PROCESS DESIGN AND CONTROL

Integrated Model for Refinery Planning, Oil Procuring, and Product Distribution

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Refinery production planning is usually performed using the refinery battery limit constraints. Two associated problems are the oil uploading and product distribution problems. These problems have traditionally been solved separately, and they have constraints that could render the solution of the planning problem infeasible or less profitable. The goal of this paper is to explore the benefits of the integration of production planning with these two models.

Introduction

With growing demand for petroleum products, increasing crude oil costs, and new environmental limits, production planning tools become key for maintaining the profit margins of refineries. Today every refinery relies on a planning department for demand forecasting, production planning, crude oil procurement, and product distribution.

Refinery planning is a well-known problem for which many mathematical programming models exist. Recent efforts to improve refinery planning models have focused on better integrating the nonlinear aspects due to the chemical reactions and the blending process, integrating the supply chain with the refinery planning, considering market uncertainty, and broadening the scope to include multiple-refineries planning in order to manage production at a strategic level. Also, several models for crude oil procurement, vessel unloading, and tanks movements upstream of the refinery exist in the literature, as well as several models for distribution pipeline scheduling for final products and truck distribution to final customers (see the overall refinery supply chain in Figure 1).

Short-term crude oil unloading and processing is a well-defined problem for which a number of models exist. The problem involves a docking station, a set of storage tanks and/or charging tanks, and a set of crude oil distillation units (CDU). Operations consist of unloading crude oil from vessels into the storage tanks and feeding the crude distillation units according to the production plan. To address this problem, several models have been developed.[1−6] In particular, Reddy et al.[7,8] presented a complete model for crude oil unloading followed by a full continuous-time formulation. These two models, together with the model developed by Mas et al.[9] are the most complete models.

Refinery production planning has been widely studied in the literature. The problem involves crude oil distillation in crude distillation units (CDU), processing through several units in order to transform the different products from the crude distillation units into more valuable products, and finally the blending, or pooling stage, where components are mixed together to obtain final products. Some important quality requirements for final products must be met at this stage. Quality constraints include the aromatic content, maximum sulfur content, vapor pressure, octane number, etc. Once the products are ready to be commercialized, they are stored into product tanks for future delivery. Kelly[10] gives an overview of the mathematical modeling of refinery planning focusing on the nonlinear aspects that arise in the objective function, quality dependencies, and blending stages. Other models have also been developed.[11−19] Some of these models focus on uncertainty but the only one that covers financial risk is the model developed by Pongsakdi et al.,[19] who use two-stage stochastic programming frameworks.

At the end of the refinery, products are sent to regional distribution centers located near the consumer markets. This is the primary transportation network, and the transportation means are ships, railroad, or pipeline. The secondary transportation network goes from the distribution center to retailers or customers, such as gas stations, airports, or other types of retailers. For this part, trucks are used. In the literature, some papers present a simple transportation network between the refinery, a set of depots, and a set of customers;[20−22] some focus on the issue of combined blending and shipping;[23,24] some focus exclusively on blending;[25] some focus on the scheduling for pipeline distribution systems;[26−28] other papers deal with product distribution by trucks;[29−32] finally, some papers deal with shipment planning.[33]

An integrated modeling approach, it is argued, would achieve better cooperation between the production plan and the inventory management upstream and downstream of the refinery, while making sure that there is no bottleneck along the crude oil supply chain. Some articles highlight the importance and discuss the links between planning, inventory management, and shipment.[34] Sarmiento et al.[35] and Chen[36] gave an extensive review of the integration of models for production and distribution. Jia et al.[15] deal in detail with the entire system for a single refinery. They argue that the overall problem could be solved either forward (from crude-oil unloading) or backward (from the production distribution) and conclude that a heuristic-based Lagrangian decomposition could be used to perform this integration. Despite all these efforts to integrate different parts of the problem, there is no integrated model for the detailed planning of crude oil unloading, production, and distribution to final customers.

The most comprehensive and advanced model for the refinery supply chain is certainly Neiro et al.[37] The paper presents a nonlinear model for refinery planning, a mixed integer linear model for storage tanks, and a simple linear model for pipelines. Then, the authors use nonlinear models for refinery units and product blending, in which several refineries are connected by a pipeline network. Although this model considers refinery planning and supply chain management for multiple sites, the model does not consider crude oil operations and distribution

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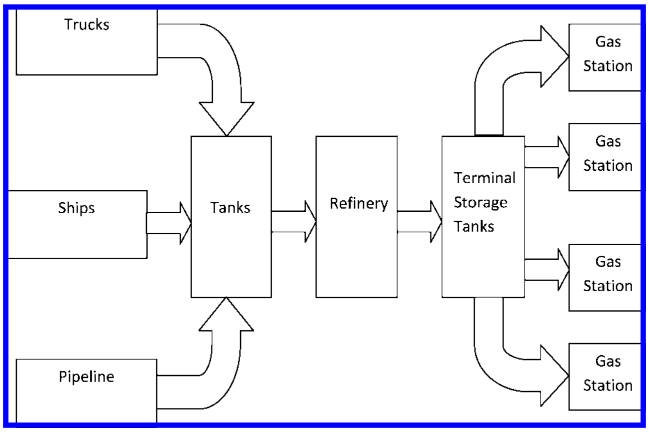

Figure 1. Overview of the oil supply chain (adapted with permission from Kenneth Grant (2006). Copyright 2006 API.

explicitly. Moreover, because of the complexity of the model, the time horizon is limited to only two time periods.

The goal of this paper is to explore the potential benefits of an integrated model involving the three parts of the crude oil supply chain (unloading, oil processing, and distribution) versus the sequential use of them in some fashion.

Writing an integrated model for such a large problem implies some difficulties. The first main problem encountered is to decide what to modelize and the scope of the model. In fact, the oil supply chain is a complex and dynamic problem which usually involves a complex network of several refineries, hundreds of distribution centers, and thousands of customers. Because we want to highlight the benefits of an integrated model we chose the simplest arrangement of one docking station, one refinery, and a set of identical distribution centers.

To build our unloading model, we use as basis the scheduling model of Reddy et al.[7] In turn, our production planning model is based on the model by Pinto et al.[12] Finally, for the distribution part, we use our own developed model for truck transportation planning on a daily basis. The three models are linked assuming the unloading section, the refinery, and the distribution center are connected by pipelines. The pipeline operations planning, however, is not addressed, so pipeline operating costs are not considered. Another reason for choosing the simplest models possible to build the MINLP model is to keep the overall integrated model small (our model already has above 770 000 equations, more than 436 000 and above 3300 binary variables). We believe that computational issues ought to be addressed separately.

The paper is organized as follows: We first present the unloading, production, and distribution models used for the integrated model. Then we discuss the integration of the different models, and finally we present an overall integrated model for the entire supply chain. In all cases we illustrate the benefits of the integration and explain where the mismatches of the models show up when they are run separately.

1. Crude Oil Unloading Model

The crude oil supply chain involves finding, extracting, and transporting crude oil to the refinery. Crude oil is transported by large tankers with capacities ranging from 100 000 deadweight tons to more than 400 000 deadweight tons for the largest ones. So, crude oil is unloaded into crude storage tanks at a docking station and then sent to the refinery for processing via pipeline, or less frequently, by railroad.

Short-term crude oil unloading and processing is a well-defined problem for which a number of models exist. However, no model exists for crude oil unloading for a time period above two weeks. Then, our approach is to build a simple planning model based on the short-term scheduling model from Reddy et al.[7]

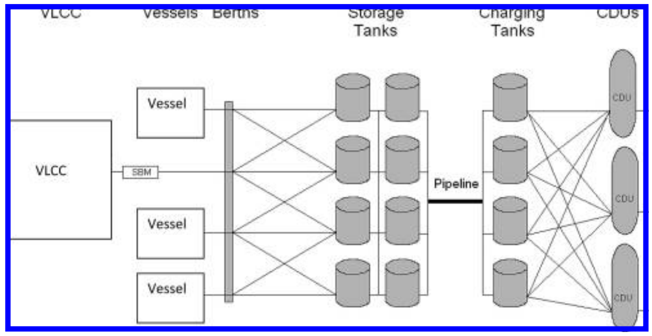

Figure 2. Schematic of the crude oil unloading problem. Adapted with permission from ref 7. Copyright 2004 AIChE.

Figure 2 gives an overview of the problem which is addressed in the unloading model. The model is different from Reddy et al.[7] in that crude oil is sent to an inland refinery through a pipeline and the model does not consider the planning for crude oil distillation, which is addressed in the production model. Although multiple connections are shown between charging Tanks and CDUs, our model will consider a simplified version of this arrangement.

Thus, the problem involves a docking station, a set of storage tanks and/or charging tanks, and a pipeline transporting crude oil to the refinery. Then the oil will be processed at the refinery through a set of crude oil distillation units (CDU). Operations consist of unloading crude oil from vessels into the storage tanks and feeding the pipeline according to the production plan. Pipeline operations are not detailed in this formulation so transportation costs between the docking station and the refinery are not considered.

Crude oil arrives at the docking station either in small single-parcel vessels or in large multiparcel tankers. A very large crude carrier (VLCC) has multiple compartments to carry several large parcels of different crude oil. Because of its huge size, a VLCC must dock offshore at a station called single buoy mooring (SBM), which connects to the crude tanks in the refinery by a SBM pipeline. In actual refineries, most of the crude is transported by large tankers, while small vessels are used only occasionally. In this formulation, only the case where crude oil arrives by large tankers that need to dock offshore is considered. The unloading from vessels is very similar to the unloading from SBM and no more difficult.

Many types of crude oil exist in the market, varying widely in properties, processability, and product yields. Crude oils are classified based on some key characteristics such as processability, yields of some premium products, impurities, or concentrations of some key components that influence the downstream processing. With such differences between crude oil, it is a common practice to segregate them in the storage tanks. So each tank will be able to store only some types of crude oil.

We first make three simplifications to the model of Reddy et al.:[7]

As stated above, the operations concerning the crude oil distillation units (CDU) will be addressed in the production model so they are not considered here.

We omit the detection of changeovers in the feed of the crude distillation unit (a changeover is a change in the feed composition of a crude distillation unit; it lasts a few hours, during which it perturbs the processing unit operation, generating off-spec products or slops).

We prohibit a vessel to stay longer than its expected departure date.

We also add two features:

We allow the model to choose the order in which the parcels are unloaded.

We allow the model to choose which crude oil to purchase when the time horizon is longer than one month.

The schedule of vessel arrival and their crude oil are given for the first month, because we assume that crude oil orders must be requested at least one month in advance. However, when the time horizon is more than one month, the model is able to decide the crude oil purchasing plan.

We assume that there are a certain number of vessels available every week and that each vessel contains a certain number of parcels of a given volume of crude oil. In the first month this information is given because we assume the order has been executed. After the first month, the model must decide the number and composition (type of oil) of the parcels to purchase each week. We now present the model.

Parcel to Single Buoy Mooring (SBM) Connections. Let XP_{pt} be a binary variable that is one when a parcel p is connected to the SBM during period t. In turn, let XF_{pt} be a binary variable that is one during the first period t in which parcel p is connected and finally let XL_{pt} be a binary variable that is one during the last period t in which parcel p is connected. Then, the following equations determine if a parcel is connected to the SBM during time period t (see Reddy et al.[7] for a detailed discussion of the equations).

$$XP_{pt} = XP_{p(t-1)} + XF_{pt} - XL_{p(t-1)} \qquad \forall p \in P, \quad \forall t \in T \tag{1}$$

$$XP_{pt} \geq XL_{pt} \qquad \forall p \in P, \quad \forall t \in T \tag{2}$$

For a parcel, there must be one and only one first connection and disconnection throughout the time horizon:

$$\sum_t XF_{pt} = \sum_t XL_{pt} = 1 \qquad \forall p \in P \tag{3}$$

We define the time at which a parcel first connects and disconnects as follows:

$$TF_p = \sum_t tXF_{pt} \qquad \forall p \in P \tag{4}$$

$$TL_p = \sum_t tXL_{pt} \qquad \forall p \in P \tag{5}$$

where TF_p the period in which parcel p first connects and TL_p is the period in which parcel p is disconnected.

A parcel must first connect before disconnecting:

$$TF_p \leq TL_p \qquad \forall p \in P \tag{6}$$

There can be at most two successive parcels connected to the SBM during a time period:

$$\sum_p XP_{pt} \leq 2 \qquad \forall t \in T \tag{7}$$

SBM to Tank Connections. A parcel is connected to a tank if and only if both the parcel and the tank are connected to the SBM. This is represented by introducing a binary variable XT_{it} that is one when tank i is connected to the docking station. Thus, tank i is connected to parcel p at time t when a variable X_{pit} = 1. The two variables are connected through the following relation:

$$XP_{pit} = XP_{pi} * XP_{pt} \qquad \forall p \in P, \quad \forall t \in T, \quad \forall i \in I \tag{8}$$

This constraint is bilinear, so we replace it with the following equivalent linearization:

$$XP_{pit} \geq XP_{pt} + XT_{it} - 1 \qquad \forall p \in P, \quad \forall t \in T, \quad \forall i \in I \tag{9}$$

$$\sum_i X_{pit} \leq 2XP_{pt} \qquad \forall i \in I, \quad \forall t \in T \tag{10}$$

$$\sum_p X_{pit} \leq 2XT_{it} \qquad \forall i \in I, \quad \forall t \in T \tag{11}$$

Indeed, consider the case where $XP_{pt} = 1$ and $XT_{it} = 1$, then eq 8 forces $X_{pit} = 1$. However, when $XP_{pt} = 0$ and $XT_{it} = 1$, then eq 8 is trivial; equation 9 forces all connections to parcel p to be zero and eq 10 is then trivial. A similar situation takes place when $XP_{pt} = 1$ and $XT_{it} = 0$. Equations 9 and 10 have however an added effect, which is that they allow only two connections. Indeed, assume two parcels (p_1 and p_2) are connected to tank i, that is, $XP_{p_1}t = XP_{p_2}t = 1$ and $XT_{it} = 1$. Then eq 10 will prevent a third one from being connected. The same happens with two tanks connected to one parcel eq 9 would prevent a third one from doing so.

