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# Novel Adaptive Sliding Mode Synchronization in a Class of Chaotic Systems

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**Abstract:** An Adaptive Sliding Mode Control (ASMC) scheme is proposed for the synchronization of two chaotic nonlinear systems. Due to sliding mode control methodology, a sliding surface is determined and the control law (synchronization method) is established. The proposed method can force an uncertain chaotic dynamic (slave dynamic) to trace another chaotic dynamic (master dynamic) without oscillating very fast and guarantee the property of asymptotical stability. The stability analysis for the proposed method is provided and simulation examples are presented to verify the effectiveness of the method.

**Key words:** Adaptive Sliding Mode Control (ASMC), chaotic nonlinear systems, synchronization

## INTRODUCTION

No one can ever deny that chaos synchronization [1] is one of the basic features in nonlinear science and is of fundamental importance in a variety of complex physical, chemical and biological systems [2]. Recently, control of chaotic systems has increasingly gained researchers interest since the pioneering work of Ott *et al*. [3]. Today, chaos has been found in many engineering systems [4]. Chaos theory and chaotic property have a wide range of useful applications in many engineering areas such as secure communication, digital communication, power electronic devices and power quality, biological systems, chemical reaction analysis and design and information processing [5]

After the publication of two relevant papers by Ott *et al*. [3] and Pecora *et al*. [6] in 1990, control and synchronization of chaos dynamics have become very important topics on the applications of nonlinear systems and a number of different methods for chaos synchronization have been presented by many researchers. Some of possible application areas are mainly in secure communications, optimization of nonlinear systems performance, modeling brain activity and pattern recognition phenomena [1-8]. It is of great importance to realize that nonlinear dynamics can play an extremely important role in resolving outstanding problems in theoretical physics [9]).

It is widely accepted that synchronization of chaotic systems is a difficult problem, mainly on the basis of the extremely sensitive characteristic of chaos to initial conditions. The cascade synchronization method [10] was presented by Pecora and Carroll, in 1990. They illustrated that the cascading of synchronized chaotic systems make it possible to reproduction of all of the signals in the original chaotic system using only one signal to monitor the synchronized motions.

Recently, the research of chaos dynamics synchronization has become a very significant research platform in the nonlinear dynamics field and researchers in this field have addressed a number of problems on chaos synchronization, such as the stability conditions for chaos synchronization, the realization for a successful synchronization, the applications of chaos synchronization etc. [11-20].

In recent 15 years, a large number of methods for chaos control and synchronization have been proposed. Such as, periodic parametric perturbation [21], drive-response synchronization [22], adaptive control [23-27], variable structure (or sliding mode) control [28-30], back-stepping control [31, 32], $H_8$ control [33], fuzzy control [34] and many others.

In [35], Dadras and Momeni developed an adaptive sliding mode control a chaotic systems. This method successfully reduces the chattering phenomenon and guarantees stability in presence of parameter uncertainties and external disturbance.

The objective of this article is to propose an appropriate adaptive sliding mode controller for synchronization of any different chaotic system, in the presence of uncertainties and external disturbance.

The organization of this paper is as follows: Section 2 addresses the system description and problem formulation. In section 3, stability analysis is given. In section 4, numerical simulations are used to demonstrate the effectiveness of the proposed method. Finally conclusion is presented.

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## SYSTEM DESCRIPTION AND PROBLEM FORMULATION

In this paper, we study a class of chaotic $n$-dimensional systems having the following system description:

Master System:

$$\begin{cases} \dot{x}_i = x_{i+1} & 1 \le i \le n-1 \\ \dot{x}_n = f(x,t) & x = [x_1, x_2, \ldots, x_n] \in R^n \end{cases} \quad (1)$$

Slave System:

$$\begin{cases} \dot{y}_i = y_{i+1}, & 1 \le i \le n-1 \\ \dot{y}_n = f(y,t) + \Delta f(y) + d(t) + u \\ \quad y = [y_1, y_2, \ldots, y_n] \in R^n \end{cases} \quad (2)$$

where, $u \in \mathbb{R}$ is the control input, $f$ is a given nonlinear function, $\Delta f(y)$ is an uncertain term representing the un-modeled dynamics or structural variation of system (2) and $d(t)$ is the disturbance of system (2).

