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Point Process Based Maintenance Modeling for Repairable Systems: A Review

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Abstract

In this article, the intricacies involved in the maintenance of the industrial repairable systems are analyzed. Based on the practical requirement, the mathematical models for repairable system maintenance which are using point process theory are listed and reviewed. The available estimation, inference and prediction methodologies are also listed. The future issues which are to be addressed in the mathematical modeling for maintenance are also pointed out.

Keywords
Point process, repairable systems, Poisson process, intensity function, multiple repairable systems, competing risks.

1. Introduction

In this highly technical modern world, the society demands the most reliable and quality products in every aspect of life. To meet this demand the production industries dependent on the complex industrial equipments which are capable of high speed and continual production capacity. In this context, the maintenance activities for these complex industrial equipments or systems not only ensure the timely delivery for profits in the business, preventing environmental hazards and safety hazards. The system which undergoes the aforementioned maintenance activities are referred in technical literature as maintained systems or repairable systems. These repairable systems are defined as, a system which after failing to perform one or more of its function satisfactorily, can be restored to fully satisfactory performance by any method other than replacement of the entire system , Ascher and Feingold [1]. Large number of technical papers and many books are published in the field of maintenance modeling for repairable system. Most of these developed models actually never applied in practical maintenance activities. Ascher and Feingold [1], Scarf [2] states that still the maintenance policy managers and engineers takes the decisions heuristically using their engineering back ground and common sense. This is because of the unrealistic assumptions of the theoretical models which do not suit the practical realistic situation of the industries. As stated by Guo et al. [3] the models developed for maintenance should be efficient, reflective of the situation and good approximating tools. And hence these mathematical models should incorporate the elements of the environment in which operation and maintenance takes place, failure causes etc. As per Kumar and Liyanage [5] if one could formulate a concise statement on the role of maintenance, it is primarily to reduce business risks on the continuous basis in cost – effective manner. All these statements in research literature leads to the development of maintenance models which can be used to plan for the spares, resources and replacement strategies which in turn help to reduce the cost of the production activities, and also to reduce safety and environmental hazards.

2. Industrial Maintenance – An Overview

With proliferation of line or mass production industries in late 1940s the concept of maintenance has changed from simple tasks of maintenance like lubrication, bolt tightening etc and Breakdown maintenance, to the more sophisticated preventive maintenance (PM) activities which are aimed to prevent the equipment failures. Though the preventive maintenance concepts reduced the failure rates of the equipments, these concepts are having many demerits. The excess maintenance, replacement of parts / components which are in good condition, disturbance of good machine alignment in the name of maintenance tasks are some of the draw backs of preventive maintenance concept. The aforementioned drawbacks of the PM tasks led to the new concepts like predictive maintenance, condition based maintenance (CBM) and reliability centered maintenance (RCM). All the Maintenance concepts and policies are aimed at one objective that is minimizing the unplanned downtime. Minimizing unplanned downtime in turn having many goals, such as business profits, reduction in safety and environmental hazards etc... To achieve these goals the maintenance managers should plan for the optimal policies for maintenance using techniques of PM, CBM, RCM etc... The Planning for getting resources and spares for the maintenance is also having its strategic importance in cost reduction. As stated by Syamsundar and Naikan [6], there are several fundamental questions regarding the failures, maintenance policy, business goals etc. and their answers will help maintenance policy makers of particular industries to decide the suitable and effective tasks in their maintenance policy. Ascher and Feingold [1] listed some very important factors in their literature, which are considered to be the influential in the failure process and also influential in subsequent maintenance policy framing. All the aforementioned discussion is leading to the research and development of tools and models which will help to deliver the optimal solutions to the challenges of maintenance.