Furthermore, the following constraint must also hold true:

$$X_{pit} \leq PT_{pt}PI_{pi} \qquad \forall i \in I, \quad \forall p \in P, \quad \forall t \in T \tag{12}$$

where PT_{pt} is one for the time period in which the vessel carrying parcel p can be at the docking station and PI_{pi} is one if tank i can have crude oil of parcel p. These two are parameters.

Tank to Refinery Pipeline Connections. The next constraint indicates that a tank cannot be connected to the pipeline while receiving crude from a parcel. It also makes sure that crude oil settles for a time period before being sent to the refinery (brine settling)

$$2XT_{it} + Y_{it} + Y_{i(t+1)} \leq 2 \qquad \forall i \in I, \quad \forall t \in T \tag{13}$$

where Y_{it} is a binary variable equal to one if tank i is connected to the refinery pipeline.

Crude Unloading. Crude oil can be transferred from a parcel to a tank only if the parcel and the tank are connected. In this case the flow must satisfy an upper limit:

$$\sum_c FCPT_{pcit} \leq FPT^U X_{pit} \qquad \forall i \in I, \quad \forall p \in P, \quad \forall t \in T \tag{14}$$

$$\sum_{c,i,p} FCPT_{pcit} \leq FPT^U \qquad \forall t \in T \tag{15}$$

where $FCPT_{pcit}$ is the flow from parcel p to tank i in period t and FTP^U is the upper bound.

The next constraint imposes that a parcel fully unloads during the time horizon:

$$\sum_{c,i,t} FCPT_{pcit} = PS_p \qquad \forall p \in P \tag{16}$$

where PS_p is the size of parcel p.

Finally, to indicate the composition of the parcels we use a binary variable PC_{pc}, which is one when parcel p is composed of crude c. We add the following inequality to ensure that a parcel contains at most one type of crude oil (note that if PC_{pc} = 0, $\forall c \in C$ then this means that parcel p is not purchased).

$$\sum_{ceC} PC_{pc} \leq 1 \qquad \forall p \in P \tag{17}$$

Crude Shipping. Crude oil flow from the docking station to the refinery ($FCTU_{ict}$) can be positive only if a tank is connected to the refinery pipeline:

$$FCTU_{ict} \le FU^{U} Y_{it} \qquad \forall c \in C, \quad \forall i \in I, \quad \forall t \in T \tag{18}$$

The total flow feeding the refinery pipeline (FU_t) is equal to the sum of flows from different tanks:

$$FU_t = \sum_{i,c} FCTU_{ict} \qquad \forall t \in T \tag{19}$$

The amount of crude fed to the pipeline must be within a lower and upper limit (FTU^L and FTU^U, respectively)

$$FU^L \le FU_t^U \le FU^U \qquad \forall t \in T \tag{20}$$

Crude Inventory. The crude level at the end of a time period (VCT_{ict}) is equal to the amount remaining from the last period $VCT_{ic(t-1)}$ plus the amount coming from a parcel $FCPT_{pcit}$ or minus the amount sent to the pipeline $FCTU_{ict}$ (a tank either receives crude oil or feeds the pipeline)

$$VCT_{ict} = VCT_{ic(t-1)} + \sum_{p} FCPT_{pcit} - FCTU_{ict} \qquad \forall c \in C, \quad \forall i \in I, \quad \forall t \in T \tag{21}$$

We also add the following constraint to make sure that crude segregation is respected:

The total crude level VU_{it} in a tank is given by

$$VU_{it} = \sum_{c} VCT_{ict} \qquad \forall i \in I, \quad \forall t \in T \tag{22}$$

In turn, this total amount must be within a lower and upper limit:

$$VU_i^L \le VU_{it} \le VU_i^U \qquad \forall i \in I, \quad \forall t \in T \tag{23}$$

This constraint is necessary because the crude is stored in floating roof tanks to minimize evaporation losses. Such a tank requires a minimum crude level (or heel) to avoid damage to the roof, when the tank goes empty. A typical situation is to have a minimum of two meters of product, which represent 15% of the tank capacity (Mas et al.[9]).

Finally, when a tank i feeds the refinery pipeline, the amounts $FCTU_{ict}$ of crude c delivered must be in proportion to the crude composition in the tank f_{ict} and therefore

$$VCT_{ict} = f_{ict} V_{it} \qquad \forall i \in I, \quad \forall c \in C, \quad \forall t \in T \tag{24}$$

$$FCTU_{ict} = f_{ict} FTU_{it} \qquad \forall c \in C, \quad \forall i \in I, \quad \forall t \in T \tag{25}$$

Constraints 24 and 25 render the model nonlinear; however, in the case where the crude oil tanks contain only one type of oil, f_{ict} becomes a fixed parameter and, thus, the model becomes mixed-integer. An example will be presented that deals with the nonlinear case (example 1.2), then we will assume that the unloading tanks are dedicated to one type of crude oil and, thus, consider a linear model (examples 1.3 and 1.4).

Production Requirements. The following constraint makes sure that crude throughput meets the minimum demand specified by the production plan (D_{ct})

$$\sum_{i} FCTU_{ict} = D_{ct} \qquad \forall c \in C, \quad \forall t \in T \tag{26}$$

Objective Function. We follow the profit maximization objective of Reddy et al.,[7] which is composed of two parts: a marginal profit obtained from distilling a crude and the operating costs related to logistics. Since, we do not consider changeovers, these are only safety stock penalties SC_{ct}.

$$\max \text{profit} = \sum_{i,c,t} FCTU_{ict} CP_{ct} - \sum_{c,i,p,t} FCPT_{i,c,p,t} CC_{c,t} - \sum_{c,t} SC_{ct} \tag{27}$$

where CP_{ct} is the perceived revenue per unit of crude sent to the refinery (Reddy et al.[7] called this "margin"), and CC_{ct} is the purchase cost of crude c.

The safety stock penalties are obtained using the following inequality:

$$SC_{ct} \ge \left(SS_c - \sum_{i} VCT_{ict}\right) SSP_{ct} \qquad \forall c \in C, \quad \forall t \in T \tag{28}$$

where SS_c is the desired safety stock level and SSP_{ct} is the unit penalty cost for crude c. That is, whenever the model renders a total amount of crude of type c in the tanks ($\sum_i VCT_{ict}$) to be larger than the safety stock, the r.h.s of constraint 28 becomes negative and the penalties are driven to zero by the objective function.

We notice that because of constraint 26, the model in reality minimizes cost. However, when constraint 26 needs to be relaxed to obtain a feasible plan (as we shall see later), then this objective function favors sending as much crude as possible to the refinery. It does not, however, take into account limitations on the charging tanks of the CDU. (This will be addressed later.)

We also notice that ordinarily, the production planning model is run also considering costs. The result of this crude demand is made equal to D_{ct}. However, this may not match the sizes of parcels of crude and therefore the unloading model may be forced to buy more crude or less crude than needed, depending on the inventory situation.

This model may be infeasible because the demand D_{ct} may not be met by combining arriving parcels and existing inventory. We discuss more on this issue later in the article when addressing the integration among models.

Finally, we note that the model does not consider any tank movements among charging units, nor does it differentiate which crude unit is fed by what tank. In fact, charging tanks are not even represented by any variable or set in the model. The assumption here is that the refinery planning model will take care of this scheduling.

2. Production Planning Model

The production model is based on the deterministic model developed by Pinto and Moro.[12] The model deals with the optimal planning at a refinery, from crude oil distillation to final product blending. The decision variables are crude oil supply purchase decisions, processing, inventory management, and blending over time periods. Crude oil is assumed to be available immediately and without limit upfront of the refinery.

With this assumption, we do not need to consider charging tanks operation at this point. Instead, an inventory management constraint for the charging tanks will be added when the unloading and production models are integrated.

The model is based on a scheme of valid paths representing a succession of operation units for the transformation of crude oil into marketable products. The product paths, as well as the blending constraints, rely on the composition of some key components, for example, sulfur and aromatic content. Each unit is represented by a set of two constraints: the flow relations

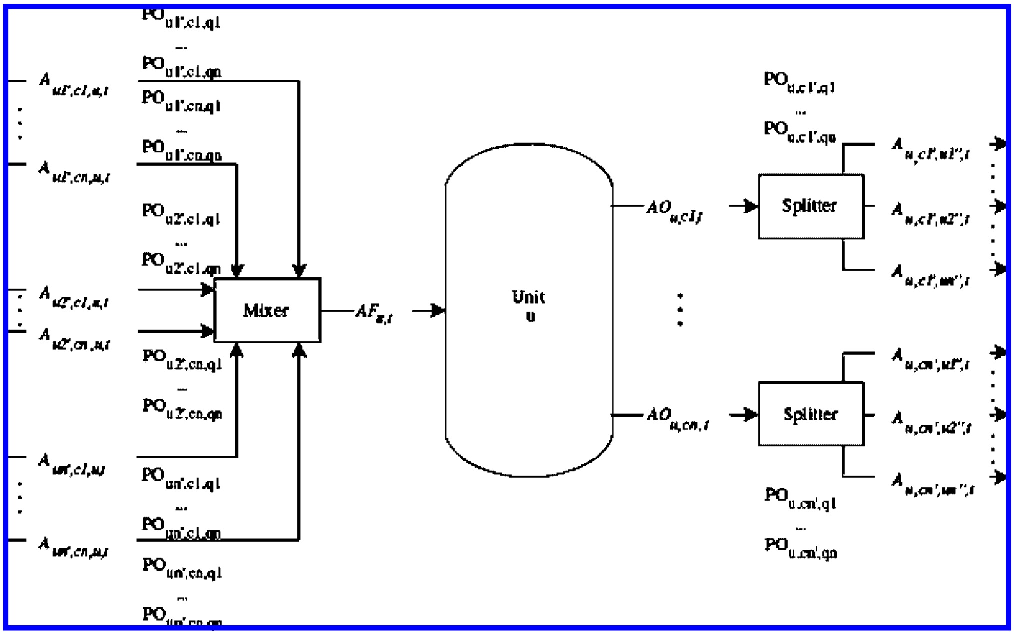

Figure 3. Processing unit model (following Moro and Pinto[12]).

and the yield properties based on the key components. Physical and chemical properties are calculated using volume and weight average (linear relations), whereas the properties that cannot be blended linearly are calculated using blending index numbers. The objective function is to maximize the total profit over the time horizon, given by the amount of product sold, minus the cost for crude oil, intermediate commodities, storage, and a penalty for unsatisfied demands.

Product demand (dem_{ct}) is obtained by aggregating the demands (DEM_{cdt}) from every demand zones in the distribution model, that is $\mathrm{dem}_{ct} = \sum_{d\in D}\mathrm{DEM}_{cdt}$.

A general representation of balancing a production unit is shown in Figure 3. Commodity c_1 is sent from unit u_1' to unit u with flow rate $A_{u_1',c_1,u,t}$ in period t. The same unit u_1' may send different commodities (c_1, c_2,..., c_n) to unit u.

Balance Equations. The following two equations represent the material balances in the mixer and splitter.

$$\mathrm{AF}_{ut} = \sum_{u'c\in C_U} A_{u'cut} \qquad \forall\, u\in U, \quad \forall\, t\in T \qquad (29)$$

$$\mathrm{AO}_{uct} = \sum_{u'} A_{ucu't} \qquad \forall\, c\in \mathrm{CO}_u, \quad \forall\, u\in U, \quad \forall\, t\in T \qquad (30)$$

Where CO_u is the set of commodities leaving unit u.

Yield Equations. The conversion of mass in unit u is represented in two ways: Using percent yields that do not depend on the feed properties, the amount of products is equal to the total inlet flow multiplied by a constant, the percent yield of that unit for the specific crude (yield_{uc}).

$$\mathrm{AO}_{uct} = \mathrm{AF}_{ut}\mathrm{yield}_{uc} \qquad \forall\, c\in \mathrm{CO}_u, \quad \forall\, u\in U, \quad \forall\, t\in T \qquad (31)$$

For percent yields that depend on the feed properties, the amount of products is equal to the sum of each inlet flow times percent yield of each inlet flow ($\mathrm{yield}_{c'c}$).

$$\mathrm{AO}_{uct} = \sum_{u',c'\in Co} A_{u'c'ut}\mathrm{yield}_{c'c} \qquad \forall\, c\in \mathrm{CO}_u, \quad \forall\, u\in U, \quad \forall\, t\in T \qquad (32)$$

Property Equations. The calculation of product properties can be accomplished in two ways:

(1) Product properties (q) leaving unit u (PO_{ucqt}) are calculated as the sum of the flow fraction times the properties of each flow as in the following equation. These are called blending equations.

$$\mathrm{PO}_{ucqt} = \frac{\sum_{u'}\sum_{c'eCO_{u'}} A_{u'c'ut}\mathrm{pro}_{u'c't}}{\sum_{u'}\sum_{c'eCO_{u'}} A_{u'c'ut}} \qquad \forall\, c\in CO_u, \quad \forall\, q\in QO_{uc}, \quad \forall\, u\in U, \quad \forall\, t\in T \qquad (33)$$

where PO_{ucqt} is the property q of commodity c from unit u and QO_{uc} is the set of properties of commodity c from unit u. Although this equation is nonlinear, we use bounds on the properties as explained below, which allows the use of a linear model.