In general, the uncertainty and the disturbance are assumed to be bounded as follows:

$$|\Delta f(y)| \le \alpha \text{ and } |d(t)| \le \beta \quad (3)$$

Where $\alpha$ and $\beta$ are positive unknown constants.

The control problem considered in this paper is that for different initial conditions of systems (1) and (2), the two coupled systems [i.e., the master system (1) and the slave system (2)] are to be synchronized by designing an appropriate control $u(t)$ in system (2) such that

$$\lim_{t \to \infty} \|y(t) - x(t)\| \to 0 \quad (4)$$

Where $\|\cdot\|$ denotes the Euclidian norm of a vector.

Let us define the state errors between the master and slave systems as

$$e_i = y_i - x_i \quad 1 \le i \le n-1 \quad (5)$$

$$\dot{e}_i = e_{i+1} \quad 1 \le i \le n-1$$

$$\dot{e}_n = \dot{y}_n - \dot{x}_n$$

Then

$$\dot{e}_n = f(y) - f(x) + \Delta f + u + d \quad (6)$$

The synchronization problem can be viewed as the problem of choosing an appropriate control law $u(t)$ such that the error states $e_i$ [$i$=1,2,...,$n$] in Eq. (6) generally converge *to zero*.

In traditional SMC, a sliding surface $S$ representing the desired system dynamics is chosen as

$$s = e_n + \sum_{i=1}^{n-1} \lambda_i e_i \quad (7)$$

The switching surface parameters $\{\lambda_i,\ i=1,\ldots,n-1\}$ are chosen based on the following two criteria. First, the values are chosen to stabilize the system during the sliding mode. Routh-Hurwitz criterion [36] is used to determine the range of coefficients $c_i$ that produce stable dynamics. That is, all the roots of the following characteristic polynomial describing the sliding surface have negative real parts with desirable pole placement,

$$P(\lambda) = \lambda^n + c_{n-1}\lambda^{n-1} + \cdots + c_2\lambda + c_1 \quad (8)$$

Second, the values are chosen such that the system during sliding mode has fast and smooth response.

Having established an appropriate switching surface, the next step is to design an adaptive sliding mode control scheme to guarantee that the system states are hitting on the sliding surface s = 0 (i.e. to satisfy the reaching condition $s\dot{s} < 0$). In order to ensure the occurrence of the sliding mode, an adaptive sliding mode law is designed. When the closed loop system is in the sliding mode, it satisfies $\dot{s} = 0$. An adaptation law is applied to construct the proposed adaptive sliding mode controller such that the chattering phenomenon, which is inherent in conventional switching-type sliding controllers, is attenuated and the steady error is also alleviated. To ensure the occurrence of the sliding motion, an adaptive control law is proposed as:

$$u = u_{eq} + u_r \quad (9)$$

Where $u_{eq}$ is equivalent control law and obtained by

$$u_{eq} = f(x,t) - f(y,t) - \sum_{i=1}^{n-1} \lambda_i e_{i+1}$$

In addition, $u_r$ is called reaching control law and it is defined as

$$u_r = -\mu\theta\psi$$

Consequently,

$$u = f(x,t) - f(y,t) - \sum_{i=1}^{n-1} \lambda_i e_{i+1} - \mu\theta\psi \quad (10)$$

where $\mu$ is a constant which should satisfy the following inequality:

$\mu>1$

$\psi$ function is defines as follows: $\psi = \hat{\alpha} + \hat{\beta}$
where $\hat{\alpha}, \hat{\beta}$ are parameters which obtained adaptively by the followings adaptive laws:

$$\dot{\hat{\alpha}} = \dot{\hat{\beta}} = |s|, \quad \hat{\alpha}(0) = \hat{\alpha}_0, \quad \hat{\beta}(0) = \hat{\beta}_0 \tag{11}$$

where $\hat{\alpha}_0$ and $\hat{\beta}_0$ are the positive and bounded initial values of $\hat{\alpha}$ and $\hat{\beta}$, respectively and $\gamma$ is a positive constant.
$\theta$ function is obtained by the below relation:

$$\theta = \frac{1 - e^{-\varphi s}}{1 + e^{-\varphi s}}$$

where in the above mentioned relation $\varphi$ is achieved adaptively by the following relation:

$$\dot{\varphi} = -\gamma\,\theta\, e^{\varphi s}$$

Due to previous definitions the main contribution of our paper is introduced in the next section.

