Black Holes with Skyrme Hair

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Abstract

This paper is intended to give a review of the recent developments on black holes with Skyrme hair.

The Skyrme model is an effective meson theory where the baryons are identified with topological solitons, so-called skyrmions. The baryon number B corresponds to the topological charge. Thus the model gives a unified description of hadron physics in terms of meson fields. The spherically symmetric $B=1$ skyrmion and axially symmetric $B=2$ skyrmion were found numerically. Upon collective quantization, they produce correct nucleon and deuteron observables respectively. Approximate solutions with higher baryon numbers were also constructed by using the rational map ansatz. Depending on the baryon number, they exhibit various discrete symmetries with striking similarities to BPS monopoles.

The Einstein-Skyrme system is also known to possess black hole solutions with Skyrme hair. The spherically symmetric black hole skyrmion with $B=1$ was the first discovered counter example of the no-hair conjecture for black holes. Recently we found the $B=2$ axially symmetric black hole skyrmion. In this system, the black hole at the center of the skyrmion absorbs the baryon number partially, leaving fractional charge outside the horizon. Therefore the baryon number is no longer conserved. We examine the $B=1,2$ black hole solutions in detail in this paper.

The model has a natural extension to the gauged version which can describe monopole black hole skyrmions. Callan and Witten discussed the monopole catalysis of proton decay within the Skyrme model. We apply the idea to the Einstein-Maxwell-Skyrme system and obtain monopole black hole skyrmions. Remarkably there exist multi-black hole skyrmion solutions in which the gravitational, electromagnetic, and strong forces between the monopoles are all in balance. The solutions turn out to be stable under spherically symmetric linear perturbations.

1 Introduction

Application of the Skyrme model to the physics of the early Universe or equivalent high-energy physics is an interesting subject.

The attempts to explain (anti-)baryon production in high-energy collisions within the Skyrme model were made in Refs. [1, 2, 3]. The scenario for the production is the same as the one for producing other topological defects such as monopoles, cosmic strings or domain walls in the early Universe via the Kibble mechanism [4, 5]. Those defects are formed after a phase transition in which global symmetry is broken spontaneously. In the case of skyrmions, broken is the chiral symmetry and as a result one of the possible orientations of the chiral field is chosen randomly in the internal symmetry space to form the defects. The use of the Skyrme model is expected to describe nonperturbative phenomena to which perturbative QCD is not accessible.

Another interesting application of the Skyrme model is the monopole catalysis of proton decay. The monopole catalysis of proton decay was studied firstly at the lepton and quark level by Callan [6], Rubakov [7] and Wilczek [8] independently. In the grand unified theory there was a symmetry between baryons and leptons at energy scale 10^{15} GeV and therefore it would have been possible for the following reaction to be driven through the anomaly

$$p + \text{Monopole} \rightarrow e^+ + \text{pions} + \text{Monopole}\,. \tag{1}$$

Interestingly this reaction occurs at a strong interaction rate unsuppressed by any small-coupling-constant effects. Callan and Witten proposed not quark-monopole but proton-monopole interactions by using the Skyrme model [9] to make a more realistic estimation of the catalysis cross section possible. The decay process can be explained because in the presence of a monopole the baryon number is no longer equal to the topological charge of the meson field. In fact, there exist non-topological solitons with non-zero baryon number which can decay without topological problems. The extension to non-abelian monopole catalysis was also studied in Ref. [10].

For the study of the high-energy physics in the early Universe, it may be important to consider the effects of gravity on baryons. With the present value of the gravitational constant, those effects are insignificant upto the Planck energy 10^{19} GeV. In fact, in the Einstein-Skyrme theory, the Planck mass is related to the pion decay constant F_π and the coupling constant α by $M_{pl} = F_\pi\sqrt{4\pi/\alpha}$. To realize the realistic value of the Planck mass, the coupling constant should be extremely small with $\alpha \sim O(10^{-39})$, which makes the theory little different from the theory without gravity. However, some theories such as scalar-tensor gravity theory and Kaluza-Klein theory predict the time variation of the gravitational constant [11, 12, 13]. And also theories with extra dimensions predict that a true Planck scale is of order a TeV. Thus there may have been an epoch in the early Universe where the gravitational effects on baryons were significant. We consider those effects worth being studied in the Skyrme model. The advantage of the Skyrme model to more realistic nucleon models is that it is straightforward to couple to gravity. Although investigating the Einstein-Skyrme system is not expected to provide any quantitatively reliable results, we cannot exclude the possibility that it may give some qualitative insight into the gravitational interaction of baryons.

The Einstein-Skyrme system has been studied by various authors. The first obtained solutions in this system are spherically symmetric black holes with Skyrme hair [14, 15, 16]. Later, regular solutions for $B = 1$ [15, 16, 17] and axially symmetric black hole and regular solutions for $B = 2$ [18] were found. The extended models to $SU(3)$ and $SU(N)$ were also studied in Refs. [19]. It has been observed that microscopic black holes can support Skyrme hair admitting fractional baryon number outside the horizon. This configuration can be interpreted as a skyrmion partially absorbed by the microscopic black hole. Therefore the model provides a semiclassical framework to study the absorption rate of a proton by a black hole of comparable size. However this process is rather insignificant because black holes of the size of a proton have large fluxes of Hawking radiation [20]. Situations in which baryon decay process might become more realistic and significant over the Hawking radiation occurs when the black hole carries electric or/and magnetic charge with which the skyrmion interacts electromagnetically as well as gravitationally. Since a charged black hole in general has an effective temperature lower than that of a Schwarzschild black hole of the same mass, the Hawking radiation effect is less. Especially interesting is the extremal black hole which has a vanishing effective temperature, so the Hawking radiation may even vanish [21]. According to this speculation, we analyzed monopole black hole solutions with Skyrme hair [22]. This model provides the semiclassical framework in which to study monopole black hole catalysis of proton decay. Although macroscopic charged black holes perhaps do not exist in nature, microscopic charged black holes may have been created in the early Universe and remain as stable relics today. Indeed the GUT monopole has a typical radius $R_M \sim M_X^{-1}$ and a mass $g^{-2}M_X$, the Schwarzschild radius for the mass is then $R_{BH} \sim 2Gg^{-2}M_X$ and thus $R_M/R_{BH} \sim 2Gg^{-2}M_X^2$ where g^2 is the coupling constant renormalized at the X-boson mass scale M_X. Hence for a sufficiently large magnetic charge $p = 1/g$ and heavy gauge boson mass, the monopoles could have undergone gravitational collapse to form monopole black holes. This monopole black hole has an additional internal degree of freedom for electric charges being a dyon black hole analogous to the 't Hooft-Polyakov monopoles being the Julia-Zee dyon [23].

In subsequent sections we review the recent developments on black holes with Skyrme hair. Section 2 contains an introduction of the Skyrme model along with its brief historical review. In section 3 we review the $B = 1$ black hole skyrmion solution and its stability analysis. In section 4 the Einstein-Skyrme model coupled to abelian gauge fields is studied to obtain $B = 1$ monopole black hole skyrmion solutions. Its stability is examined in detail and the technique to study monopole black hole catalysis of baryon decay is discussed briefly. $B = 2$ black hole skyrmions are examined in section 5. It is shown that the energy density and baryon density of the solutions are torus in shape having a horizon at the center. Conclusion and Discussion are in section 6.

Throughout this paper we use the metric signature $(-, +, +, \cdots)$ and the Einstein summation convention. Greek indices are used to denote spacetime components of a tensor, while Italian indices are used to denote purely spatial components.

2 The Skyrme Model and Skyrmions

In this section we give an introduction of the Skyrme model, reviewing its historical development. A more detailed and complete review of the Skyrme model is provided by Zahed and Brown in Ref. [24].

2.1 Historical Review

It has been known that in the large-N_c effective meson theory, baryons emerge as solitons. This recognition has a long history.

In the Standard Model QCD is the theory for strong interactions. However it is $SU(3)$ gauge theory and there is no hope of solving it exactly and therefore an approximation method is necessary for the qualitative estimation of QCD. The most convenient approximation method would be a perturbation which, in fact, worked very well for QED. However it seemed that there was no small parameter to expand in low-energy QCD since the coupling constant is no longer small at low energies. The unexpected possible parameter was proposed by 't Hooft [25]. Although in the real world hadrons have definite color degrees of freedom $N_c = 3$, he suggested considering the number of colors N_c as a free parameter and obtaining the series expansion in powers of $1/N_c$. Then a detailed analysis of the diagrams shows that the limit of $N_c \to \infty$ fixing $g_3^2 N_c$ is equivalent to the effective chiral field theory for mesons where g_3 is a coupling constant. Thus 't Hooft succeeded to prove that the chiral effective Lagrangian is indeed low energy QCD. The next question was, how can baryons be incorporated in this large N_c effective meson theory?

Witten gave a rigorous answer to this question [26]. In his paper he showed that baryon mass is of order N_c which is inverse of the expansion parameter $1/N_c$. Hence in the large-N_c limit, they become singular. This is a typical feature seen in solitons. From this fact, Witten conjectured that baryons are solitons in the large-N_c meson theory.

Much earlier than Witten, in the flow of development of the effective theory of QCD, Skyrme attempted to construct a unified theory of strong interactions based on the meson field alone [27]. He explicitly constructed the minimal Lagrangian if the non-linear sigma model in $d = 1 + 3$ that supports topological soliton solutions and suggested that they are baryons [28]. However this remarkable idea was neglected until the 80s.

A unification of the ideas came in 1983. Witten studied solitons in the current algebra effective Lagrangian, which is the $SU(N_c)$ chiral models with the QCD anomalous term (Wess-Zumino term [29]) [30]. In QCD, the decay process $K\bar{K} \to \pi^+ + \pi^- + \pi^0$ is anomalous in the sense that it is observed in nature but is forbidden in the chiral model because of the symmetry $U \to U^\dagger$ which conserves the parity of the number of mesons. By including the Wess-Zumino term in the action, the anomalous process can be correctly described [31]. Witten showed that the anomalous piece of the current is responsible for the soliton solutions and their quantum numbers such as spin and baryon number. In particular for the $N_c = 2$ case which is the chiral model with N_c flavor degrees of freedom, the Wess-Zumino term vanishes. However from the homotopy argument $\pi_3(SU(2)) = Z$, there are still solitons. Besides $\pi_4(SU(2)) = Z_2$ suggests that in a suitably compactified space-time, there are two topological classes of maps from space-time to $SU(2)$ and hence the solitons can be spin-$\frac{1}{2}$ particles [32]. A detailed analysis for the property of the skyrmion as a nucleon was performed in Ref. [33] upon quantization of its collective coordinates.

