Modified Maxwell-slip Model of Presliding Friction

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Abstract: The distributed Maxwell-slip model provides a convenient way to describe the presliding friction behavior. The modified single-state Maxwell-slip (MMS) model is proposed with the main benefit to require two concentrated parameters only when describing the smooth hysteresis of the presliding friction. The model is rate-independent at both, saturated and unsaturated hysteresis, and is consistent with the generalized empirical friction model structure. Some novel perceptions on the frictional memory and drift are considered and proved in experiments. The evaluation performed on an actuator system with multiple coupled frictional surfaces reveals the proposed model as easily identifiable and accurate in prediction.

Keywords: friction, hysteresis, nonlinearity, damping, nonlinear models

1. INTRODUCTION

The attention paid to the friction modeling and compensation is substantial. The dynamic friction with its miscellaneous complex characteristics appears to be inherently difficult to describe in a simple and accurate way, despite that multiple control-oriented friction models are available up to date. A former overview of modeling and compensation approaches developed to master the friction in machines is given by Armstrong-Helouvrty et al. [1994]. The recent detailed analysis of the modeled friction dynamics and its properties provided by Armstrong-Helouvrty and Chen [2008] and by Al-Bender and Swevers [2008] mention several advanced, empirically motivated friction models widespread in the system and control community. Among, are Dahl (see Dahl [1968]), LuGre described and revisited by Canudas de Wit et al. [1995] and Aström and Canudas de Wit [2008], Leuven (Swevers et al. [2000]), single-state elastoplastic (Dupont et al. [2002]), and Generalized Maxwell-Slip (Al-Bender et al. [2005]) friction models. The novel two-state dynamic friction model with elastoplasticity was recently proposed by Ruderman and Bertram [2010] and further evaluated on two different experimental systems (Ruderman et al. [2010]). Multiple aspects of the frictional impact on the control performance can be found in the literature, e.g. by Hensen et al. [2003] and Putra et al. [2007], in terms of the induced limit cycles and overcompensated friction correspondingly.

This paper contributes towards better understanding and describing the presliding friction phenomenon and introduces the modified single-state Maxwell-slip (MMS) model with two concentrated parameters. The comparison of the proposed MMS model is done considering as reference the standard approach of distributed Maxwell-slip elements connected in parallel which is widely used when describing the pre-sliding friction. Beyond that, the general aspects of the presliding friction behavior are reviewed and some novel perceptions on the frictional memory and drift are provided and discussed based on experimental observations. The model evaluation performed on an actuator system with multiple coupled frictional surfaces approves its suitability and ease of identification.

2. PRESLIDING FRICTION MODELING

2.1 Presliding Hysteresis

In the presliding regime, in which the adhesion and deformation forces that arise due to an interaction of surface asperities predominate, the friction depends on the relative displacement rather than relative velocity. The most notable frictional phenomenon during presliding is the position dependent hysteresis. The reason for its occurrence is that the asperity junctions deform elasto-plastically, thus behaving as nonlinear hysteretic springs (see e.g. Al-Bender and Swevers [2008]). As the relative displacement increases and the elasto-plastic deflection of interacting asperities saturates the contacting surfaces slide on each other until a motion reversal. At one closed motion cycle the asperities dissipate their total elastic energy, thus giving rise to hysteresis losses. The shape of the presliding hysteresis depends on the material and surface properties, e.g. roughness, average height and distribution of asperities, tangential and normal stiffness, as well as the used lubrication medium. An accurate modeling of the presliding friction requires an explicit consideration of basic material/surface properties, e.g. using Finite Element Methods. However, this is rather unfeasible for the fast simulation and control purposes. For that reason an empirically motivated friction modeling pursues the objective to find a possibly compact analytic form which maps the observed frictional phenomena, without lose the accuracy and generality at the same time. At this, the approximated presliding friction hysteresis can be described as rate-dependent or rate-independent and exhibit a drifting or non-drifting effect. For more details see the
Z-properties chart proposed by Armstrong-Helouvry and Chen [2008]. Apart from the Maxwell-slip (MS) formalism, first proposed by Iwan [1966] and the recent MMS approach multiple modeling strategies can be applied, in order to capture the presliding hysteresis. The Dahl, LuGre, and friction model with elasto-plasticity (Ruderman and Bertram [2010]) employ single-state nonlinear differential equations so as to saturate the presliding friction state towards some predefined steady-state level. This type of the modeling hysteresis does not provide the nonlocal memory and exhibits consistently the drifting effect. Being mostly compact in parametrization the implemented differential-based hysteresis models can implicate instabilities by an increasing input velocity (see e.g. by Lu et al. [2009]). The reason for this is a non-saturated dynamic state after pre-sliding, so that the velocity-dependent state equation can lead to certain divergence depending on the applied integrators and numerical solver type. Alternatively, the Preischach-type hysteresis models (see e.g. by Brokate and Sprekels [1996]) allow to map arbitrary hysteresis functions with nonlocal memory, though at price of a large number of spatial distributed parameters. The latest one impedes the control-oriented implementation and identifiability of this type of hysteresis models.

