

# Dissipativity-based Distributed Nonlinear Predictive Control for Cascaded Systems

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**Abstract:** Developing centralized controllers for large-scale systems, e.g. complex chemical processes, electrical power networks, is an important, yet challenging problem. Typically, it would be of advantage if one could divide the problem into smaller parts (or subproblems), and to opt for a decentralized or distributed alternative. On the other hand, due to physical interconnections between subsystems, it is often complicated to find such a solution. Additionally, performance loss or even instability of the overall system can arise if the decentralization is not done properly. It is, therefore, fundamental to develop suitable distributed/decentralized techniques in order to avoid these issues.

In this paper, we propose a decentralized dissipativity-based nonlinear model predictive control strategy for cascades of physically interconnected systems, e.g. cascades of hydroelectric power plants, multi-cell batteries. Under mild assumptions, we prove that the control method guarantees overall stability by independently enforcing stability of the local subsystems. This allows for efficient decentralized solutions. Furthermore, the local controllers only exchange limited information and no model of the neighbor systems is required, facilitating decoupled local controller design. Two interconnected water tanks are used to demonstrate the effectiveness of our approach.

*Keywords:* distributed control, nonlinear predictive control, dissipativity-based control, continuous time systems

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## 1. INTRODUCTION

One of the main reasons for which decentralized/distributed approaches have become interesting is the need of controlling large-scale systems, composed by many components or distributed on a wide geographical area. In this case, it is difficult to obtain a centralized controller, i.e. a regulator which supervises the complete system. This is due to the fact that such a controller needs to have complete reliable knowledge of the overall state of the plant as well as models of all parts—hard to achieve for a large system—; centralized solutions require great computational resources to be implemented in real-time—difficult even for nowadays powerful computers—; it can suffer from communication limitations or extreme structural complexity. A *dividi-et-impera* strategy, in which several local regulators are used instead of centralized one—see Šiljak (1978); Bakule (2008) and the references therein for a review—is the most promising solution to tackle such complex problems. The local controllers are responsible for regulating only a part of the overall system (or *local subsystems*); they have usually at disposition only *local measurements*, important variables and model, but they might be able to exchange pieces of information, e.g. current state, satisfied/unsatisfied constraints. However, interactions between the subsystems cannot be neglected since they might have great influence on the overall system

performance, e.g. contribute to obtain better solutions, or, they could destabilize the complete system.

In addition to the attention for decentralized and distributed systems, model predictive control (MPC) techniques, have become relevant for the industry. This is due to the fact that MPC can cope well with practical industrial requirements, such as handling nonlinear dynamics, hard/soft constraints and uncertainties, or providing optimality with respect to a desired performance cost (see Camacho and Bordons (2007); Rawlings and Mayne (2009)). Different MPC algorithms have been developed both for linear and nonlinear systems (NMPC), cf. sampled-data NMPC, Findeisen (2006); Findeisen et al. (2004), passivity-based MPC, Raff et al. (2004, 2007), and input-to-state-stability-based MPC, Limon et al. (2009); Lazar et al. (2008); Magni et al. (2006). See e.g. Findeisen et al. (2007); Magni et al. (2009); Rawlings and Mayne (2009), for overviews on recent results on MPC is presented. Although, MPC has shown to be effective in many practical applications, one of its main drawbacks is that it requires an accurate model of the system under control. As a consequence, it is hard to apply to large-scale systems, where an overall model can be difficult to obtain or solve in real-time. Therefore, decentralized and distributed MPC algorithms are of large interest, cf. Scattolini (2009); Rawlings and Mayne (2009); Dunbar (2007). For sake

of clearness, as inter alia presented in Scattolini (2009). With *decentralized* we refer to algorithms where the local regulators do not exchange any information, while with *distributed* the case when some information is exchanged in order to decide the right control action to apply. Other decentralized solutions were presented in Keviczky et al. (2004); Mercangöz and Doyle (2006); Richards and How (2004); Venkat et al. (2007); Alessio and Bemporad (2008); Dunbar (2007), where mainly attention on linear systems is paid. In Raimondo et al. (2007); Magni and Scattolini (2006), a decentralized NMPC solution for nonlinear discrete time systems is described, whereas in Liu et al. (2009) a decentralized Lyapunov NMPC algorithm for nonlinear continuous time systems is introduced. In Alessio and Bemporad (2008), stability conditions for distributed MPC under uncertain information exchange are presented.