2.2 The Model

In the lowest order, the chiral Lagrangian is given by

$$\mathcal{L} = \frac{F_\pi^2}{16}\,\mathrm{tr}\,(U^\dagger \partial_\mu U U^\dagger \partial_\mu U)\ , \quad U = e^{2i\vec{\tau}\cdot\vec{\pi}/F_\pi} \tag{2}$$

in the exponential parameterization where $F_\pi = 186$ MeV is the pion decay constant, $\vec{\tau}$ are the Pauli matrices and $\vec{\pi}$ are pion fields. The static energy is then given by

$$E = \int d^3x\, \frac{F_\pi^2}{16}\,\mathrm{tr}\,(U^\dagger \partial_i U U^\dagger \partial_i U)\,. \tag{3}$$

One can show that this model does not support a topological soliton as follows. Let us introduce the dimensionless variable $\tilde{\mathbf{x}}$ defined by $\mathbf{x} = \alpha\tilde{\mathbf{x}}$. Then for a static configuration, the energy can be written as

$$E = \alpha \int d^3\tilde{x}\, \frac{F_\pi^2}{16}\,\mathrm{tr}\,(U^\dagger \partial_i U U^\dagger \partial_i U)\,. \tag{4}$$

The integrand on the right-hand side is non-negative and hence the energy is minimized at $\alpha = 0$ with $E = 0$. Therefore the solution is trivial.

Skyrme introduced a new term in the fourth order derivative which retains the chiral symmetry of the model but supports a soliton solution [28]. The so-called Skyrme model is defined by

$$\mathcal{L}_S = \frac{F_\pi^2}{16}\,\mathrm{tr}\,(U^\dagger \partial_\mu U U^\dagger \partial_\mu U) + \frac{1}{32a^2}\,\mathrm{tr}\,[\partial_\mu U U^\dagger, \partial_\nu U U^\dagger]^2 \tag{5}$$

where a is a dimensionless parameter whose value can be fixed by experiment. The static energy is then given by

$$E = \int d^3x \left\{ \frac{F_\pi^2}{16}\,\mathrm{tr}\,(U^\dagger \partial_i U U^\dagger \partial_i U) + \frac{1}{32a^2}\,\mathrm{tr}\,[\partial_i U U^\dagger, \partial_j U U^\dagger]^2 \right\}\,. \tag{6}$$

One can see that this model indeed supports a nontrivial soliton solution by following the earlier argument. Expressing the static energy in terms of the rescaled variable $\tilde{\mathbf{x}}$, one obtains

$$E = \alpha \int d^3\tilde{x}\, \frac{F_\pi^2}{16}\,\mathrm{tr}\,(U^\dagger \partial_i U U^\dagger \partial_i U) + \frac{1}{\alpha}\int d^3\tilde{x}\, \frac{1}{32a^2}\,\mathrm{tr}\,[\partial_i U U^\dagger, \partial_j U U^\dagger]^2\,. \tag{7}$$

Since both integrands on the right-hand side are nonnegative, the energy is minimized at $\alpha \neq 0$.

Let us find a topological soliton in the Skyrme model. We introduce $A_\mu = U^\dagger \partial_\mu U$ and write the energy in terms of A_μ

$$E = \int d^3x \, \mathrm{tr} \left\{ \frac{F_\pi^2}{16} A_i A_i^\dagger + \frac{1}{32a^2} (\epsilon_{ijk} A_j A_k)(\epsilon_{ilm} A_l A_m)^\dagger \right\} . \tag{8}$$

The boundary condition for E to be finite is

$$A_i \to 0 \qquad \text{as } |\mathbf{x}| \to \infty \tag{9}$$

which is equivalent to saying that U approaches some constant matrix at infinity. Without loss of generality, we define this constant as the unit matrix

$$U \to I \qquad \text{as } |\mathbf{x}| \to \infty \, . \tag{10}$$

Then the spacetime is compactified to the three-sphere S^3.

There is a lower bound for the energy (8). From the Cauchy-Schwarz inequality,

$$\left(\frac{F_\pi}{4} A_i - \frac{1}{4a} \epsilon_{ijk} A_j A_k \right)^2 \geq 0 \, , \tag{11}$$

one can obtain

$$E \geq \frac{F_\pi}{8a} \int d^3x \, |\mathrm{tr} \, (\epsilon_{ijk} A_i A_j A_k)| \, . \tag{12}$$

The topological current of the skyrmion is

$$B^\mu = - \frac{\epsilon^{\mu\nu\rho\sigma}}{24\pi^2} \, \mathrm{tr} \, (A_\nu A_\rho A_\sigma) \tag{13}$$

with $\partial_\mu B^\mu = 0$. The topological charge is given by the zeroth component of the current

$$B = \int d^3x \, B^0 = - \frac{1}{24\pi^2} \int d^3x \, \mathrm{tr} \, (\epsilon_{ijk} A_i A_j A_k) \tag{14}$$

which corresponds to the winding number of the map $S^3 \to S^3$ and characterized by an integer as $\pi_3(S^3) = Z$. In the Skyrme model, topological charge is identified with the baryon number and hence skyrmions are interpreted as baryons [28].

From Eqs. (12) and (14) one finds the Bogomol'nyi bound

$$E \geq \frac{3\pi^2 F_\pi}{a} |B| \, . \tag{15}$$

At present, no soliton solution saturating this bound is found. But the skyrmions which have been obtained numerically probably represent the global minimum of the energy for given B.

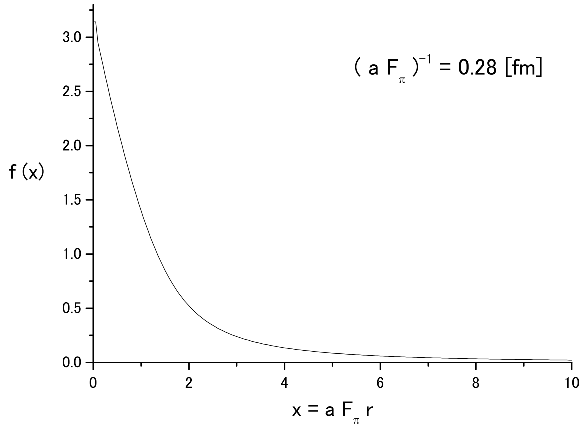

Figure 1: Profile f as a function of radial distance $x = aF_\pi r$.

The Skyrmion can be found by using the hedgehog ansatz

$$U = e^{if(r)\hat{\mathbf{x}}\cdot\tau} = \cos f(r) + i\vec{n}\cdot\vec{\tau}\sin f(r) \tag{16}$$

where $\vec{n} = \mathbf{x}/r$ with the boundary conditions

$$f(0) = \pi\,, \quad f(\infty) = 0\,. \tag{17}$$

Inserting the ansatz into the energy functional (6), one gets

$$E = 4\pi \int_0^\infty dr r^2 \left[\frac{F_\pi^2}{8}\left(f'^2 + \frac{2\sin^2 f}{r^2}\right) + \frac{1}{2a^2}\frac{\sin^2 f}{r^2}\left(\frac{\sin^2 f}{r^2} + 2f'^2\right)\right] \tag{18}$$

where a prime denotes derivative with respect to r. The static solution can be given as an extremum of the energy. Hence it satisfies $\delta E/\delta F = 0$. Equivalently

$$\left(\frac{x^2}{4} + 2\sin^2 f\right) f'' + \frac{x}{2}f' + f'^2 \sin 2f - \frac{\sin^2 f \sin 2f}{x^2} = 0 \tag{19}$$

where we introduced a dimensionless variable $x = aF_\pi r$. The solution of the equation (19) satisfying the boundary conditions (17) is shown in Fig. 1. For this solution, we have

$$B = -\frac{1}{2\pi}[2f - \sin 2f]_\pi^0 = 1\,. \tag{20}$$

Thus this solution represents a nucleon. For higher baryon numbers, $B = 2$ skyrmions were obtained numerically and shown to be axially symmetric [34, 35]. Braaten *et al.* constructed skyrmions upto $B = 6$ by descretizing the model on a cubic lattice [36]. Interestingly, it has been shown that multi-skyrmions with $B > 2$ exhibit various discrete symmetries analogously to multi-BPS monopoles in the use the rational map ansatz proposed in Ref. [37].

3 $B = 1$ Black Hole Skyrmions

As stated earlier, it is straightforward to couple the Skyrme model to gravity. The Skyrme Lagrangian and topological current are written in a covariant manner. After imposing the suitable ansatz and boundary conditions on both the chiral field and metric, one can solve numerically the Einstein equations coupled to the chiral field to obtain black hole solutions with Skyrme hair. The Einstein-Skyrme system was firstly studied by Luckock and Moss [14] where the Schwarzschild black hole with Skyrme hair was obtained numerically. This is a counter example of the no-hair conjecture for black holes [38]. They observed that the presence of the horizon in the core of skyrmion unwinds the skyrmion, leaving fractional baryonic charge outside the horizon. The full Einstein-Skyrme system was solved later to obtain spherically symmetric black holes with Skyrme hair [15, 16] and regular gravitating skyrmions [16, 17]. In this section we review the $B = 1$ black hole skyrmion solution and analyze its stability following Refs. [14, 15, 16, 17, 39].

3.1 Field Equations and Static Solutions

The Einstein-Skyrme system is defined by the Lagrangian

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_G + \mathcal{L}_S \\ &= \frac{R}{16\pi G} + \frac{F_\pi^2}{16} g^{\mu\nu}\mathrm{tr}\,(L_\mu L_\nu) + \frac{1}{32a^2} g^{\mu\nu} g^{\rho\sigma}\mathrm{tr}\,([L_\mu, L_\rho][L_\nu, L_\sigma])\end{aligned} \tag{21}$$

For $B = 1$, let us impose the hedgehog ansatz on the chiral field

$$U(r) = \cos f(r) + i\vec{n}\cdot\vec{\tau}\sin f(r)\,. \tag{22}$$

Correspondingly, we shall impose the spherically symmetric ansatz on the metric

$$ds^2 = -N^2(r)C(r)\,dt^2 + \frac{1}{C(r)}dr^2 + r^2 d\Omega^2 \tag{23}$$

where we have defined

$$C(r) = 1 - \frac{2Gm(r)}{r}\,.$$

At the horizon $r = r_h$, we have $C(r_h) = 0$, that is, $m(r_h) = r_h/(2G)$. Inserting these ansatz into the Lagrangian (21), one obtains the static energy density for the chiral field

$$E_S = \frac{4\pi F_\pi}{a}\int_{x_h}^{\infty}\left\{\frac{1}{8}\left(Cf'^2 + \frac{2\sin^2 f}{x^2}\right) + \frac{\sin^2 f}{2x^2}\left(2Cf'^2 + \frac{\sin^2 f}{x^2}\right)\right\} Nx^2\,dx\,. \tag{24}$$

where we have introduced dimensionless variables

$$x = aF_\pi r = r/0.28\,\mathrm{fm}\,, \qquad \hat{m}(x) = aF_\pi G m(r)\,.$$

It should be noted that in the presence of a black hole, the matter field is defined only outside the horizon and therefore the integral over the space is performed from the horizon to infinity.

The covariant topological current is defined by

$$B^{\mu} = -\frac{\epsilon^{\mu\nu\rho\sigma}}{24\pi^2}\frac{1}{\sqrt{-g}}\mathrm{tr}\left(U^{-1}\partial_\nu U U^{-1}\partial_\rho U U^{-1}\partial_\sigma U\right). \tag{25}$$

whose zeroth component corresponds to the baryon number density

$$B^0 = -\frac{1}{2\pi^2}\frac{1}{N}\frac{f'\sin^2 f}{r^2}. \tag{26}$$

We impose the boundary conditions on the profile function

$$f(\infty) = 0\,, \tag{27}$$

which determines the value at the horizon $f(r_h) = f_h$. Then the baryon number becomes

$$B = \int \sqrt{-g}\,B^0\,d^3x = -\frac{2}{\pi}\int_{f_h}^{0}\sin^2 f df = \frac{1}{2\pi}(2f_h - \sin 2f_h)\,. \tag{28}$$

This shows that the solution possesses fractional baryonic charge when $f_h < \pi$.