## STABILITY ANALYSIS

**Theorem 1:** Consider the system dynamics, if this system is controlled by u(t) in Eq. (9) with adaptation law (11), then the closed loop system is globally asymptotically stable.

**Proof:** Let

$$\tilde{\alpha} = \hat{\alpha} - \alpha, \tilde{\beta} = \hat{\beta} - \beta \tag{12}$$

$\alpha$ and $\beta$ are unknown constants. Thus the following expression holds.

$$\dot{\tilde{\alpha}} = \dot{\hat{\alpha}}, \dot{\tilde{\beta}} = \dot{\hat{\beta}} \tag{13}$$

Consider the following Lyapunov function candidate

$$V(t) = \frac{1}{2}(s^2 + \theta^2 + \tilde{\alpha}^2 + \tilde{\beta}^2) \tag{14}$$

Then, the derivative of *V* is

$$\dot{V}(t) = s\dot{s} + \theta\dot{\theta} + \tilde{\alpha}\dot{\tilde{\alpha}} + \tilde{\beta}\dot{\tilde{\beta}} \tag{15}$$

Where, by direct computation, we have

$$\dot{\theta} = \frac{d\theta}{d\varphi}\frac{d\varphi}{dt} = \frac{2se^{-\varphi s}}{(1 + e^{-\varphi s})^2}\dot{\varphi} \tag{16}$$

In the above equation, if $\dot{V}$ is negative for all $s \neq 0$, then the so-called reaching condition [36] is satisfied. That is, the control *u* is designed to guarantee that the states are hitting on the sliding surface s = 0.

$$\begin{aligned}\dot{V}(t) &= s\dot{s} + \theta\dot{\theta} + \tilde{\alpha}\dot{\hat{\alpha}} + \tilde{\beta}\dot{\hat{\beta}} \\ &= s(\Delta f + d - \mu\theta\psi) + \theta\dot{\theta} + \tilde{\alpha}\dot{\hat{\alpha}} + \tilde{\beta}\dot{\hat{\beta}} \\ &= s(\Delta f + d - \mu\theta\psi) + \theta\frac{2se^{-\varphi s}}{(1 + e^{-\varphi s})^2}\dot{\varphi} \\ &\quad + \tilde{\alpha}\dot{\hat{\alpha}} + \tilde{\beta}\dot{\hat{\beta}}\end{aligned} \tag{17}$$

As we define $\theta$ in Eq(10),we have

$$\begin{aligned}\theta &= \frac{1 - e^{-\varphi s}}{1 + e^{-\varphi s}} \times \frac{e^{\frac{\varphi s}{2}}}{e^{\frac{\varphi s}{2}}} = \frac{e^{\frac{\varphi s}{2}} - e^{\frac{-\varphi s}{2}}}{e^{\frac{\varphi s}{2}} + e^{\frac{-\varphi s}{2}}} \\ &= \tanh(\frac{\varphi s}{2})\end{aligned} \tag{18}$$

and

$$-1 < \tanh(\frac{\varphi s}{2}) < 1 - |s| < |s|\tanh(\frac{\varphi s}{2}) < |s| \tag{19}$$

According to Eq(19),Eq(17) changes to

$$\begin{aligned}\dot{V} &\le \alpha|s| + \beta|s| - \mu\psi|s| + \theta\frac{2se^{-\varphi s}}{(1 + e^{-\varphi s})^2}\dot{\varphi} + \tilde{\alpha}\dot{\hat{\alpha}} + \tilde{\beta}\dot{\hat{\beta}} \\ &= \alpha|s| + \beta|s| - \mu\psi|s| - 2|s|\gamma\frac{(1 - e^{-\varphi s})^2}{(1 + e^{-\varphi s})^4} + \\ &\quad \psi|s| - \psi|s| + (\hat{\alpha} - \alpha)\dot{\hat{\alpha}} + (\hat{\beta} - \beta)\dot{\hat{\beta}} \\ &= -(\hat{\alpha} - \alpha)|s| - (\hat{\beta} - \beta)|s| - \mu\psi|s| + \psi|s| \\ &\quad - 2\gamma\eta|s| + (\hat{\alpha} - \alpha)\dot{\hat{\alpha}} + (\hat{\beta} - \beta)\dot{\hat{\beta}}\end{aligned}$$