(2) Product properties from unit u that can be determined over average values obtained from plant data, for example, isomerate from isomerization unit and reformate from reformer unit:

$$\mathrm{PO}_{ucqt} = \mathrm{pro}_{ucq} \qquad \forall\, c\in CO_u, \quad \forall\, q\in QO_{uc}, \quad \forall\, u\in U, \quad \forall\, t\in T \qquad (34)$$

Bounds. The stream flowing to each unit should be within established minimum and maximum values:

$$\mathrm{un}_u \le \mathrm{AF}_{ut} \le \mathrm{ux}_u \qquad \forall_u\in U, \quad \forall\, t\in T \qquad (35)$$

The quantity of each crude oil refined AC_{ct} in each time period is bounded:

$$\mathrm{on}_c \le \mathrm{AC}_{ct} \le \mathrm{ox}_c \qquad \forall\, c\in C_o, \quad \forall\, t\in T \qquad (36)$$

where C_o is the set of crude oils. The allowable quantity of finish product stored in each time period is also limited:

$$\mathrm{AS}_{ct} \le \mathrm{stox}_c \qquad \forall\, c\in C_p, \quad \forall\, t\in T \qquad (37)$$

where C_p is the set of finished products.

Quality constraints. Product quality is within certain specifications:

$$\mathrm{pn}_{cq} \le \mathrm{PO}_{ucqt} \le \mathrm{px}_{cq} \qquad \forall c \in CO_u, \quad \forall q \in QO_{uc}, \quad \forall u \in U, \quad \forall t \in T \qquad (38)$$

Substitution of PO_{ucqt} as defined in eq 33 and multiplication by the denominator of eq 33 renders a linear expression.

Objective Function. The objective is to maximize the total profit over the time horizon. The profit is given by the amount of product sold (regular sales plus discount sales), minus the cost for crude oil, intermediate commodities, storage, and unsatisfied demands. To build the objective function the following is defined:

(1) AC_{ct} is equal to the amount of crude oil refined in that time period and given by

$$\mathrm{AC}_{ct} = \sum_{u \in U_c} \mathrm{AO}_{uct} \qquad \forall c \in C_o, \quad \forall t \in T \qquad (39)$$

where AO_{uct} is the amount of crude oil flow out from crude oil storage tank in each time period.

(2) AI_{ct} is the amount of purchased intermediate added in that time period and given by

$$\mathrm{AI}_{ct} = \sum_{u \in U_c} \mathrm{AO}_{uct} \qquad \forall c \in C_{IA}, \quad \forall t \in T \qquad (40)$$

where AO_{uct} is the amount of intermediates flowing out from their storage tank in each time period. In turn C_{IA} is the set of purchased intermediates.

(3) AL_{ct} is the product volume that cannot satisfy its demand (lost demand). The demand of each product must be equal to the volume of that product sale plus the volume of lost demand of that product:

$$\mathrm{dem}_{ct} = \mathrm{sales}_{ct} + \mathrm{AL}_{ct} \qquad \forall c \in C_p, \quad \forall t \in T \qquad (41)$$

In this equation, C_p is the set of commercial products.

(4) MANU_{ct} is equal to the amount of product produced in that time period.

$$\mathrm{MANU}_{ct} = \sum_{u} \mathrm{AO}_{uct} \qquad \forall c \in C_p, \quad \forall t \in T \qquad (42)$$

where AO_{uct} is the amount of commercial products flowing out from a product storage tank in each time period.

(5) AS_{ct} represents the closing stock and $\mathrm{AS}_{c(t-1)}$ represents the opening stock. In the equation, the financial cost incurred relates to the average stock level over the period. Unless the stock levels are known, they are assumed that the average stock level is equal to the arithmetic mean of the opening and closing stock. The balance of product storage can be found in the following equation:

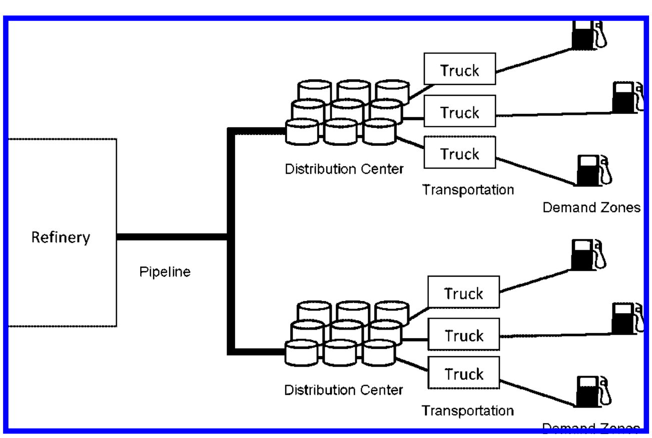

Figure 4. Product distribution system. Adapted from ref 28. Copyright 2006 American Chemical Society.

$$\mathrm{AS}_{ct} = \mathrm{AS}_{c(t-1)} + \mathrm{MANU}_{ct} - \mathrm{sales}_{ct} - \mathrm{AD}_{ct} \qquad \forall c \in C_p, \quad \forall t \in T \qquad (43)$$

where AD_{ct} represents the amount of product c that is sold at a cheaper price. This is because sometimes production exceeds demand. and therefore production needs to be sold at a cheaper discounted price.

Moreover, maximum product tanks capacity ASU_c must be respected:

$$\mathrm{AS}_{ct} \le \mathrm{ASU}_c \qquad (44)$$

We now write the objective function as follows:

$$\mathrm{profit} = \sum_{t,c \in C_P} [\mathrm{sales}_{ct}\mathrm{CP}_{ct} + \mathrm{AD}_{ct}\mathrm{CP}_{ct}(1 - \mathrm{disc}_{ct})] + -\sum_{t,c \in C_O} \mathrm{AC}_{ct}\mathrm{CC}_{ct} - \sum_{t,c \in C_{IA}} \mathrm{AI}_{ct} * \mathrm{ci}_{ct} - \sum_{t,c \in C_P} \left[\left(\frac{\mathrm{AS}_{ct} + \mathrm{AS}_{c(t-1)}}{2}\right) * \mathrm{CP}_{ct}\mathrm{int} - \mathrm{AL}_{ct}\mathrm{CL}_{ct}\right] \qquad (45)$$

where int represents the average interest rate payable in that period and disc_{ct} is the discount rate for product c at time period t, CI_{ct} is the unit purchase price of intermediate commodity c at time period t, CL_{ct} is the unit penalty cost for lost (unsatisfied) demand of product c in time period t, and CP_{ct} is the unit sale price of product c in time period t.

3. Distribution Model

Good distribution models exist for the distribution of petroleum products (as discussed above), but no model is available for road distribution planning by truck on a daily basis for a long time horizon. So, a new model for product distribution downstream of the refinery was developed.

At the end of the refinery process, daily production is stored into tanks at the refinery. Then, products are sent to several regional distribution centers located near the consumer markets. This is the primary transportation network, and transportation means are ship, railroad, or pipeline. Then the secondary transportation network goes from the distribution center to customers, such as airports, gas stations, or other types of retailers. For this part, trucks are usually used (see Figure 4).

We formulate the distribution problem from a distribution center (DC) to several customers. As in the case of the production model, time is discretized into time periods, typically a day or a week. Each day, the distribution center receives lots of products through one pipeline connected to a refinery. Products are segregated at the distribution center; therefore a tank can have only one type of product throughout the time horizon. Finally, delivery to each customer could be formulated as a routing problem where the goal is to minimize the total driving distance. But if there are a large number of customers, the size of the problem is a concern. Thus, to keep the model simple, we aggregate the retailers into demand zones following the approach of Sear.[21] With this approach, a demand zone represents a geographical cluster of customers. For instance, a cluster may represent 10 gas stations in a given city (see Figure 5). It follows that the total demand of a cluster is typically larger than a truck capacity.

The first consequence is that it may be necessary to do more than one trip to a demand zone during a time period. A second consequence is that a truck will service only one demand zone during a trip. That is, a truck can make more than one trip during a time period, but it will always go back to the distribution center for refill before servicing a second customers' zone.

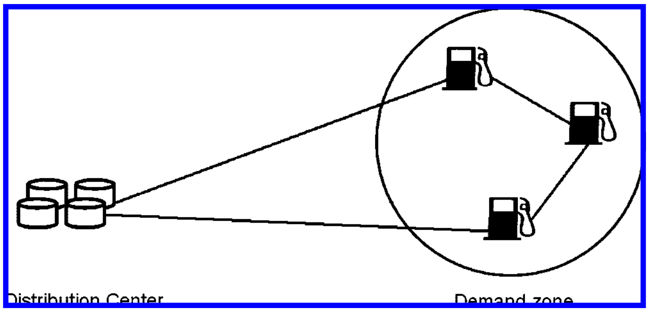

Figure 5. Clusters of customers (following Sear[21]).

With this model, the data needed are the driving time for the round trip between the distribution center and each demand zone. The driving time should include the time necessary to load products into the truck at the distribution center, and also an interdrop time for driving within the demand zone.

Typically, trucks can transport different products at the same time in different tanks. However, a truck must transport similar products every day in order to avoid cleaning its tanks. Therefore, products are classified into groups of similar quality, like gas station products, fuel products, or liquefied gas products, and each truck is assigned to transport only one group of product throughout the time horizon.

For the replenishment of gas stations or others retailers, different operational modes can take place: (1) Either the retailer issues a replenishment order to the distribution center which should be honored in the next x hours (24 h for instance) (this first case can happen for small independent retailers) or (2) some sensors are installed into the tanks at the retailers, and the distribution center is responsible for the replenishment so that the retailers do not run out of product (this second case is more common for large brand stations such as Conoco-Phillips or Seven-Eleven). Another possible case is that (3) some independent retailers (family owned business or large superstores) come directly to the distribution center to pick up some products.

A planning model for the first case relies on a daily forecast of the retailer demand for product delivery. On the other hand, the second case is based on a forecast of the retailer sales. In the following model, we focus on the second case, where the distribution center is responsible for the delivery. However, a similar model can be applied for the first case, just by changing the demand.

The goal of the model is to maximize the profit by selling product to the retailers so that they do not run out of product. However, it can happen that the inventory level at the distribution center is not sufficient to meet all the demands. In this case, there is a penalty when a customer falls below a given safety stock, plus a bigger penalty if a customer is not able to satisfy all its demands. The penalty represents the additional cost to fulfill the demand. Additional cost arises when products must be purchased from another depot or from a competitor.

In summary, the problem can be described as follows. Given

1. the initial inventory level at the tank farm,
2. the inventory level and demand forecast of each demand zone,
3. the driving time for the round trip to each demand zone,
4. a sufficient fleet of trucks and their operating cost.

Determine

1. the production purchased from the refinery for each time period,
2. the inventory level at the distribution center and at each retailer,
3. a delivery schedule for retailers' replenishment.

We now present the equations of the model:

Product Reception. A product lot from the refinery is characterized by its size and type of product. The total amount of product lots that flows through the pipeline at each time period ($REC_{c,t}$) must be within a lower and an upper bound (REC^L and REC^U respectively).

$$REC^L \le \sum_{c \in C_p} REC_{c,t} \le REC^U \qquad \forall t \in T \qquad (46)$$

where C_p is the set of final products.

Inventory Tracking. The amount of product at the end of a time period (VD_{ct}) is given by the previous inventory level plus the amount of product received (REC_{ct}) minus the sum of amount lifted for delivery (DEL_{cdt}).

$$VD_{ct} = VD_{c(t-1)} + REC_{ct} - \sum_d DEL_{cdt} \qquad \forall c \in C_p, \quad \forall t \in T \qquad (47)$$

Floating roof tanks are used for the storage of final products. These tanks require a minimum amount of product (VD^L_{ct}) to prevent the structure from being damaged. Maximum tank capacity (VD^U_{ct}) must also be respected. Thus, we write

$$VD^L_{ct} \le VD_{ct} \le VD^U_{ct} \qquad \forall c \in C_p, \quad \forall t \in T \qquad (48)$$

Customer Delivery. It is assumed that each truck will transport only similar products throughout the entire time horizon, like gas station products, or fuel products for instance. Customers are grouped according to the type of product they need. Thus, we define sets of products g composed of these needed products. All these sets are subsets of a bigger set G. Then, the total number of tours to a customer zone d in a time period t (Z_{dt}) is limited by the number of trucks transporting the type of products NBK(g) times the number of tours per truck per time period NBR:

$$\sum_{c \in g} Z_{dt} \le NBK(g) * NBR \qquad \forall g \in G, \quad \forall t \in T \quad (49)$$

The maximum truck capacity TCU must not be exceeded.

$$\sum_{c \in C} DEL_{cdt} \le Z_{dt} * TCU \qquad \forall d \in D, \quad \forall t \in T \quad (50)$$

The total time driven DTgt by the trucks of group g during a time period t is given by

$$DT_{gt} = \sum_{d \in g} Z_{dt} * TIM_d \qquad \forall g \in G, \quad \forall t \in T \qquad (51)$$

where TIM_d is the average time for the round trip to demand zone d.

Finally, the total time of two tours per day and per trucks regardless of the demand zone serviced during a time period must not exceed the maximum available time MDT.

$$DT_{gt} \le MDT \times NKBK(g) \qquad \forall g \in G, \quad \forall t \in T \quad (52)$$

Customer Inventory Tracking. The inventory level for each customer VOL_{cdt} is equal to the precedent inventory level plus the amount delivered DEL_{cdt} minus the amount sold or used SOL_{cdt}.

$$VOL_{cdt} = VOL_{cd(t-1)} + DEL_{cdt} - SOL_{cdt} \qquad \forall c \in C_p, \quad \forall d \in D, \quad \forall t \in T \quad (53)$$

The amount stored should not exceed the storage capacity of each customer.

$$\mathrm{VOL}_{cdt} \le \mathrm{VOLU}_{cdt} \qquad \forall c \in C_p, \quad \forall d \in D, \quad \forall t \in T \tag{54}$$

The amount sold (or used) by the customer cannot exceed the forecasted sale DEM_{cdt} for product c in each time period.

$$\mathrm{SOL}_{cdt} \le \mathrm{DEM}_{cdt} \qquad \forall c \in C_p, \quad \forall d \in D, \quad \forall t \in T \tag{55}$$

Objective Function. The objective is to maximize the profit over the time horizon, given by the total amount of sales minus the transportation cost, the penalties incurred by a stock below the safety stock, and the demand that cannot be satisfied. To define this objective the following are introduced:

•Product purchase cost

$$\mathrm{PC}_t = \sum_{c \in C_p} \mathrm{REC}_{ct} * \mathrm{COST}_{ct} \qquad \forall t \in T \tag{56}$$

where REC_{ct} is the amount of product received from the refinery and COST_{ct} is the unit cost of product c (which can be set to zero when the refinery and the distribution center are owed by the same company).