3. Different Approaches in Mathematical Modeling for Maintenance

The maintenance modeling of industrial maintained systems which are also referred as repairable systems are modeled traditionally using their failure times. The failure time of a repairable system is a random (uncertain) variable, and in a continuum (here it is a time line) the collection of this random variable form a stochastic process. There have been many approaches used in the research literature to mathematically model the aforementioned stochastic process of failure times. There are more than a thousand research articles available using the semi-Markhov and Markhov process for maintenance modeling. Some of the other commonly used stochastic process models are Renewal process, Non-Homogenous Poisson process (NHPP), and Homogenous Poisson process (HPP). The renewal process models are based on a strong assumption that the system is brought to the condition of a new system, after it was repaired for failure i.e. the maximum possible repair, also known as maximal repair. The drawback of this model is that the assumption made is not practically possible in most of the cases. In contrast to the renewal process, the NHPP models which look at the trend of the failures in a monotonous increasing or decreasing trends. Here these models use the concept of minimal repair which represents the repair as minimum as possible to put the system back in to the function. This is also known as minimal repair models. The afore mentioned models are discussed in detail by Ascher and Feingold [1], Rigdon and Basu [7] for renewal process and Poisson process, Cox [8] for a comprehensive analysis of renewal process, and Birolini [10] for the Markhov models for repairable systems.

4. Stochastic Point Process Models

A stochastic point process model is explained as occurrence of highly localized events that are randomly distributed in a continuum, Ascher and Feingold [1]. Here in the context of repairable systems maintenance modeling the localized events are taken as failures of the system and continuum as the time. In this point process the number of failures in the time interval *0* to *t* is denoted by *N(t)* which is a random variable. The *N(t)* can be called as the count process. An intensity function $\lambda(t \mid H_{t^-})$ of the count process *N(t)* is a stochastic process which is used to model and study the failure in repairable system. It is given by,

$$\lambda(t \mid H_{t^-}) = \lim_{\Delta t \to 0} \frac{P((N(t+\Delta t) - N(t)) = 1 \mid H_{t^-})}{\Delta t}$$

$$\lambda(t \mid H_{t^-}) = \lim_{\Delta t \to 0} \frac{P(\Delta N(t) = 1 \mid H_{t^-})}{\Delta t}$$

Where H_{t^-} denotes the data available prior to time *t* and also said as the history of failure process through time *t*. Here the assumption we make is that there is a zero probability for the occurrences of simultaneous failures. The intensity function $\lambda(t \mid H_{t^-})$ which is also called as the complete intensity function or conditional intensity process forms the comprehensive basis for modeling the failure events for maintained systems. Here we use the base line intensity function $\lambda_0(t)$, which is the hazard function of the time to the first failure, in describing models by means of their conditional intensity function $\lambda(t \mid H_{t^-})$.

4. Models with Repair Effects

4.1.1. Basic Models:

The basic models for the repairable systems are renewal process, NHPP and HPP; these can be specified by their conditional intensity functions. These are most common and simple models. In the renewal process, the conditional intensity function depends on the prior data (H_{t^-}) through time *t*, the time since the most recent failures (i.e. the Backward recurrence time)

$$\lambda(t \mid H_{t^-}) = \lambda_0(t - T_{N(t^-)}) .$$

The renewal process is modeled with the time between failures identically and independently distributed. The distributions which are commonly used are exponential, gamma, Weibull, log-normal etc. In the NHPP models the intensity function is the non-constant one. The conditional intensity function for this model is the function of chronological age of the system.

$$\lambda(t \mid H_{t^-}) = \lambda_0(t)$$

and When the conditional intensity function of NHPP takes the following form, then we call it as Power law process.

$$\lambda(t \mid H_{t^-}) = \alpha\beta.t^{\beta-1}$$

For the log-linear process the conditional intensity function will be of the following form

$$\lambda(t \mid H_{t^-}) = \exp(\alpha + \beta.t)$$

When the time between failures of a renewal process is exponentially distributed and the intensity function of NHPP is a constant, then both the process will give rise to a special case called as HPP. The conditional intensity function of HPP is given by,

$$\lambda(t \mid H_{t-}) = \lambda_0$$

4.1.2. Extension of the basic models

With extension and generalization of NHPP and renewal process models, Lindqvist [11] proposed a new model termed as trend renewal process (TRP), whose conditional intensity function is given by,

$$\lambda(t \mid H_{t^-}) = z(\Lambda_0(t) - \Lambda_0(T_{N(t^-)}))\lambda_0(t)$$