The field equations for the gravitational fields $N(x)$ and $\hat{m}(x)$ can be derived from the Einstein equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{29}$$

which read

$$N' = \frac{\alpha}{4}\left(x + \frac{8\sin^2 F}{x}\right)Nf'^2 \tag{30}$$

$$\hat{m}' = \frac{\alpha}{8}\left[(x^2 + 8\sin^2 f)Cf'^2 + 2\sin^2 f + \frac{4\sin^4 f}{x^2}\right] \tag{31}$$

where we have defined the coupling constant $\alpha = 4\pi G F_\pi^2$. The variation of the static energy (24) with respect to the profile $f(x)$ leads to the field equation for matter

$$f'' = \frac{1}{NC(x^2+8\sin^2 f)}\left[-(x^2+8\sin^2 f)N'Cf' + \left(1+\frac{4\sin^2 f}{x^2}+4Cf'^2\right)\right.$$
$$\left.\times N\sin 2f - 2(x+4\sin 2f f')NCf' - 2\left(1+\frac{8\sin^2 f}{x^2}\right)(\hat{m}-\hat{m}'x)Nf'\right]. \tag{32}$$

To solve these coupled field equations, let us consider the boundary conditions for the gravitational fields. Expanding the fields $f(x), \hat{m}(x), N(x)$ around the horizon

$x = x_h$ and substituting into the field equations (30)-(32), one obtains upto second order

$$
\begin{aligned}
f(x) &= f_h + f_1(x - x_h) + O((x - x_h)^2) \\
\hat{m}(x) &= \frac{x_h}{2} + \hat{m}_1(x - x_h) + O((x - x_h)^2) \\
N(x) &= N_h + N_1(x - x_h) + O((x - x_h)^2)
\end{aligned}
$$

where f_h and N_h are shooting parameters which should be determined so as to satisfy the boundary conditions at infinity $f(\infty) = 0$ and $N(\infty) = 1$, and

$$\hat{m}_1 = \frac{\alpha}{4}\left(\sin^2 f_h + \frac{2\sin^4 f_h}{x_h^2}\right) \tag{33}$$

$$f_1 = \frac{(x_h^2 + 4\sin^2 f_h)\sin 2f_h}{x_h(x_h^2 + 8\sin^2 f_h)(1 - 2\hat{m}_1)} \tag{34}$$

$$N_1 = \frac{\alpha}{4}\left(x_h + \frac{8\sin^2 f_h}{x_h}\right) N_h f_1^2 \,. \tag{35}$$

The dependence of the profile function on the coupling constant and the horizon size is shown in Fig. 2. There are two branches of solutions for each value of the coupling constant. Let us call the solution with larger (smaller) values of f_h as upper (lower)-branch. The skyrmion shrinks as α or/and x_h increase for upper-branch while it expands for lower-branch. Fig. 3 shows the dependence of the value of the profile function at the horizon on the horizon size. One can see that for $\alpha \neq 0$, all black hole solutions converge to globally regular solutions as $x_h \to 0$. As α approaches to zero, the lower-branch solutions converge to the Schwartzschild black hole with Skyrme hair. On the other hand, the upper-branch solutions converge to the $n = 1$ colored black hole solution [40]. The maximum value of the coupling constant with which black hole solutions exit is 0.126.

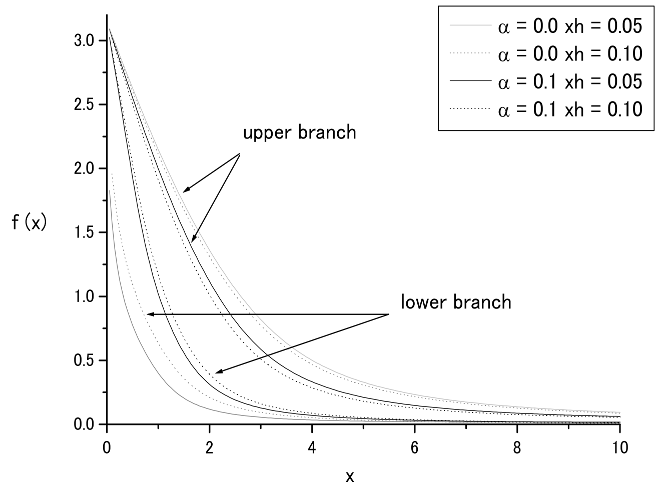

Figure 2: Profile function $f(x)$ with $\alpha = 0.00, 0.10$ and $x_h = 0.05, 0.10$ for upper and lower branch.

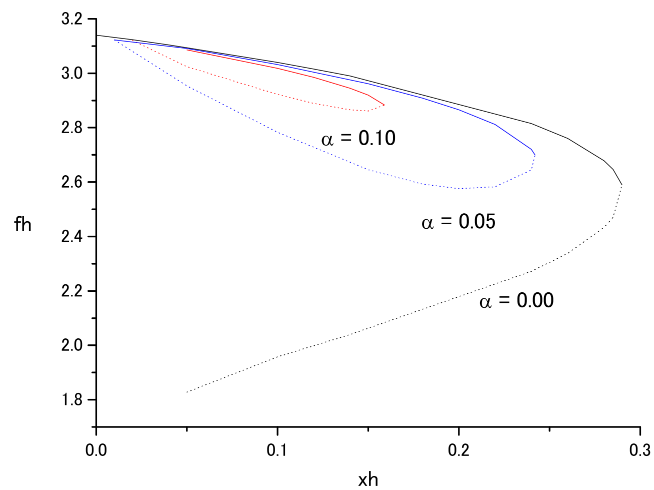

Figure 3: The shooting parameter f_h as a function of the horizon size x_h with $\alpha = 0.00, 0.05, 0.10$. Solid line shows the stable branch and dotted line the unstable branch.

3.2 Linear Stability Analysis

The linear stability of the $B = 1$ black hole skyrmion was studied in detail in Refs. [39, 17]. In order to examine the stability, let us consider the time-dependent Skyrme action given by

$$\mathcal{S} = -\frac{\pi e^2 F_\pi^4}{2} \int \left[(-\frac{1}{e^\delta C}\dot{f}^2 + Cf'^2)u + v \right] e^\delta dx \tag{36}$$

where we have defined

$$\delta = \log N \,, \quad u = x^2 + 8\sin^2 f \,, \quad v = \sin^2 f(x^2 + 2\sin^2 f) \tag{37}$$

Then time-dependent field equation can be obtained by taking variation with respect to f as

$$(e^\delta C u f')' + \frac{1}{2}\left(\frac{1}{e^\delta C}\dot{f}^2 - e^\delta C f'^2\right) u_f - \frac{e^\delta v_f}{x^2} = \frac{1}{e^\delta C} u \ddot{f} \tag{38}$$

where $u_f = \delta u/\delta f$ and $v_f = \delta v/\delta f$. From the action (36), the time-dependent Einstein equations are derived as

$$G_{00} = 8\pi G T_{00} \quad \rightarrow \quad 1 - C - C'x = \frac{\alpha^2}{4}\left[\left(\frac{1}{e^{2\delta}C}\dot{f}^2 + Cf'^2\right)u + \frac{2v}{x^2}\right] \tag{39}$$

$$G_{11} = 8\pi G T_{11} \quad \rightarrow \quad -1 + C + \frac{(e^{2\delta}C)'}{e^{2\delta}}x = \frac{\alpha^2}{4}\left[\left(\frac{1}{e^{2\delta}C}\dot{f}^2 + Cf'^2\right)u - \frac{2v}{x^2}\right] \tag{40}$$

which read to the following two equations for the gravitational fields

$$\delta' = \frac{\alpha^2}{4x}\left(\frac{1}{e^{2\delta}C^2}\dot{f}^2 + f'^2\right)u \tag{41}$$

$$-(Cx)' + 1 = \frac{\alpha^2}{2x^2}v + C\delta' x \,. \tag{42}$$

Let us consider the small radial fluctuations of the profile and gravitational fields around the static classical solutions denoted by f_0, N_0 and C_0 as

$$f(x,t) = f_0(x) + f_1(x,t) \tag{43}$$

$$\delta(x,t) = \delta_0(x) + \delta_1(x,t) \tag{44}$$

$$C(x,t) = C_0(x) + C_1(x,t) \,. \tag{45}$$

Substituting into Eqs. (41) and (42) gives the linearized equations

$$\delta_1' = \frac{\alpha^2}{2x}(2u_0 f_0' f_1' + u_{f_0} f_0'^2 f_1) \tag{46}$$

$$-(e^{\delta_0} C_1 x)' = \frac{\alpha^2}{2x^2} e^{\delta_0} v_{f_0} f_1 + e^{\delta_0} C_0 \delta_1' x \,. \tag{47}$$

Eq. (46) and the classical field equation derived from Eq. (38)

$$\frac{e^{\delta_0} v_{f_0}}{x^2} = (e^{\delta_0} C_0 u_0 f_0')' - \frac{1}{2} e^{\delta_0} C_0 u_{f_0} f_0'^2 \tag{48}$$

are inserted into Eq. (47) and resultantly one gets

$$-(e^{\delta_0} C_1 x)' = \frac{\alpha^2}{2} (e^{\delta_0} C_0 u_0 f_0' f_1)' \tag{49}$$

which can be integrated immediately

$$C_1 = -\frac{\alpha^2}{2x} C_0 u_0 f_0' f_1 \,. \tag{50}$$

Similarly we shall linearize the field equation (38). Using Eqs. (46), (48) and (50), one arrives at

$$(e^{\delta_0} C_0 u_0 f_1')' - U_0 f_1 = \frac{1}{e^{\delta_0} C_0} \ddot{f}_1 \tag{51}$$

where

$$\begin{aligned} U_0 \;\; = \;\; & -(e^{\delta_0} C_0 u_{f_0} f_0')' + \left(\frac{\alpha^2}{2x} e^{\delta_0} C_0 u_0^2 f_0'^2 \right)' - \frac{\alpha^2}{2x} e^{\delta_0} C_0 u_0 u_{f_0} f_0'^3 \\ & + \frac{1}{2} e^{\delta_0} C_0 u_{ff_0} f_0'^2 + \frac{e^{\delta_0} v_{ff_0}}{x^2} \,. \end{aligned} \tag{52}$$

Setting $f_1 = \xi(x) e^{i\omega t} / \sqrt{u_0}$, Eq. (51) becomes

$$-(e^{\delta_0} C_0 \xi')' + \left[\frac{1}{2\sqrt{u_0}} \left(e^{\delta_0} C_0 \frac{u_0'}{\sqrt{u_0}} \right)' + \frac{1}{u_0} U_0 \right] \xi = \omega^2 \frac{1}{e^{\delta_0} C_0} \xi \,. \tag{53}$$

Let us introduce the tortoise coordinate x^* such that

$$\frac{dx^*}{dx} = \frac{1}{e^{\delta_0} C_0} \tag{54}$$

with $-\infty < x^* < +\infty$. Eq. (53) is then reduced to the Strum-Liouville equation

$$-\frac{d^2 \xi}{dx^{*2}} + \hat{U}_0 \xi = \omega^2 \xi \tag{55}$$

where

$$\hat{U}_0 = e^{\delta_0} C_0 \left[\frac{1}{2\sqrt{u_0}} \left(e^{\delta_0} C_0 \frac{u_0'}{\sqrt{u_0}} \right)' + \frac{1}{u_0} U_0 \right] \,. \tag{56}$$

If the black hole skyrmion is stable, this equation has no negative mode which induces exponential grow in ξ. Numerically solving the Eq. (53) or (56), one finds no negative mode in the upper branch and one negative mode in the lower branch. Therefore, it is concluded that the black hole skyrmion in the upper branch is stable and unstable in the lower branch.