Regards to introduce adaptive law in (11) the above relations changes to:

$$\dot{V} = (1 - \mu)\psi|s| - 2\gamma\eta|s| \tag{20}$$

Where

$$\eta = \frac{(1 - e^{-\varphi s})^2}{(1 + e^{-\varphi s})^4} > 0 \tag{21}$$

Since $\mu>1$ has specified in Eq. (9) and $\gamma>0$, we obtain the following inequality

$$\dot{V} \le -[(\mu - 1)\psi + 2\gamma\eta]|s| < 0 \tag{22}$$

## NUMERICAL SIMULATIONS

This section of the paper presents illustrative examples to verify and demonstrate the effectiveness of the proposed method. The simulation results are carried out using the MATLAB software. A time step size 0.001 was employed.

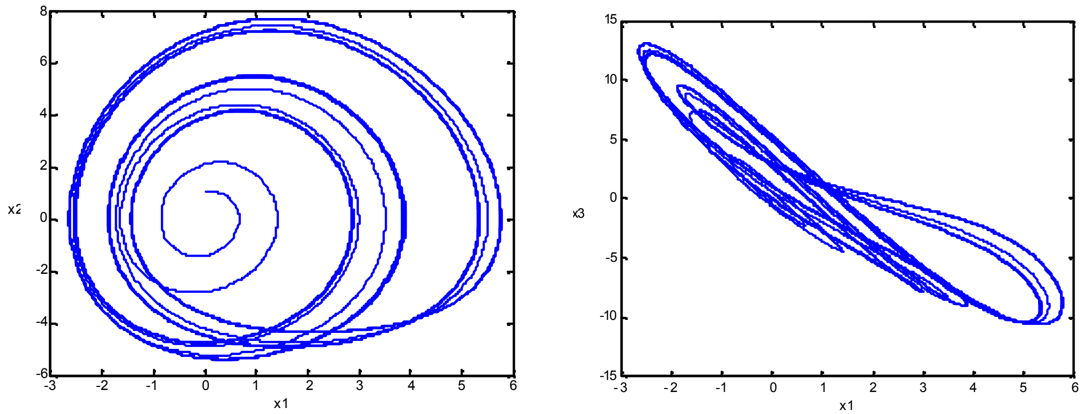


Fig. 1: The chaotic trajectory of master dynamic

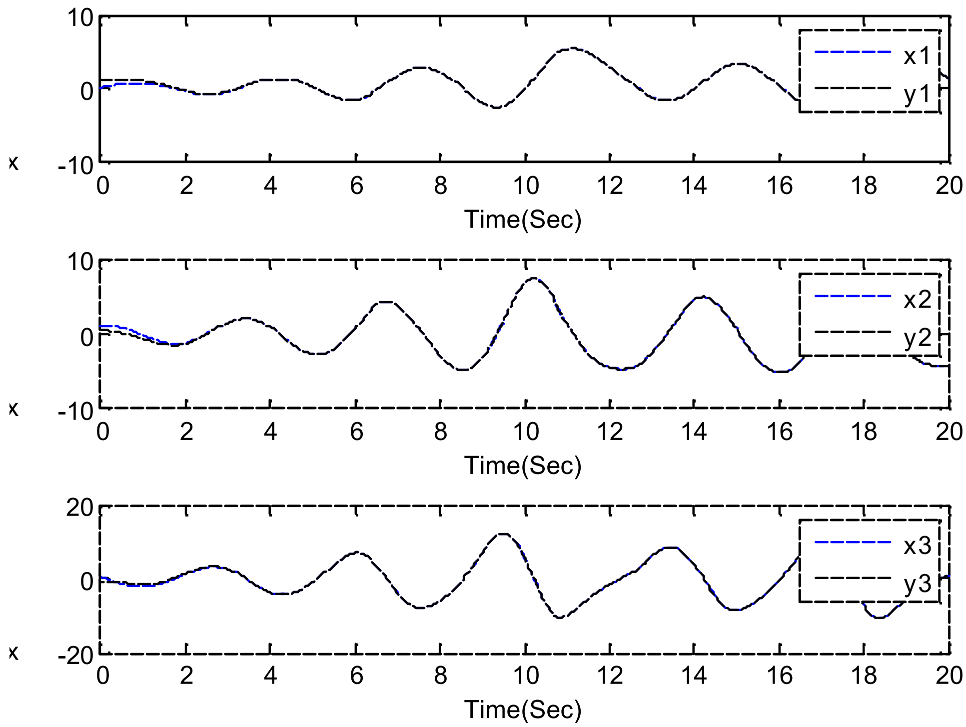


