Analytical Evaluation of Nonlinear Distortion Effects on OFDMA Uplink Signals

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Abstract—In this paper we consider the evaluation of nonlinear distortion effects on the uplink transmission of systems employing OFDMA signals (Orthogonal Frequency Division Multiple Access). Our results allow an analytical spectral characterization of the transmitted signals, as well as the computation of the nonlinear interference levels on the received signals. It is shown that the power allocated to each user has a key impact on the nonlinear distortion effects. It is also shown that nonlinear distortion levels are lower when just a small fraction of the subcarriers is used (i.e., when the system load is small).\footnote{This work was partially supported by FCT (Fundação para a Ciência e Tecnologia), under the plurianual funding, and C-MOBILE project IST-2005-27423.}

I. Introduction

OFDMA schemes (Orthogonal Frequency Division Multiple Access) [1], [2] are OFDM modulations (Orthogonal Frequency Division Multiplexing) [3], [4] where a different set of subcarriers is assigned to each user. Therefore, they combine an OFDM modulation with an FDMA scheme (Frequency Division Multiple Access). Moreover, they are suitable for severely time-dispersive channels and allow a flexible and efficient management of the spectrum. For these reasons, they were selected for future broadband wireless systems [5], [6]. OFDMA is used in wireless broadband access technologies IEEE 802.16a/d/e, commonly referred to as WiMAX, and is currently a working assumption in 3GPP Long Term Evolution downlink, named High Speed OFDM Packet Access (HSOPA). It is also the access method candidate for Wireless Regional Area Networks (WRAN).

As with OFDM and other multicarrier modulations, the transmitted signals have large envelope fluctuations and high PMEPR (Peak-to-Mean Envelope Power Ratio), leading to amplification difficulties. For this reason, several techniques have been proposed to reduce the envelope fluctuations of OFDMA signals namely through suitable pre-processing schemes [7], [8], [9]. As an alternative, we can employ clipping and filtering techniques, already shown to be effective for conventional OFDM signals [10], [11], [12], as well as MC-CDMA schemes [13].

This paper deals with the analytical evaluation of nonlinear distortion effects on OFDMA signals for the uplink transmission that can result from nonlinear power amplification. For this purpose, we take advantage of the Gaussian-like nature of OFDMA signals with a large number of subcarriers as extend results of [12], [14], [15] to OFDMA schemes. Our results allow an analytical spectral characterization of the transmitted signals, as well as the computation of the nonlinear interference levels on the received signals. They can also be used to compute the corresponding BER (Bit Error Rate). This allows an efficient approach for studying aspects such as the type of nonlinear device, the impact of the system load (fraction of subcarriers used), the set of subcarriers assigned to each user (continuous, randomly spaced or regularly spaced subcarriers), etc.

This paper is organized as follows: Sec. II describes the OFDMA schemes considered in this paper. The analytical evaluation of nonlinear distortion effects on uplink transmission OFDMA signals is made in Sec. III and a set of performance results is presented in Sec. IV. Finally, Sec. V is concerned with the conclusions of this paper.

II. OFDMA Systems

In this paper we consider an OFDMA system with $P$ users and $N_p$ subcarriers assigned to the $p$th user, as depicted in Fig. 1. The total number of subcarriers is $N'$, the number of in-band subcarriers (i.e., the number of subcarriers that can be assigned to users) is $N$ (it is assumed that $N \geq \sum_{p=1}^{P} N_p$) and we have $N' - N$ subcarriers that are always idle. The idle subcarriers are used to simplify the design of the reconstruction filter; they can also be used for unused regions of the spectrum (e.g., when the transmission band is fragmented [5]).