4 Monopole Black Hole Skyrmions

In this section, charged black hole solutions with Skyrme hair are discussed. We find stable non-topological skyrmion solutions as well as topological ones in a background of charged black holes. The Skyrme model is valid at a greater scale than the quark confinement scale where the gauge fields can be assumed to be abelian. Although more complicated non-abelian monopole solutions are possible, depending on the details of the Higgs sector [10], we shall restrict attention to the simplest case. The black hole mass for abelian monopoles has the lower bound

$$M = \frac{p}{\sqrt{G}} = pM_{pl} \approx 2.54 \times 10^{-7} \ \text{Kg}\,, \tag{57}$$

where we denoted the magnetic charge $p \approx 11.7$ and the Planck mass $M_{pl} \approx 2.1768 \times 10^{-8}$ Kg. We shall also examine the stability of our solutions. The main obstacle to proton decay around the monopole black hole is electric charge conservation. The black hole cannot swallow the proton whole because this would tip it over the extremal limit. It should come as no surprise, therefore, that the non-topological solutions are stable. The introduction of charged fermions, which can carry the electric charge away, will be also briefly discussed.

This work was carried out with the collaboration of I. Moss and E. Winstanley.

4.1 Field Equations and Static Solutions

The Lagrangian is based on a gauged version of the original Skyrme Lagrangian constructed by Callan and Witten [9]. The natural extension to the charged $SU(2)$ chiral field is gauging the Skyrme model. For a gauge transformation

$$U \to U + ie\alpha\,[Q, U]\,, \qquad A_\mu \to A_\mu + \partial_\mu \alpha\,, \tag{58}$$

one can define the covariant derivative

$$D_\mu U = \partial_\mu U - ieA_\mu\,[Q, U] \tag{59}$$

where A_μ is the photon field, e is the charge of proton electric charge in unrationalized units, and

$$Q = \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix} \tag{60}$$

is the charge matrix of quarks. Replacing merely the derivatives in the Lagrangian with the covariant derivatives $D_\mu U = \partial_\mu U - ieA_\mu\,[Q, U]$ is, however, not sufficient in the sense that QCD anomalous processes, such as $\pi^0 \to \gamma\gamma$ or the $\gamma\pi^+\pi^-\pi^0$ vertex, are not included. This is a manifestation of the chiral symmetry breaking due to the presence of the $U(1)$ gauge fields. Thus the correct Lagrangian should include an anomalous term $\mathcal{L}_A$. The anomalous term arises from the Wess-Zumino term which vanishes in the $SU(2)$ Skyrme model. It has been worked out in Refs. [9, 31] that the gauge invariant form of the Wess-Zumino term $\Gamma(U)$ has additional terms given by

$$\hat{\Gamma}(U, A_\mu) = \Gamma(U) + e \int d^4x \, A_\mu J^\mu - ie^2 \int d^4x \, F_{\mu\nu} A_\rho T_\sigma \tag{61}$$

which reads

$$\mathcal{L}_A = \frac{1}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} [eA_\mu \, \mathrm{tr} \, (QL_\nu L_\rho L_\sigma - QR_\nu R_\rho R_\sigma) - ie^2 F_{\mu\nu} A_\rho T_\sigma] \tag{62}$$

where the left current $L_\mu = U^\dagger \partial_\mu U$ and right current $R_\mu = U \partial_\mu U^\dagger$ and

$$T_\sigma = \mathrm{tr} \, (Q^2 L_\sigma - Q^2 R_\sigma + \frac{1}{2} QU^\dagger QUL_\sigma - \frac{1}{2} QUQU^\dagger R_\sigma) \tag{63}$$

have been defined. Consequently, we have the following gauged Einstein-Skyrme Lagrangian

$$\mathcal{L} = \mathcal{L}_S + \mathcal{L}_A + \mathcal{L}_{em} + \mathcal{L}_G \tag{64}$$

where

$$\begin{aligned}
\mathcal{L}_S &= \frac{F_\pi^2}{16} \mathrm{tr} \, (U^\dagger D_\mu U U^\dagger D_\mu U) + \frac{1}{32a^2} \mathrm{tr} \, ([U^\dagger D_\mu U, U^\dagger D_\nu U]^2) \\
\mathcal{L}_A &= \frac{e}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} A_\mu \mathrm{tr} \, [Q(U^\dagger \partial_\nu U U^\dagger \partial_\rho U U^\dagger \partial_\sigma U + \partial_\nu U U^\dagger \partial_\rho U U^\dagger \partial_\sigma U U^\dagger)] \\
&\quad + \frac{ie^2}{8\pi^2} \epsilon^{\mu\nu\rho\sigma} (\partial_\mu A_\nu) A_\rho \\
&\quad \times \mathrm{tr} \, [Q^2 (\partial_\sigma U) U^\dagger + Q^2 U^\dagger (\partial_\sigma U) + \frac{1}{2} Q (\partial_\sigma U) Q U^\dagger - \frac{1}{2} QUQ(\partial_\sigma U^\dagger)]
\end{aligned}$$

and the free actions are

$$\mathcal{L}_{em} = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu}, \qquad \mathcal{L}_G = \frac{1}{16\pi G} R \, .$$

The gauge-invariant baryon current is then given by

$$\begin{aligned}
B^\mu &= \frac{\epsilon^{\mu\nu\rho\sigma}}{24\pi^2} [\mathrm{tr} \, (L_\nu L_\rho L_\sigma) + 3ieA_\nu \mathrm{tr} \, Q(L_\rho L_\sigma - R_\rho R_\sigma) + 3ie\partial_\nu A_\rho \mathrm{tr} \, Q(L_\sigma - R_\sigma)] \\
&= \frac{\epsilon^{\mu\nu\rho\sigma}}{24\pi^2} \mathrm{tr} \, (L_\nu L_\rho L_\sigma) + \frac{\epsilon^{\mu\nu\rho\sigma}}{24\pi^2} \, \partial_\nu \, [3ieA_\rho \mathrm{tr} \, Q(L_\sigma - R_\sigma)]
\end{aligned} \tag{65}$$

which shows the additional current is a total divergence.

In the spherically symmetric case with a magnetic charge the gauge field has the form

$$A = p \, (1 - \cos\theta) \, d\phi + \Phi \, dt \tag{66}$$

where p is a magnetic charge and the Dirac quantization condition is $pe = 1$. The usual skyrmion has a magnetic moment which would interact with a magnetic monopole and break spherical symmetry. We use instead a non-topological ansatz for the chiral field

$$U = e^{if(r,t)\tau^3} . \tag{67}$$

One can see that this field is made up of neutral pions and commutes with the charge matrix $Q = \frac{1}{6} + \frac{1}{2}\tau^3$. Nevertheless it has a non-zero total electric and baryonic charge due to the effects of anomalies as we shall see below. After inserting the ansatz (67) into the zeroth component of the baryonic current (65), one obtains

$$n_B = \frac{ep}{2\pi}[f(\infty) - f(0)] . \tag{68}$$

The solution with the boundary conditions $f(0) = 0$ and $f(\infty) = 2\pi$ possesses unit baryon number and hence it can be interpreted as a baryon surrounding the monopole.

If the field ansatz (67) is substituted into the meson and electromagnetic interaction terms in the Lagrangian, they become

$$\mathcal{L}_S = -\frac{F_\pi^2}{8}(\partial f)^2 \tag{69}$$

$$\mathcal{L}_A = -\frac{e^2}{8\pi^2}E_i B^i f \tag{70}$$

$$\mathcal{L}_{em} = \frac{1}{8\pi}(E^2 - B^2) \tag{71}$$

where the index $i = 1, 2, 3$ and the electromagnetic fields E_i and B_i are defined by $F_{0i} = \sqrt{-g_{tt}}\, E_i$ and $F_{ij} = \epsilon_{ijk}B^k$. When combined

$$\mathcal{L}_A + \mathcal{L}_{em} = \frac{1}{8\pi}\left(E - \frac{e^2}{2\pi}Bf\right)^2 - \frac{1}{8\pi}B^2\left(1 - \frac{e^4}{4\pi^2}f^2\right) . \tag{72}$$

The extrema of the action occur when the electric field is given by

$$E = \frac{e^2}{2\pi}Bf . \tag{73}$$

This situation is reminiscent of the factorization of the Lagrangian that occurs for a BPS monopole [41].

The electric field implies a total charge

$$q = \frac{e^2 p}{2\pi}f \tag{74}$$

or asymptotically

$$q_\infty = n_B e \tag{75}$$

and the $n_B = 1$ solution can therefore be interpreted as a proton. If a black hole appears in the background, the inner boundary condition for the field f should be imposed not at the origin but at the event horizon $r = r_+$. Thus the baryon number in the presence of an event horizon will be defined as

$$n_B = \frac{ep}{2\pi}\left[f(\infty) - f(r_+)\right], \tag{76}$$

which implies the baryon number swallowed by the black hole is

$$n_B^- = \frac{ep}{2\pi}\left[f(r_+) - f(0)\right]. \tag{77}$$

Thus the total baryon number can be recovered as the sum

$$n_B^{tot} = n_B + n_B^- . \tag{78}$$

If $f(r_+) = 0$, the baryon number is still an integer and conserved. This configuration represents a proton tightly bound to the black hole. On the other hand, if f takes some positive value at the horizon the baryon number is not an integer and the skyrmion carries fractional baryon number and electric charge. This configuration will be interpreted as a proton partially swallowed by the black hole. In particular, $f(r_+) = 2\pi$ means that the black hole has swallowed a whole proton, leaving nothing outside the horizon.

It is interesting to observe that, while the baryon number disappears inside the horizon, the electric charge of the black hole can still be measured outside, turning the monopole black hole into a dyon black hole. Therefore, while the baryon number conservation is violated, charge conservation is not violated.