In this paper, we consider systems which can be represented as a cascade of nonlinear input-affine physically interconnected continuous time systems, such for instance a cascade of interconnected tanks, or of continuous stirred tank reactors where endothermic reactions take place (see Figure 1). Each system  $\Sigma_i$  is controlled by a local controller  $C_i$ , in our case an NMPC controller. We assume that no model mismatch in the model used in the controllers with reference to the locally controlled subsystem, and that the controllers do not have a model of the overall system. Additionally, the controllers exchange information about the predicted output  $\bar{y}_i(t)$  of their corresponding local subsystem. Under rather mild conditions, we prove that by utilizing this distributed NMPC algorithm overall stability is achieved by enforcing local stability. Differently from standard approaches, which require the solution of a decentralized robust problem, e.g. min – max, max – min, tube-based, our solution exploits the cascade structure and passivity properties of the system to solve a sequence of merely minimization problems. Therefore, it is less computationally demanding, less conservative, and it can be easily applied to nonlinear systems, e.g. cascades of hydroelectric power-plants.

In the following section, we first give a formal detailed description of the problem under investigation; then, a short introduction on dissipativity-based NMPC is provided. In Section 3, the main results of this work are presented; finally, a small example is used to show the effectiveness of our approach for decentralized control.

## 2. PROBLEM STATEMENT

We consider a cascade system composed of  $n$  nonlinear input-affine systems  $\Sigma_i$ :

$$\dot{x}_i(t) = f_i(x_i(t)) + g_i(x_i(t))u_i(t) + y_{i-1}(t), \quad (1a)$$

$$y_i(t) = h_i(x_i(t)), \quad (1b)$$

where  $x_i \in \mathbb{R}^m$ ,  $u_i \in \mathbb{R}^m$ ,  $y_i \in \mathbb{R}^m$ , are respectively state, input, and output of  $\Sigma_i$ , while  $y_{i-1}(t)$  represents the effects of the system  $\Sigma_{i-1}$  on  $\Sigma_i$ , cf. Figure 1.

In this work, we assume that each subsystem satisfies the following assumptions.

*Assumption 1.* For each system  $\Sigma_i$ ,  $f_i(\cdot)$ ,  $g_i(\cdot)$ , and  $h_i(\cdot)$  are Lipschitz continuous, and without loss of generality the origins of the local subsystems are equilibrium points, i.e. for  $x_i = u_i = 0$ ,  $f_i(0) = 0$ .

*Assumption 2.* For  $u_i(t) = y_{i-1}(t) = 0$ ,  $\Sigma_i$  is zero-state detectable (ZSD), cf. Brogliato et al. (2007), i.e. the solution of  $\dot{x}_i(t) = f_i(x_i(t))$  is such that  $\lim_{t \rightarrow \infty} |x_i(t)| = 0$ , for  $y_i(t) = 0$ ,  $\forall t \geq 0$ .

Before introducing in details the problem under investigation, we recall here a few definitions, which are helpful for understanding the main result.

*Definition 3.* (Dissipative System). A system  $\Sigma$  with input  $u(\cdot)$  and output  $y(\cdot)$  is said to be *dissipative*, cf. Brogliato et al. (2007), if there exists a continuously differentiable function  $V(\cdot) \geq 0$  (*storage function*) such that

$$V(x(t)) - V(x(0)) \leq \int_0^t w(u(s), y(s)) ds$$

holds for every solution  $x(t)$  of  $\Sigma$ ,  $\forall x(0)$ ,  $t \geq 0$ , and all admissible controls. The function  $w(u, y) : \mathbb{U} \times \mathbb{Y} \rightarrow \mathbb{R}$  is the *supply-rate*,  $\int_{t_1}^{t_2} |w(u, x)| < \infty$ , for any  $t_1 < t_2 \in \mathbb{R}^+$ .