Fig. 2: The time response of states of master and slave dynamics

The simulation is done with the initial value $[x_1\ x_2\ x_3]^T = [0\ 1\ 0.5]^T$, $[y_1\ y_2\ y_3]^T = [1\ 0.5\ 1]^T$, $\hat{\alpha}_0 = 1$, $\hat{\beta}_0 = 11$, $\varphi_0 = 0.1$, $\theta_0 = 0.1$, $\gamma = 0.1$, $\mu = 50$. $f(x,t)$ and $f(y,t)$ are defined as follows

$$f(x,t) = -cx_1 - bx_2 - ax_3 + mx_1^2$$

$$f(y,t) = -cy_1 - by_2 - ay_3 + my_1^2$$

The system is perturbed by an uncertainty term

$\Delta f(x,t) = 0.5\sin(\pi x_1)\sin(2\pi x_2)\sin(3\pi x_3)$ and $d(t) = 0.2\cos(t)$, where $\Delta f(x,t) \le \alpha = 0.5$ and $d(t) \le \beta = 0.2$.

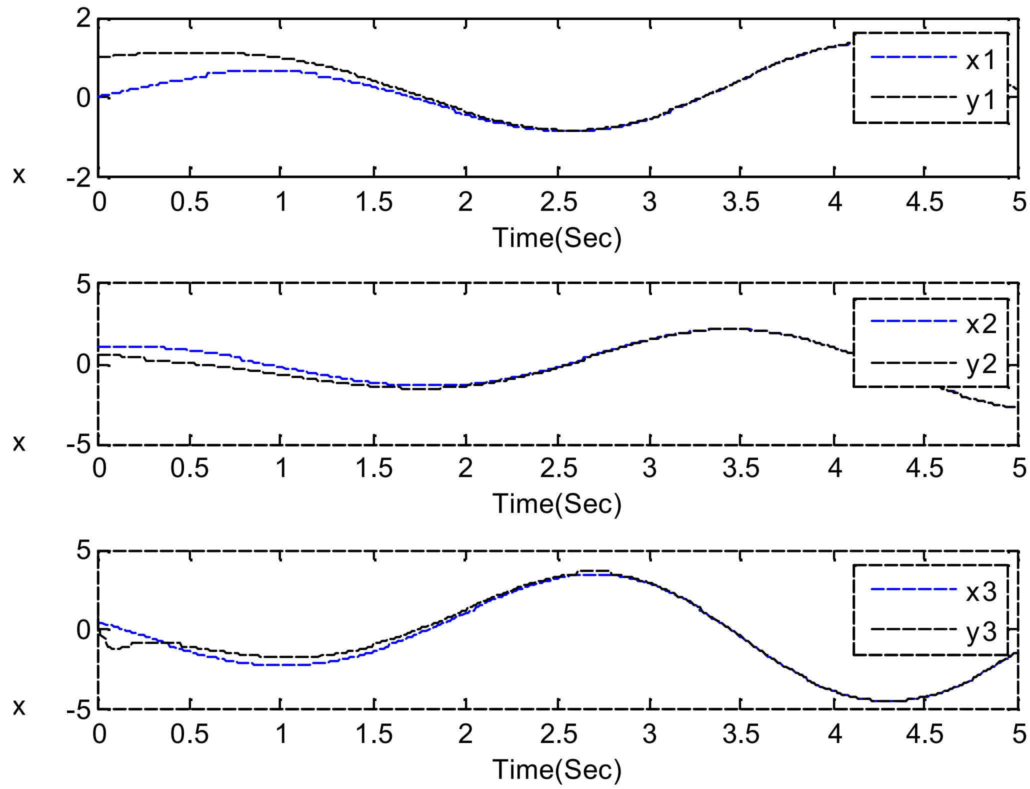


Fig. 3: The time response of states of master and slave dynamics (first 5 seconds)