4.1.1 Extremal black hole solutions

In the extremal case we can obtain a general solution based on the Papapetrou-Majumdar metrics [42, 43]. We begin with the background metric fixed and later generalize to solve the full Einstein equations with chiral matter. The Papapetrou-Majumdar metrics have the form

$$ds^2 = -H^2 dt^2 + H^2(dx^2 + dy^2 + dz^2) \tag{79}$$

where

$$H = 1 + \sum_{n=1}^{n_M} \frac{GM_n}{R_n} \tag{80}$$

and R_n is the ordinary Euclidean distance from the point mass M_n located in three-dimensional space. We also associate these point masses with magnetic charges $P_n = G^{1/2} M_n$, and the magnetic field

$$B = G^{-1/2} H^{-1} \partial H . \tag{81}$$

The matter Lagrangian obtained earlier (69) has the form

$$\mathcal{L} = -\frac{1}{8}F_\pi^2\,(\partial f)^2 - \frac{1}{8\pi}B^2\,(1+\alpha^2 f^2) \tag{82}$$

where we will set

$$\alpha = \frac{e^2}{2\pi}\,, \qquad \mu^2 = \pi G F_\pi^2\,. \tag{83}$$

The Skyrme field equation obtained from the Lagrangian on this background becomes

$$-\,\mu^2\partial^2 f + \alpha^2 H^{-2}(\partial H)^2 f = 0\,. \tag{84}$$

We lose no generality by taking equal charges $P_n = p$. The solution with baryon number $n_B = n_M$ is then

$$f = 2\pi H^{-s} \tag{85}$$

where

$$s = -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{\alpha^2}{\mu^2}}\,. \tag{86}$$

Since $F_\pi \ll m_{pl}$, we can use $s \approx \alpha/\mu$ for the pion model.

For a single black hole, the Reissner-Nordström coordinate r is related to R by $R = r - r_+$ and we obtain

$$f = 2\pi\left(1 - \frac{r_+}{r}\right)^s\,. \tag{87}$$

The field is effectively expelled by the black hole and vanishes on the horizon $r = r_+$.

The mass of the chiral field configuration can be obtained by integrating the Lagrangian (82),

$$m_f = \frac{1}{8}F_\pi^2\int_\Sigma f\partial_i f\,dS^i \tag{88}$$

where Σ is a large surface containing all of the masses. This gives

$$m_f = 2\pi^3 s F_\pi^2 G\sum_n M_n \approx \pi^{3/2} n_B\, e\, F_\pi\,. \tag{89}$$

The total mass in the chiral field is much less than one baryon mass per mass point. We can see how it is energetically favorable for a free skyrmion to change its internal configuration from the original Skyrme form to the simpler form used here when it comes into contact with a black hole monopole. The total topological description of this transformation for a single monopole is exactly as described in [9].

It is interesting to see that the electrostatic energy cancels due to the factorization occurring in the Lagrangian (72). The chiral field mass is independent of the separation of the holes and therefore there are no forces between them. This is similar to the situation for BPS monopole solutions [41], and suggests that there is a solution of the full Einstein-matter system. This existence of the solution will now be demonstrated.

The spatial part of the Einstein tensor for the metric (79) is

$$G_{ij} = -2H^{-2}(\partial_i H)(\partial_j H) + H^{-2}(\partial H)^2 g_{ij} \tag{90}$$

and the Ricci scalar is

$$R = -2H^{-1}\partial^2 H\,. \tag{91}$$

Substituting the Einstein tensor for the Lagrangian (82) into the Einstein equations gives

$$H^{-1}\partial^2 H = -\mu^2(\partial f)^2 \tag{92}$$

$$H^{-2}(\partial_i H)(\partial_j H) = -\mu^2(\partial_i f)(\partial_j f) + GB_iB_j(1+\alpha^2 f^2)\,. \tag{93}$$

These make up a complete system of equations when we include the Maxwell equation

$$\partial(HB) = 0\,. \tag{94}$$

The second Einstein equation implies that ∂f, ∂H and B are all parallel. We therefore impose a condition $f \equiv f(u)$, $B = b(u)H^{-1}\partial H$, where

$$u = -\mu^{-1}\log H\,. \tag{95}$$

The system of equations becomes equivalent to an ordinary differential equation with independent parameter u,

$$f'' + \mu(1+f'^2)f' - \alpha^2 b^2 f = 0 \tag{96}$$

where

$$b^2 = \frac{1+f'^2}{1+\alpha^2 f^2}\,. \tag{97}$$

The horizon corresponds to $u \to -\infty$ and the far region to $u \to 0$. The horizon must therefore be at a critical point of the first-order system corresponding to (96). There is only one critical point, $(f, f') = (0,0)$, hence

$$(f, f') \to (0,0) \quad \text{as} \quad u \to -\infty\,. \tag{98}$$

Since the critical point is a saddle, the solution is unique and exists for all values of μ. Having obtained the unique solution to (96), we then define

$$V(u) = 1 + \mu \int_u^0 b(x) e^{-\mu x} dx \,. \tag{99}$$

It is easily seen from (95) that

$$\partial_i V = V' \partial_i u = HB \,. \tag{100}$$

Hence the Maxwell equation implies $\partial^2 V = 0$ and we can write

$$V = 1 + \sum_n \frac{GM_n}{R_n} \,. \tag{101}$$

Inverting (99) gives $u(V)$.

4.1.2 Spherically symmetric solutions

In the non-extremal case we shall consider spherically symmetric metrics which can be parameterized in the form

$$ds^2 = -\frac{\Delta}{r^2} e^{2\delta} dt^2 + \frac{r^2}{\Delta} dr^2 + r^2 d\Omega^2 \tag{102}$$

where Δ and δ are functions of r and t. After inserting the metric and the other field ansatz (66) and (67) into the Einstein field equations, one obtains

$$(\Delta e^{\delta} f')' - \frac{\lambda^2}{r^2} e^{\delta} f = -\frac{2r^4}{\Delta^3} e^{-\delta} \dot{\Delta} \dot{f} - \frac{r^4}{\Delta} e^{-\delta} \dot{\delta} \dot{f} + \frac{r^4}{\Delta^2} e^{-\delta} \ddot{f} \tag{103}$$

$$\delta' = \mu^2 r \left(\frac{r^4}{\Delta} e^{-2\delta} \dot{f}^2 + f'^2 \right) \tag{104}$$

$$e^{-\delta} \left(\frac{\Delta e^{\delta}}{r} \right)' = 1 - \frac{\mu^2 \lambda^2}{r^2} f^2 - \frac{Gp^2}{r^2} \tag{105}$$

where μ and λ are constants,

$$\mu^2 = \pi F_\pi^2 G \,, \quad \lambda^2 = \frac{e^4 p^2}{4\pi^3 F_\pi^2} \,. \tag{106}$$

The electric charge within a sphere of radius r is given by equation (74).

For very small μ, which is the case for pions, the chiral field has little effect on the background metric and we may take $\delta = 0$ and express Δ in terms of the mass M, electric charge Q and magnetic charge p of the black hole as

$$\Delta = r^2 - 2GMr + G(Q^2 + p^2) \,. \tag{107}$$

The Skyrme field equation (103) on this background is therefore

$$(\Delta f')' - \frac{\lambda^2}{r^2} f = 0\,. \tag{108}$$

This should be solved subject to the boundary condition on f_∞ which fixes the total charge,

$$q_\infty = \frac{e^2 p}{2\pi} f_\infty = n_B e\,. \tag{109}$$

The non-extremal black hole possesses two horizons at $r = r_-$ and $r = r_+$ ($r_+ > r_-$), related to the mass and charge by

$$GM = \frac{1}{2}(r_- + r_+)\,, \quad GQ^2 = r_- r_+ - Gp^2\,. \tag{110}$$

The solution to Eq. (108) can be obtained analytically,

$$f = 2\pi n_B \frac{P_q((r_+ + r_-)/(r_+ - r_-) - [2r_+ r_-/(r_+ - r_-)]/r)}{P_s((r_+ + r_-)/(r_+ - r_-))} \tag{111}$$

where $P_s(z)$ is a Legendre function and

$$s = -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{\lambda^2}{r_+ r_-}}\,. \tag{112}$$

The black hole become a dyon with electric charge related to the value of f at the event horizon,

$$Q = \frac{n_B e}{P_s((r_+ + r_-)/(r_+ - r_-))}\,. \tag{113}$$

This relation can be solved, together with (110), to obtain $Q \equiv Q(M)$, showing the existence of a one-parameter family of solutions (n_B and p being regarded as fixed). In particular, $Q \to 0$ as M approaches the extremal limit pM_{pl} and the meson field is expelled from the hole.

Larger values of μ may be realized for a hypothetical model where U is unrelated to pions, and this is discussed below. We will consider a static solution. We can replace Δ by a mass function $m(r)$, defined implicitly by the relation

$$\Delta = r^2 - 2Gmr + G(p^2 + q^2) \tag{114}$$

where the charge is given by equation (74). The static equations become

$$m' = \frac{\Delta}{2Gr}\delta' + \mu^2\lambda^2 f f' \tag{115}$$

$$\delta' = \mu^2 r (f')^2 \tag{116}$$

$$f'' + \left(\frac{\Delta'}{\Delta} + \delta'\right) f' - \frac{\lambda^2}{r^2\Delta} f = 0\,. \tag{117}$$

Suitable boundary conditions are $f \to 2\pi$ and $\delta \to 0$ as $r \to \infty$.

In the numerical results the fields are scaled to the horizon size,

$$\hat{r} = \frac{r}{r_+}\,, \quad \hat{m} = \frac{Gm}{r_+}\,. \tag{118}$$

The solutions are parameterized by a parameter $\hat{p}$, defined by

$$\hat{p}^2 = \frac{Gp^2}{r_+^2} \tag{119}$$

which is restricted to $\hat{p} \leqslant 1$.

The extremal black hole solutions have $\Delta_+ = \Delta'_+ = 0$. The regular solution to equation (117) has

$$f_+ = 0\,, \quad Q = \frac{e}{2\pi} f_+ = 0\,, \quad \hat{p} = 1\,. \tag{120}$$

Hence the proton lies fully outside the black hole, as we saw before. The numerical solution for f is shown in Fig. 4. This agrees with well with the result (87), because the value of μ used here is still quite small. The results are still qualitatively similar for chiral models with μ of order one.

For the non-extremal solution, we begin the integration of the field equations close to the horizon, with

$$\hat{m} = \hat{m}_0 + \hat{m}_1(\hat{r}-1) + \hat{m}_2(\hat{r}-1)^2 + \cdots \tag{121}$$

$$\delta = \delta_+ + \delta_1(\hat{r}-1) + \cdots \tag{122}$$

$$f = f_+ + f_1(\hat{r}-1) + \cdots \tag{123}$$

where δ_+ and f_+ are shooting parameters determined so as to satisfy the boundary conditions at infinity.

As can be seen from the above expansion, the Skyrme field must have a nonzero value at the horizon otherwise the only allowed solution is the trivial one. Consequently the nonextremal black hole acquires an electric charge

$$Q = \frac{\mathrm{e}}{2\pi} f_+, \tag{124}$$

and allows the skyrmion to have fractional electric charge. The numerical results for this solution are shown in Figs. 5 and 6. Again, these agree well with the fixed background for small values of μ.

We have a single one parameter family of solutions with $\hat{p} \leq 1$. In Figs. 7 and 8, we plot the horizon radius r_+ and skyrmion mass m_f as functions of black hole mass M. Figure 7 is related to the entropy of the black hole. The entropy of the black hole can be defined in the form

$$S = \frac{1}{4G} \mathcal{A}_{bh} \tag{125}$$

where $\mathcal{A}_{bh}$ is the area of black holes, which is (πr_+^2). The research in the entropy of black holes has revealed that the entropy of an extremal Reissner-Nordstrom black hole is zero, despite its finite size and it cannot be reached from the non-extremal state [44]. Thus the extremal black hole is a thermodynamically different object from the non-extremal one.