•Inventory cost

$$\mathrm{IC}_t = \sum_{c \in C_p} \mathrm{VD}_{ct} * \mathrm{PRI}_{ct} * \mathrm{int} \qquad \forall t \in T \tag{57}$$

•Transportation cost

$$\mathrm{TRC}_t = \sum_{g \in G} \mathrm{DT}_{gt} * \mathrm{DTC} \qquad \forall t \in T \tag{58}$$

•Total safety stock penalty at the distribution center

$$\mathrm{TSSC}_t = \sum_{c \in C_p} \mathrm{SSC}_{ct} \qquad \forall t \in T \tag{59}$$

where

$$\mathrm{SSC}_{ct} \ge (\mathrm{SS}_c - \mathrm{VD}_{ct})\mathrm{SSP}_{ct} \qquad \forall c \in C_p, \quad \forall t \in T \tag{60}$$

•Total safety stock penalty at the customers

$$\mathrm{TSSCC}_t = \sum_{c \in C_p, d \in D} \mathrm{SSCC}_{cdt} \qquad \forall t \in T \tag{61}$$

where

$$\mathrm{SSCC}_{cdt} \ge (\mathrm{VOLL}_{cd} - \mathrm{VOL}_{cdt})\mathrm{SSP}_{ct} \qquad \forall c \in C_p, \quad \forall d \in D, \quad \forall t \in T \tag{62}$$

•Total unsatisfied demand penalty is given by

$$\mathrm{TUDC}_t = \sum_{c \in C_p, d \in D} \mathrm{UDC}_{cdt} \qquad \forall t \in T \tag{63}$$

where

$$\mathrm{UDC}_{cdt} \ge (\mathrm{DEM}_{cdt} - \mathrm{SOL}_{cdt})\mathrm{UDP}_{ct} \qquad \forall c \in C_p, \quad \forall d \in D, \quad \forall t \in T \tag{64}$$

With all these definitions the objective is written as follows:

$$\mathrm{profit} = \sum_{c \in C_p, d, t} \mathrm{DEL}_{cdt}\mathrm{PRI}_{ct} - \sum_t (\mathrm{PC}_t + \mathrm{TSSC}_t + \mathrm{IC}_t + \mathrm{TRC}_t + \mathrm{TUDC}_t + \mathrm{TSSCC}_t) \tag{65}$$

4. Integrated Unloading and Production Model

Most refineries get their crude oil supply by a pipeline connected to a docking station where crude oil arrives by tankers. Typically, the docking station sends crude oil to a crude oil terminal, or crude oil hub, which dispatches oil to one or several refineries. In our model, we consider that the refinery is directly linked to the docking station. So, all the commodities sent from the docking station through the pipeline arrive in some crude oil tanks at the refinery. Moreover, we assume that it takes one day for crude oil to go from the docking station to the refinery.

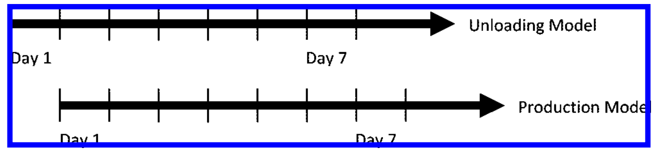

Figure 6. Offset of periods between models.

To explore the benefit of an integrated unloading and production model, we compare two plans one given by running the production model first and then trying to supply the refinery with enough crude oil to realize the production plan, and the other given by running the unloading and production models in one single integrated model.

In the nonintegrated model, production is run first assuming that the quantity of crude oil available during the first month is constrained by the amount available in the initial inventory plus the total amount arriving in the vessels during the first month.

$$\sum_{t \in T_1} \mathrm{AC}_{ct}^{\mathrm{prod}} \le \mathrm{inv}_c + \sum_{t \in T_1, p \in \{PC_{pc}=1\}} \mathrm{PS}_p^{\mathrm{unload}} \qquad \forall c \in C_o \tag{66}$$

where $\mathrm{AC}_{ct}^{\mathrm{prod}}$ is the amount of crude oil c used by the refinery in time period t, inv_c is the initial inventory level at both docking station and the refinery, $\mathrm{PSs}_p^{\mathrm{unload}}$ is the size of the parcels, with $\{\mathrm{PC}_{pc} = 1\}$ is the set of parcel containing crude oil c oil and T_1 is the set of time periods in the first month. In turn, inv_c is given by

$$\mathrm{inv}_c = \sum_i \mathrm{VCT}_{ict_0} \qquad \forall c \in C_o \tag{67}$$

where t_0 is the period before the planning is considered.

As stated above, $\mathrm{PS}_p^{\mathrm{unload}}$ is a parameter for periods belonging to T_1. After that period, the planning model will determine $\mathrm{AC}_{ct}^{\mathrm{prod}}$, without any constraint.

Then the crude oil requirements from the production model are passed along to the unloading model in order to try to find a feasible supply plan

$$D_{ct}^{\mathrm{unload}} = AC_{c(t+1)}^{\mathrm{prod}} \qquad \forall c \in C_o, \quad \forall t \in T \tag{68}$$

One may note that we assume that the crude oil takes one day to travel from the docking station to the refinery; that is why the unloading model should satisfy the production requirement of the following day ($\mathrm{AC}_{c(t-1)}^{\mathrm{prod}}$). This implies that there is one time period shift between the unloading and production models (Figure 6).

In some cases, the unloading model could be unable to satisfy 100% of the crude oil requirement from the production model rendering the unloading model infeasible. This situation is due to the limited capacity of the docking station (SBM and tanks) and to the time a parcel needs to be unloaded and sent to the refinery (as it will be illustrated in the examples). In this case, managers still need a feasible plan for unloading and production. Thus, the following procedure is followed: (1) The production plan is run using the initial inventory constraints 66 and 67 and fixed values of $\mathrm{PS}_p^{\mathrm{unload}}$. (2) The unloading model is run using

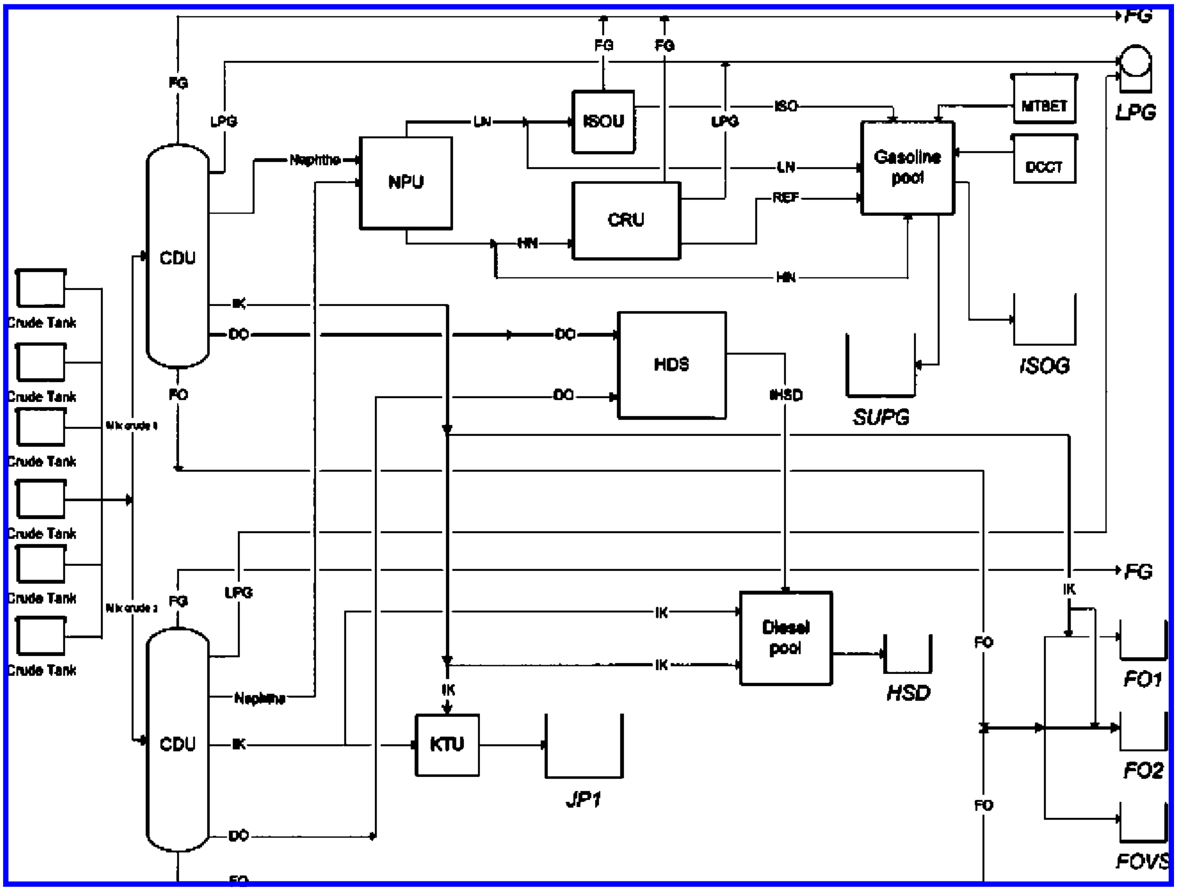

Figure 7. Simplified scheme of refinery used in examples.

Table 1. Refinery Product Initial Inventory Level

LPG	SUPG	ISOG	JP-1	HSD	FO1	FO2	FOVS
1500	14 100	8 400	15 400	54 000	0	0	0

Table 2. Crude Distillation Units Limits

	min	max
CDU2	3 180	6 360
CDU3	6 360	12 720

constraint 68. (3) If the model is feasible, then the production plan, as well as the unloading plan, are set and orders for new crudes are obtained for periods after T_1. These orders are in the form of parcels that will be arriving and are determined by the unloading model. (4) If the unloading model is infeasible, then a new unloading plan is computed by reducing the crude oil requirement from production by a percentage x of its original value using the following:

$$D_{ct}^{\text{unload}} \geq AC_{c(t+1)}^{\text{prod}}(1-x) \qquad \forall c \in C_o, \quad \forall t \in T \quad (69)$$

Once this inequality is added, the model is free to send different crudes, and moreover, purchase different crudes in different proportions. Thus, it may try to purchase too much of a particular crude. Therefore, we limit this by adding the following constraint.

$$D_{ct}^{\text{unload}} \leq AC_{c(t+1)}^{\text{prod}}(1+x) \qquad \forall c \in C_o, \quad \forall t \in T \quad (70)$$

(5) Then, a new production plan is computed taking into account the limited crude oil supply of the unloading plan. However, some inventory management must take into account the limited capacity of the charging tanks:

$$VR_{ct} = VR_{c(t-1)} + FCTU_{c(t-1)}^{\text{unload}} - AC_{ct}^{\text{prod}} \qquad \forall c \in C_o, \quad \forall t \in T \quad (71)$$

where VR_{ct} is the inventory level of crude oil c at the refinery, $FCTU_{c(t-1)}^{\text{unload}}$ is the amount received from the docking station, and AC_{ct}^{prod} is the amount distilled. VR_{ct} is in turn limited by a maximum capacity.

$$VR_{ct} \leq VR_c^{\max} \qquad \forall c \in C_o, \quad \forall t \in T \quad (72)$$

Steps 4 and 5 show the limitation of this approach, limitations we intend to illustrate in the examples. Indeed, reducing the crude units demand by a fraction may be much more inefficient than finding some other set of demands that would make the refinery utilization more efficient.

For the integrated model, the two problems are simply merged into one single model using constraints 68, 71, and 72.

To make meaningful comparisons, we will compute operating costs of both models in the nonintegrated models, revenues obtained by the planning model, and purchasing costs computed by the unloading model.