![Fig. 1. OFDMA system with $p$ users.](image)

frequency-domain block transmitted by the $p$th user on the uplink is \( \{S_k^{(p)}; k = 0, 1, \ldots, N' - 1\} \), where \( S_k^{(p)} = \xi_p A_k^{(p)} \) if \( k \in \Psi_p \) and 0 otherwise. The set \( \{A_k^{(p)}; k \in \Psi_p\} \) is associated to the $p$th user (it is assumed that $\Psi_p \cap \Psi_{p'} = \emptyset$, etc.)
i.e., different sets are assigned to different users) and $\xi_p$ is an appropriate weighting coefficient that accounts for power control. The time-domain signal associated to the $p$th user is
\[ s^{(p)}(t) = \sum_n s_n^{(p)} r(nT/N') h_T(t - nT/N') \tag{1} \]
where $T$ is the duration of the useful part of the block, $r(t)$ is a suitable time-domain window (used to reduce the out-of-band radiation levels [12]) and $h_T(t)$ is the impulse response of the reconstruction filter. The block of time-domain samples $\{s_n^{(p)}; n = 0, 1, \ldots, N' - 1\}$ is the IDFT of the frequency-domain block $\{g^{(p)}(k); k = 0, 1, \ldots, N' - 1\}$ (as usual, it is assumed that the time-domain and frequency-domain blocks are periodic, with period $N$, i.e., $S^{(p)}_{k+N} = S^{(p)}_k$ and $S^{(p)}_{n+N} = S^{(p)}_n$ for any integer $n$ and $k$).

If $E|S^{(p)}_n| = 0$ (denotes 'ensemble average') and $E|S^{(p)}_n g^{(p)}(k)| = G^{(p)}_S(k) \delta(k-k')$ for $k = k'$ and 0 otherwise, then it can be shown that the PSD of $s^{(p)}(t)$ is given by
\[ G^{(p)}_s(f) = \left| H_T(f) \right|^2 \sum_{k=-N/2}^{N/2-1} G^{(p)}_S(k) \left| R^{eq} \left( f - \frac{k}{T} \right) \right|^2, \tag{2} \]
where $H_T(f) = F\{h_T(t)\}$ (denotes 'Fourier Transform'). $T_B$ is the duration of the block and $R^{eq}(f) = \sum_{l=-\infty}^{\infty} R(f - lN/T)$, with $R(f) = F\{r(t)\}$. When the number of subcarriers is high ($N >> 1$) the time-domain coefficients $s_n^{(p)}$ can be approximately regarded as samples of a zero-mean complex Gaussian process.\footnote{We consider an even number of subcarriers and the subcarrier with $k = N/2$ associated to $f = 0$.} It can be easily demonstrated that $E|s_n^{(p)}| = 0$ and that its autocorrelation is given by
\[ R^{(p)}_s(n-n') = E[s^{(p)}_n s^{(p)*}_{n'}] = \frac{1}{(N')^2} \sum_{k=-N/2}^{N/2-1} G^{(p)}_S(k) \exp \left( j2\pi \frac{k(n-n')}{N'} \right), \tag{3} \]
where $\{R^{(p)}_s(n); n = 0, 1, \ldots, N' - 1\} = \frac{1}{N'} \text{IDFT} \{G^{(p)}_S(k); k = 0, 1, \ldots, N' - 1\}$. Moreover $R^{(p)}_s(0) = E[|s^{(p)}|^2] = 2\sigma_p^2 = 1/N^2 \sum_{k=-N/2}^{N/2-1} G^{(p)}_S(k)$, with $\sigma_p^2$ denoting the variance of the real and imaginary parts of $s^{(p)}_n$.