The other figure shows how the proportion of the skyrmion which is swallowed by the black hole increases with the black hole mass.

Figure 9 shows how the horizon value of f changes as the coupling constant μ changes. As can be seen from the figure, for small μ the electromagnetic interaction is dominant so that the skyrmion is absorbed by the black hole to a lesser extent. On the other hand for large μ the gravitational interaction is dominant so that most of the skyrmion is absorbed by the black hole.

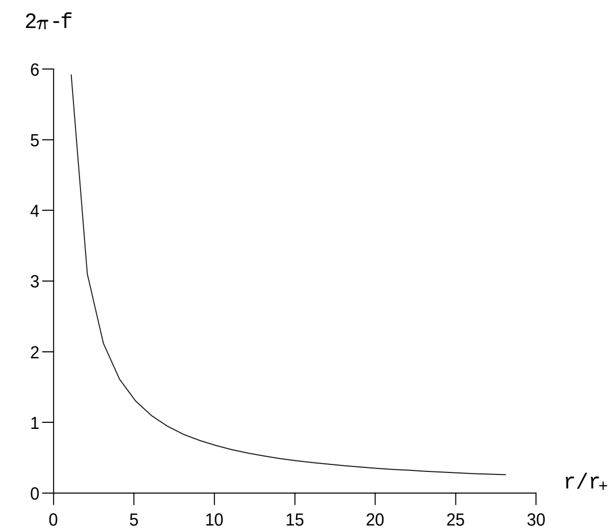

Figure 4: Profile function f as a function of $\hat{r} = r/r_+$ for an extremal hole and $\mu = 10^{-4}$.

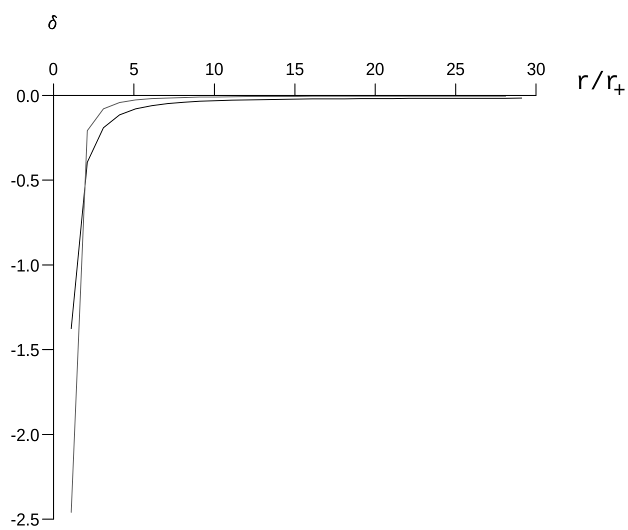

Figure 5: Backreaction δ $(\times 10^3)$ as a function of $\hat{r} = r/r_+$ for a non-extremal black hole, $\hat{p} = pm_{pl}/r_+ = 0.9$. Results for $\mu = 10^{-3}$ and $\mu = 10^{-4}$ are shown.

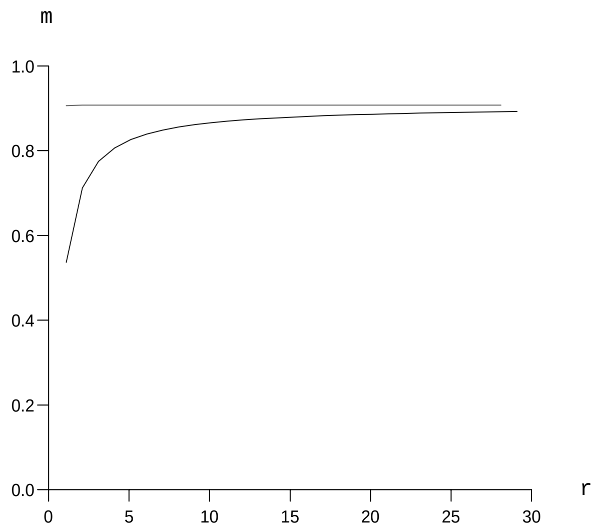

Figure 6: Mass function m as a function of $\hat{r}$ for $\hat{p} = 0.9$. Results for $\mu = 10^{-3}$ and $\mu = 10^{-4}$ are shown

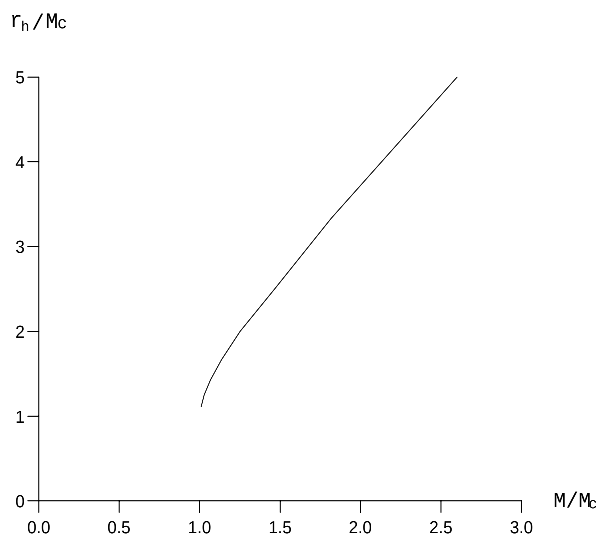

Figure 7: Horizon radius r_+/M_c as a function of the black hole mass M/M_c for $\mu = 10^{-4}$.

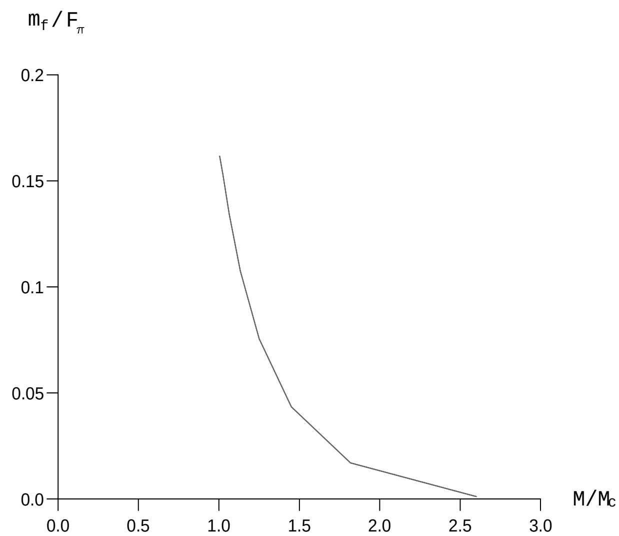

Figure 8: Skyrmion mass m_f as a function of the black hole mass M/M_c for $\mu = 10^{-4}$.

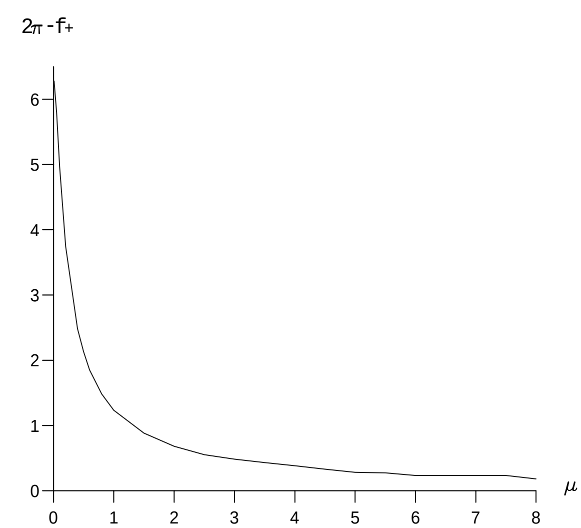

Figure 9: The value of f at the horizon for various values of μ.

4.2 Stability Analysis

In this section we show that the skyrmion solutions which we have obtained are stable under spherically symmetric linear perturbations. We shall begin with the analysis of a skyrmion on the fixed background.

In the fixed-background case the skyrme field is the only perturbed field and can be expanded about the skyrmion solution f_0 by writing

$$f(r,t) = f_0(r) + e^{i\omega t}\xi(r). \tag{126}$$

Equation (126) is inserted into equation (103) with $\delta = 0$ to obtain the eigenvalue equation

$$-\left(\Delta_0 \xi'\right)' + \frac{\lambda^2}{r^2}\xi = \frac{r^4}{\Delta_0}\omega^2\xi, \tag{127}$$

where the background equation has been used.

If ω is real and $\omega^2 > 0$ the solution is stable, and if ω is imaginary and $\omega^2 < 0$ it is unstable since the mode can grow or decay exponentially under the small perturbation. To show which is the case we multiply both sides of equation (127) by ξ and integrate in r from the horizon to infinity

$$\int_{r_+}^{\infty} \left[\frac{\Delta_0}{2}\xi'^2 + \frac{\lambda^2}{r^2}\xi^2\right] dr = \omega^2 \int_{r_+}^{\infty} \frac{r^4}{\Delta_0}\xi^2 \, dr \ , \tag{128}$$

where integration by parts and boundary conditions have been used. It can be seen that the integrands of both sides are positive definite, which means that $\omega^2 > 0$. Hence the skyrmion on the fixed background is linearly stable .

Next we analyze the stability of the skyrmion with backreaction. In this case we have to expand the metric as well as the skyrme field around the classical solutions f_0, δ_0 and Δ_0

$$\begin{aligned} f(r,t) &= f_0(r) + f_1(r,t) \\ \delta(r,t) &= \delta_0 + \delta_1(r,t) \\ \Delta(r,t) &= \Delta_0 + \Delta_1(r,t) \ . \end{aligned}$$

These are substituted into Eqs. (103)-(105) to obtain the following coupled equations up to first order

$$\left[\left(\Delta_0\delta_1 f_0' + \Delta_0 f_1' + \Delta_1 f_0'\right) e^{\delta_0}\right]' - \frac{\lambda^2}{r^2}\left(\delta_1 f_0 + f_1\right) e^{\delta_0} = \frac{r^4}{\Delta_0} e^{-\delta_0}\ddot{f}_1 \tag{129}$$

$$\delta_1' = 2\mu^2 r f_0' f_1' \tag{130}$$

$$\left(\frac{2\mu^2\lambda^2}{r^2} f_0 f_1 + \frac{\Delta_0}{r}\delta_1'\right) e^{\delta_0} = -\left(\frac{\Delta_1}{r} e^{\delta_0}\right)'. \tag{131}$$

Equation (131) can be integrated with the help of the static field equation,

$$\Delta_1 = -2\mu^2 r \Delta_0 f_0' f_1 \ . \tag{132}$$

Substituting Eqs. (130) and (132) into Eq. (129) one obtains the first order equation for f_1

$$\left(\Delta_0 e^{\delta_0} f_1'\right)' - \left[2\mu^2 \left(r\Delta_0 e^{\delta_0} f_0'^2\right)' + \frac{\lambda^2 (M_c)^2}{r^2} e^{\delta_0}\right] f_1 = \frac{r^4}{\Delta_0} e^{-\delta_0} \ddot{f}_1 \,. \tag{133}$$