The presliding friction hysteresis behaves as rate-independent. That is the periodic input-output hysteresis map provides the same shape for all input frequencies.

\[ F_i = \begin{cases} k_i \dot{x}, & \text{if } |F_i| < B_i \\ 0, & \text{else} \end{cases} \]

According to superposition of single Maxwell elements the total friction force is given by

\[ F = \sum_{i=1}^{L} F_i. \]

The smoothness of the friction force transition after each motion reversal depends on the number of Maxwell-slip elements assumed in the entire model. Being a piece-wise linear approximation of a hysteresis function the Maxwell-slip friction model is more accurate, more elements TAR Maxwell slip elements are assumed. On the other hand, an increasing number of distributed elements implicate a non-uniqueness of the parameter solution by identification. The MS model with more than one element offers the nonlocal memory characteristic. The latest one means that an outer hysteresis loop follows its previously trajectory since an internal hysteresis loop is closed at the same point at which the outer loop was quit. Figure 2 illustrates the presliding hysteresis with nonlocal memory described by means of the MS model with \( L = 5 \). The corresponding "zig-zag"-shaped position profile with a set of alternating local minima and maxima offers a characteristic reversion point \( X_m \) as shown in Fig. 2. At this particularly shown position the actual hysteresis state is memorized for three times – twice at \( F_m'' \), and once at \( F_m' \). It is obvious, that each of three minor hysteresis loops closes up at the same operating point as it sheers out. Well understandable, the nonlocal memory effect arises from the distributed MS model structure, wherever each singe element offers its proper state as being either sticking or slipping.

2.2 Maxwell-slip Approach

The Maxwell-slip approach for representing hysteresis functions consists in a parallel connection of \( L \) elasto-plastic Maxwell (also known as Jenkin) elements subject to the common input velocity \( \dot{x} \) (see Fig. 1 on the left). Each element \( i \) is characterized by its stiffness \( k_i \) and breakaway condition \( B_i \) which, in simplest case, reflects the constant stiction force. The element remains sticking until the spring deflection force \( F_i \) exceeds the breakaway condition upon which the element begins to slide. As each single element is assumed to be massless it offers a static relationship between the spring deflection and the resulting force. Once the element begins to slide the deflection force remains constant until the motion changes the direction. The state representation of a single Maxwell element can be written as

\[ \dot{x}_i = \begin{cases} k_i \ddot{x}, & \text{if } |F_i| < B_i \\ 0, & \text{else} \end{cases} \]

Fig. 2. Presliding hysteresis with nonlocal memory

The novel presliding friction model we propose in this work requires several assumptions to be made, in order to be conform to existing friction models and do not violate the main well-established friction properties.

**Assumption 1.** The presliding friction hysteresis behaves as rate-independent. That is the periodic input-output hysteresis map provides the same shape for all input frequencies.
The rate-independent property of the presliding friction rests upon former works of considering the geometrical deformation of asperities on the near-surface of contacting materials (see e.g. Al-Bender et al. [2004] for details). As inertia forces at the macroscopic level can be neglected during presliding, the nonlinear friction map \( f : x \mapsto F \) is invariant with respect to affine transformations on the time scale. That is the input-output pair \((\dot{x}(t), F(t))\) is equivalent to that one \((\dot{x}(a + b), F(a + b))\) for all \( a \in \mathbb{R} \) and \( b \in \mathbb{R}^+ \). As summarized by Armstrong-Helouvry and Chen [2008] some dynamic friction models, e.g. Dahl and MS one, provide the rate-independent, and some of them, e.g. LuGre, Leuven, single-state elastoplastic and GMS one the rate-dependent hysteresis behavior. Considering a total force balance of the moving body, we assume the rate-independent presliding friction, so that the rate-dependent force contribution will be sourced out to the lumped system inertia.

