Novel Blind CFO Estimator for Uplink Interleaved OFDMA Systems

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Abstract

In orthogonal frequency-division multiple access (OFDMA) systems, closely spaced and overlapped sub-carriers are divided into groups and assigned to multiple users for simultaneous transmission. However, carrier frequency offsets (CFOs) between the transmitter and the receiver destroy the orthogonality and introduce inter-carrier interference (ICI), resulting in multiple-access interference (MAI). By the inner structure of the received signals, this paper presents a novel CFO estimation algorithm for interleaved OFDMA uplink systems. The proposed algorithm exploits the idea of whitening the spatial covariance matrix of received signals to obtain the CFO estimates of all users. The proposed approach relies only on stationary second-order statistics that are based on a joint diagonalization of a set of covariance matrices. Simulation results illustrate the efficacy of the proposed algorithm.

Keywords: Carrier Frequency Offset (CFO), Orthogonal Frequency-Division Multiple-Access (OFDMA), Uplink System, Joint Diagonalization.

1. Introduction

As a promising technique to fulfill the high data rate demand for next-generation broadband wireless network, orthogonal frequency-division multiple access (OFDMA) has gained increased interest recently. This OFDM-based multiplexing technique possesses advantages such as high spectrum efficiency, robustness against multi-path fading channel, resistance to multi-access interference (MAI), etc [1, 2]. However, inherited from OFDM, OFDMA is extremely vulnerable to carrier frequency offsets (CFOs), which will destroy the orthogonality among sub-carriers and introduce inter-carrier interference (ICI) and MAI. In an OFDMA uplink system, CFO estimation becomes a multi-parameter estimation problem and is considered to be a crucial challenge, since different users have possibly different CFOs, and all CFOs have to be simultaneously estimated at the base station.

Some CFO estimation algorithms in the uplink of OFDMA systems have been developed. In [3], the CFOs are estimated by maximizing the mean likelihood function via exploiting an important sampling technique. In [4], an iterative scheme, which is referred to as the alternating-projection frequency estimator (APFE), is proposed. Although the algorithm can achieve CRB with fast convergence rate, it requires an exhaustive grid searching to estimate each CFO and an iterative procedure to refine the estimates, which results in unattractive complexity. A suboptimal approach [5], which replaces the matrix inversion in APFE with an approximated matrix extension, is reported. Its computational complexity is lower than APFE. However, it requires a large number of sub-carriers. In [6], a simple iterative algorithm is derived via exploiting the fact that the tile structure in 802.16e [7] provides inherent MAI power compression in the frequency domain. In [8], a CFO estimation scheme is reported by minimizing the MAI by making use of the repetitive structure of the users’ training sequences. The CFO estimators in [3-6, 8] are all based on the inserted training sequences in front of each data packets.

However, when training sequences are not available, the CFO for each user cannot easily be obtained. Fortunately, this problem can be overcome with a proper carrier assignment scheme (CAS), such as subband-based CAS or interleaved CAS. The methods in [9, 10], which consider the sub-band CAS, exploit the filter bank to separated the received signals, and thus CFOs can be easily obtained.
separately. However, Subband-based OFDMA systems are sensitive to frequency selective fading. With the interleaved CAS, the blind CFO estimators [11, 12] estimate the CFOs via applying MUSIC and ESPRIT, respectively. Due to the spectrum peak searching involved, the computational complexity of the MUSIC estimator is not attractive. The ESPRIT estimator can avoid the complex peak searching, and then obtain lower computation complexity. However, its performance is not as well as that of MUSIC algorithm. In [13], an optimum CFO estimator is proposed by truncating the series expansion of the correlation matrix in maximum likelihood function.

In this paper, a new CFO blind estimator is proposed for the interleaved OFDMA uplink systems. The idea of whitening the spatial covariance matrix of received signals is exploited to obtain the estimation of the matrix including all CFOs’ information. Furthermore, the joint diagonalization technique of the multiple whitened covariance matrices is used to improve the estimation performance.

The outline of the paper is organized as follows. Section 2 briefly introduces the data model of the interleaved OFDMA uplink systems. The proposed method is derived in Section 3. Section 4 describes the simulation results. Section 5 provides a concluding remarks to summarize the paper.

