + tr(B \cdot \bar{V}_k^H \bar{V}_k + T_k^H P_k^L T_k) \tag{ix}$$

Here  $P_k^L = P_k^L$ ,  $P_k^L P_k^L = P_k^L$  and the first term is zero because of  $\square_\square^L \bar{V}_\square = 0_{\square \square} \times \square_\square$  and  $\bar{V}_\square \square_\square^L = 0_{\square \square} \times \sqrt{B} \bar{V}_\square$  is contributes the data transmission,  $P_k^L \square_\square$  is used in PAPR reduction by minimizing the peak amplitude of transmission signals; it is removed by the wireless channel.  $\sqrt{\square} \bar{V}_\square$ , is also used in PAPR reduction at the lower bound  $\square_\square \square_\square$  in equation (vii). From equation (viii) precoder and receiver  $\square_\square = \square_\square^H$ , then the received signal is given as

$$y_k = \sqrt{B} \sum_k S_k + U_k^H n_k \tag{x}$$

Where  $\sqrt{B} \bar{V}_\square$  is denotes the first  $\square_\square$  column vectors of  $\square$  and  $\square_\square^L \square_\square F_k = \sqrt{B} \sum_k I$ . The second term in equation (viii) is not effect on the received signal.

**PAPR MINIMIZATION**

From the design parameter  $\square_\square$  and is given in equation (viii) then reformulate the problem defined in equations (vi), (vii) and it's are shown in below equations that are

$$minimize \max_{T_k, \forall k, \forall j, \forall i} \left[ \left| \tilde{x}_j^{(i)}(n) \right|^2 \right]_{(xi)}$$

subject to

$$P_{avg}^- \leq E[S_k^H F_k^H F_k S_k] \leq P_{avg}^+ \tag{xii}$$

Where  $\tilde{X}^{(i)} = Q^{IDFT} [f_{1,i}^r, S_{1,i}, \dots, f_{N_c,i}^r, S_{N_c,i}]^T$  **(xiii)**

Here  $\square_\square \square_\square \square_\square$  denotes the  $\square_\square \square_\square \times \square_\square \square_\square$  IDFT matrix. From equation (viii),  $\square_\square$  is not in function of the  $\square_\square \times 1$  target vector variable  $\square_\square$ , but these are contained in row vector for the precoding matrix  $\square_\square$ , it can be defined as

$$f_{k,i}^r \sqrt{B} \cdot \bar{v}_{k,1}^r + [P_{k,i}^L, t_{k,1}, \dots, P_{k,i}^L, t_{k,d_k}] \tag{xiv}$$

Substitute equation (viii) in equation (xii) then it is given as

$$P_{avg}^- \leq B(p_s/d_k) \text{tr}(\bar{V}_k^H \bar{V}_k) + (p_s/d_k) \sum_{l=1}^{d_k} t_{k,l}^H P_k^- t_{k,l} \leq P_{avg}^+ \quad (xv)$$

The average power consumption is larger than  $P_{avg}^-$  because the average power is minimized when the design parameters are  $\alpha_{1,1}, \dots, \alpha_{1,d_k} = 0$ . We assume that the  $P_{avg}^- = P_{avg}$ , and  $P_{avg}^+ = P_{avg}$ . Here  $P_{avg}$  is the transmission power for carrying the data signals. In equation (xv) second boundary is neglected then the average power consumption is maintained in between  $P_{avg}^-$  and  $P_{avg}^+$ . So we can have an opportunity to minimize PAPR is close to 0.5.

Let us consider scalar parameter  $\alpha$ . We can intuitively predict that the constant  $\alpha$  would be selected close to 0.5 when PAPR negligibly affects the error rate performance so that MIMO pre-coding gain is more critical than PAPR performance. However, it is noted that minimizing the maximum amplitude at the expense of the received SNR loss does not always guarantee the *peak-to-average power ratio* reduction due to the variation of the average transmission power. It implies that there is  $\alpha$  which provides the best tradeoff between the PAPR performance and SNR loss given system parameters such as, modulation order, dynamic range and  $\alpha$ .