*Definition 4.* (Passive System). A system  $\Sigma$  with input  $u(\cdot)$  and output  $y(\cdot)$  is *passive* if there exists a continuously differentiable function  $V(\cdot) \geq 0$  such that the inequality

$$V(x(t)) - V(x(0)) \leq \int_0^t u(s)^T y(s) ds$$

holds for  $t \geq 0$ , i.e. it is a dissipative system with supply-rate function  $w(u, y) = u^T y$ .

*Definition 5.* (IF-OFP). A system  $\Sigma$  with input  $u(\cdot)$  and output  $y(\cdot)$  is called input-feedforward output-feedback passive (IF-OFP) if there exists a continuous function  $V(\cdot) \geq 0$  such that the inequality  $V(x(t)) - V(x(0)) \leq \int_0^t (u^T y - \nu u^T u - \rho y^T y) ds$ , holds for  $t \geq 0$ ,  $\nu \in \mathbb{R}$  and  $\rho \in \mathbb{R}$ . In this case, we say the system is IF-OFP( $\nu, \rho$ ).

Notice that in general finding a storage function for a system is difficult. This majorly depends on the structure of the problem under investigation. Additionally to Assumptions 1-2, we suppose that the following statements hold for each system  $\Sigma_i$ .

*Assumption 6.* For  $u_i(t) = y_{i-1}(t) = 0$ ,  $\Sigma_i$  is IF-OFP( $\nu_i, \rho_i$ ), i.e. for each system there exists a continuously differentiable local function  $V_i(\cdot) \geq 0$  such that  $\dot{V}_i \leq u_i(t)^T y_i(t) - \nu_i u_i(t)^T u_i(t) - \rho_i y_i(t)^T y_i(t)$ , where this represents the differential version of IF-OFP.

The ideas behind our work are based on the *passivity-based NMPC* approach firstly introduced in Raff et al. (2004, 2007). The basic idea is to exploit the cascade structure of the system such that each NMPC controller solves the optimal control problem (OCP):

$$\min_{u_i(\cdot)} \int_{t_k}^{t_k+T_p} F_i(\bar{x}_i(\tau), \bar{u}_i(\tau)) d\tau + E_i(\bar{x}_i(t+T_p)) \quad (2a)$$

$$\text{s.t. } \dot{\bar{x}}_i(t) = f_i(\bar{x}_i(t)) + g_i(\bar{x}_i(t))\bar{u}_i(t) + y_{i-1}(t), \quad (2b)$$

$$\bar{y}_i(t) = h_i(\bar{x}_i(t)), \quad \bar{x}_i(t_k) = x_i(t_k), \quad (2c)$$

$$\bar{x}_i(t) \in \mathbb{R}^m, \quad \bar{u}_i(t) \in \mathbb{R}^m, \quad \bar{y}_i(t) \in \mathbb{R}^m, \quad (2d)$$

$$\bar{x}_i(t+T_p) \in \Omega_i, \quad (2e)$$

$$\tilde{u}_i^T \tilde{y}_i - \rho_i \tilde{y}_i^T \tilde{y}_i - \nu_i \tilde{u}_i^T \tilde{u}_i \leq 0, \quad (2f)$$

where  $\tilde{u}_i = [y_{i-1}, \bar{u}_i]^T$ ,  $\tilde{y}_i = [y_{i-1}, \bar{y}_i]^T$ , are extended input/output vectors. The complete state  $x_i(t)$  is assumed