Setting $f_1(r,t) = \xi(r) e^{i\omega t}$ one obtains an eigenvalue equation for ξ,

$$-\left(\Delta_0 e^{\delta_0} \xi'\right)' + \left[2\mu^2 \left(r\Delta_0 e^{\delta_0} f_0'^2\right)' + \frac{\lambda^2 (M_c)^2}{r^2} e^{\delta_0}\right] \xi = \omega^2 \frac{r^4}{\Delta_0} e^{-\delta_0} \xi \,. \tag{134}$$

We introduce the tortoise coordinate r^* such that

$$\frac{dr^*}{dr} = \frac{1}{\Delta_0 e^{\delta_0}} \tag{135}$$

and r^* runs from $-\infty$ to $+\infty$ as r runs from r_+ to $+\infty$. Then Eq. (134) is reduced to the Sturm-Liouville equation

$$-\frac{d^2\xi}{dr^{*2}} + U\xi = \omega^2 r^4 \xi, \tag{136}$$

where

$$U = \left[2\mu^2 \left(r\Delta_0 e^{\delta_0} f_0'^2\right)' + \frac{\lambda^2}{r^2} e^{\delta_0}\right] \Delta_0 e^{\delta_0}. \tag{137}$$

On the left-hand side we have r^4, which makes the equation different from the previous eigenvalue equation with the fixed background. However, since r^4 remains positive through the whole space, the same conditions for stability as the ordinary eigenvalue equation can be applied. As can be seen by examining U, $U \to 0$ as $r \to r_+$, (i.e. $r^* \to -\infty$), $U \to U_\infty > 0$ as $r \to \infty$, and $U > 0$ in between. In addition the solution does not change its shape for any value of the coupling constant μ. Therefore we can safely conclude that a skyrmion with backreaction is also linearly stable.

When the solutions are stable, proton decay can only take place when extra particle fields are included in the model to carry the electric charge away. Since the underlying $SU(5)$ theory does not require lepton charge conservation, it should satisfy $\Delta B = \Delta L$. The configuration we concern here is spherically symmetric and thus the theory is reduced to one space and one time dimension. In this case the bosonization technique can be applied in order to include leptons [45, 46]. According the technique, we simply replace fermionic current to a real scalar field as

$$j^\mu = \bar{\chi}(r,t)\gamma^\mu \chi(r,t) \to -\frac{1}{\sqrt{\pi}} \epsilon^{\mu\nu} \partial_\nu \phi\,(r,t)\,. \tag{138}$$

Then following the procedure described in Ref. [9], we obtain the equations for the fixed background (approximately)

$$\left(\Delta f'\right)' - \frac{\lambda^2}{r^2}(f - \phi) = -\frac{r^4}{\Delta} \ddot{f}\,, \tag{139}$$

$$\left(\Delta \phi'\right)' + \frac{\lambda^2}{r^2}(f - \phi) = -\frac{r^4}{\Delta} \ddot{\phi}\,, \tag{140}$$

The stability arguments no longer apply. The dynamical process of a black hole swallowing a proton can be examined by solving these time-dependent field equations numerically. For a flat-space time, these were solved by Chemtob in Ref. [47] where the catalysis cross section was estimated approximately to be $1\text{mb}/\beta$ with velocity β, which confirms that the monopole catalysis proceeds at a strong-interaction scale without any suppression.

5 $B = 2$ Black Hole Skyrmions

In this section black hole solutions of $B = 2$ skyrmions with axisymmetry are studied. The study by Hartmann *et al.* showed the Einstein-Yang-Mills-Higgs theory possesses axially symmetric monopole and black hole solutions [48, 49]. We follow their numerical technique to solve the Einstein-Skyrme model with axisymmetry. The obtained solution exhibits a torus shape in the energy and baryon density with a black hole at the center. Similarly to the case of $B = 1$, the baryon number is not conserved and takes fractional values outside the horizon.

Recent studies of theories with large extra dimensions indicate that a true Planck scale is of order a TeV and the production rate of black holes massive than the Planck scale become quite large [50, 51]. Therefore, if a skyrmion represents a proton, this kind of black hole may be created in p-p collisions at the LHC in future.

This work was carried out with the collaboration of T. Torii and K. Maeda.

5.1 Axially Symmetric Configurations

The Lagrangian for the Einstein-Skyrme system has been already given in Eq. (21). Since the $B = 2$ skyrmion is axially symmetric, we impose axially symmetric ansatz for the metric and chiral fields. We use the ansatz for the metric given in Ref. [48]

$$ds^2 = -f dt^2 + \frac{m}{f}(dr^2 + r^2 d\theta^2) + \frac{l}{f} r^2 \sin^2\theta d\varphi^2 \tag{141}$$

where $f = f(r,\theta),\ m = m(r,\theta),$ and $l = l(r,\theta)$.

The axially symmetric chiral fields can be parameterized by

$$U = \cos F(r,\theta) + i\vec{\tau}\cdot\vec{n}_R \sin F(r,\theta) \tag{142}$$

with $\vec{n}_R = (\sin\Theta\cos n\varphi, \sin\Theta\sin n\varphi, \cos\Theta)$ and $\Theta = \Theta(r,\theta)$. The integer n corresponds to the winding number of the Skyrme fields and for $B = 2$ we have $n = 2$.

In terms of F and Θ, the Lagrangian takes the form

$$\mathcal{L}_S = \mathcal{L}_S^{(1)} + \mathcal{L}_S^{(2)} \tag{143}$$

where

$$\begin{aligned}
\mathcal{L}_S^{(1)} &= -\frac{F_\pi^2}{8}\frac{f}{m}\left[(\partial_r F)^2 + \frac{1}{r^2}(\partial_\theta F)^2 + \left\{(\partial_r\Theta)^2 + \frac{1}{r^2}(\partial_\theta\Theta)^2\right\}\sin^2 F \right.\\
&\quad \left. + \frac{n^2}{r^2\sin^2\theta}\frac{m}{l}\sin^2\Theta\sin^2 F\right]\\
\mathcal{L}_S^{(2)} &= -\frac{1}{2a^2r^2}\left(\frac{f}{m}\right)^2\left[\left(\partial_{[r}F\partial_{\theta]}\Theta\right)^2 + \frac{n^2}{\sin^2\theta}\frac{m}{l}\left\{(\partial_r F)^2 + \frac{1}{r^2}(\partial_\theta F)^2\right\}\sin^2\Theta \right.\\
&\quad \left. + \frac{n^2}{\sin^2\theta}\frac{m}{l}\left\{(\partial_r\Theta)^2 + \frac{1}{r^2}(\partial_\theta\Theta)^2\right\}\sin^2 F\sin^2\Theta\right]\sin^2 F
\end{aligned}$$

and we have defined the notation

$$\partial_{[r}F\partial_{\theta]}\Theta = \partial_r F \partial_\theta \Theta - \partial_\theta F \partial_r \Theta \,. \tag{144}$$

The baryon current in curved spacetime is defined in Eq. (25). The baryon number is then given by integrating B^0 over the hypersurface $t = 0$,

$$\begin{aligned} B &= \int dr d\theta d\varphi \sqrt{g^{(3)}}\, B^0 \\ &= -\frac{1}{\pi}\int dr d\theta \, (\partial_{[r}F\partial_{\theta]}\Theta)\sin\Theta(1-\cos 2F) \\ &= -\frac{1}{\pi}\int dF d\Theta \sin\Theta(1-\cos 2F) \\ &= \frac{1}{2\pi}(2F - \sin 2F)\cos\Theta \Big|_{F_0,\Theta_0}^{F_1,\Theta_1} \end{aligned} \tag{145}$$

where (F_0, Θ_0) and (F_1, Θ_1) are the values at the inner and outer boundary respectively. In flat spacetime we have

$$(F_0, \Theta_0) = (\pi, 0) \;\; \text{and} \;\; (F_1, \Theta_1) = (0, \pi),$$

which gives $B = 2$. In the presence of a black hole, the integration should be performed from the horizon to infinity, which change the values of F_0 and allow the B to take fractional values of less than two. This situation can be interpreted as the black hole absorbing a skyrmion as was seen in the $B = 1$ case.

The energy density is given by the zero-zero component of the stress-energy tensor

$$\begin{aligned} -T_0^0 &= \frac{F_\pi^2}{8}\frac{f}{m}\left[(\partial_r F)^2 + \frac{1}{r^2}(\partial_\theta F)^2 + \left\{(\partial_r\Theta)^2 + \frac{1}{r^2}(\partial_\theta\Theta)^2\right\}\sin^2 F \right. \\ &\left. + \frac{n^2}{r^2\sin^2\theta}\frac{m}{l}\sin^2 F\sin^2\Theta\right] + \frac{1}{2a^2r^2}\frac{f^2}{m^2}\Big[(\partial_{[r}F\partial_{\theta]}\Theta)^2 \\ &+ \frac{n^2}{\sin^2\theta}\frac{m}{l}\left\{(\partial_r F)^2 + \frac{1}{r^2}(\partial_\theta F)^2\right\}\sin^2\Theta \\ &+ \frac{n^2}{\sin^2\theta}\frac{m}{l}\left\{(\partial_r\Theta)^2 + \frac{1}{r^2}(\partial_\theta\Theta)^2\right\}\sin^2 F\sin^2\Theta\Big]\sin^2 F \,. \end{aligned} \tag{146}$$

5.2 Boundary Conditions

Let us consider the boundary conditions for the chiral fields and metric functions. At the horizon $r = r_h$, the zero-zero component of the metric satisfies

$$g_{00} = -f(r_h, \theta) = 0 \,. \tag{147}$$

Regularity of the metric at the horizon requires

$$m\left(r_h,\theta\right)=l\left(r_h,\theta\right)=0\,. \tag{148}$$

The boundary conditions for $F(r,\theta)$ and $\Theta(r,\theta)$ at the horizon are obtained by expanding them at the horizon and inserting into the field equations which are derived from $\delta\mathcal{L}_S/\delta F=0$ and $\delta\mathcal{L}_S/\delta\Theta=0$ respectively. Consequently we obtain

$$\partial_r F\left(r_h,\theta\right)=\partial_r\Theta\left(r_h,\theta\right)=0\,. \tag{149}$$

The condition that the spacetime is asymptotically flat requires

$$f\left(\infty,\theta\right)=m\left(\infty,\theta\right)=l\left(\infty,\theta\right)=1\,. \tag{150}$$

The boundary conditions for F and Θ at infinity remain the same as in flat spacetime

$$F\left(\infty,\theta\right)=0\ ,\qquad \partial_r\Theta\left(\infty,\theta\right)=0\,. \tag{151}$$

For the solution to be axially symmetric, we have

$$\partial_\theta f\left(r,0\right)=\partial_\theta m\left(r,0\right)=\partial_\theta l\left(r,0\right)=0, \tag{152}$$

$$\partial_\theta f\left(r,\frac{\pi}{2}\right)=\partial_\theta m\left(r,\frac{\pi}{2}\right)=\partial_\theta l\left(r,\frac{\pi}{2}\right)=0. \tag{153}$$

Regularity on the axis and axisymmetry impose the boundary conditions on F and Θ as

$$\partial_\theta F\left(r,0\right)=\partial_\theta F\left(r,\frac{\pi}{2}\right)=0, \tag{154}$$

$$\Theta\left(r,0\right)=0,\ \Theta\left(r,\frac{\pi}{2}\right)=\frac{\pi}{2}. \tag{155}$$

Under these boundary conditions, we shall solve the Einstein equations and the matter field equations numerically.