## RESULTS AND ANALYSIS

The PAPR dB scale performance of the proposed scheme is compared with conventional scheme and PMP for  $4 \times 2, 8 \times 2$  MIMO is shown in below figure.

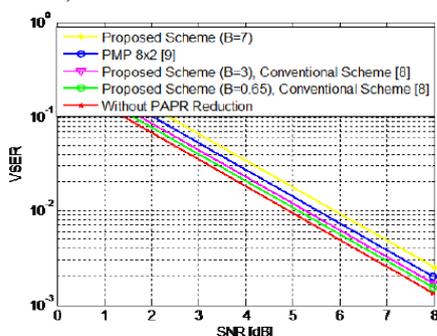


Fig: PAPR dB scale performance of the proposed scheme, conventional scheme and PMP.

Let us consider simulation parameters ( $N_c, N_s, N_{sc} = N_{sp}, N_{sp}$ ) is assumed like as (4, 2, 2, 64), (8, 2, 2, 64) with 16-QAM. As the baseline approaches for PAPR performance evaluation, we focus on the convex optimization approaches of an Error Vector Magnitude (EVM) based optimization scheme [3], proposed symbol optimization based PMP [9] and a MIMO OFDM signal without PAPR reduction, where the maximum EVM is -30 dB and -20 dB from [3].

Here the EVM is defined as

$$\sqrt{d_k/p_s \cdot \sum_{k=1}^{N_c} \|e_k\|^2}$$

Where  $e_k$  is difference between the original and distorted signal. -30 dB and -20 dB EVM are equivalent to -15 dB and -10 dB SNR loss [10], and their corresponding  $\alpha$  in the proposed scheme are 0.65, 3.

Then compare algorithm performance with [3] and [9]. We evaluate PAPR performance for a system parameter  $4 \times 2$  MIMO of 16-QAM and its corresponding VSER performance. As a reference for SU-MIMO transmission scheme, we consider a transmit Zero Forcing Beam Forming (ZF-BF) without the PAPR reduction scheme with  $R_k = I$ . Also the PAPR reduction scheme [3] and [9] are applied to SU-MIMO based on ZF-BF with  $R_k = I$ . The VSER performance of our proposed scheme is degraded only depending on  $\alpha$  compared to the reference case.

From Fig, it can be observed that the PAPR value ranges from 0.25 dB to 2.9 dB according to  $\alpha$  and  $N_{sc}$ . In consideration of 10-3 percentage of PAPR performance with  $B = 0.65$ , the proposed scheme outperforms EVM-based conventional scheme [3] and [9] by more than 4.3 dB and 1 dB respectively.

The vector symbol error rate performance of the proposed scheme is compared with the conventional scheme [3] and PMP [9] for  $4 \times 2$  is shown in below Figure.3.

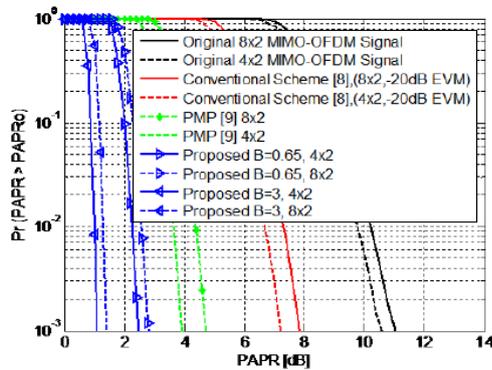


Fig: VSER performance of the proposed scheme, conventional scheme and PMP.

From Fig, it can be observed that the proposed method also show the good VSER performance over PMP [9] for  $\beta = 0.65$  and  $\beta = 3$ . When  $\beta = 0.65$ , the proposed scheme shows PAPR gain is 3.2 dB and  $\beta = 3$ , the proposed scheme shows VSER gain is 0.9 dB over [9], it can be seen in figures.2 and 3. Recall that the transmission power for data transmission is maintained as  $P_{avg}$ , so that the received SNR loss of the proposed scheme will be marginal close to 0.5. Since the PMP scheme [9] also uses the null space of the MIMO channel, but without the constraint of the desired signal space power, relatively large power consumption is allowed to minimize peak power, which may degrade VSER performance compared to the proposed scheme. The proposed scheme and conventional scheme [4] show the same VSER performance due to the same cost for both schemes. The PAPR performances of the proposed scheme for different MIMO configurations are shown in below fig.