With the exception of constraints 24 and 25 the rest of the constraints are linear. In the following examples, only one case will use these constraints (example 1.2). In the rest, the assumption was made that the crude oil tanks contain only one type of crude and so the model becomes linear. The linear cases were solved using GAMS (version 2.0)/CPLEX (version 9.0) and the nonlinear case was solved using GAMS(version 2.0) /DICOPT(version 2x-C (2005)) with CONOPT (version 3) as a nonlinear solver. All models were run using a Windows

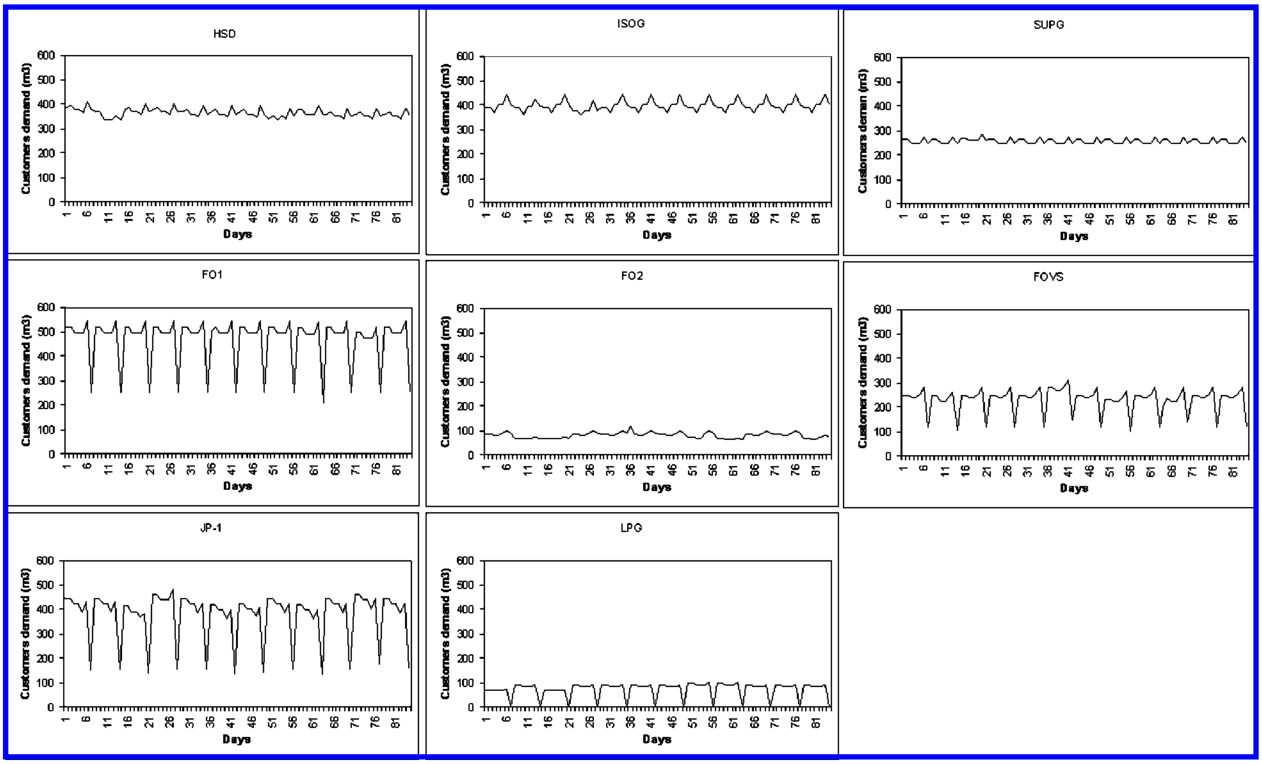

Figure 8. Product customer demand lumped byproduct.

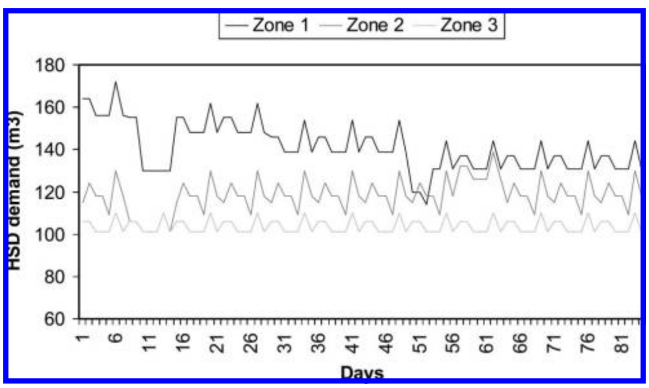

Figure 9. Customer demand for HSD throughout the time horizon.

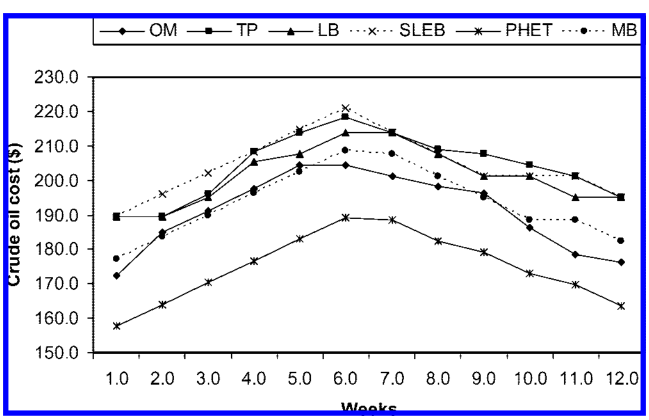

Figure 10. Variations of crude oil prices over the time horizon ($\$/m^3$).

Server 2003 on a Dell PowerEdge 2850 Intel Xeon Dual 2.8 GHz with 2GB of RAM. The solution algorithm used in CPLEX is the default branch-and-cut algorithm for the MILP formulation.

Example 1. The production model was applied to refinery data from Pongsakdi et al.[19] (Figure 7). The refinery has eight processing units: two atmospheric distillation units (CDU), two naphtha pretreating units (NPU), one light naphtha isomerization unit (ISOU), two catalytic reforming units (CRU), one kerosene treating unit (KTU), one gas oil hydrodesulphurization (GO-HDS), and one deep gas oil hydrodesulphurization (DGO-HDS). Final products are liquefied petroleum gas (LPG), gasoline RON 91 (SUPG), gasoline RON 95 (ISOG), jet fuel (JP-1), high speed diesel (HSD), fuel oil 1 (FO1), fuel oil 2 (FO2), and low sulfur fuel oil (FOVS). Moreover, the entire production of fuel gas (FG) and part of FOVS has been used as energy source for the plant. Finally, six crude oils are available for purchase: Oman (OM), Tapis (TP), Labuan (LB), Seria light (SLEB), Phet (PHET), and Murban (MB).

The inventory of crude oil is zero at the beginning of the time horizon while the inventory of final product is given in Table 1. Crude distillation units processing limits are given in Table 2. Figure 8 presents the customer demand aggregated byproduct throughout the entire time horizon. As an example, Figure 9 shows the demand of high speed diesel (HSD) day by day and customer by customer. Demands have been generated assuming a daily average with random variations with a weekly pattern that presents a reduced demand on Sunday. Finally, prices for the unloading, production, and distribution models are given in Figures 10 and 11.

For the unloading model, we consider a docking station where very large crude carriers unload through a single-buoy mooring (SBM) and nine tanks. Tanks 1, 2, 3, and 4 can have crude oils OM, TP, and LB; tanks 5, 6, and 7 can have crude oils SLEB and PHET; while tanks 8 and 9 can only contains MB crude oil. We show this particular scheme in Figure 12. The general data are presented in Table 3. The initial inventory at the docking stations is given in Table 4. For the unloading model, we will use the information of products demands lumped byproduct given in Figure 7. Except the cost of crude oil and the price of

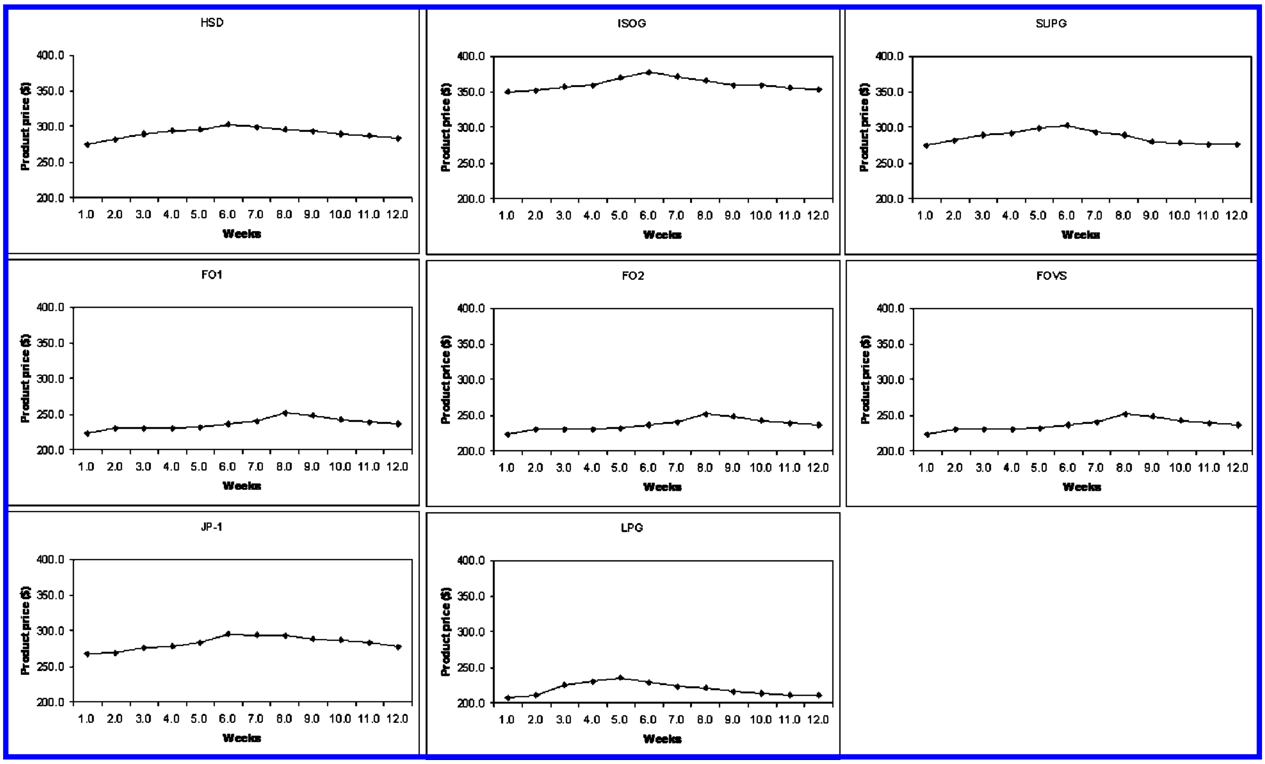

Figure 11. Product prices over the time horizon ($/m^3).

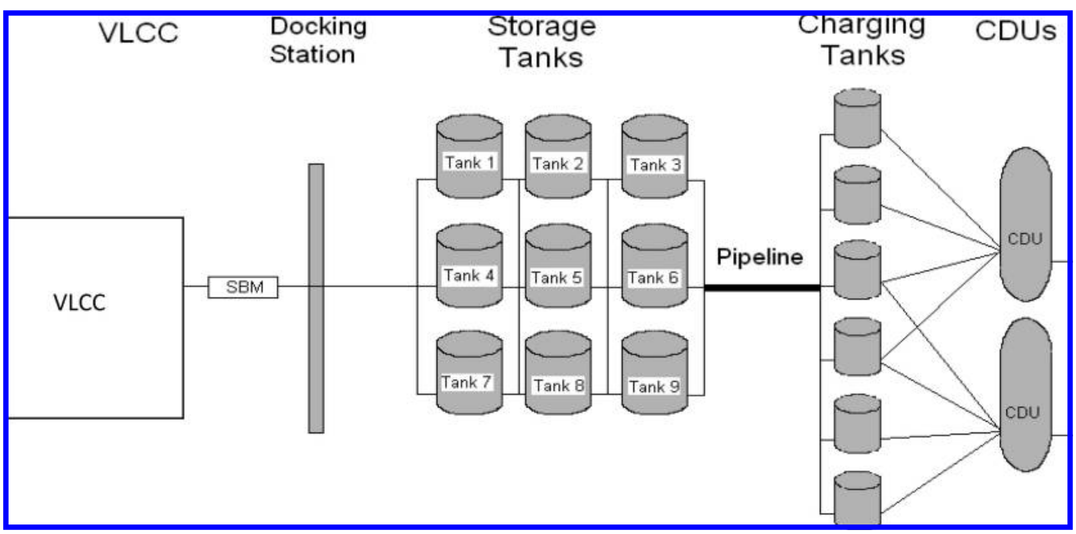

Figure 12. Unloading system configuration.

final product which are given in Figures 9 and 10, the rest of the parameters of the two models are the same as in Pongsadki et al.[19]

Example 1.1. The first example illustrates a case where the nonintegrated model and the integrated one give the exact same plan. The time horizon is 1 week. All parcels arriving and their crude type are known (PS_p and PC_{pc} are known). Each tank contains only a single type of crude oil, so the nonlinear constraints (crude oil composition in the tanks) become linear. Table 5 shows the arrival of crude oil parcels (data) which matches the demand from the production plan presented in Table 6 (demand from the production plan is a result for the production model but a data for the unloading model). The two models give the exact same plan, and, thus, the same profit. The plan of the amount of crude oil sent to the refinery (result) is exactly

Table 3. General Data−Unloading

parameter	value (m^3)
FPT^U (upper limit on amount transferred from a parcel to a tank)	30 000
FU^L_{ut} (lower flow rate of crude distillation unit 1/2, respectively)	3180/6360
FU^U_{ut} (upper flow rate of crude distillation unit 1/2, respectively)	6360/12720
SS_c (desired safety stock of crude oil at the docking station)	
OM	10 000
TP & LB	0
SLEB	10 000
PHET	10 000
MB	10 000
SSP_{ct} (unit safety stock penalty $/m^3)	$CP_{ct}/2$

Table 4. Initial Inventory at Docking Stations (Examples 1.1, 1.3, and 1.4)

tank	min capacity (m^3)	max capacity (m^3)	crude	initial inventory (m^3)
tank 1	9 000	60 000	OM	35 000
tank 2	9 000	60 000	OM	24 000
tank 3	6 000	40 000	OM	10 000
tank 4	6 000	40 000	LB	25 000
tank 5	9 000	60 000	SLEB	20 000
tank 6	9 000	60 000	PHET	30 000
tank 7	6 000	40 000	PHET	18 000
tank 8	9 000	60 000	MB	35 000
tank 9	9 000	60 000	MB	25 000

Table 5. Parcel Arrival—Example 1.1

parcel	p1	p2	p3	p4	p5
arrival day	1	1	1	4	4
leaving day	3	3	3	5	5
type	OM	PHET	MB	OM	PHET
size (m^3)	20 000	20 000	20 000	20 000	20 000

Table 6. Crude Oil Requirement from Production (AC) (m^3)—Example 1.1

CDU	crude	day 1	day 2	day 3	day 4	day 5	day 6	day 7
CDU2	OM	729	728	706	706	743	806	584
CDU2	PHET	3240	3234	3138	3138	3303	3580	2596
CDU3	OM	5155	5091	5152	5152	5047	4871	5496
CDU3	MB	3959	3910	3957	3957	3876	3741	4221

Table 7. Results for Example 1.1

model	eq	cont. var.	int. var.	CPU time	costs ($)	revenue ($)	profit ($)
unloading	2686	2281	287	0.3 s	15,318,234	0	
production	1470	1694	0	0.1 s	249,425	16,274,015	16,024,590
nonintegrated					15,567,659	16,274,015	706,356
unloading part					15,318,234	0	
production part					249,425	16,274,015	16,024,590
integrated	4073	3976	287	1.1 s	15,567,659	16,274,015	706,356

equal to the demand from the production plan. Table 7 shows the result of the two models which have the same profit.