### Notation
Superscripts \((\cdot)^T\), \((\cdot)^H\) and \((\cdot)^T\) denote the conjugate, the conjugate transpose and the transpose operators, respectively. \(\cdot\) is the modulus operator and \(\mathbb{E}\{\cdot\}\) represents the expected value. \(\text{diag}(a,\cdots,a)\) is the diagonal matrix with diagonal entries \(a,\cdots,a\). \(\odot\) is the Schur product. Bold face small letters are used for vectors and bold face capital alphabets are for matrix representation. \(I\) stands for the identity matrix.

### 2. Signal models for interleaved OFDMA uplink

Consider the interleaved uplink of an OFDMA system with \(N\) sub-carriers in which \(K\) active users simultaneously communicate with the base station. The \(N\) sub-carriers are evenly divided into \(Q\) \((Q > K)\) sub-channels, each of which has \(P = N/Q\) sub-carriers. Each user only occupies one sub-channel, and the \(q\)th sub-channel is assigned to the \(k\)th user, whose sub-carrier indices is defined as \(\{qQ + q,\cdots,(P-1)Q + q\} \ (q = 0,1,\cdots,Q-1)\). After the cyclic prefix (CP) is removed, the signal sample of the \(n\)th sub-carrier of one OFDMA block at the base station (BS) can be described as

\[
y(n) = \sum_{k=1}^{K} y_k(n) + z(n) \quad (n = 0,1,\cdots,N-1)
\]

where \(K\) denotes the number of all active users. \(z(n)\) is modeled as a white additive Gaussian noise with zero-mean and equal variance \(\sigma^2\). \(y_k(n)\) is the signal sample of the \(n\)th sub-carrier from the \(k\)th user at the uplink receiver given by

\[
y_k(n) = \sum_{p=0}^{P-1} H_{k,p} X_{k,p} e^{j2\pi(nP + q_k)}
\]

where \(H_{k,p}\) stands for the channel frequency response on the \(p\)th sub-carrier of the \(k\)th user’s channel during one OFDMA block. \(X_{k,p}\) denotes the data symbol on the \(p\)th sub-carrier of the \(k\)th user with \(\mathbb{E}\{|X_{k,p}|^2\} = 1\). \(\xi_k = \Delta f_k / \Delta f\) is the normalized CFO of the \(k\)th user. \(\Delta f\) and \(\Delta f_k\) are the sub-carrier spacing and the CFO between the \(k\)th user and the BS, respectively. \(|\Delta f_k|\) is assumed to be less than half of OFDMA sub-carrier spacing, i.e., \(|\xi_k| < 0.5\).

From (1), we can construct a \(Q \times P\) matrix \(Y\) by a data stacking technology. The matrix \(Y\) has the following form
\[
Y = \begin{bmatrix}
y(0) & y(1) & \cdots & y(P-1) \\
y(P) & y(P+1) & \cdots & y(2P-1) \\
\vdots & \vdots & \ddots & \vdots \\
y(N-P) & y(N-P+1) & \cdots & y(N-1)
\end{bmatrix} = V_{Q,P}S_{K,P} + Z_{Q,P} \tag{3}
\]

where \( S = C_{K,P} \otimes (B_{K,P}W_{p,p}) \). The \( k \)th row of \( S \) is \([y_k(0), y_k(1), \cdots, y_k(P-1)]\), which denotes the received signals of the \( k \)th user. The \( k \)th column of \( V \) is \( v_k = [1, e^{j2\pi k}, e^{j4\pi k}, \cdots, e^{j2\pi (P-1)k}]^T \), where \( \theta_k = (q + \xi_k)/Q \) is the effective CFO of the \( k \)th user. \( C = [c_1^T, c_2^T, \cdots, c_K^T] \) with \( c_i = [1, e^{j2\pi i/P}, \cdots, e^{j2\pi (P-1)k}]^T \), and \( B = [b_1^T, b_2^T, \cdots, b_K^T] \) with \( b_k = [H_{k,1}X_{k,1}, H_{k,2}X_{k,2}, \cdots, H_{k,P}X_{k,P}] \). \( W \) is an IFFT matrix and \( Z \) is the white Gaussian noise matrix.