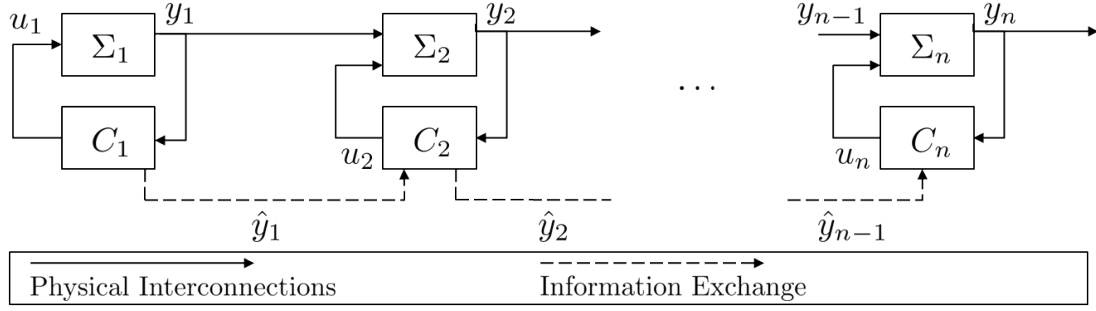


Fig. 1. Scheme of the cascade system considered in this work.

to be locally available. With  $\bar{\cdot}$  we denote the controller internal variables. The solution is an optimal control signal  $u^*(\cdot)$ , over the finite prediction horizon  $T_p$ , which is applied for the time-span  $[t_k, t_{k+1})$ , where  $t_k$  denotes the sampling instants. Closed loop stability is achieved by properly choosing the cost functional  $F_i(\cdot)$ , the terminal cost  $E_i(\cdot)$ , the terminal region  $\Omega_i \subset \mathbb{R}^m$ , and the prediction horizon  $T_p$ . The main difference with standard NMPC problems is the presence of condition (2f) (*dissipativity condition*), which is used to ensure local dissipativity with respect to both the control inputs and the physical interconnection with the other system.

Note that  $y_{i-1}(t)$  is required. In general, in similar situations, the physical interconnections  $y_{i-1}(t)$  are considered as external disturbances. Therefore, min-max or tube-based methods are used. These kinds of algorithms are, however, computationally demanding and challenging. The idea behind our method is to exploit the structure of the cascade interconnection in order to forward propagate the predicted information  $\bar{y}_i(t)$  to the following controller. In other words, once the first system solves the optimization problem, it provides the predicted information about the output to the following one, which then has everything to solve its local OCP. The operation is then repeated throughout the system cascade. Notice that our method requires the solution of a simpler/smaller minimization problem. Thus, it is less demanding than standard robust approaches, e.g. min-max or set-based methods, and less conservative, since it uses the information exchange to compensate for the physical interconnections. However, it requires a special system structure.

### 3. MAIN RESULTS

Before presenting the main results of this work, we introduce a few definitions and results about diagonal stability and quasi-dominant matrices.

**Definition 7.** (Quasi-dominant Matrix). A square matrix  $A$  is called *quasi-dominant* if there exists a positive diagonal matrix  $P = \text{diag}\{p_1, \dots, p_n\}$  such that  $a_{i,i}p_i \geq \sum_{j \neq i} |a_{i,j}|p_j, \forall i$ , respectively,  $a_{j,j}p_j \geq \sum_{i \neq j} |a_{j,i}|p_i, \forall j$ .

If  $P$  can be chosen as the identity matrix, then the matrix  $A$  is called *row- or column-diagonally dominant* (see Lady (1996)).

**Remark 8.** Notice that the definition of quasi-dominant matrix implies the elements on the main diagonal to be positive. A more general definition of quasi-dominant

matrix for which such condition is not required can be utilized instead (see Lady (1996)).

**Definition 9.** (Hurwitz Diagonally Stable). We say that a matrix is *Hurwitz diagonally stable* (Kaszakurewicz and Bhaya (2000)) if there exists a diagonal matrix  $P > 0$  such that  $A^T P + P A < 0$ . We refer to the space of Hurwitz diagonally stable matrices as  $\mathcal{D}_c(P)$ , or simply  $\mathcal{D}_c$ , where  $P$  is omitted whenever not necessary.

Additionally, the following results hold:

**Theorem 10.** (Kaszakurewicz and Bhaya (2000)). If  $-A$  is quasi-dominant, then  $A \in \mathcal{D}_c$ .

**Corollary 11.** (Taussky (1949)). Every symmetric quasi-dominant matrix is also in  $\mathcal{D}_c$ .

**Lemma 12.** (Kaszakurewicz and Bhaya (2000)). If a matrix  $A$  is in  $\mathcal{D}_c$ , then  $A^T \in \mathcal{D}_c$ .