Example 1.2. This example illustrates the case where the mix in the storage tanks constraints the feed to the refinery. The model is in this case nonlinear.

The configuration is different than in the other case. We consider a marine access refinery with only one set of tanks that are storage and charging tanks at the same time (see Figure 13), and the crude distillation charging plan is addressed in the unloading model.

Table 8. Initial Inventory at Docking Station (m^3)—Example 1.2

	capacity (m^3)					
tank	min	max	crude 1	initial inventory (m^3)	crude 2	initial inventory (m^3)
tank 1	9 000	60 000	OM	35 000		
tank 2	9 000	60 000	OM	24 000	TP	6 000
tank 3	6 000	40 000	OM	10 000	LB	2 000
tank 4	6 000	40 000	LB	25 000		
tank 5	9 000	60 000	SLEB	20 000		
tank 6	9 000	60 000	PHET	30 000		
tank 7	6 000	40 000	PHET	18 000	SLEB	8 000
tank 8	9 000	60 000	MB	35 000		
tank 9	9 000	60 000	MB	25 000		

Table 9. Results for Example 1.2

model	CPU Time	cost	revenue	profit
nonintegrated				infeasible
unloading		$15,318,234	0	
production		$254,601	$16,274,015	$16,019,414
integrated	10.7 s	$15,572,835	$16,274,015	$701,180

Table 10. Production Crude Oil Utilization (m^3)—(AC): Example 1.2, Integrated Model

	crude	day1	day2	day3	day4	day5	day6	day7
CDU2	OM	929	929	495	502	920	622	499
CDU2	LB			99			40	32
CDU2	SLEB			796	824			815
CDU2	PHET	4127	4127	1791	1854	4090	2746	1834
CDU3	OM	4524	4524	5639	5357	5069	5572	5359
CDU4	TP			366		691	359	
CDU5	MB	3475	3475	3542	4114	2403	4113	4116

Table 11. Parcel Arrival —Example 1.3

	p1	p2	p3	p4	p5	p6	p7	p8	p9	p10
arrival day	1	1	1	8	15	15	15	22	22	22
leaving day	3	3	3	10	17	17	17	24	24	24
type	OM	PHET	MB	MB	OM	PHET	MB	OM	PHET	MB
size (Km^3)	30	30	30	30	30	30	30	30	30	30

The time horizon also spans one week. The schedule of parcel arrivals as well as production requirement is the same as in example 1.1. Table 8 shows the initial inventory level at the docking station. Some tanks have more than one type of crude oils so nonlinear composition constraints 24 and 25 are needed. The solver Dicopt (version 2) for MINLP is used with GAMS (version 2.0).

In the nonintegrated model case a feasible solution that satisfies both the unloading and production plans (that is $x = 0$) cannot be found. The mix in the tanks forces the model to send some crude oils that are not requested by the production

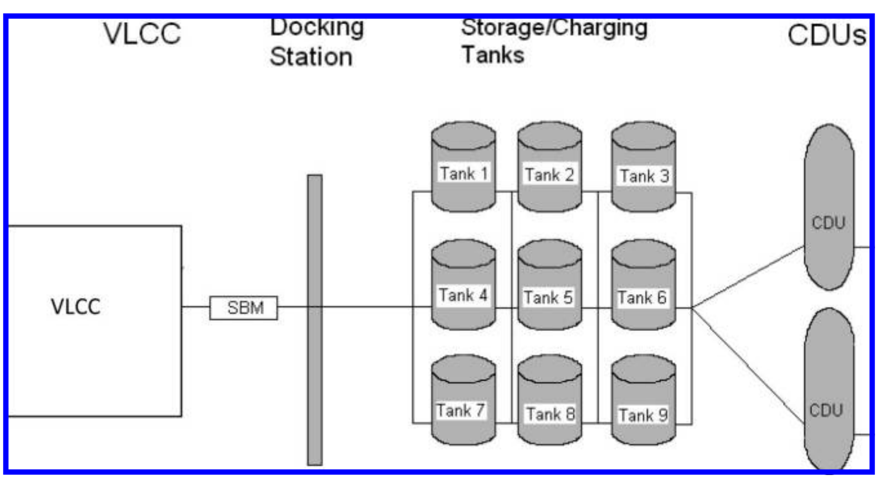

Figure 13. Unloading system configuration—example 1.2.

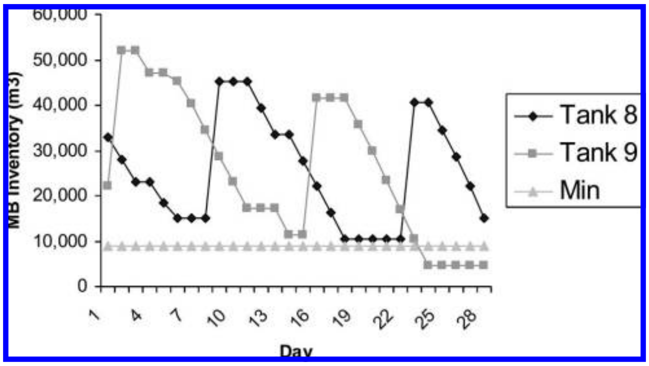

Figure 14. Tank levels for example 1.3 (plan is unfeasible in days 25 to 28).

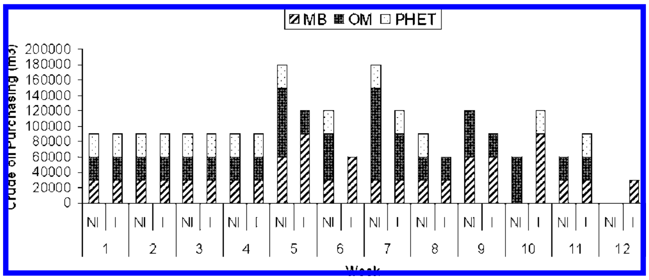

Figure 15. Crude oil purchasing plan (m^3)—nonintegrated (NI) and integrated (I) models.

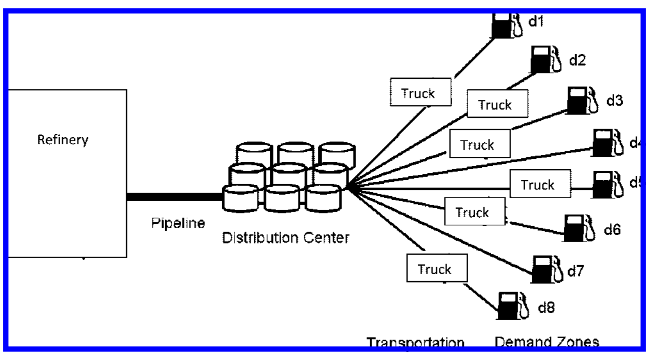

Figure 16. Distribution system configuration.

plan. A feasible solution for the unloading model can be found for $x = 0.1$ (and also by allowing some crude oil that is not requested with a maximum of 1000 m^3 of nonrequested crude oil per time period). However, the production model is not feasible with this crude oil supply. After several attempts, it has not been possible to find a feasible plan for both unloading and production.

For the integrated model, a feasible solution is found in 10.7 s, with a profit slightly lower than in the previous example ($701,180 instead of $706,356 in example 1.1). Results are presented in Table 9 and the feasible plan for production crude oil utilization is presented in Table 10.

Example 1.3. Example 1.3 illustrates the case where the production plan is not feasible because the unloading model cannot supply enough crude oil because of the limited capacity of the tanks. It has a time horizon of one month; the initial inventory level is the same as in example 1.1, and we keep each tank devoted to the same crude as in example 1.1. The parcel arrivals are given in Table 11. Thus, the problem is a MILP.

In the nonintegrated model, the production model is run first and its consumption of crude oil is passed along to the unloading

Table 14. Distribution System Trucks

group	products	demand zone	no. trucks
gasoline/diesel	HSD, ISOG, SUPG	d1, d2, d3	16
fuel products	JP-1, FO1, FO2, FOVS	d4, d5, d6, d7	18
liquefied gas	LPG	d8	2

Table 15. Distribution System Driving Times[a]

demand zone	driving time (h)
d1	3.0
d2	3.5
d3	4.5
d4	3.0
d5	4.0
d6	4.5
d7	3.5
d8	4.5

[a] Assume 9 h driving time/day.

Table 16. Distribution Center Storage Data

product	min level (m^3)	safety stock (m^3)	max level (m^3)
HSD	1 500	3 000	10 000
ISOG	1 500	3 000	10 000
SUPG	1 500	3 000	10 000
JP-1	3 000	6 000	20 000
FO1	3 000	6 000	20 000
FO2	1 200	2 400	8 000
FOVS	1 200	2 400	8 000
LPG	1 500	3 000	10 000
total	14 400	28 800	96 000

Table 12. Results for Example 1.3

model	equ.	cont. var.	int. var.	CPU time (s)	cost ($)	revenue ($)	profit ($)
unloading	31 315	17 991	1 316	22.2	57,587,098	0	
production	5 859	6 755	0	0.5	2,787,887	66,480,113	63,692,226
nonintegrated					60,374,985	66,480,113	6,105,128
unloading					57,587,098	0	
production					2,560,480	66,480,113	63,919,633
integrated	37 511	24 747	1 316	17.7	60,147,578	66,480,113	6,332,535

Table 13. Results for Example 1.4

model	equ.	cont. var.	int. var.	CPU time	costs ($)	revenue ($)	profit ($)
unloading	737 525	384 987	2 676	6.5 min	223,378,641	0	
					12,827,872		
production	18 067	20 839	0	0.7 s	236,206,513	261,829,327 261,829,327	249,001,455
nonintegrated							25,622,814
unloading					199,021,933	0	
production					11,255,747	265,427,106	254,171,359
integrated	754 567	405 827	2 676	17.7 s	210,277,680	265,427,106	55,149,427

Table 17. Demand Zones Storage Data

product	safety stock (m^3)				max capacity (m^3)			
	HSD	ISOG	SUPG		HSD	ISOG	SUPG	
c1	200	150	150		500	400	400	
c2	200	100	150		500	300	400	
c3	200	100	150		500	300	400	
	JP-1	FO1	FO2	FOVS	JP-1	FO1	FO2	FOVS
c4	80	150	100	70	200	400	250	300
c5	150	150	100	100	400	400	250	300
c6	150	150			400	300		
c7	150	150			400	300		
	LPG				LPG			
c8	150				400			

Table 18. Delivery Plan for Example 2.1—Nonintegrated Model

product	day 1 (m^3)	day 2 (m^3)	day 3 (m^3)	day 4 (m^3)	day 5 (m^3)	day 6 (m^3)	day 7 (m^3)	total (m^3)
LPG	71	71	67	67	67	74	0	417
SUPG	262	262	250	250	250	274	250	1,798
ISOG	389	389	369	403	403	444	403	2,800
JP-1	447	447	425	425	391	431	151	2,717
HSD	385	394	375	375	366	412	375	6,235
FO1	520	520	496	496	496	545	252	3,325
FO2	84	84	80	80	89	90	89	605
FOVS	250	250	238	238	254	281	119	1,630

Table 19. Delivery Plan for Example 2.1—Integrated Model

product	day 1 (m^3)	day 2 (m^3)	day 3 (m^3)	day 4 (m^3)	day 5 (m^3)	day 6 (m^3)	day 7 (m^3)	total (m^3)
LPG	68	70	35	35	70	35	104	417
SUPG	262	249	244	247	248	298	250	1,798
ISOG	617	151	136	613	379	410	495	2,801
JP-1	447	447	844	219	219	219	321	2,716
HSD	341	387	425	260	481	412	375	2,681
FO1	300	600	175	680	699	424	447	3,325
FO2	31	31	186			256	101	605
FOVS	250	166	214	149	158	318	375	1,630

model. The unloading model is unable to satisfy all the MB crude oil demand because of constraints for the tank movements. In fact, whenever a parcel is unloading into a tank at the docking station, this tank cannot send product to the refinery during the next day because crude oil needs to settle for brine settling. Part of the unloading infeasible plan is presented in Figure 14. The graph shows the inventory level of the tanks carrying MB crude oil (tanks 8 and 9). The infeasibility arises during the last week. Tank 8 receives a parcel of 30 000 m^3 at day 23 so it cannot send crude oil to the refinery until day 25. During these 2 days, tank 9 is the only tank available to furnish MB crude oil to the refinery but its inventory level is under the minimum allowed inventory level.

Thus, the unloading model is not able to satisfy all the demand of production. Still, a feasible solution which satisfies 90% of the production plan's requirement can be found ($x = 0.1$). This solution is passed along to the production model and a new feasible production plan is found with a benefit of $6,105,128. On the other hand, the integrated model is able to find a feasible plan with a profit of $6,332,535 which is 3.7% better than the nonintegrated one. The reason is that the nonintegrated model is forced to purchase more intermediate commodities to make up for the imperfect crude oil supply. All results are summarized in Table 12.

The last situation where the nonintegrated model for unloading and production can be infeasible is when the production model does not consume enough crude oil. This leads to a situation where there is no more space to store the new parcels arriving.