**Assumption 2.** The elasto-plastic asperity deformation saturates at the Coulomb friction level which is the upper boundary of the presliding friction force.

The considered friction behavior describes solely the presliding phase, and neither transient friction dynamic nor steady-state Stribeck and viscous friction effects are taken into account. The simple Coulomb friction law remains valid as the normal load is assumed to be constant, and no transient response in the velocity affects significantly the friction behavior. The latest one requires low acceleration and low velocity motions so as to eliminate the non-modeled transient friction phenomena.

**Assumption 3.** The adhesion coefficient which characterizes the maximal tangential force remains constant.

The adhesion coefficient, which is otherwise the function of the contact time, also known as the dwell time, determines the adhesion force that must be overcome before the moving body begins to slide. This assumption supposes a time invariant consideration of the frictional mechanisms, that is most common for control-oriented friction modeling.

The assumptions made above reduce the complex friction dynamics to a special case of the presliding friction only. At this, the proposed formulation is consistent with the generalized empirical friction model structure introduced by Al-Bender and Swevers [2008]. According to the latest one the presliding friction force is a hysteresis function of relative displacement, with nonlocal memory characteristics. The unique discrepancy with the proposed modeling approach consists in the (non)local memory effect and corresponding drift which will be further addressed in 2.4. When overall friction dynamics is required, the mentioned assumptions can be lifted without lose the model validity. Thereby, the MMS model can be easily integrated into the total friction model structure, similar or such as done by Ruderman and Bertram [2010] and Ruderman and Bertram [2011].

The introduced presliding friction model can be represented like a single Maxwell-slip element, though with a pivotal difference in the connecting spring behavior (compare in Fig. 1 on the left and right). The reversible nonlinear spring exhibits a saturating elasto-plastic deflection until the motion direction changes. Assuming the exponentially decreasing stiffness \( K(x) \) by an increasing relative displacement after reversal, the overall hysteresis force converges towards the constant Coulomb friction. The saturated instantaneous friction indicates the onset of plastic sliding, at which the deformed contact asperities provide an approximately constant level of the resistive tangential force. The corresponding friction dynamic is introduced as a first-order nonlinear differential equation

\[
\dot{F} = \Omega \dot{x} K \exp(-K|x_r|),
\]

with

\[
\Omega = \text{sgn}(\dot{x}) F_c - F
\]

when motion reversal occurs. The variable stiffness capacity \( \Omega \) of the elasto-plastic asperities interaction memorizes the last reversal state, at which the asperity junctions are released and reloaded again, though in the opposite direction. At this, the integrated relative displacement \( x_r \) is reset to zero. Two parameters only, the constant Coulomb friction \( F_c \) and the initial stiffness \( K \), determine the overall hysteresis behavior from which an exemplarily taken loop is shown in Fig. 1 lower on the right. The exponential position dependent stiffness map ensures the rate-independency of both, major (saturated) and minor (unsaturated) hysteresis loops as illustrated in Fig. 3. Here, the up-chirp position input with a frequency range 0.1–10 Hz is applied, once with the normalized amplitude \( |x|_{\text{max}} = 5 \), and once with that one \( |x|_{\text{max}} = 1 \). Note, that the time derivative of the chirp position signal provides a relative motion within a large velocity range as depicted in Fig. 3 (c), (f). The resulting position dependent hysteresis retains the equal shape independent of the applied input frequency. For the unsaturated case, the differing trajectory of the first curve in Fig. 1 (e), also known as a virgin hysteresis curve, reveals the drifting effect caused by the initial elasto-plasticity of asperity contacts.

