Revenue Optimization in the Generalized Second-Price Auction

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joint work with David R. M. Thompson
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Introduction

Despite years of research into novel designs, search engines have held on to (quality-weighted) GSP.

Question

How can revenue be maximized within the GSP framework?

Various (reserve price; squashing) schemes have been proposed.

We do three kinds of analysis:

- theoretical: single slot, Bayesian
- computational, perfect information: enumerate all pure equilibria; consider best and worst
- computational: consider the equilibrium corresponding to a DS truthful mechanism with the appropriate allocation rule
Outline

1. Model and auctions
2. Theoretical analysis, single-slot auctions
3. What happens in the multi-slot case?
4. Equilibria corresponding to DS truthful mechanisms
Modeling advertisers

Definition (Varian’s model [Varian 07])

Each advertiser $i$ has a valuation $v_i$ per click, and quality score $q_i$. In position $k$, $i$’s ad will be clicked with probability $\alpha_k q_i$, where $\alpha_k$ is a position-specific click factor.
“Vanilla” GSP

- rank by \( b_i q_i \), charge lowest amount that would preserve position in the ranking.

1 slot, 2 bidders, quality scores \( q_1 = 1 \) and \( q_2 = 0.5 \):
GSP with Squashing

- rank by $b_i(q_i)^s$, $s \in [0, 1]$ [Lahaie, Pennock 07].
  - $s = 1$: vanilla GSP
  - $s = 0$: no quality weighting
- used in practice by Yahoo!, according to media reports

1 slot, 2 bidders, quality scores $q_1 = 1$ and $q_2 = 0.5$, $s = 0.19$. 
GSP with unweighted reserves (UWR)

- Vanilla GSP with global minimum bid and payment of $r$
  - UWR was common industry practice; now replaced by QWR.

1 slot, 2 bidders, quality scores $q_1 = 1$ and $q_2 = 0.5$, $r = 0.549$. 
GSP with quality-weighted reserves (QWR)

- Vanilla GSP with per-bidder minimum bid and payment $r/q_i$
- UWR was common industry practice; now replaced by QWR.

1 slot, 2 bidders, quality scores $q_1 = 1$ and $q_2 = 0.5$, $r = .375$. 
1 slot, 2 bidders, quality scores $q_1 = 1$ and $q_2 = 0.5$, $r = 0.505$, $s = 0.32$. 

**GSP with unweighted reserves and squashing (UWR+sq)**
GSP: quality-weighted reserves and squashing ($\text{UWR}+\text{sq}$)

1 slot, 2 bidders, quality scores $q_1 = 1$ and $q_2 = 0.5$, $r = 0.472$, $s = 0.24$. 
Considering Varian’s valuation model, our main findings:

- **QWR** is consistently the lowest-revenue reserve-price variant, and substantially worse than UWR.

- **Anchoring**: a new GSP variant that is provably optimal in some settings, and does well in others

- First systematic investigation of the interaction between reserve prices and squashing

- First systematic investigation of the effect of equilibrium selection on the effectiveness of revenue optimization
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Revenue-optimal position auctions

- The auctioneer is selling impressions. A bidder’s per-impression valuation is $q_i v_i$, where:
  - the auctioneer knows $q_i$
  - the auctioneer knows the distribution from which $v_i$ comes

- Thus, even if per-click valuations are i.i.d., each bidder has a different per-impression valuation distribution, and the seller knows about those differences.

- Strategically, it doesn’t matter how $q$’s are distributed, because it is impossible for a bidder to participate in the auction without revealing this information.
Proposition

Consider any one-position setting where each agent $i$'s per-click valuation $v_i$ is independently drawn from a common distribution $g$. If $g$ is regular, then the optimal auction uses the same per-click reserve price $r$ for all bidders.

Proof.

- Because $g$ is regular, we must maximize virtual surplus.
- $i$’s value per-impression is $q_i v_i$.
- Transforming $g$ into a per-impression valuation distribution $f$ gives: $f(q_i v_i) = g(v_i)/q_i$ and $F(g_i v_i) = G(v_i)$.
- Substituting into the virtual value function gives:

$$
\psi_i(q_i v_i) = q_i \left( v_i - \frac{1 - G_i(v_i)}{g_i(v_i)} \right)
$$

- Optimal per-click reserve $r_i$ is solution to $\psi_i(q_i r_i) = 0$, which is independent of $q_i$. 

Revenue Optimization in the GSP

Leyton-Brown (joint work with David Thompson)
Definition (Anchoring GSP)

Bidders face an unweighted reserve $r$, and those who exceed it are ranked by $(b_i - r)q_i$.

Proposition

When per-click valuations are drawn from the uniform distribution, anchoring GSP is optimal.