The parametric inference of the TRP model is studied by Elvebakk et al. [14]. Kristov [12] extended the NHPP model into non- homogenous renewal process (NHRP). In this model the hazard function of the renewal process has taken as its intensity function and inter-failure times replaced with times to failure. The two NHRPs based on log-normal and Weibull are postulated to provide a good fit to the practical trends observed. Berman [13] modeled an extension of NHPP with a new concept that each system experiences the shocks with NHPP intensity $\lambda(t)$ and the failure occurs only at the K^{th} shock. If *K<1* after repair then the system is said to be deteriorating and *K>1* after repair then the system is termed as improving. This model is termed as Inhomogeneous gamma model. A special form of inhomogeneous gamma model termed as modulated gamma process is also proposed by Berman [13] with the conditional intensity function

$$\lambda(t \mid H_{t^-}) = \rho \exp(\beta' z(t))$$

The modulated gamma process discussed above is extended for cyclic trend of fixed frequency model whose conditional intensity function is given by $\lambda(t \mid H_{t^-}) = \rho \exp(\beta_1 \cos(\omega t) + \beta_2 \sin(\omega t))$. Here ω represents the periodicity of cyclic trend and β_1 and β_2 are the regression coefficients of the covariates. Kijima [15] introduced a model for the concept of age recovery or age loss. He incorporated the effect of imperfect repair using the concept of virtual age. The model is represented by, $\lambda(t \mid H_{t^-}) = = \lambda_0(\varepsilon(t))$ where $\varepsilon(t)$ is the effective age of the unit. Doyen and Gaudoin [16] proposed two models, arithmetic reduction of age (ARA) and arithmetic reduction of intensity (ARI) models. These models more accurately represent the conditional distribution of inter-failure times. The ARA Model is represented by

$$\lambda(t \mid H_{t^-}) = \lambda_0(t - \sum_{i=1}^{N_{t^-}} s_i)$$

And the ARI model is represented by $\lambda(t \mid H_{t^-}) = = \lambda_0(t) - \sum_{i=1}^{N_{t^-}} s_i$ where s_i reflect age or intensity reduction factors.

4.2. Models which considers other conditions

In the repairable system maintenance there always exist some special conditions apart from the regular ones. To reflect these special conditions in the model building, some of the basic models are extended and synthesized to get more practical models with good fit for the available data. The branching Poisson process (BPP), generalized Weibull renewal process (GWRP), bounded intensity process (BIP), The superimposed power law process and superimposed renewal process are the some extended models of Renewal process, HPP, and NHPP. In some maintained systems because of the inadequate repair after failure (Primary failure), there will be subsequent failure (subsidiary failure) due to same reason as primary failures, the BPP model account for this phenomenon. The BPP is the superimposition of primary and subsidiary failure series, with the assumption that both the failure types are indistinguishable. Ascher and Feingold [1] and Hansen and Ascher [17] dealt the BPP model in their publications. To Model the non-monotonic failure occurrences of different types, various generalization methods of Weibull distribution is published by Murthy et al. [18] and more recently by Pham and Lai [19]. In the repairable system as the age increases the failures also increases, with the subsequent repair action and substitution of parts in the system, the system as a whole consists of parts with non-uniform ages which will lead to the constant failure intensity. To model such system Pulcini [20] proposed a BIP Model, which initially evolves as power law process with the beta value of 2 and converges to HPP asymptotically, with the conditional intensity function of

$$\lambda(t \mid H_{t^-}) = \alpha\left(1-\exp\left(-t/\beta\right)\right)$$

Pulcini [21] also proposed a model to analyze the complex repairable system which can exhibit the bathtub curve behavior of the intensity function. In this model superimposition of two power law process are done to analyze the failure pattern. This model is termed as SPLP model with conditional intensity function of

$$\lambda(t \mid H_{t^-}) = \lambda_1(t) + \lambda_2(t)$$

$$\lambda(t \mid H_{t^-}) = \alpha_1\beta_1 t^{\beta_1-1} + \alpha_2\beta_2 t^{\beta_2-1}$$

As per Ascher and Feingold [1] the SRP models are formed by superimposing the *n* independent renewal process. This model can be used to analyze the system with mix of components each of which are represented by renewal process. To encounter the other practical difficulties in modeling for the maintained system, discrete processes other than Poisson process also used. One such model, Geometric Process (GP) model proposed by Leung and Fong [22] to represent the wear or age which in turn causes the decrease in operating times due to the increase in repair time (due to rectification of the accumulated wear and age related problem). Yang et al. [23] presented the maximum likelihood function for this GP model.