![Fig. 3. Rate-independent saturated (a–c) and unsaturated (d–f) hysteresis behavior; (a),(d) up-chirp position input (0.1–10 Hz), (b),(e) friction force versus position, (c),(f) friction force versus relative velocity](image-url)
Let note, that a minor hysteresis loop is considered, so and no friction drift can occur during a periodic excitation. Since an increasing relative deflection force $F_{rel}(x)$ is a function of relative displacement, the mechanical work $W$ of an elasto-plastic asperity deformation is equal to the integral area, gray-shaded for the descending and diagonal-dashed for the ascending hysteresis branch. Since the same mechanical work when moving in both directions is required, no residual plastic deformation occurs on a closed $x_1$-$x_2$-$x_1$ cycle, and it can be written

$$W_{non} - W''_{non} = \int_{x_1}^{x_2} F_{rel}(x) \, dx - \int_{x_2}^{x_1} F_{rel}(x)'' \, dx = 0. \quad (5)$$

That means, the system reconstructs fully the memorized departure state that is equivalent to the nonlocal memory property. In this case, two consecutive reversal points are well symmetrical from the energy dissipation point of view, and no friction drift can occur during a periodic excitation. Let note, that a minor hysteresis loop is considered, so that the reversal point $x_2$ does not mandatory correspond to the fully saturated plastic sliding.

![Fig. 4. Nonlocal (a) and local (b) hysteresis memory](image)

Further, consider the hysteresis with local memory, at which the closed $x_1$-$x_2$-$x_1$ cycle does not return to the point of departure as depicted in Fig. 4 (b). The similar consideration of the mechanical work reveals

$$W_{loc}' - W_{loc}'' = \int_{x_1}^{x_2} F_{rel}(x)' \, dx - \int_{x_2}^{x_1} F_{rel}(x)'' \, dx = \delta, \quad (6)$$

in which the area $\delta$ corresponds to the energy unbalance due to residual plastic deformation. In other words, less mechanical work is required to bring the system back to $x_1$ after the motion reversal at $x_2$. Hence, the friction drift occurs since the onset of saturated plastic sliding is shifted on the right, behinds the $x_1$-limit of the closed position cycle. During unsaturated behavior the asperities deformed elasto-plastically can not fully realize the previously states after being reloaded in opposite direction. From the system point of view the local hysteresis memory stores the last reversal state only, whereas the previous reversal states are erased from the system memory.

The mentioned considerations of the frictional drift, which we associate with the local hysteresis memory, are affirmed to certain extent by the following experimental observations. The slow (1 Hz) sinusoidal excitation is applied to the experimental testbed (described further in 3.1), once with a zero phase and once with a paraphrase, so as to conduct the initial motion in opposite directions. At this, the excitation amplitude is set below the Coulomb friction level preestimated afore.

The measured system response depicted in Fig. 5 shows an obvious position drift in (a) and (c), and the corresponding thickening presliding hysteresis in (b) and (d). Remarkable is the fact, that the drifting appears to provide a saturating behavior that argues against the former perceptions on the friction drift phenomenon. Mostly, the friction drift was considered as a source of a continuously increasing displacement when a periodic force below the breakaway limit is applied (see e.g. by Dupont et al. [2002]). Often, the friction drift is designated as a spurious friction phenomenon (Al-Bender and Swevers [2008]). The observed saturating drift can be presumably explained by some limiting state of asperity deflections, towards which the system converges as an oscillating excitation below the Coulomb friction is applied.

Note, that the drive experiments were performed with the warmed-up system, so that the kind of thermal drifting sources can be excluded a priori. Ditto, the nearly symmetrical drift in both directions depending on the initial motion conditions argues rather for the structural than for thermal nature of the observed drifting. It appears that at the saturated limiting state the adhesion and elastic forces predominate on the contacting surfaces, and no further drift occurs under applied continuous excitation conditions.
3. EXPERIMENTAL MODEL EVALUATION

3.1 Testbed

The experimental evaluation of the proposed presliding friction model is performed on an electro-mechanical actuator system composed of a BLDC (brushless direct current) motor and a high reduction gear. A schematic representation of the experimental testbed is depicted in Fig. 6. No significant additional elasticities are assumed to be present in the system, so that the total system inertia \( J \) is composed as a lumped rotationally symmetrical load. The specific mechanical assembly provides multiple coupled frictional surfaces \( f_1-f_5 \), from which \( f_4 \) represents the teeth interaction of the gear, and the residual ones are of the ball bearing type. The integrated low-level fast current control of the motor reveals a nearly reactionless power source which allows to neglect the overall control dynamics. The angular velocity and the angular position of the motor shaft are measured with a 15-bit resolver. All input and output signals are available via xPC Target realtime platform from the MathWorks Inc.