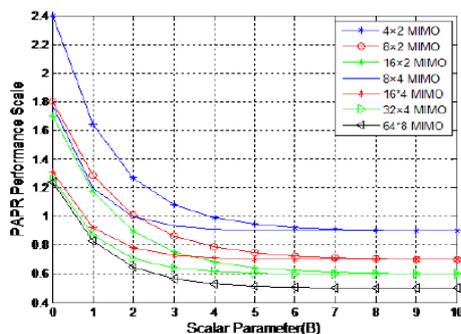


Fig: PAPR performance of the proposed scheme with different MIMO configurations.

The PAPR performance is significantly improved with small reduction of  $\beta$  when PAPR close to 0.5. In contrast,  $\beta$  decreases. From figure.4, it can be also observed that the PAPR performance is marginally improved even if  $\beta$  is doubled. However, the PAPR is significantly reduced as scalar parameter  $\beta$  increases then PAPR close to 0.5 and PAPR converge to a certain level. The proposed method reducing PAPR since the maximum amplitude of the time domain signal is minimized while keeping the average transmission power at a certain level. From figure.4, the performance curves of  $13 \times 2$ ,  $32 \times 4$  and  $64 \times 8$  MIMO, it can be observed that PAPR performance approaches the same level as  $\beta$  increase in case that  $\beta_{10}/\beta_0$  ( $\beta_{10}/\beta_0$ ) is the same, where  $\beta_0 = \beta_{10}$ . When  $\beta$  and  $\beta_{10}/\beta_0$  are simultaneously increased, the limiting PAPR can be achieved with relatively large  $\beta$ . Thus, it is observed that the proposed scheme can be achieve a PAPR close to 0.5 with increasing number of transmitters ( $N_t$ ) and number of transmitters or number of receivers ( $N_r/N_t$ ). The amplitude levels of different MIMO configuration performance of the proposed scheme is shown in below Table.2.

	Scalar Parameter(B)										
	0	1	2	3	4	5	6	7	8	9	10
4*2 MIMO	2.4	1.62	1.28	1.1	1	0.97	0.92	0.9	0.9	0.9	0.9
8*4 MIMO	1.79	1.2	1	0.93	0.92	0.91	0.9	0.9	0.9	0.9	0.9
8*2 MIMO	1.8	1.3	1	0.88	0.8	0.76	0.74	0.71	0.7	0.7	0.7
16*4 MIMO	1.3	0.96	0.8	0.74	0.72	0.7	0.7	0.7	0.7	0.7	0.7
16*2 MIMO	1.7	1.19	0.9	0.77	0.68	0.64	0.62	0.61	0.6	0.6	0.6
32*4 MIMO	1.26	0.89	0.7	0.64	0.62	0.6	0.6	0.6	0.6	0.6	0.6
64*8 MIMO	1.25	0.84	0.64	0.58	0.54	0.52	0.51	0.5	0.5	0.5	0.5

Table.2: PAPR performance of the proposed scheme

### ALGORITHM FOR THE GENERALIZED INVERSE OF A MATRIX

An algorithm for finding the generalized inverse of a matrix is as follows, according to Adetunde et al;

**Step 1:** In A of rank r, find any non-singular minor of order r call it M

**Step 2:** Invert M and transpose the inverse (M)

**Step3:** In A replace each element of M by the corresponding element of (M) That is  $a_{st} = M_{st}$

element of  $m$ , then replace a  $b$   $M$ , the  $(t,s)$  element of  $M$  equivalent to the  $(s, t)$  element of the transpose of  $M$   
**Step4:** Replace all the other elements of  $A$  by zero  
**Step5:** Transpose the resulting matrix and the result is  $G$  a generalized inverse of  $A$ .

## CONCLUSION

The proposed MIMO pre-coding scheme consists of two parts, one is minimizing PAPR and other one is obtaining the multiplexing gain. The PAPR is minimized when the maximum amplitude of time domain signals are minimized while keeping the average power at certain level. The PAPR performance is close to 0.5, the proposed method minimizes the errors and it gives effective data rate transmission.

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