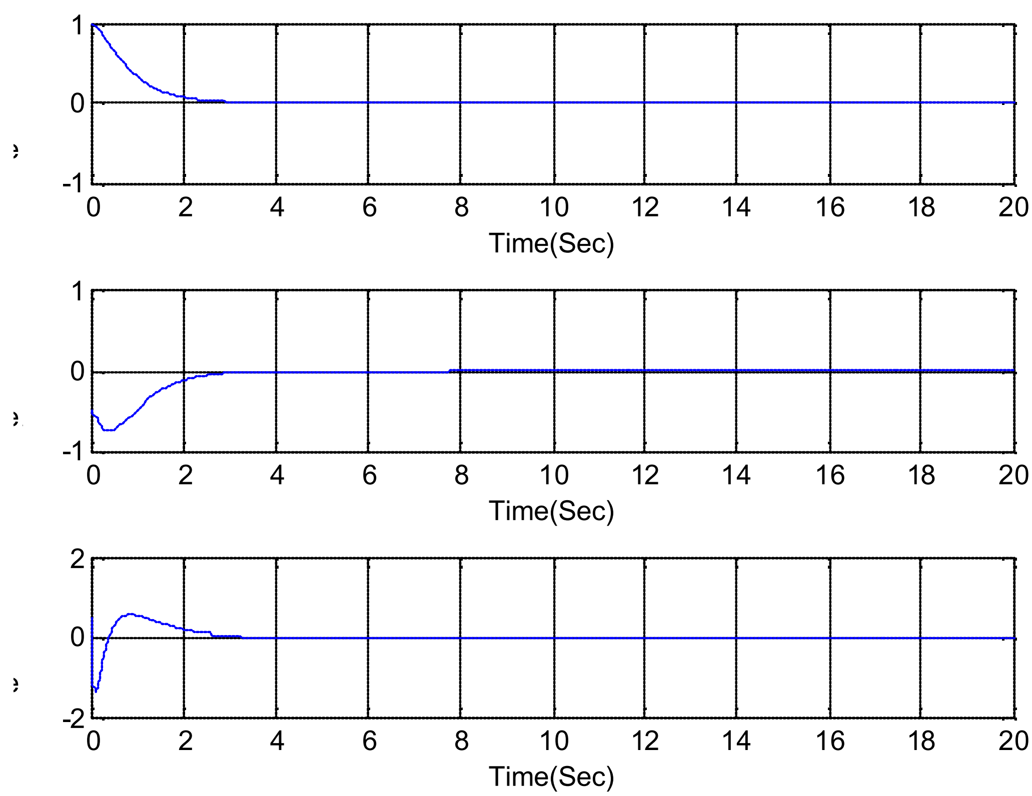


Fig. 4: The time response of the tracking errors (between states of master and slave dynamics)