4.3. Proportional Intensity Models

The failure times of maintained systems depends on many factors such as operating environment, prior history of repairs and maintenance, components, material and design of the system, Kumar [24]. Some factors are constant over the period of time and some are time dependent. With the basis of regression model proposed by Cox [9], taking some of the factors as covariates and concomitant variables, two models with time varying covariates has been proposed. Time varying covariates given by

$$\lambda(t \mid H_{t^-}) = \lambda_0(t) f(\gamma'.Z(t))$$

Time invariant covariates given by

$$\lambda(t \mid H_{t^-}) = \lambda_0(t) f(\gamma'.Z)$$

Where, $\lambda_0(t)$ is the baseline intensity function, γ' regression coefficient of time varying covariates, *Z(t)* time varying covariates. The above models are called as proportional intensity models. In the above model the covariates strength can make the baseline intensity function to increase or decrease. The link function *f(.)* may take the form of exponential, log or logistic. For the systems having general repairs, based on the prior repair history a proportional intensity model with log-linear baseline intensity has been proposed by Guo et al. [4]. This model considers simultaneously the time trends and repair effects, with the assumption of cumulative number of failure captures the age, use behavior and repair history of the system. It is represented by

$$\begin{aligned}\lambda(t \mid H_{t^-}) &= \lambda_0(t) f(\gamma'.Z(t)) \\ &= \lambda_0(t) f(\gamma.E(N_{t^-}))\end{aligned}$$

For the imperfect repair and accumulating number of failures with proportional intensity Pena and Hollander [28] proposed a general class of models $\lambda\left(t \mid H_{t^-}\right) = \lambda_0(\varepsilon(t))\rho\left(N\left(t^-\right)f\left(\gamma' Z(t)\right)\right)$. Combining the preventive maintenance effects and variables of prediction a model termed as generalized proportional intensities model (GPIM) has been proposed by Percy and Alkali [26]. They incorporated intensity scaling factors for preventive and corrective maintenance. It is represented as, $\lambda\left(t \mid H_{t^-}\right) = \lambda_0(t)\ \{\prod_{i=1}^{M(t)} r_i\}\{\prod_{j=1}^{N(t)} s_j\}\ exp\ (\gamma' Z(t))$. Where r_i and s_j are intensity scaling factor for preventive and corrective maintenance respectively, *M(t)* and *N(t)* represents total number of preventive and corrective maintenance actions respectively.

4.4 Marked Point Process models

In the actual industrial environment with multiple systems, the following situations may arise many times. Afore discussed mathematical models (also known as univariate point process) may not do justice to reflect the practical difficulties. The situations may be the systems are similar or dissimilar with heterogeneity, multiple failures and failures of same or different types, varying effects with various failures and variety of maintenance activities to tackle various failures. To handle the aforementioned conditions and to reflect the originality of the situation, we can use the multivariate point process. A multivariate point process is a collection of *k* univariate point processes, which may of course be dependent on each other. While another method of handling failures with multi dimensions such as above mentioned is that, the basic failure events should be included in a univariate point process with other features placed as marks at each failure event forming a random variable (a single mark) or a random vector (multiple marks) leading to a marked point process. Here marks represent number of failures, maintenance types required for particular failure, degradation level of a system etc. To deal with the multiple systems and heterogeneity between them, various models have been developed. The heterogeneous trend renewal process (HTRP), negative binomial process (NBP) and other general class of process are some of the models developed to deal the aforementioned situations.

Bain and Wright [26] applied the NBP model to deal with multiple maintained systems, which are having different intensity though they are similar in nature. These different intensities reflects the heterogeneity between the systems and to reflect this heterogeneity, the Poisson parameter λ has been considered as the random variable which is following gamma distribution and a compound Poisson model has been developed. Bain and Wright [26] used methods of moments and Savani and Zhigljavsky [27] used generalized moment based estimates for the estimation of the parameters of the NBP model. Some unobserved heterogeneity among the multiple systems has been dealt by Elvebakk et al. [14] in the HTRP model, where unobserved heterogeneity is introduced as a multiplicative factor which is a random variable depends on the intensity. The imperfect repair and proportional intensities, and accumulating number of failures are reflected in the general class model proposed by Pena and Hollander [28]. In this model, the heterogeneity is represented by a multiplicative factor W_i working on the intensity and is represented by

$$\lambda\left(t \mid H_{t^-}\right) = W_i\lambda_0(\varepsilon(t))\rho\left(N\left(t^-\right)f\left(\gamma' Z(t)\right)\right)$$