5.3 Numerical Results

Let us introduce dimensionless coordinate and coupling constant

$$x=aF_\pi r\ ,\qquad \alpha=\pi G f_\pi^2\,.$$

Then the free parameters are the horizon x_h and the coupling constant α for this system. We shall take $\alpha=0$ as decoupling of gravity from the matter, effectively $G=0$.

In Figs. 10, 11 are the energy densities of the black hole solutions with $\alpha=0.0$, 1.5 respectively. As α becomes larger, the energy density becomes smaller and sparse. This can be interpreted that the black hole absorbs more skyrmions for a larger coupling constant. The shape is slightly distorted in the background of the

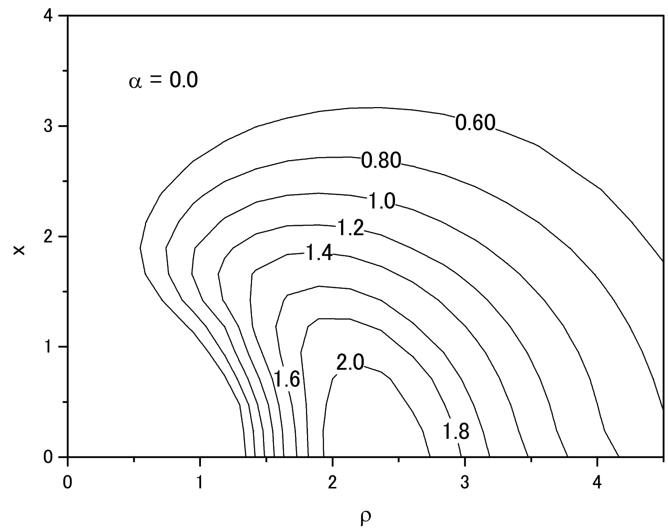

Figure 10: The energy density ϵ in cylindrical coordinates ρ and z with $x_h = 1.0, \alpha = 0.0$.

black hole so that one can see the spherically symmetric horizon at the center of the skyrmion. Fig. 12 is the baryon density around the black hole. The dependence of the baryon density on the value of the coupling constant is small. It can be checked that the energy and baryon density vanish at $\rho = 0$. Inserting the metric functions as well as the profile functions expanded around the horizon instead, one can also see that the energy and baryon density vanish at the horizon.

The domain of existence of the black hole solution is shown in Fig. 13. There exist minimum and maximum value of x_h and α beyond which no black hole solutions exist. Therefore the regular skyrmion solutions can not be recovered from the black hole solutions by taking the limit of $x_h \to 0$ unlike the case of $B = 1$ [17]. In Fig. 14 is the dependence of the baryon number on x_h. One can see that the baryon number decreases as the black hole grows in size. This figure confirms that the baryon number is no longer conserved due to the black hole absorbing the skyrmion.

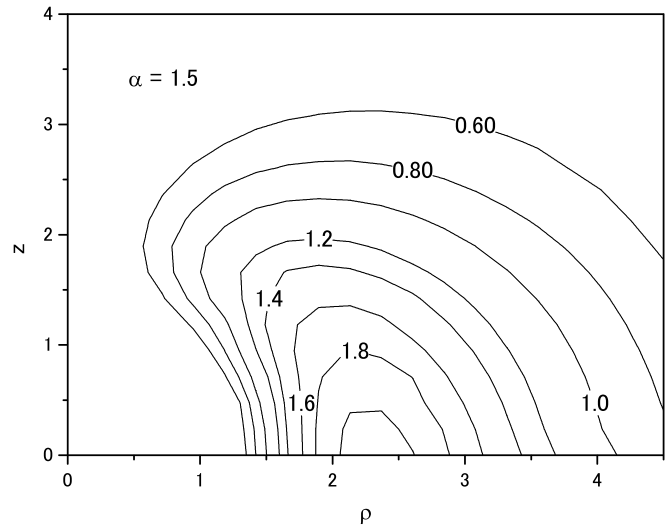

Figure 11: The energy density ϵ in cylindrical coordinates ρ and z with $x_h = 1.0, \alpha = 1.5$.

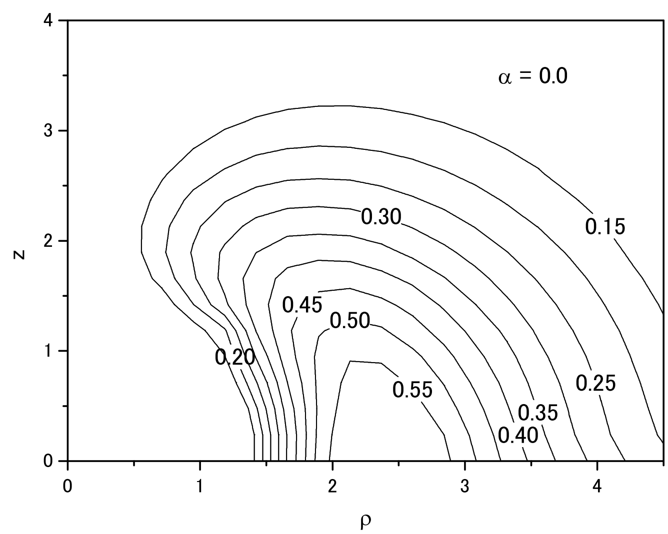

Figure 12: The baryon density b with $x_h = 1.0, \alpha = 0.0$.

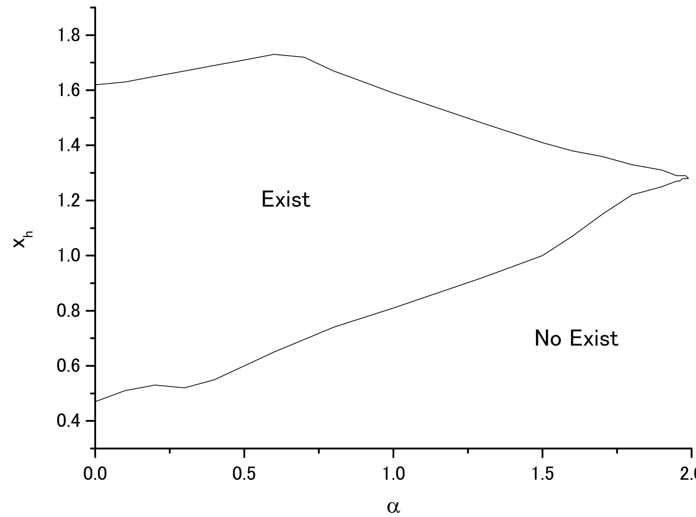

Figure 13: The domain of existence of the solution. For $\alpha \gtrsim 2.0$, there exists no non-trivial solution.

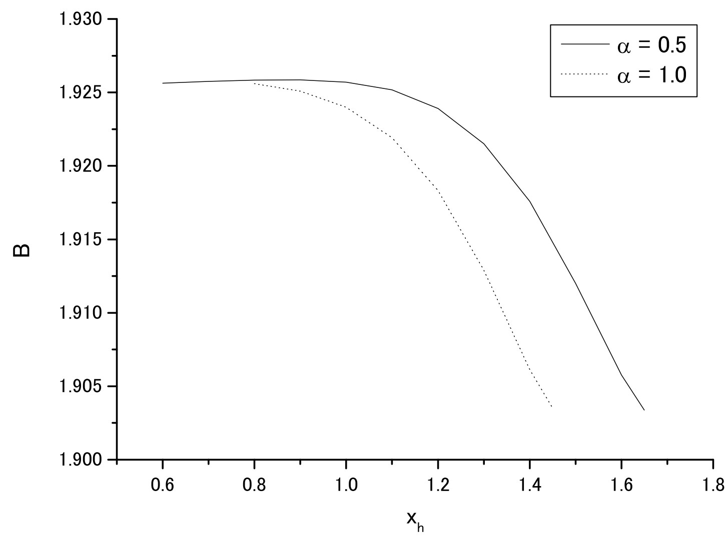

Figure 14: The dependence of the baryon number on the size of the horizon.

6 Conclusion and Discussion

In this paper black hole solutions with Skyrme hair are reviewed. Sec. 3 is devoted to the study of the spherically symmetric black hole with $B = 1$ Skyrme hair and its stability analysis. In Sec. 4 the gauged Einstein-Skyrme system is constructed and the monopole black hole with $B = 1$ Skyrme hair is obtained. The extended Einstein-Skyrme system to $B = 2$ axially symmetric configuration is studied in Sec. 5. The black hole solution with $B = 2$ Skyrme hair is obtained and it is shown that the energy and baryon density exhibit a torus shape with the spherically symmetric horizon at the center.

The common feature in those solutions is that they can support fractional baryonic charges outside the horizon, violating baryon number conservation. The study of the monopole black hole skyrmion shows that although global charge conservation such as baryon number is violated, gauge charge conservation such as electric or magnetic charge is still hold. It is remarkable that the black hole and monopole black hole solution with $B = 1$ Skyrme hair turned out to be stable under linear perturbations even with the non-integer baryonic charge. Obviously it is important to study the stability of the $B = 2$ solutions. We expect that the stability analysis may be performed by applying the catastrophe theory for black holes with non-linear hair [52].

For these microscopic black holes, however, we cannot ignore the quantum effects. In fact, they are stable only classically and will decay to the unstable solutions due to quantum transitions. Besides black holes of the size of a proton should have large fluxes of Hawking radiation [20]. Hence situations in which baryon decay process might become more realistic and significant over the Hawking radiation occurs only when the black hole carries electric or/and magnetic charge with which the skyrmion interacts electromagnetically as well as gravitationally. Especially interesting is the extremal black hole which has a vanishing effective temperature, so the Hawking radiation may even vanish. The free skyrmion has a magnetic moment and, if it has the correct orientation, it will be attracted to the monopole. When the proton approaches the black hole monopole, the fields rearrange themselves into the energetically preferred configuration of skyrmion hair solutions described in Sec. 4. For the stable solutions, baryon decay can only takes place when extra particle fields are included. We have given a rigorous argument to study dynamical proton decay to a lepton by a monopole black hole. The bosonization technique is conveniently introduced and the system is reduced to two coupled time-dependent field equations. Solving these equations with appropriate boundary conditions at the horizon will be our future work.

Recently we performed collective quantization of a $B = 1$ gravitating skyrmion and calculated various nucleon observables under the gravitational effects in Ref. [53]. It will be interesting to extend the work to the black hole skyrmion. Also interesting but hard is to consider black holes with $B \geq 3$ Skyrme hair which has discrete symmetries. The rational map ansatz will be a powerful tool for obtaining multi-skyrmion black hole solutions. The inclusion of gauge fields may also be possible to study the interaction between a monopole black hole and multi-skyrmion. Extending the model to higher dimensions will be also an exciting problem. It is known that multi-extremal black hole solutions can be generalized to p-brane objects by coupling to an antisymmetric tensor A_{p+1} with $p+1$ indices. In brane theory context, Skyrme fields are axions interacting with branes. The generalization of our model to brane theory will be worth studying in future.

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