### III. Nonlinear Amplification of OFDMA Uplink Signals

In this section we present an analytical approach for studying the impact of nonlinear amplification of OFDMA uplink signals. As usual, the power amplifier is modelled as a bandpass memoryless nonlinear device [16]. The basic uplink transmitter structure considered in this section is depicted in Fig. 2. This means that the complex envelope of the signal transmitted by the $p$th user at the power amplifier output can be written as
\[ s^{(p)}_{pa}(t) = g(|s^{(p)}_n(t)|) \exp(j \arg(s^{(p)}_n(t))), \tag{4} \]
with $|g(\cdot)|$ and $\arg(g(\cdot))$ denoting the so-called AM-to-AM and AM-to-PM conversion characteristics. The signal at the input of the power amplifier is $s^{(p)}(t)$, with PSD and autocorrelation given by (2) and (3), respectively. It can be shown [12], [14], [15] that the complex envelope of the signal at the power amplifier output $s^{(p)}_{pa}(t)$ can be decomposed into uncorrelated useful and self-interference components, as follows:
\[ s^{(p)}_{pa}(t) = \alpha_p s^{(p)}(t) + d^{(p)}(t), \tag{5} \]
where $\alpha_p$ is given by (18). The average power of the useful component is $P_u = |\alpha_p|^2\sigma_p^2$ and the average power of the self-interference component is given by $I_p = P^{(p)} - P_u$, where $P^{(p)}$ denotes the average power of the signal at the nonlinearity output and is given by (19), as shown in the appendix. Using the results of the appendix, the autocorrelation of the complex envelope at the amplifier output is given by
\[ R^{(p)}_{s_{pa}}(\tau) = \sum_{\gamma=0}^{\infty} 2P^{(p)}_{2\gamma+1}\frac{R^{(p)}_{R_{s_{pa}}}^{(p)}(\tau)}{2\gamma+1} G^{(p)}_G(f), \tag{6} \]
with input given by (3) and $R^{(p)}_{s_{pa}}^{(p)}(\cdot)$ and $P^{(p)}_{2\gamma+1}$ given by (21) and (22), respectively. The corresponding PSD is given by
\[ G^{(p)}_s(f) = F\{R^{(p)}_{s_{pa}}(\tau)\} = \sum_{\gamma=0}^{\infty} 2P^{(p)}_{2\gamma+1}\frac{R^{(p)}_{R_{s_{pa}}}^{(p)}(0)}{2\gamma+1} G^{(p)}_G(f), \tag{7} \]
with $G^{(p)}_G(f)$ given by (24).

The power of the transmitted signal is $P^{(p)}_{s_{pa}} = \sum_{\gamma=0}^{\infty} P^{(p)}_{2\gamma+1}$ and it can be decomposed as the sum of useful and self-interference components. The power, autocorrelation and PSD of the self-interference component are $I^{(p)} = \sum_{\gamma=1}^{\infty} P^{(p)}_{2\gamma+1}$,
\[ R^{(p)}_{d_{pa}}(\tau) = \sum_{\gamma=1}^{\infty} 2P^{(p)}_{2\gamma+1}\frac{R^{(p)}_{R_{d_{pa}}}^{(p)}(\tau)}{2\gamma+1} f^{2\gamma+1}_{G^{(p)}_G(f)}(\tau), \tag{8} \]
and
\[ G^{(p)}_d(f) = F\{R^{(p)}_{d_{pa}}(\tau)\} = \sum_{\gamma=1}^{\infty} 2P^{(p)}_{2\gamma+1}\frac{R^{(p)}_{R_{d_{pa}}}^{(p)}(0)}{2\gamma+1} f^{2\gamma+1}_{G^{(p)}_G(f)}(f), \tag{9} \]
respectively. The PSD of the overall signal received at the BS is illustrated in Fig. 3 and is given by

$$G_p(f) = \sum_{p=1}^{P} G_{s_{Tx}}^{(p)}(f)|H^{(p)}(f)|^2,$$

(10)

with $H^{(p)}(f)$ denoting the channel impulse response associated to the $p$th user.