Pena et al. [30] and Pena et al. [29] have given the non-parametric inference and semi-parametric inference respectively to the above models. According to Lehmann [31], whenever a system undergoes a repair after each failure, the level of degradation of the system is pushed back to a point which lies between the levels of new system and the system just before the failure. In this regard a model for degradation of maintained system is developed based on the marked point process. Here, the failure events and the inspection time are taken as events, and the degradation level, repair levels, etc. is taken as marks. No practical application was shown in the literature for this model. To asses the failure rate of the competing modes of reliability data base Cooke [32, 33] proposed models for independent competing risks and for competing risks with dependency. Bunea et al. [34, 35] also proposed many models for competing risks and also proposed methods for selecting model based on various factors of models. For a system which fails due to one of the series of independently acting competing failure mechanism, and the system also have imperfect repair, moreover the system is also under the preventive or CBM based maintenance activities, Langseth and Lindqvist [36] proposed an intensity proportional repair alert model. The model assumption is that the conditional density of CBM is proportional to the intensity of the imminent failure process. The repair alert function of the model, proposed a metric to test the level of alertness of the maintenance group of industry to check impending failures by applying CBM measures. Also the Langseth and Lindqvist [37], Lindqvist et al. [38] proposed, tools to test the intensity proportional assumption, the use power law process for repair alert function, respectively. For complex repairable system, Doyen and Gaudoin [39] proposed a general frame work for modeling the process of corrective and CBM preventive maintenance along with imperfect repair models for the failure process. Using the Weiner process Lindqvist and Skogrud [40] proposed idea of modeling a degradation process for maintained system.

4.5. Alternate time scale models

Most of the stochastic process models use the calendar or Global time, alternatively, some of the models has to be built upon the alternative time scales to suit the practical applicability. For example, the number of miles the automobile has driven can be used as the time scale. In the case of aircraft the number of flying hours, take-offs, landings, time since the last overhaul of aircraft engine may be taken as the alternate time scale models. External and environmental factors such as temperature, stresses, operating speed can also be taken as the alternate scale to model the failure process. Depending on the stochastic process and its application, single time scale or multiple scales leading to mixed scale can be used to model the failure procedure. A cumulative hazard rate which depends on the environmental changes that affect the failure has been taken as the time scale, Ozekici [41], developed a model with the concept of intrinsic age of the system. By deploying the local time (time since the last failure) and global time (age of the equipment) together in the proportional intensity model, Percy and Kobbacy [42] proposed a model for failure process. Again, in the proportional intensity model by taking the loads, usage, number of stops and starts (all are external factors) as covariates, Duchesne and Lawless [43] postulated alternate time scales for modeling of failure, for which the conditional intensity function is given by,

$$\lambda(t \mid H_{t^-}) = \lambda_0(t, Z(t))$$

Where the base line intensity (λ_0) is the function of calendar time and alternative time scales are based on the external influences of environment.

For the external environmental factors which affect the failure process, an accelerated time assumption is made and the conditional intensity function is given with a multiplicative factor which accelerates the time scales and it is given by

$$\lambda(t \mid H_{t^-}) = \lambda_0(\exp(\gamma' t Z(t)))$$

This accelerated time with respect to failure modeling for the industrial maintained systems are studied by Guida and Giorgio [44], Guerin et al. [45], Yun and Kim [46] and Yun et al. [47]. Gertsbakh and Kordonsky [48] in their research literature suggested a heuristic principle that the best time scale was the one which provided the smallest coefficient variation. To find out the optimum inspection interval and replacement strategy for the subsystems in a system a "repair and maintenance indicator (RMI)" was proposed by Lugtigheid et al. [49]. This RMI is the linear combination of weighted subsystem ages which replaces the time scale in the baseline intensity function of a proportional intensity model.