3. Algorithm Formulation

Let \( y_p, s_p, z_p \) \((p = 1, 2, \cdots, P)\) denote the \( p \)th column of \( Y, S, Z \), respectively. We have the following equation

\[
y_p = x_p + z_p = Vs_p + z_p \tag{4}
\]

Notice that the matrix \( V \) includes the CFO information, we may be derived the CFOs of all users via the estimated matrix \( V \). Without any loss of generality, assume that \( \mathbb{E}\{s_p s_p^H\} = I \), so that the dynamic range of \( s_p \) is accounted for by the magnitude of the corresponding columns of \( V \). Applying to the signal part \( x_p \) of the observation a whitening matrix \( K \times Q \) \( W \), i.e.,

\[
\mathbb{E}\{WX_p x_p^H W^H\} = WVV^H W^H = I . \tag{5}
\]

The above equation shows that if \( W \) is a whitening matrix, then \( WV \) is a unitary matrix, namely, there must exist a \( K \times K \) unitary matrix \( U \) which satisfies \( WV = U \). As a consequence, matrix \( V \) can be derived as

\[
V = WV^H U . \tag{6}
\]

The new CFO estimation algorithm is mainly developed as follows.

3.1. Estimate the whitening matrix \( W \)

Consider the covariance matrix \( R(0) \) of \( y_p \), we have

\[
R(0) = \mathbb{E}\{y_p y_p^H\} = \mathbb{E}\{x_p x_p^H\} + \sigma^2 I = \mathbb{E}\{s_p s_p^H\} V^H + \sigma^2 I = WV^H + \sigma^2 I , \tag{7}
\]

so, \( VV^H = R(0) - \sigma^2 I \). It implies that a whitening matrix \( W \) can be obtained from \( R(0) \), provide that the noise covariance is known or can be estimated. Exploiting the eigenvalue decomposition (EVD) to \( R(0) \), it yields

\[
R(0) = U A U^H + U A U^H . \tag{8}
\]
where $A = \text{diag} [\lambda_1, \lambda_2, \cdots, \lambda_K]$ and $A_s = \text{diag} [\lambda_{s1}, \lambda_{s2}, \cdots, \lambda_{sK}]$ with $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{s1} = \cdots = \lambda_0 = \sigma^2$ are the $Q$ eigenvalues in descending order. $U_s = [u_1, u_2, \cdots, u_K]$ and $U_s = [u_{s1}, u_{s2}, \cdots, u_{sK}]$, where $u_i$ is the eigenvector corresponding to the $i$th eigenvalue. It is readily to know that $\{(\lambda_i - \sigma^2)^{-1/2} u_i, (\lambda_i - \sigma^2)^{-1/2} u_{s1}, (\lambda_i - \sigma^2)^{-1/2} u_{s2}, \cdots, (\lambda_i - \sigma^2)^{-1/2} u_{sK}\}^H$ is a whitening matrix, i.e.,

$$W = \{(\lambda_i - \sigma^2)^{-1/2} u_i, (\lambda_i - \sigma^2)^{-1/2} u_{s1}, (\lambda_i - \sigma^2)^{-1/2} u_{s2}, \cdots, (\lambda_i - \sigma^2)^{-1/2} u_{sK}\}^H.$$  \hspace{1cm} (9)

### 3.2. Estimate the unitary matrix $U$

Let $R(l) = \mathbb{E}\{y_p y_{p+l}^H\} = \mathbb{E}\{s_p s_{p+l}^H\} V^H = VR(l)V^H$, and consider the spatially whitened covariance matrix defined as $\overline{R}(l) = WR(l)W^H$. Invoking $WV = U$, we obtain

$$\forall l \neq 0 \overline{R}(l) = WR(l)V^H VW = U R(l) U^H.$$ \hspace{1cm} (10)

In Eq. (10), $U$ is unitary and $R(l)$ is diagonal, which means that any whitened covariance matrix is diagonalized by the unitary transform $U$. Thus, the unitary matrix $U$ may be derived as a unitary diagonalizing matrix of $\overline{R}(l)$.

Starting from the EVD of $\overline{R}(l)$, $\overline{R}(l)$ can be expressed as follows

$$\overline{R}(l) = \overline{U} \overline{A} \overline{U}^H.$$ \hspace{1cm} (11)

According to [14,15], it is to know that $\overline{U}$ is essentially equal to $U$, namely, $\overline{U} = UP$, where $P$ has exactly one nonzero entry in each row and column, where these entries have unit modulus. However, it is not necessarily true, since an unfortunate choice of $l$ may result in unidentifiability of $U$ from $\overline{R}(l)$.

In the next derivation, the idea of joint diagonalization in [14] is exploited, which intends to reduce the unidentifiable probability of $U$, and more importantly, increases the statistical of the procedure by inferring the value of $U$ from a larger set of statistics.