Example 1.4. The fourth example illustrates the use of the unloading model over a long time horizon of 3 months in which the model has to decide which type of crude oil to purchase. The arrival of vessels is scheduled for the first month as in the example 1.3 and is used as data. However, these arrivals are a a decision variable for the following months.

We assume that there are two vessels available every week and that each vessel contains three parcels of 30 000 m^3 of crude oil. The model can choose not to purchase a parcel or to purchase it; in this case it must decide the type of crude oil of the parcel.

In this example, the unloading model is unfeasible because there is not enough capacity at the docking station to store all the crude oil. Actually, the crude oil requirement from the production plan in the second month exceeds the maximum unloading capacity of the docking station. Then, as in the previous example, a feasible plan is computed for the nonintegrated model by relaxing the demand from production ($x = 0.3$). Then a new production plan is computed according to the unloading plan. The unloading plan costs are $223,378,641 and the production plan has a profit of $249,001,455 so the overall profit is $25,622,814.

The integrated model, in turn, is able to find a feasible plan in 24 min with a profit of $55,149,427 (more than double compared to the nonintegrated model) (Table 13). The unloading plan costs $199,021,933 and the production plan has a profit of $254,171,359. The penalty for lost demand is only $225,474 compared to $4,113,184 for the nonintegrated model. Figure 14 shows the plans for crude oil purchasing for the nonintegrated and integrated models.

5. Integrated Production and Distribution Model

For the integration of the production and distribution models, we consider a case where the refinery is connected to one distribution center via one pipeline. In the industry, a refinery would actually be linked to several big terminals, distribution centers to final customers, as well as other refineries of the company. Nevertheless, this model and its implication (the cooperation mode achieves a better solution than when the two models are run separately) remain valid for industrial cases, where the refinery is connected to several distribution centers and other refineries via a pipeline network. It is assumed that the distribution center is located at a distance such that two days are necessary for a product lot to go from the refinery to the distribution center.

To establish a comparison between the integrated and the nonintegrated model, we first find the optimum plan when the two models are run separately. The following procedure is used.

Table 20. Results for Example 2.1

model	equ.	cont. var.	int. var.	CPU time	costs ($)	revenue ($)	profit ($)
production	1 470	1 701	0	0.2 s	3,455,779	4,761,907	1,306,128
distribution	1 604	2 479	56	3.1 s	74,671	4,761,907	4,687,236
nonintegrated							1,231,457
production part					3,453,436	0	
distribution part					74,665	4,761,907	4,687,242
integrated	3 076	4 238	56	5.4 s	3,528,101	4,761,907	1,233,806

Table 21. Production Plan for Example 2.2—Nonintegrated Model

	week 1	2	3	4
FG	136	130	127	125
LPG	776	700	651	629
SUPG	2 039	2 070	1 740	1 744
ISOG	5 120	2 744	1 615	1 054
JP-1	1 791	2 643	2 816	2 544
HSD	8 164	5 791	4 479	4 381
FO1	3 902	3 317	3 325	3 621
FO2	2 295	1 427	1 602	974
FOVS	2 597	2 674	1 581	1 532
total	26 821	21 496	17 934	16 604
grand total				82 856

Table 22. Production Plan for Example 2.2—Integrated Model

week	1	2	3	4
FG	134	125	125	123
LPG	741	632	629	597
SUPG	1 731	1 742	1 744	1 764
ISOG	4 380	1 057	1 055	1 033
JP-1	1 536	1 536	1 536	1 536
HSD	7 525	5 482	5 524	5 592
FO1	4 811	1 334	667	1 889
FO2	1 776	3 180	3 794	2 403
FOVS	1 581	1 581	1 537	1 080
total	24 214	16 668	16 610	16 015
total				73 507

The production planning problem is solved using the information on demand and price only, without considering the distribution part, which is assuming the distribution system can deliver any product at any time and has infinite tank capacity to hold inventory. In addition, penalties for unsatisfied demand are not considered in the production cost and no product can be sold at a discounted price (although it is still possible to get rid of extra production with no profit). Then the result is used to find a distribution plan.

One could try to solve the distribution problem first, and pass the resulting demand to the production model. Although this is possible, if the distribution model is run first, it may happen that the production model is not able to satisfy the distribution plan (because of maximum production capacity). In this case, the overall plan would be infeasible. The other way around, the distribution model can always adjust because it can use other sources of products or pay penalties.

After running the distribution model using the daily production plan as input, the total profit is computed as the sum of profits from distribution minus the costs from production.

$$\text{profit} = \sum_{c\in C_p,d,t} \text{DEL}_{cdt}\text{PRI}_{ct} - \sum_t (\text{PC}_t + \text{TSSC}_t + \text{IC}_t + \text{TRC}_t + \text{TUDC}_t + \text{TSSCC}_t) - \sum_{t,c\in C_o} \text{AC}_{ct}\text{CC}_{ct} - \sum_{t,c\in C_{IAo}} \text{AI}_{ct}\text{CI}_{ct} - \sum_{t,c\in C_p}\left[\left(\frac{\text{AS}_{ct}+\text{AS}_{c(t-1)}}{2}\right)\text{CP}_{ct}\text{int}\right] \quad (73)$$

In the integrated model, the output of the production model is directly linked to the distribution model. This is accomplished by the following equation:

$$\text{REC}^{\text{dist}}_{ct} = \text{sales}^{\text{prod}}_{c(t-2)} \qquad \forall c\in C_p, \quad \forall t\in T \qquad (74)$$

As in the nonintegrated problems, the penalty for unsatisfied demand is set to zero in the production model; and no product can be sold at a discounted price. In addition, we assume that the refinery and the distribution belong to the same company (which is usually the case in industry), so product sale prices in the production model and product purchase costs in the distribution model are set equal.

$$\text{CP}^{\text{prod}}_{ct} = \text{COST}^{\text{dist}}_{ct} \qquad \forall c\in C_p, \quad \forall t\in T \qquad (75)$$

We do not expect any infeasibility to arise while running the nonintegrated models in the way described.

Example 2. The production model used in this example is the same as in example 1, except that we assume an infinite capacity of crude oil supply at the beginning of the refinery.

For the distribution model, we consider one refinery linked to one distribution center servicing eight demand zones. The eight products from the refinery model are being used: HSD, ISOG, SUPG, JP-1, FO1, FO2, FOVS, and LPG. Products are segregated into three groups: gasoline and diesel products (HSD, ISOG, SUPG) for gas stations (demand zone 1, 2, 3), fuel products (JP-1, FO1, FO2, FOVS for zones 4, 5, 6, 7), and liquefied gas (LPG for zone 8). The distribution center owns 36 trucks: 18 for the gasoline and diesel products, 18 for the fuel products, and 2 for the liquefied gas. Figure 16 shows the system. Product prices are given above in Figure 11, truck availability is given in Table 14 and driving times are given in Table 15. The driving cost is $35/hr. Tables 16 and 17 provide information of the storage data for the distribution center and the customers, respectively.

To illustrate the different situations where the integrated model performs better than when the two models are run separately, four variations of this example are presented.

Example 2.1. For this case the time horizon is 1 week. The initial inventory level at the distribution center is set equal to the safety inventory, and the inventory level at each customer is at their maximum capacity. In this particular case, the amount that should be produced and delivered every day is exactly the amount sold by the customers. In this case, the integration of the two models does not lead to any significant improvement ($2,350 or ~0.2%). This small difference is because the nonintegrated model is forced to satisfy the exact customer demand every day, while the integrated model has more flexibility to send products to the distribution center any day (see Tables 18 and 19). In both models, the distribution model delivers exactly the demand to each customer. Final results on profit are shown in Table 20.

Example 2.2. This second example illustrates the case where the integrated model achieves a better profit by producing less than the nonintegrated model because of a high inventory level at the distribution center and at each customer. The time horizon

Table 23. Results for Example 2.2

model	equ.	cont. var.	int. var.	CPU time	costs ($)	revenue ($)	profit ($)
production	5 859	6 783	0	0.5 s	15,032,016	0	
distribution	6 392	9 892	224	6.2 s	364,835	19,316,121	18,951,287
nonintegrated					15,396,851	19,316,121	3,919,271
production					13,630,216	0	
distribution					232,697	19,313,736	19,081,038
integrated	3 076	4 238	224	5.4 s	13,862,913	19,313,736	5,450,823

Table 24. Results for Example 2.3

model	CPU time	cost ($)	revenue ($)	profit ($)
production	0.5 s	14,599,175	0	
distribution	22.4 s	495,565	19,542,887	19,047,322
nonintegrated	22.9 s	15,094,740	19,542,887	4,448,147
production part		14,768,107	0	
distribution part		282,274	20,696,522	20,414,248
integrated	19.0 s	15,050,381	20,696,522	5,646,141

is one month. The inventory level for each customer is still at the maximum capacity, but this time, the inventory level at the distribution center is set to a high level (twice the safety stock level).

Tables 21 and 22 present the production plans for the nonintegrated and integrated models, respectively. (The two plans are represented week by week for clarity but the models are still run on a day by day basis.) One can see that in the integrated model, the production level is lower than in the nonintegrated model. Over the month, production level is 82 856 m^3 for the nonintegrated model while it is only 73 507 m^3 for the integrate model.

In the case of the nonintegrated model, the refinery still produces enough to satisfy all customer demand. On the other hand, the integrated model adjusts the production level to the high inventory level at the distribution center and at the customers. This forces a reduction in production so the integrated model saves almost $1,400,000 in crude oil purchasing over a month, which represents almost 3 days of production. This leads to a better profit (39% improvement) compared to the nonintegrated model (see results in Table 23).

Example 2.3. Example 2.3 is the opposite situation of example 2.2: the inventory level at the distribution center and at each customer is set at the minimum inventory level so the integrated model achieves a better profit by producing and selling more than the nonintegrated model. The time horizon is still 1 month,

The production plan for the nonintegrated model is the same as in the previous example, that is, just the amount requested by the customers. However, the integrated model produces more than the demand and, thus, is able to sell more products to the customers who replenish their low inventory level. The production cost is higher ($ 150,000 increase), but the revenue from the distribution part is much better ($ 1,300,000 increase), so the overall profit increases by 27% (Table 24).

Example 2.4. The last example illustrates the use of the distribution model over a 3 month′s time horizon. It represents a scenario where there is a high inventory of gasoline and diesel, while the inventory of fuel products is low (see Table 25). For instance, such a situation can arise if there is unexpected cold weather; in this case, the demand for fuel products will be high for heating, while people will tend to travel less and, thus,

Table 25. Initial Inventory—Example 2.4 (Safety Stock) (m^3)

dem. zone	HSD	ISOG	SUPG	JP-1	FO1	FO2	FOVS	LPG
DC	4.5(3.0)	3.8(3.0)	5.5(3.0)	5.6(6.0)	4.5(6.0)	2.0(2.4)	2.1(2.4)	3.2(3.0)
1	4.5(2.0)	3.0(1.5)	2.5(1.5)	0	0	0	0	0
2	4.0(2.0)	2.5(1.0)	3.0(1.5)	0	0	0	0	0
3	3.0(2.0)	2.0(1.0)	3.0(1.5)	0	0	0	0	0
4	0	0	0	1.0(0.8)	2.0(1.5)	1.0(1.0)	1.0(0.7)	0
5	0	0	0	2.0(1.5)	1.5(1.5)	1.0(1.0)	1.5(1.0)	0
6	0	0	0	1.5(1.5)	1.0(1.5)	0	1.0(1.0)	0
7	0	0	0	1.0(1.5)	1.0(1.5)	0	0.8(1.0)	0
8	0	0	0	0	0	0	0	2.0(1.5)

Table 26. Results for Example 2.4

model	equ.	cont. var.	int. var.	CPU time	costs ($)	revenue ($)	profit ($)
production	17 563	20 335	0	3.0 s	46,762,477	0	0
distribution	19 160	29 660	672	40.0 s	2,101,049	59,716,37	57,615,322
						157,615,322	
nonintegrated	36 723	49 995	672	43.0 s	46,762,477		10,852,845
production					46,489,080		
distribution					1,459,057	60,927,700	59,468,644
						60,927,700	
integrated	36 725	50 669	672	133.4 s	47,948,137		12,979,564

Table 27. Results for Example 3.1

model	equ.	cont. var.	int. var.	CPU time	costs ($)	revenue ($)	profit ($)
unloading	31 315	17 991	1 3160	0.7 s	57,587,098	0	
production	5 859	6 783	224	0.4 s	2,787,887	0	
distribution	6 392	9 892		8.7 s	7,132,640	95,598,965	88,466,325
nonintegrated				9.8 s	67,507,625	95,598,965	28,091,340
unloading					57,587,098	0	
production					3,129,603	0	
distribution					3,631,020	101,689,485	98,038,295
integrated	43 568	34 892	1 540	60.0 s	64,347,721	101,689,485	37,321,594

Table 28. Results for Example 3.2

model	equ.	cont. var.	int. var	CPU time	costs ($)	revenue ($)	profit ($)
unloading	737 525	384 987	2 676	16.6 min	223,378,641	0	
production	18 067	20 839	0	12 s	8,714,688	0	
distribution	19 160	29 660	672	34.4 s	11,828,685	265,727,385	253,898,705
non-integrated							21,805,376
unloading					198,565,292	0	
production					11,014,727	0	
distribution					5,848,465	269,081,255	263,232,790
Integrated	773,728	436,160	3,348	43.7 min	215,428,484	269,081,255	53,652,772

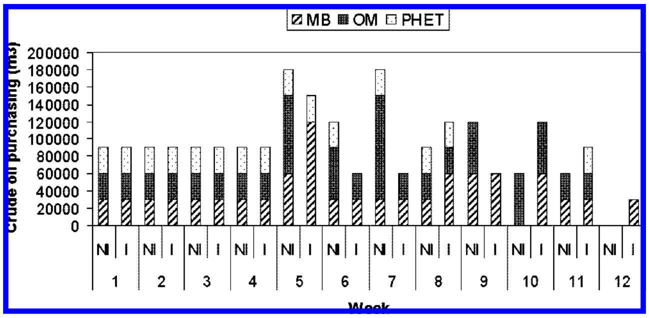

Figure 17. Crude oil purchasing plan (m^3). Fully integrated model vs nonintegrated model.