As presented in the following theorem, diagonal stability is fundamental to guarantee overall stability of the complete cascade. In fact, as known from passivity theory, while the parallel of passive systems is passive, this is not in general the case for cascades. Thus, although the NMPC controllers enforce local dissipativity, additional conditions are required to guarantee stability of the whole cascade.

**Theorem 13.** (Distributed Dissipativity-Based NMPC). Consider the cascade of  $n \geq 2$  IF-OFP  $(\nu_i, \rho_i)$  systems  $\Sigma_i$ , with  $\nu_i, \rho_i \in \mathbb{R}$ , where the input  $u_i^*(t)$  is obtained from the solution of the local OCP (2). If  $\hat{\nu}, \hat{\rho} \in \mathbb{R}$  are such that the matrix

$$A = \begin{bmatrix} A_{uu} & \frac{1}{2}A_{uy} \\ \frac{1}{2}A_{uy}^T & A_{yy} \end{bmatrix} \quad (3)$$

where

$$A_{uu} = \begin{bmatrix} \nu_1 - \hat{\nu} & 0 & \dots & 0 \\ 0 & \nu_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \nu_n \end{bmatrix}, \quad A_{uy} = \begin{bmatrix} -1 & 0 & \dots & 1 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & -1 \end{bmatrix},$$

$$A_{yy} = \begin{bmatrix} \rho_1 + \rho_2 + \nu_2 - 1 & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \rho_{n-1} + \rho_n + \nu_n - 1 & 0 \\ 0 & \dots & 0 & \rho_n - \hat{\rho} \end{bmatrix}$$

is quasi-dominant. And, the  $n$  local OCP (2) supply-rate functions  $V_i \in C^1$  are such that

$$\dot{V}_i \leq \tilde{u}_i^T \tilde{y}_i - \rho_i \tilde{y}_i^T \tilde{y}_i - \nu_i \tilde{u}_i^T \tilde{u}_i \quad (4)$$

is feasible. Then,

i) The overall cascade system is IF-OFP( $\hat{\nu}, \hat{\rho}$ ),  $\hat{\nu}, \hat{\rho} \in \mathbb{R}$ ,  
 i.e.  $\exists V \in C^1$  such that

$$\dot{V} \leq u_1^T y_n - \hat{\nu} u_1^T u_1 - \hat{\rho} y_n^T y_n. \quad (5)$$

ii) And,  $\forall i \in \{1, \dots, n\}$ ,  $\lim_{t \rightarrow \infty} |x_i(t)| = 0$ , i.e. each system  $\Sigma_i$  is stable.

**Proof.** As in many NMPC proofs, we need to show two properties: feasibility, and convergence.

*Feasibility:* Assume there exists an initial time  $t_0 \in \mathbb{R}^+$  such that the OCPs (2) are feasible. If the problem admits an initial solution, from Assumption 6, and conditions (3)-(4), there always exists at least one  $u_i(t)$  such that the dissipativity constraint (2f) is verified at any following recalculation time. Thus, recursive feasibility follows similarly to Fontes (2001); Raff et al. (2007).

*Convergence:* To show convergence, and thus stability, we need to prove that  $\exists V \in C^1$  such that (5) holds.

We can consider the sum of the singular storage functions  $V_i$  as general storage function  $V$  for the overall cascade, i.e.  $V = \sum_{i=1}^n V_i$ . It follows that the system is IF-OFP( $\hat{\nu}, \hat{\rho}$ ) if and only if it has a supply-rate function which is upper-bounded by (5), i.e.

$$\begin{aligned} \sum_{i=1}^n \dot{V}_i &\leq u_1^T y_n - \hat{\nu} u_1^T u_1 - \hat{\rho} y_n^T y_n \\ \text{iff } \sum_{i=1}^n \dot{V}_i - u_1^T y_n + \hat{\nu} u_1^T u_1 + \hat{\rho} y_n^T y_n &\leq 0. \end{aligned} \quad (6)$$