Revenue Optimization in the GSP

Leyton-Brown (joint work with David Thompson)
### Optimizing GSP variants by grid search: uniform, 2 bidders

<table>
<thead>
<tr>
<th>Auction</th>
<th>Revenue ($\pm 1e-5$)</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCG/GSP</td>
<td>0.208</td>
<td>—</td>
</tr>
<tr>
<td>Squashing</td>
<td>0.255</td>
<td>$s = 0.19$</td>
</tr>
<tr>
<td>QWR</td>
<td>0.279</td>
<td>$r = 0.375$</td>
</tr>
<tr>
<td>UWR</td>
<td>0.316</td>
<td>$r = 0.549$</td>
</tr>
<tr>
<td>QWR+Sq</td>
<td>0.321</td>
<td>$r = 0.472, s = 0.24$</td>
</tr>
<tr>
<td>UWR+Sq</td>
<td>0.322</td>
<td>$r = 0.505, s = 0.32$</td>
</tr>
<tr>
<td>Anchoring</td>
<td>0.323</td>
<td>$r = 0.5$</td>
</tr>
</tbody>
</table>

- Anchoring’s $r$ agrees with [Myerson 81] and QWR’s with [Sun, Zhou, Deng 11].
- Optimal parameters for other variants don’t correspond to recommendations from the literature.
Optimal auction for the log-normal distribution

Anchoring is not always optimal
(but perhaps it is always a good approximation?)

Optimal auction for log normal, 1 slot, 2 bidders, quality scores
$q_1 = 1$ and $q_2 = 0.5$. Anchoring shown for comparison.
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Revenue Optimization in the GSP

Leyton-Brown (joint work with David Thompson)
Computing equilibria

- Action-graph games (AGGs) exploit structure to represent games in exponentially less space than the normal form [Bhat, LB 04; Jiang, LB 06; Jiang, LB, Bhat 11].
- Games involving GSP and Varian’s preference model have such structure [Thompson, LB 09].
- Heuristic tree search can enumerate all pure-strategy Nash equilibria of an AGG [Thompson, Leung, LB 11].
Investigating multiple slots with grid search

- Leverage AGGs to consider more than a single slot, and to examine different equilibria of GSP variants to determine impact of equilibrium selection
  - Sample perfect-information games from the distribution over values and quality scores
    - 5 bidders; 26 bid increments each; 3 slots; uniform valuations
  - Enumerate pure-strategy equilibria
  - Consider statistics over their best and worst (conservative) NE.
- Identify optimal parameter settings by performing fine-grained grid search.
Any reserve scheme **dramatically improves** vanilla GSP’s worst-case revenue (look at reserves of $0$).

Optimal **unweighted reserves** are higher than quality-weighted.

High bidding can do the work of high reserve prices. Thus, worst-case reserve prices tend to be higher than best case.
Equilibrium Selection and Squashing

- Squashing can improve revenue in best- and worst-case equilibrium. (Recall: $s = 1$ is vanilla GSP.)
- Smaller impact, lower sensitivity than reserve prices.
- Gap between best and worst is consistently large ($\sim 2.5 \times$).
Comparing variants optimized for best/worst case

<table>
<thead>
<tr>
<th>Auction</th>
<th>Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vanilla GSP</td>
<td>3.814</td>
</tr>
<tr>
<td>Squashing</td>
<td>4.247</td>
</tr>
<tr>
<td>QWR</td>
<td>9.369</td>
</tr>
<tr>
<td>Anchoring</td>
<td>10.212</td>
</tr>
<tr>
<td>QWR+Sq</td>
<td>10.217</td>
</tr>
<tr>
<td><strong>UWR</strong></td>
<td>11.024</td>
</tr>
<tr>
<td><strong>UWR+Sq</strong></td>
<td>11.032</td>
</tr>
</tbody>
</table>

Worst-case equilibrium

<table>
<thead>
<tr>
<th>Auction</th>
<th>Revenue</th>
</tr>
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<tbody>
<tr>
<td>Vanilla GSP</td>
<td>9.911</td>
</tr>
<tr>
<td>Squashing</td>
<td>10.820</td>
</tr>
<tr>
<td>QWR</td>
<td>11.534</td>
</tr>
<tr>
<td>UWR</td>
<td>11.686</td>
</tr>
<tr>
<td><strong>Anchoring</strong></td>
<td>12.464</td>
</tr>
<tr>
<td>QWR+Sq</td>
<td>12.627</td>
</tr>
<tr>
<td><strong>UWR+Sq</strong></td>
<td>12.745</td>
</tr>
</tbody>
</table>

Best-case equilibrium

- **Worst case**: 2-way tie (UWR+Sq, UWR)
- **Best case**: 3-way tie (UWR+Sq, QWR+Sq, Anchoring)
- UWR’s worst case is better than QWR’s best case.
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Equilibrium Selection

With vanilla GSP, it’s common to study the equilibrium that leads to the efficient (thus, VCG) outcome. Many reasons why this is an interesting equilibrium:

- Existence, uniqueness, polytime computability [Aggarwal et al 06]
- Envy-free, symmetric, competitive eq [Varian 07; EOS 07]
- Impersonation-proof [Kash, Parkes 12]
- Doesn’t predict that GSP gets more revenue than Myerson (“Non-contradiction criterion”) [ES 10]

Analogously, can compute the equilibrium corresponding to a DS truthful mechanism with the appropriate allocation rule.