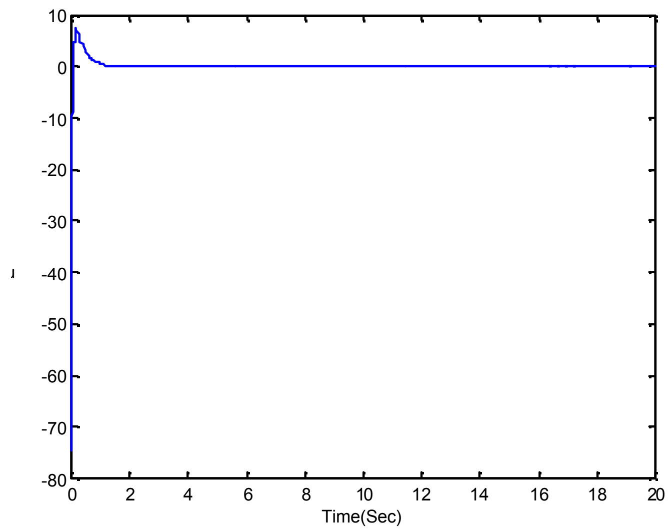


Fig. 5: The control input of slave dynamic

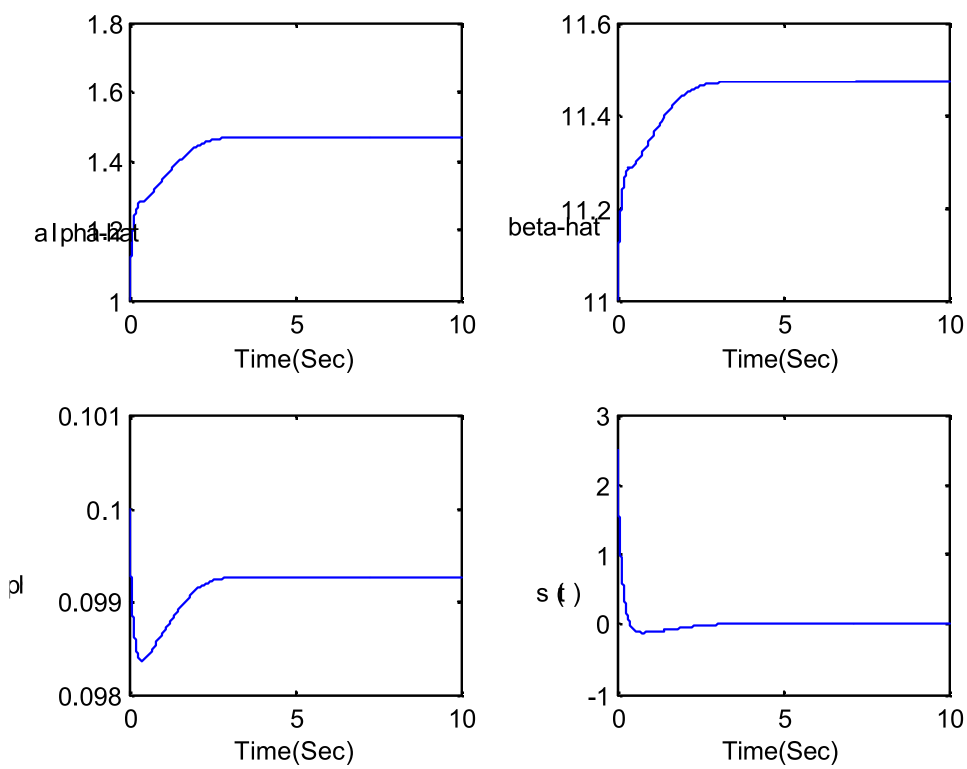


Fig. 6: The response of adaptation parameters and s (t) versus time (firs 10 seconds)

Both master and slave dynamics with the parameters a = 1.2, b = 2.92 and c = 6 are chaotic and retains one positive Lyapunov exponent as shown in Fig. 1. The simulation results are shown in Fig. 2-6 with the adaptation algorithm and under the given ASMC. Figure 2 represents respectively the state time responses of master and slave dynamics. This figure obviously shows that the slave dynamic trace the master dynamic successfully. Figure 3 illustrates state time responses for the first 5 seconds. Figure 4 shows the tracking errors of state time responses and Fig. 5 represents the control input. The adaptation parameters and the sliding surface dynamic (s(t)) are shown in Fig. 6. It can be seen that chattering does not appear, due to continuous control.

From the simulation results, it is clear that the obtained theoretic results are feasible and efficient for synchronization of two uncertain chaotic dynamical systems.

## CONCLUSION

In this paper, synchronization of two adaptive nonlinear systems is studied, using Lyapunov method. The proposed control scheme can be implemented without requiring the bounds of unknown parameters, disturbance and uncertainties to be known in advance and the chattering phenomenon is eliminated. In comparison with the other method presented in the past, our proposed scheme showed better synchronization performance and is more comprehensive.

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