Fig. 6. Schematic representation of the experimental testbed with multiple coupled frictional surfaces \( f_1-f_5 \)

3.2 Identification of Presliding Friction

To identify the presliding friction the slow down-chirp excitation with the frequency range 1–0.5 Hz and low amplitude is applied. The excitation is chosen so as to induce a decreasing position response which fades into the micro-cycles as depicted in Fig. 7. Note, that the generated motion does not drop out completely and the micro-oscillations of the same frequency occur after the time about 10 sec as indicated by the arrow in Fig. 7 (b). The overall system dynamics is given by

\[
J \ddot{x} + F(\dot{x}) = u, \tag{7}
\]

in which the driving input torque \( u \) is assumed to be linear to the controlled motor current. As the first and second time derivative of the output position \( x \) are required the measured signals are smoothed with a small-window moving average filter. Subsequently, the system dynamics is identified using the standard least-squares algorithm. Note, that the total system inertia is identified together with the friction parameters. The minimized least-squares error

\[
\text{LSE} = \sum_{i=1}^{N} (u_i - \hat{u}_i(x_i))^2 \tag{8}
\]

is computed from overall \( N \) data tuples of the measured excitation and response, at this using the closed system dynamic provided in (7). The inverse model prediction \( \hat{u}(x) \) is computed in the numerical simulation using the fixed step solver with a step size equal to the sampling time of the control framework.

Fig. 7. Measured presliding system behavior; (a) down-chirp excitation (1–0.5 Hz), (b) position response, – arrow indicates the onset of micro-cycles

The measured and identified presliding behavior is shown in Fig. 8, whereas the driving torque is depicted as a function of relative displacement. Despite the high noisy torque values, the measured and predicted system responses coincide well with each other. At this, not only the main hysteresis loops, but equally the multiple micro-cycles show a good agreement between the model and measurement.

Fig. 8. Measured and identified presliding system behavior with multiple hysteresis loops including micro-cycles

In order to obtain an adequate measure of the MMS model performance, the well-established MS model with five elements is taken as a reference and ditto identified on the same data set using the same identification algorithm. The least-squares convergence is compared in Fig. 9 for both friction models. Note, that the LSE is depicted logarithmic over the total number of iterations. The parameter search
of the MMS based dynamics converges already after 20 iterations, and the residual LSE behaves quasi-constant up to the interrupt of the identification procedure. Unlike, the converged estimate of the MS model parameters (nearly up from 110 iterations) provides a periodic oscillating pattern, thus indicating a possible presence of a local minima in the found parameter solution. This comes as not surprising since the distributed structure of the MS model allows an ambiguity of the \((k, B)\)-distribution over the multiple elements.

![Fig. 10. Error histogram of MS and MMS friction models](image)

The error histograms of both identified models are shown in Fig. 10. The prediction errors are nearly normal distributed, that reveals the general model suitability and leads back to the limited quality of experimental data. However, the MMS model performs as slightly superior while showing a lower error spreading and lower maximal error. At the same time the MMS model requires the five times fewer parameters to be determined than the applied MS one.

4. CONCLUSIONS

This paper describes the novel modified Maxwell-slip (MMS) model of presliding friction. The model is derived based on the consideration of elasto-plastic asperities deflection which saturates at the Coulomb friction level. The main benefit compared to former Maxwell-slip based approaches is the request of two concentrated parameters only when describing the smooth hysteresis of presliding friction. The model is rate-independent over the total amplitude range and is consistent with the generalized empirical friction model structure. The novel perceptions on the frictional memory and drift are provided from the energy balance point of view, together with some supporting experimental observations. The performed experimental evaluation discloses the ease of parameter identification and fairly accurate model prediction. The MMS model of presliding friction is easily integrable in more complex model structures to describe the overall friction dynamics as e.g. shown by Ruderman and Bertram [2011]. Thus, the proposed model can be used either as stand-alone or as a sub-model within more expanded modeling frameworks for the fast simulation and control applications.

REFERENCES