Let us consider the transmission of the OFDMA uplink signal over a time-dispersive channel. It is clear that the received symbol on the $k$th subcarrier is

$$Y_k = \sum_{p=1}^{P} \left( \alpha_p \xi_p A_k^{(p)} H_k^{(p)} + \xi_p D_{eq}^{(p)} H_k^{(p)} \right) + N_k$$

(11)

with $N_k$ denoting the corresponding channel noise. We can calculate a signal-to-interference ratio (SIR) for the $p$th user on each subcarrier, given by

$$\text{SIR}_k^{(p)} = \frac{\alpha_p^2 \xi_p^2 E[|A_k^{(p)}|^2]}{D_k^{eq}},$$

(12)

with

$$D_k^{eq} = \sum_{p'=1}^{P} \xi_{p'}^2 E[|D_k^{(p')}|^2]$$

(13)

and also an equivalent signal to noise plus self-interference ratio (ESNR) for the $p$th user on each subcarrier, given by

$$\text{ESNR}_k^{(p)} = \frac{\alpha_p^2 \xi_p^2 |H_k^{(p)}|^2 E[|A_k^{(p)}|^2]}{D_k^{Heq} + E[|N_k|^2]}$$

(14)

with

$$D_k^{Heq} = \sum_{p'=1}^{P} \xi_{p'}^2 |H_k^{(p')}|^2 E[|D_k^{(p')}|^2].$$

(15)

IV. Performance Results

In this section we present a set of results concerning the performance evaluation of nonlinear power amplification. We consider a Solid State Power Amplifier (SSPA) with $p = 1$ and saturation level $s_M/\sigma = 2.0$. The considered OFDMA system has $N = 256$ subcarriers and a PSK constellation, with a Gray mapping rule, on each subcarrier. The system has $P = 4$ users with an equal number of contiguous subcarriers assigned to each one, i.e., $N_p = 64$, $p = 1, 2, 3, 4$.

We first consider an AWGN channel. Fig. 4 shows the evolution of $D_k^{eq}$, defined in (13), for equal power users with 1, 2 or 4 active users. Figs. 5 and 6 compare the evolution of $E[\alpha_p^2 |S_k^{(p)}|^2]$, $D_k^{eq}$ and SIR$^{(p)}$ for different attribution of the power control coefficients \{0, 5, 10, 15\} dB. These coefficients are assigned in two different ways: no particular order (in this case we consider a low power user followed by a high power user) or in increasing order.

Let us now consider the transmission over a time-dispersive channel. Fig. 7 shows the evolution of $E[|H_k^{(2)}|^2]$, $E[|D_k^{(p)}|^2]$ and ESNR$^{(2)}$ with increasing power attribution for user 2. Fig. 8 compares the evolution of $E[|S_k^{(p)}|^2]$ (solid line) and $D_k^{eq}$ (dashed line) with power control coefficients \{ξ1, ξ2, ξ3, ξ4\} = \{0, 5, 10, 15\} dB (A) and \{ξ1, ξ2, ξ3, ξ4\} = \{0, 5, 15, 10\} dB (B).

ESNR$_k$ for user 2 with different attribution of power control
coefficients. We include the evolution of ESNR$_k^{(2)}$ with equal power users for the sake of comparison. Similar results can be obtained for the other users.

V. Conclusions

In this paper we presented an analytical tool to evaluate nonlinear distortion effects on the uplink transmission of systems employing OFDMA signals. Our results allow an analytical spectral characterization of the transmitted signals, as well as the computation of the nonlinear interference levels on the received signals. They can also be used to find the corresponding BER. A set of performance results was presented, showing that the power allocated to each user has a key impact on the nonlinear distortion effects. Users with smaller allocated power face stronger interference levels and increasing attribution of power control coefficients can lead to lower interference levels. It is also shown that nonlinear distortion levels are significantly different when just a small fraction of the subcarriers is used (i.e., when the system load is small).