For each system  $\Sigma_i$ , we can explicitly write the supply-rate as

$$\begin{aligned} \dot{V}_i &\leq \tilde{u}_i^T \tilde{y}_i - \nu \tilde{u}_i^T \tilde{u}_i - \rho_i \tilde{y}_i^T \tilde{y}_i \\ &= u_i^T y_i - \nu_i u_i^T u_i - \rho_i y_i^T y_i + (1 - \nu_i - \rho_i) y_{i-1}^T y_{i-1} \end{aligned}$$

(from the definition of virtual-inputs/outputs). This allows us to re-formulate (6) as

$$\begin{aligned} \sum_{i=1}^n (u_i^T y_i - \nu_i u_i^T u_i - \rho_i y_i^T y_i) + \sum_{i=1}^{n-1} (1 - \rho_{i+1} - \nu_{i+1}) y_i^T y_i \\ - u_1^T y_n + \hat{\nu} u_1^T u_1 + \hat{\rho} y_n^T y_n \leq 0. \end{aligned} \quad (7)$$

If we then define the vector  $z = [u_1, \dots, u_n, y_1, \dots, y_n]^T$ , then (7) can be expressed in the matricial form

$$z^T B z = z^T \begin{bmatrix} B_{uu} & \frac{1}{2} B_{yu} \\ \frac{1}{2} B_{uy}^T & B_{yy} \end{bmatrix} z,$$

where

$$B_{uu} = \begin{bmatrix} -\nu_1 + \hat{\nu} & 0 & \dots & 0 \\ 0 & -\nu_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & -\nu_n \end{bmatrix}, \quad B_{yu} = \begin{bmatrix} 1 & 0 & \dots & -1 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & 1 \end{bmatrix},$$

$$B_{yy} = \begin{bmatrix} -\rho_1 + 1 - \rho_2 + \nu_2 & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & -\rho_{n-1} + 1 - \rho_n + \nu_n & 0 \\ 0 & \dots & 0 & -\rho_n + \hat{\rho} \end{bmatrix}.$$

From Theorem 10, if there exists  $\hat{\nu}, \hat{\rho}$  such that that  $-B = A$  is quasi-dominant, then  $B$  is Hurwitz diagonally stable. It follows that the supply-rate function  $V$  is negative definite and the overall cascade is dissipative, or IF-OFP( $\hat{\nu}, \hat{\rho}$ ). From Assumptions 2 and 6, together with the dissipativity condition (2f), since a zero-state detectable IF-OFP system is stable, it follows that

$$\lim_{t \rightarrow \infty} |x_i(t)| \rightarrow 0.$$

*Remark 14.* Notice that when the cascade is composed by a large number of systems, dissipativity can be tested in terms of the feasibility of the LMI

$$B_{uu} < 0, \text{ and} \quad (8a)$$

$$B_{yy} - \frac{1}{2} B_{uy}^T \cdot B_{uu}^{-1} \cdot \frac{1}{2} B_{uy} < 0, \quad (8b)$$

where  $\hat{\nu}, \hat{\rho} \in \mathbb{R}$  can be arbitrary chosen.

*Remark 15.* Notice that the asymptotic stability is enforced by the dissipativity condition (2f), while the cost function (2a) plays merely an "economic role", i.e. we can arbitrary choose (2a) such that the optimal desired goal is achieved.

#### 4. SIMULATION EXAMPLE

To underline the effectiveness of our approach, in this section we consider a simple example where two tank systems  $\Sigma_1$  and  $\Sigma_2$  are physically interconnected. This can be seen as the simplified model of a cascade of hydroelectric power plants, where the level of the water in the basins must maintain a specific value. Although only two systems are considered, the example can be easily generalized to a sequence of  $n$  interconnected tanks. The overall cascade  $\Sigma$  is represented by the following equations:

$$\Sigma \begin{cases} \Sigma_1 \begin{cases} \dot{x}_1(t) = -\frac{a_1}{A_1} \sqrt{2gx_1(t)} + \frac{1}{A_1} u_1(t), \\ y_1(t) = x_1(t) + u_1(t), \end{cases} \\ \Sigma_2 \begin{cases} \dot{x}_2(t) = -\frac{a_2}{A_2} \sqrt{2gx_2(t)} + \frac{1}{A_2} u_2(t) + \frac{a_1}{A_2} y_1(t), \\ y_2(t) = x_2(t) + 2u_2(t), \end{cases} \end{cases}$$

where  $a_i, A_i, g$ , are positive constant values;  $x_i \in \mathbb{R}^+$ ,  $u_i \in \mathbb{R}$ ,  $y_i \in \mathbb{R}$  are the state, the input and the output, respectively. In particular,  $A_i$  represent the tank cross sections;  $a_i$  are the sections of the outgoing pipes; and  $g = 981 \text{ cm/sec}^2$  is the gravitational constant. The interconnection between the two tanks is given by the term  $\frac{a_1}{A_2} y_1(t)$ . It easy to show that the function  $V_i(x_i(t)) = \frac{A_i}{2} x_i(t)^2$  is a supply-rate function for each system, when the interconnections are neglected. In particular, we have

$$\begin{aligned} \dot{V}_i(x_i(t)) &= A_i x_i(t) \left( -\frac{a_i}{A_i} \sqrt{2gx_i(t)} + \frac{1}{A_i} u_i(t) \right) \\ &= u_i(t) x_i(t) - \underbrace{a_i \sqrt{2gx_i(t)} x_i(t)}_{\geq 0} \\ &\leq u_i(t) x_i(t) \\ &= u_i(t) y_i(t) - \nu_i u_i(t)^2, \end{aligned}$$

with  $\nu_1 = 1$ ,  $\nu_2 = 2$ , respectively for  $\Sigma_1$ , and  $\Sigma_2$ . This means that each system is dissipative with IF-OFP( $\nu_i$ ,

0). From Theorem 13, we can test the dissipativity of the system by checking the quasi-dominance of the matrix

$$-B = \begin{bmatrix} (1-\hat{\nu}) & 0 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 2 & 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 & 1 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 & -\hat{\rho} \end{bmatrix}. \quad (9)$$

From (9), we therefore obtain that the overall cascade is dissipative with IF-OFP( $\hat{\nu}$ ,  $\hat{\rho}$ ),  $\hat{\nu} \leq -2$ ,  $\hat{\rho} \leq -1$ ; that there exists two local dissipativity-based NMPC controllers which stabilize the cascade system; and, that the origin of the overall cascade is asymptotically stable. In our case, given  $A_1 = 50 \text{ cm}^2$ ,  $A_2 = 30 \text{ cm}^2$ ,  $a_1 = 0.2 \text{ cm}^2$ ,  $a_2 = 0.1 \text{ cm}^2$ , our objective is to obtain the following level profiles:

$$x_{1r} = \begin{cases} 14 \text{ cm for } t \in [0, 45] \text{ sec} \\ 10 \text{ cm for } t \in [45, 90] \text{ sec} \end{cases}$$

$$x_{2r} = \begin{cases} 12 \text{ cm for } t \in [0, 30] \text{ sec} \\ 10 \text{ cm for } t \in [30, 60] \text{ sec} \\ 12 \text{ cm for } t \in [60, 90] \text{ sec} \end{cases}$$

respectively in Tank 1, and in Tank 2, by using the local dissipativity-based controllers presented before. It is assumed that in the first tank the initial level is  $x_{10} = 8 \text{ cm}$ , while on the second one  $x_{20} = 6 \text{ cm}$ . Both controllers use the cost function

$$\int_t^{t+T_p} (500(x_i(\tau) - x_{ir})^2 + u_i^2(\tau)) d\tau,$$

with  $T_p$  equal to 10 sec. For simplicity of implementation, instead of exchanging the complete output trajectory  $y_1(t)$ , only the  $|y_1(\tau)|_\infty$ ,  $\tau \in [t, t + T_p]$  is used. This has the advantage of reducing the information exchange and simplifying the coding of the algorithm. The corresponding results for the two tanks are presented in Figure 2. As shown, the two controllers behave qualitatively well and can easily and independently steer the two tanks to the desired set-point in short time. Although for such a small system, quantitatively might not be advantageous to use such a distributed solution, if the number of systems is large, finding a centralized solution would be very difficult, if not almost impossible. In this situation, solving  $n$  small problems is preferable than solving a big one.