$$RMI(t) = \sum_{i=1}^{P} \omega_i s_i(t)$$

Where ω_i is the weight of the i^{th} subsystem and $s_i(t)$ is the accumulated operating time of the i^{th} subsystem at time t. The conditional intensity function with *RMI(t)* is given by

$$\lambda(t \mid H_{t^-}) = \lambda_0(RMI(t))\exp(\gamma' Z(t))$$

5. Inference for Point Process Models

For the available models which are mentioned above (in the section 4), for their effective utilization, the proper tools are required to apply this models to the data generated by the maintenance process. After the proposal of a model, we have to apply techniques to estimate the unknown parameters of the model and using these parameters the fit of the model to the available data set is to be checked. Further, the required quantities of interest for the maintenance to be predicted using the models. After the aforementioned important steps, then the model can be utilized to solve the industrial maintenance problem. The methods for inferring the parameters of the models are listed as, Method of Maximum likelihood, Methods of moments, Method of least square and Expectation-Maximization (EM) algorithm. Among these, the method of maximum likelihood is the generally used estimation technique, while others are used for specific cases and models. Testing of the model for its fit for a particular data set is done in many ways. The simplest one is the graphical fit, where estimated and observed failures are plotted against time and observed visually. Check of fit can also be obtained by calculating the sum of squares of the distance between the expected and observed values, where the least sum of the square will give best fit. Similarly highest maximum likelihood value of maximum likelihood estimation gives the best fit. Some more methods for goodness-of-fit is also published in various research literatures, D'Agostino and Stephens [50] for various renewal processes, Ascher and Feingold [1] and Rigdon and Basu [7] for various basic models, Kvaloy and Lindqvist [51,52], Vaurio [53], Kvaloy et al. [54] and Lindqvist [55] provided goodness-of-fit tests for point process models for repairable systems. For multiple systems and heterogeneity, the goodness-of –fit tests are given by Kvaloy [56}, Kvaloy and Lindqvist [51], and Rigdon and Basu [7]. After the estimation and testing, the models can be used to predict the future quantities of interest for maintenance. Few following relationships can be utilized to estimate these quantities of interest. Expected numbers of failures are calculated using the relation

$$\Lambda(t) = E\left(\Lambda(t) \mid H_{t^-}\right) = E\left(\int_0^t \lambda\left(y \mid H_{y^{t-}}\right)dy\right)$$

in the time interval [*0, t*].

Conditional reliability of the system is given by,

$$R_i\left(t_i \mid H_{t^-}\right) = \exp\left(-\int_{t_{i-1}}^{t} \lambda\left(y \mid H_{y^-}\right)dy\right)$$

where i is the i^{th} segment of failure process after the $(i-1)^{th}$ failure.

Conditional failure distribution is given by

$$f_i\left(t_i \mid H_{t^-}\right) = \lambda\left(t \mid H_{t^-}\right) \exp\left(-\int_{t_i-1}^{t} \lambda\left(y \mid H_{y^-}\right)dy\right)$$

6. Future Requirements in Modeling for Maintenance

After reviewing the models which are developed in last few decades, it is observed that the following few features should be incorporated so that modeling can be more effective and relevant to the practical industrial needs. The models should reflect the practical features of maintenance on the sustained basis, models should accommodate the environmental changes and parameter changes due to increase in data set, which leads to multi-phase models. Models should be developed for dealing with multiple failures and common cause failures. Models which are dealing with CBM is to be given more attention as little work has been done in that area. Also the further work has to be done on models with repair consideration, with small data sets and with censored data sets.

7. Conclusions

To avoid the failure or in unavoidable circumstances to increase the time between failures of industrial maintained systems, the maintenance policy makers try to estimate the magnitudes of factors which are governing the maintenance. Identification and monitoring of crucial parameters for failure and planning the maintenance activities based on this are required for the effective maintenance of repairable systems. From the business angle the planning for the availability of the system at crucial time (like at peak seasonal demands) by identifying the critical maintenance parameters of the system based on the estimated parameters is another advantage of the mathematical modeling of the repairable systems. From the equipment manufacturer's point of view the identification of crucial parameters governing the failure will certainly help to improve the equipment quality at design stage itself and later in manufacturing (of equipment) stage also. This will definitely improve the reliability of the system. In this paper we attempted to review the models which are available in the research literatures of last few decades with an eye on the practical requirements of repairable system of the modern complex industries, to deploy appropriate model for effective maintenance planning and implementation.

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