- see previous analyses of squashing [LP 07] and reserves [ES 10].
Distributions

For these experiments, we used two distributions:

- **Uniform** \( v_i \)'s drawn from uniform \((0, 25)\); \( q_i \)'s drawn from uniform \((0, 1)\).

- **Log-Normal** \( q_i \)'s and \( v_i \)'s drawn from log-normal distributions; \( q_i \) positively correlated with \( v_i \) by Gaussian copula. (Similar to [LP07]; new parameters based on personal communication.)

We compute equilibrium following recursion of [Aggarwal et al 06]. We optimize parameters by grid search.
## Revenue across GSP variants, optimal parameters

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<tr>
<td>VCG</td>
<td>7.737</td>
</tr>
<tr>
<td>Squashing</td>
<td>9.123</td>
</tr>
<tr>
<td>QWR</td>
<td>10.598</td>
</tr>
<tr>
<td>UWR</td>
<td>12.026</td>
</tr>
<tr>
<td>QWR+Sq</td>
<td>12.046</td>
</tr>
<tr>
<td>Anchoring</td>
<td>12.2</td>
</tr>
<tr>
<td>UWR+Sq</td>
<td>12.220</td>
</tr>
</tbody>
</table>

### Uniform distribution

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<tbody>
<tr>
<td>VCG</td>
<td>20.454</td>
</tr>
<tr>
<td>QWR</td>
<td>48.071</td>
</tr>
<tr>
<td>Squashing</td>
<td>53.349</td>
</tr>
<tr>
<td>QWR+Sq</td>
<td>79.208</td>
</tr>
<tr>
<td>UWR</td>
<td>80.050</td>
</tr>
<tr>
<td>Anchoring</td>
<td>80.156</td>
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<tr>
<td>UWR+Sq</td>
<td>81.098</td>
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### Log-Normal Distribution

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Reserve Prices

- All three reserve-based variants (anchoring, QWR and UWR) provide substantial revenue gains (compare to reserve 0).
- Anchoring very slightly better than UWR; both substantially better than QWR.
Squashing + UWR

- Adding squashing to UWR provides small marginal improvements (compare to $s = 1$) and does not substantially affect the optimal reserve price.
Squashing + QWR

- Adding squashing to QWR yields big improvements (compare to $s = 1$); high sensitivity.
- But, the higher the squashing power ($s \to 0$), the less reserve prices are actually weighted by quality.
- Log-normal: optimal parameter setting ($s = 0.0$) removes quality scores entirely and is thus equivalent to UWR.

Revenue Optimization in the GSP

Leyton-Brown (joint work with David Thompson)
Does squashing help QWR via reserve or ranking?

Squashing applied to reserve only (log normal)

- Applying squashing only to reserve prices can dramatically increase QWR’s revenue (compare to $s = 1$).
  - However, there has to be a lot of squashing (i.e., $s$ close to 0)
  - optimal reserve is very dependent on squashing power
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  - However, there has to be a lot of squashing (i.e., $s$ close to 0)
  - optimal reserve is very dependent on squashing power
  - optimal parameter setting is $s = 0$: identical to UWR
- Applying squashing **only to ranking**, the marginal gains from squashing over QWR (with optimal reserve) are very small.
Because equilibrium computation is cheap, we can scale up.

Top 4 mechanisms are still nearly tied. Squashing and QWR are consistently below.

As \( n \) increases, squashing gains on QWR.

For log normal, squashing substantially outperforms QWR.
Conclusions

We optimized revenue in GSP-like auctions under Varian’s valuation model, conducting three different kinds of analysis.

- QWR was consistently the lowest-revenue reserve-price variant, and substantially worse than UWR.
- Anchoring does well; optimal in simple settings
- Equilibrium selection: vanilla GSP, squashing have big gaps between best and worst case
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Why do search engines prefer QWR to UWR? Possible explanations:
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- Analysis should consider cost of showing bad ads
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Why do search engines prefer QWR to UWR? Possible explanations:

- Whoops—they should use UWR.
- Analysis should consider long-run revenue
- Analysis should consider cost of showing bad ads
- Actually, they do some other, secret thing, not QWR.