Here, the “off” of a $K \times K$ matrix $\Gamma$ with entries $\gamma_{i,j}$ is defined as

$$\text{off}(\Gamma) \overset{\Delta}{=} \sum_{i \neq j \in [K]} |\gamma_{i,j}|^2.$$ \hspace{1cm} (12)

In numerical analysis, we know the unitary diagonalization of $\Gamma$ is equivalent to zeroing $\text{off}(G^H \Gamma G)$ by some unitary matrix $G$. If a matrix $\Gamma$ can be written in the form $\Gamma = UD\overline{U}^H$, where $U$ is unitary and $D$ is diagonal with distinct diagonal elements, then it may be unitarily diagonalized only by unitary matrices which is essentially equal to $U$ [14], that is, if $\text{off}(G^H \Gamma G) = 0$, then $G$ is essentially equal to $U$.

Having the above analysis, we consider $M$ distinct values of $l$, such as $l_1, l_2, \cdots, l_M$, and obtain a set consists of $M$ whitened covariance matrices, namely, $\mathcal{R} = \{\overline{R}(l_1), \overline{R}(l_2), \cdots, \overline{R}(l_M)\}$. The “joint diagonalization” (JD) criterion is defined, for any $K \times K$ matrix $G$, as the following nonnegative function of $G$

$$\square (\mathcal{R}, G) \overset{\Delta}{=} \sum_{m=1 \cdots M} \text{off}(G^H \overline{R}(l_m) G).$$ \hspace{1cm} (13)
A unitary matrix is referred as a joint diagonalizer of the set \( \mathfrak{R} \) if it minimizes the above JD criterion over the set of all unitary matrices.

Having \( W \) and \( G \) which is essentially equal to \( U \), we have \( \tilde{V} = W^* G = W^* U \tilde{F} = V \tilde{P} \), where \( \tilde{P} \) possesses the same feature with \( P \). It shows that \( \tilde{V} \) is essentially equal to \( V \). Let \( \tilde{V} = [\tilde{v}_1, \tilde{v}_2, \ldots, \tilde{v}_K] \), where \( \tilde{v}_k = \left[1, e^{j2\pi \xi_k}, e^{j4\pi \xi_k}, \ldots, e^{j2\pi (q-1)\xi_k}\right]^T \) \( (k \in [1, 2, \ldots, K]) \), \( \tilde{\xi}_k \) denotes the CFO of one of the active users. Since distinct user have different CFO range, once \( \tilde{V} \) is obtained, the effective CFO of each user can be estimated via

\[
\tilde{\xi}_k = \arg \left( \prod_{q=1}^{Q-1} \tilde{v}_{q,k} \right) (k \in [1, 2, \ldots, K]) ,
\]

where \( \tilde{v}_{q,k} \) is the \((q,k)\)-entry of the matrix \( \tilde{V} \).

3.3. Summary of the proposed method

The proposed algorithm can be summarized as follows:

1) According to equation (3), formulate the received signal samples into matrix \( Y \) and estimate the covariance matrix \( \hat{R}(0) = YY^H / \hat{P} \).

2) Compute the eigendecomposition of \( \hat{R}(0) \), i.e., \( \hat{R}(0) = \hat{U} \Lambda \hat{U}^H \), where \( \Lambda = \text{diag} \{ \lambda_0, \lambda_1, \ldots, \lambda_Q \} \) in which \( \lambda_0 \geq \cdots \geq \lambda_K \geq \lambda_{K+1} \geq \cdots \geq \lambda_Q \) and \( \hat{U} = [\hat{u}_1, \hat{u}_2, \ldots, \hat{u}_Q] \).

3) The average value of the last \( Q - K \) smallest eigenvalues of \( \hat{R}(0) \) is denoted as an estimate \( \hat{\sigma}^2 \) of the noise variance. The whitening matrix \( W \) can be estimated by the following expression:

\[
\hat{W} = [(\lambda_1 - \hat{\sigma}^2)^{-1/2} \hat{u}_1, (\lambda_2 - \hat{\sigma}^2)^{-1/2} \hat{u}_2, \ldots, (\lambda_K - \hat{\sigma}^2)^{-1/2} \hat{u}_Q]^H.
\]

4) Calculate the whitened covariance matrices \( \overline{\hat{R}}(l_m) \) \( (m = 1, 2, \ldots, M) \), and then a unitary matrix \( \hat{U} \) is obtained as joint diagonalizer of the set \( \{ \overline{\hat{R}}(l_1), \overline{\hat{R}}(l_2), \ldots, \overline{\hat{R}}(l_M) \} \).