Appendix

Let us consider a Gaussian signal whose complex envelope $x(t)$ has zero mean and autocorrelation $R_x(t)$. If this signal is submitted to a bandpass memoryless nonlinearity then the complex envelope of the signal at the power amplifier output can be written as

$$y(t) = g(|x(t)|) \exp(j \arg(x(t))).$$  \hfill (16)

It can be shown [12], [14], [15] that $y(t)$ can be decomposed into uncorrelated useful and self-interference components, as follows:

$$y(t) = \alpha x(t) + d(t),$$  \hfill (17)

where $E[y(t)x(t-\tau)] = 0$ and

$$\alpha = \frac{E[y(t)x(t)^*]}{E[|x(t)|^2]} = \frac{E[Rg(R)]}{E[R^2]} =$$

$$= \frac{1}{2\sigma^2} \int_0^{+\infty} RA(R) \exp(j \Theta(R)) \frac{R}{\sigma^2} \exp \left(-\frac{R^2}{2\sigma^2}\right) dR, \hfill (18)$$

with $\sigma^2 = E[|x(t)|^2]/2$, $R = |x(t)|$, $A(R) = |g(R)|$ and $\Theta(R) = \arg(g(R))$. The average power of the signal at the nonlinearity output is given by

$$P_{out} = \frac{1}{2} E[g^2(R)] = \frac{1}{2} \int_0^{+\infty} g^2(R) \frac{R}{\sigma^2} \exp \left(-\frac{R^2}{2\sigma^2}\right) dR. \hfill (19)$$
It can also be shown [12], [14], [15] that

\[ R_y(\tau) = E[y(t)y(t-\tau)^\gamma] = \sum_{\gamma=0}^{+\infty} 2P_{2\gamma+1} f_{2\gamma+1}^R(R_x(\tau)), \]  

where

\[ f_{2\gamma+1}^R(R(\tau)) \triangleq \frac{(R(\tau))^{\gamma+1}(R^*(\tau))^{\gamma}}{(R(0))^{2\gamma+1}} \]  

and \( P_{2\gamma+1} \) denotes the total power associated to the IMP (Inter-Modulation Product) of order \( 2\gamma + 1 \), which can be obtained as follows [17]:

\[ P_{2\gamma+1} = \frac{|v_{2\gamma+1}|^2}{2\gamma!(\gamma + 1)!}, \]  

where \( v_{2\gamma+1} = \int_0^{+\infty} R g_C(R) W_{2\gamma+1}(R/\sqrt{2}\sigma^2) dR \), with \( W_{2\gamma+1}(x) = \gamma!/2e^{-x^2}xL^{(1)}_\gamma(x^2) \), where \( L^{(1)}_\gamma(\cdot) \) denotes a generalized Laguerre polynomial of order \( \gamma \) [18].

Therefore, the PSD of \( y(t) \) is

\[ G_y(f) = F\{R_y(\tau)\} = \sum_{\gamma=0}^{+\infty} \frac{2P_{2\gamma+1}}{(R_x(0))^{2\gamma+1}} f_{2\gamma+1}^G(G_x(f)), \]  

where

\[ f_{2\gamma+1}^G(G(f)) \triangleq G(-f) \ast \cdots \ast G(-f) \ast G(f) \ast \cdots \ast G(f). \]

Clearly \( R_y(\tau) = |\alpha|^2 R_x(\tau) + R_d(\tau) \), with the autocorrelation of the self-interference component given by

\[ R_d(\tau) = E[d(t)d(t-\tau)^\gamma] = \sum_{\gamma=1}^{+\infty} 2P_{2\gamma+1} f_{2\gamma+1}^R(R_x(\tau)) \]  

and \( G_y(f) = |\alpha|^2 G_x(f) + G_d(f) \), with

\[ G_d(f) = F\{R_d(\tau)\} = \sum_{\gamma=1}^{+\infty} \frac{2P_{2\gamma+1}}{(R_x(0))^{2\gamma+1}} f_{2\gamma+1}^G(G_x(f)). \]

The average power of the useful component is \( S = |\alpha|^2 \sigma^2 \) and the average power of the self-interference component is \( I = P_{\text{out}} - S = 1/2 R_d(0) = \sum_{\gamma=1}^{+\infty} \).

References