## 5. CONCLUSIONS AND FUTURE WORK

In this paper, we presented a distributed dissipativity-based predictive control solution to asymptotically stabilize a cascade of interconnected systems. The main idea is to use a *dividi-et-impera* approach: instead of solving the overall problem by using a centralized solution — difficult to realize for large systems —, the control problem is split into several smaller subsystems. Each one is then controlled by a local dissipativity-based NMPC controller, which utilizes an additional *dissipativity condition* to enforce local stability under the influence of the other systems. It was proved that by utilizing the formerly described framework, under rather mild conditions, dissipativity of the overall cascade is obtained. The distributed dissipativity-based NMPC possesses computational

advantages. In general, computationally demanding robust methods such as min – max or set-based approaches are required for this kind of problems. These solutions are, however, typically conservative and difficult to solve in real-time. Our algorithm, instead, requires the solution of  $n$  small (in terms of states to be considered) simple minimization problems that can be efficiently solved in real-time. A two-tank system was used to explain the effectiveness of the proposed methodology.

Future work will consider more general interconnecting structures, not limited to cascade systems. Additionally, direct study about the influence of the exchanged information on the closed-loop performance is of interest. Furthermore, the computational delays as well as information uncertainties, e.g. information losses of delayed information, should be considered in order to improve its efficacy.

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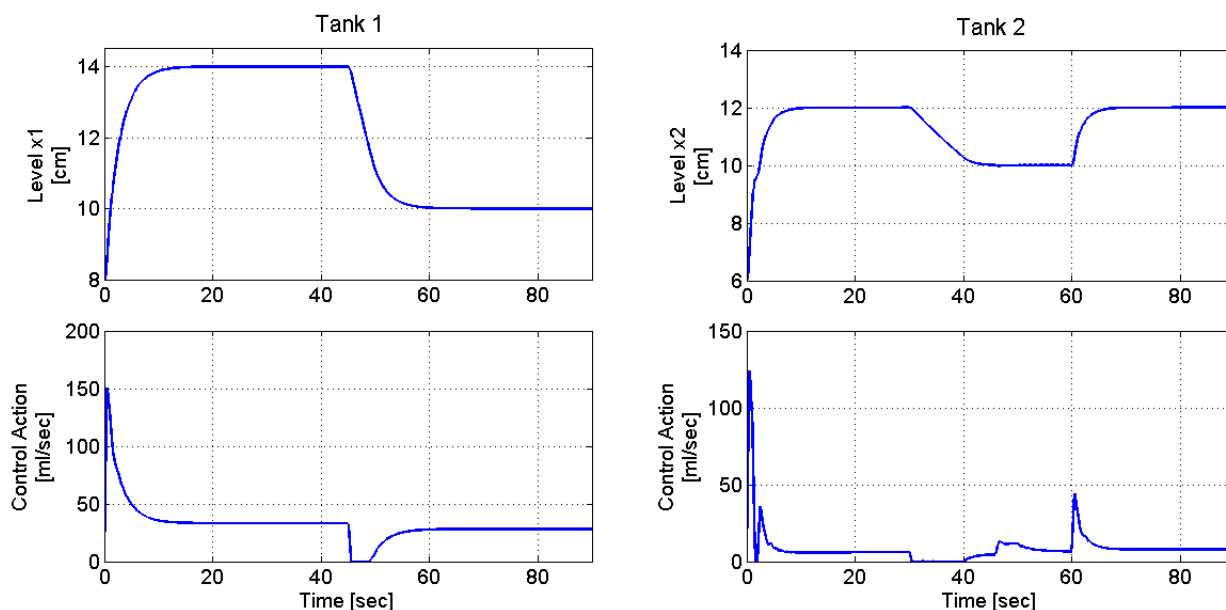


Fig. 2. Simulation results for the two-tank system.

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