5) The matrix \( V \) including the CFO information is estimated by \( \hat{V} = \hat{W}^* \hat{U} \), and thus the effective CFOs can be obtained via \( \hat{\theta}_k = \frac{1}{\pi Q(Q-1)} \arg \left( \prod_{q=1}^{Q-1} \hat{v}_{q,k} \right) (k \in [1, 2, \ldots, K]) \), where \( \hat{v}_{q,k} \) is the \((q,k)\)-entry of the matrix \( \hat{V} \).

4. Simulation Results

In this section, the performance of the proposed algorithm is evaluated using computer simulation. The considered interleaved uplink OFDMA system has \( N = 512 \) sub-carriers which are divided into \( Q = 16 \). The length of CP is 64. The normalized CFOs of all active users \( \xi_k \) \( (k = 1, 2, \ldots, K) \) are generated as random variables that are uniformly distributed in \((-0.5, 0.5)\) with 500 random trial runs. All user data symbols used for CFO estimation are modulated with a quadrature phase-shift keying scheme. The Rayleigh channel with 4 paths with exponentially decaying power delay profiles is selected to model the multi-path fading channel. The performance of the proposed algorithm can be assessed via the normalized root mean square error (NRMSE) defined as:

\[
\text{NRMSE} = \sqrt{\frac{1}{K \Pi} \sum_{m=1}^{M} \sum_{v=1}^{N} \left( \hat{\xi}_{v,k} - \xi_{v,k} \right)^2} ,
\]
where $K$ denotes the number of all users, $\Pi$ is the amount of Monte Carlo runs, $\hat{\xi}_{\rho,\lambda}$ is the estimate of $\xi_{\rho,\lambda}$, $\rho$ is the index of the Monte Carlo runs.

Firstly, we evaluate the effect of the number of the jointly diagonalizing spatial whitened covariance matrices on the performance, and compare the performance of the proposed algorithm and ESPRIT [12]. Fig.1 shows that the NRMSE curves of the CFO estimation as a function of SNR of the proposed algorithm and the ESPRIT algorithm, respectively. It is clear from Fig.1 that using multiple covariance matrices can improve the performance of estimation. The performance of using diagonalizing five matrices or three matrices outperforms that of using only one. Furthermore, it can be seen that the performance of the proposed algorithm with $M = 3$ is inferior to that of ESPRIT. While the performance with $M = 5$ outperforms that of ESPRIT, basically. In the following simulation, the performance is assessed by jointly five diagonalizing covariance matrices.

Figure 1. Performance comparison between the proposed algorithm and the ESPRIT algorithm.

In the second experiment, we examine the performance of the proposed method with various subchannels. Fig.2 shows the NRMSE of CFO estimation as a function of SNR with $Q = 8, 16$ users, respectively. It can be seen that increasing the subchannel number properly will improve the performance of the proposed algorithm.

Figure 2. Performance comparison for various $Q$. 

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In the third experiment, we examine the performance of the proposed method with various users. Fig.3 shows the NRMSE of CFO estimation as a function of SNR with $K = 1, 2, 4$ users, respectively. We observed that the NRMSE decreases when the SNR increases. As expected, the NRMSE of the proposed algorithm increases when the number of users in one OFDMA block decreases.

![Figure 3. Performance comparison for various $K$.](image)

In the fourth experiment, we compare the performance of the proposed method with different OFDMA blocks. Fig.4 shows the NRMSE of CFO estimation as a function of SNR with 2, 4, 8 blocks, respectively. As can be seen from Fig.4, the better performance is achieved by increasing the number of blocks. This is because the multiple OFDMA block diversity can increase the amount of signal samples and noise samples which will result in the improved performance of CFO estimation.

![Figure 4. Performance comparison for different blocks.](image)

5. Conclusions

A new CFO blind estimator is proposed for the interleaved OFDMA uplink systems in this paper. The proposed algorithm exploits the idea of whitening the spatial covariance matrix of received signals to estimate the CFOs of all users and makes use of the joint diagonalization techniques of the multiple whitened covariance matrices to improve the parameter estimation performances. Simulation results indicate that it performs well. The proposed estimator shows its great potential for the interleaved
uplink OFDMA system when training sequences do not work.

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6. References