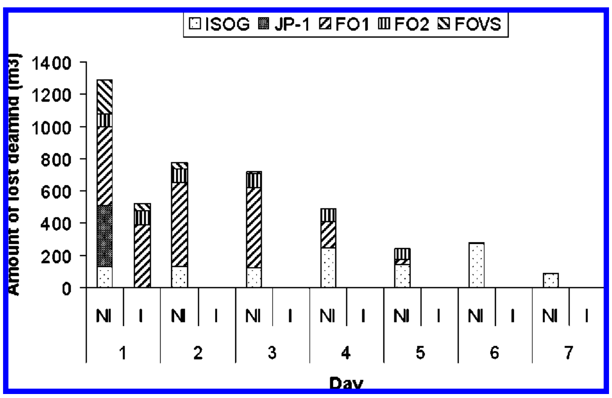

Figure 18. Amount of lost demand (m^3). Fully integrated model vs nonintegrated model.

demand for gasoline will be low. As in the previous examples, the integrated model achieves a better cooperation between the two parts, and leads to a 19.6% improvement compared to the separate case (Table 26).

6. Integrated Unloading, Production, and Distribution Model

The three models are linked together following the strategies presented in sections 4 and 5.

Data from Pongsakdi et al.[19] are used for the production model, and data from the example 2.4 are used for the distribution part. For the unloading model, data from the example 1.3 are used in the first example (1 month) while data from the example 1.4 are used for the second example (3 months).

In both examples, the methodology for the nonintegrated model follows exactly what have been described in the integration of unloading and production on one hand, and production and distribution on the other hand

(1) Run the production model with the customer demand, without considering the distribution part, and with a limited crude oil supply for the first month and assume an infinite crude oil supply for the rest of the time horizon.

(2) Run the unloading model to try to satisfy the crude oil requirement from the production plan. (a) If the unloading plan is feasible, then the production plan is feasible too, so go to step 3. (b) Otherwise, find a feasible unloading plan by relaxing the demand, that is, find a value x such that the unloading plan is feasible, then run the production model again with the crude oil supply found in the feasible unloading plan in order to determine a feasible production plan.

(3) Run the distribution model using the production output of the feasible production plan.

Example 3. We consider one refinery receiving crude oil from one docking station by a pipeline and sending the final products to five identical distribution centers by five pipelines.

Example 3.1. This first example covers a 1 month horizon. For the nonintegrated model, the unloading and production plans from example 1.3 are used. The production plan is suboptimal so the distribution plan is not optimal either. It has a penalty for unsatisfied demand of $205,270.

The integrated model gives a feasible solution in 1 CPU minute with a profit of $37,321,594, which is 32.9% better than when each model is solved individually. This comes from a better unloading plan but especially from a better production plan which takes into account the inventory level downstream (see example 2.4). The result is presented in the Table 27.

Example 3.2. This case is an application of the full model over a 3 months horizon. The main goal is to determine the crude oil purchasing plan which is the main source of profit improvement in this example. For the nonintegrated model, the unloading and production plans are the plans found in example 1.4, and then the distribution plan is computed using the initial inventory level of example 2.4. Because of the bad crude oil supply (crude supplies are depicted in Figure 17), the production plan is very suboptimal so the distribution plan has a profit 13% lower than when the production plan is optimal (see production plan of the nonintegrated model example 2.4). This results in a penalty for unsatisfied demand of $5,304,500, which happens entirely during the first week (see Figure 18 and Table 28).

In turn, the integrated model finds an optimal crude oil purchasing plan which costs $24,813,349 less than in the nonintegrated model and is well suited to produce what is needed by the customers. This leads to a good production and distribution plans so the penalty for unsatisfied demand is only $583,865.

Conclusions

The main point of this paper has been to establish that integrating the different parts of the refinery supply chain achieves better results than trying to solve each part in a sequential push or pull manner. This has been demonstrated through several examples showing that the integrated model guarantees (1) that the plan is feasible along the entire supply chain and (2) that the profit is optimized regarding the entire system and not only subparts of it.

The integrated model fully uses price variations for both crude oils and final products. However, price forecasting is a very difficult exercise in the petroleum industry, so the introduction of uncertainty would render the model more robust and reliable.

The difference in profit between the fully integrated and nonintegrated models is significantly high. Results pinpoint at the high reduction in penalties in the distribution portion of the supply chain. Even if those do not exist, or are lower because penalties prices are smaller or because data on demand accommodates better to results of a nonintegrated model, there are still significant changes in procuring different crudes at a smaller cost (10%, which is significant).

Finally, the integrated model needs to be applied to a larger more complex industrial case using some actual data (we made some estimations). Most petroleum companies own many refineries and tens or even hundreds of distribution centers. Moreover, the companies are constantly trading crude oils, intermediate, and final products among each others. So modeling the overall picture can be really challenging. In such a complex environment, it would be necessary to relax some of the constraints involving integer variables and nonlinearities, at the cost of losing some of the precision of the actual model for the benefit of computational time.

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Nomenclature.

Unloading Model.

Sets

$p \in P$ = set of parcels

$c \in C_o$ = set of crude oils

$i \in I$ = set of storage tanks

$t \in T$ = set of time periods

Parameters

IC_{ic} = 1 if tank i may contain crude oil c during time horizon, 0 otherwise

PT_{pt} = 1 if vessel containing parcel p can be at the docking station in period t, 0 otherwise

PS_p = size of parcel p

$\mathrm{FPT}^{\mathrm{U}}$ = upper limit flow rate between a parcel and a tank during a time period

$\mathrm{FU}^{\mathrm{L/U}}$ = lower and upper limit of flow rate to the pipeline during a time period

$\mathrm{VU}_i^{\mathrm{L/U}}$ = lower and upper capacity of tank i

D_{ct} = production requirement of crude oil c in time period t

SS_c = desired safety stock of crude oil c

SSP_{ct} = safety stock penalty for crude oil c at time period t (\$ per m^3)

CC_c = unit cost of crude oil$c$ in time period t (\$ per m^3)

CP_{ct} = perceived revenue per unit of crude oil $c$sent to the refinery in time period t (\$ per m^3)

Binary variables

$\mathrm{XP}_{\mathrm{pt}}$ = 1 if parcel p is connected to the SBM pipeline in time period t, 0 otherwise

XT_{it} = 1 if tank i is connected to the docking station in time period t, 0 otherwise

YT_{it} = 1 if tank i is connected to the refinery pipeline in time period t, 0 otherwise

PC_{pc} = 1 if parcel p is composed of crude oil c, 0 otherwise

PI_{pi} = 1 if parcel p can be connected to tank i sometime during the time horizon, 0 otherwise

Continuous Variables (Some Assume Only 0−1 Values Due to Constraints)

$f_{ict} = \mathrm{VCT}_{ict}/\mathrm{VU}_{it}$ = the volume fraction of crude oil c in tank i during time period t

XP_{pt} = 1 if parcel p is connected to the SBM pipeline in time period t, 0 otherwise

XL_{pt} = 1 if parcel p leaves the SBM pipeline in time period t, 0 otherwise

X_{pit} = 1 if both parcel p and tank i are connected to the SBM pipeline in time period t, 0 otherwise

Continuous Positive Variables

profit = objective function

FCPT_{cpit} = amount of crude c flowing from parcel p to tank i during time period t

FU_t = total amount of crude fed to the pipeline during period t

FCTU_{ict} = amount of crude c flowing from tank i to the pipeline during time period t

VT_{it} = total amount of crude in tank i at the end of time period t

VCT_{ict} = amount of crude c in tank i at the end of time period t

SC_{ct} = safety stock penalty for time period t

TF_p = time period in which parcel p first connects to the SBM

TL_p = time period in which parcel p is disconnected

Production Model.

Sets

$c \in C$ = set of commodities

q = set of properties

$u \in U$ = set of production units

$t \in T$ = set of time periods

U_c = set of units that produce commodity c

$\mathrm{QO}_{u,c}$ = set of properties of commodities c leaving unit u

C_p = set of commercial products

C_o = set of crude oils

C_{IA} = set of purchased intermediate

$\mathrm{UC}_{u,c}$ = set of ordered pairs of unit and commodity u,c that feeds unit u

$\mathrm{UO}_{u,c}$ = set of units that are fed by commodity c of unit u

CO_u = set of commodities leaving unit u

c_{tank} = set of crude oil storage tanks

CDU = set of crude distillation units

CRU = set of catalytic reforming units

NPU = set of naphtha pretreating units

HDS = set of hydrodesulphurization units

GSP = set of gasoline pool units

INT = set of gasoline intermediate tanks

AV_q = set of properties on volume basis

AW_q = set of properties on weight basis

Parameters

$\mathrm{pro}_{u,c,q}$ = Property q of commodity $c$from unit u

$\mathrm{px}_{c,q}$ = maximum property q of product c

$\mathrm{pn}_{c,q}$ = minimum property q of product c

$\mathrm{yield}_{c,c'}$ = percent of component c in crude oil c' (%)

$\mathrm{yield}_{u,c}$ = percent yield of commodity c from unit u (%)

$\mathrm{dem}_{c,t}$ = demand of product c in time period t (m^3)

ux_u = maximum capacity of unit u (m^3)

un_u = minimum capacity of unit u (m^3)

ox_c = maximum monthly purchase of crude oil c (m^3)

on_c = minimum monthly purchase of crude oil c (m^3)

stox_c = maximum storage capacity of product c (m^3)

$\mathrm{CP}_{c,t}$ = unit sale price of product c in time period t (\$/m^3)

$\mathrm{CC}_{c,t}$ = unit purchase price of crude oil c in time period t (\$/m^3)

$\mathrm{CI}_{c,t}$ = unit purchase price of intermediate c in time period t (\$/m^3)

$CL_{c,t}$ = unit of lost demand penalty for product c in time period t ($/m^3)
$density_u$ = density of feed to unit c (ton/m^3)
$fuel_u$ = percent energy consumption for unit u based on tFOE (%)
dis_c = percent discount from normal price (%)
ASU_c = maximum tanks capacity of product c
PO_{ucqt} = product properties q of crude c leaving unit u

Continuous Variables

$PO_{u,c,q,t}$ = property q of commodity c from unit u in time period t
$AF_{u,t}$ = amount of feed to unit u in time period t (m^3)
$AO_{u,c,t}$ = amount of outlet commodity c from unit u in time period t (m^3)
$A_{u,c,u',t}$ = amount of commodity c flow between unit u and unit u' in time period t (m^3)
$MANU_{c,t}$ = amount of product c produced in time period t (m^3)
$AC_{c,t}$ = amount of crude oil c refined in time period t (m^3)
$AI_{c,t}$ = amount of intermediate c added in time period t (m^3)
$AS_{c,t}$ = amount of product c stored in time period t (m^3)
$AL_{c,t}$ = amount of lost demand for product c in time period t (m^3)
$AD_{c,t}$ = amount of discount product c sold in time period t (m^3)
$burned_{c,t}$ = amount of product c burned in time period t (m^3)
$used_t$ = amount of fuel used in time period t(tFOE)
$sales_{c,t}$ = sales of product c in time period t (m^3)

Distribution Model.

Sets

$d \in D$ = set of demand zone, i.e. cluster of customers
$g(c) \in G$ = set of demand zone asking for products that can be transported in the same trucks (example g_1 are the gas stations, g_2 are the customers asking for fuel products, etc)
$c \in C_p$ = set of final products
$t \in T$ = set of time periods

Parameters

NBK_g = number of trucks available for delivery to group g
NBR = maximum number of tours that a truck can do during time period
$REC^{L/U}$ = lower and upper limit on products flowing from the refinery during a time period
$COST_{ct}$ = purchase cost of product c from the refinery at time period t
$VD_c^{L/U}$ = lower and upper capacity of product c at the distribution center
SS_c = desired safety stock of product c at the distribution center
SSP_{ct} = safety stock penalty for product c at time period t ($ per m^3)
int = interest rate per time period for inventory cost (dead capital)
PRI_{ct} = sale price of product c at time period t ($ per m^3)
TC^U = truck capacity
TIM_d = driving time needed to service demand zone d (hours)
MDT = available time per truck and per time period
DTC = driving cost ($ per hour)
$VOLU_{cd}$ = maximum storage capacity of product c for demand zone d
$VOLL_{cd}$ = safety inventory level of product c for demand zone $_d$
UDC_{ct} = penalty for unsatisfied demand of product c at time period t ($ per m^3)

Integer variables

Z_{dt} = number of tours servicing demand zone d in time period t

Continuous positive variables

profit = total profit over time horizon ($)
VD_{ct} = inventory level of product $c$at the distribution center at the end of time period t
IC_t = inventory cost at time period t
SSC_{ct} = distribution center safety stock penalty for product c ($)
$TSSC_t$ = total distribution center safety stock penalty for time period t ($)
REC_{ct} = amount of product p received from the refinery during time period t
DEL_{cdt} = amount of product c delivered to demand zone d during time period t
DT_{gt} = total driving time during time period t by trucks in g
TRC_t = transportation cost during time period t ($)
SOL_{cdt} = forecasted amount of product c sold (or used) by demand zone d in time period t
$SSCC_{cdt}$ = safety stock penalty for demand zone d for product c ($)
$TSSCC_t$ = total customer safety stock penalty for time period t ($)
VOL_{cdt} = inventory level of of product c for demand zone d in time period t
UDC_{cdt} = cost of unsatisfied demand for for demand zone d for product c ($)
$TUDC_t$ = total cost of unsatisfied demand for time period t ($)
VD_{ct} = inventory level at the distribution center of product c at time period t

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