Hybrid SADOA/TDOA mobile positioning for cellular networks

R.-T. Juang, D.-B. Lin and H.-P. Lin

Abstract: A signal attenuation difference of arrival (SADOA) scheme is proposed to combine with the time difference of arrival (TDOA) method for mobile location estimation. On the basis of ratio of distances between the mobile and base stations derived from differences of signal attenuations, each SADOA measurement yields a circle on which the mobile may lie. Meanwhile, each TDOA measurement defines a hyperbola on which the mobile may reside. The proposed hybrid SADOA/TDOA scheme uses Taylor-series expansion to linearise the circles and hyperbolas and iteratively computes the mobile position based on least-squares estimation. Without perfect path loss modelling and hardware modification, the proposed scheme reduces location errors compared with either technique separately. Simulations demonstrate encouraging performance with 50% improvement over the conventional TDOA method in shadowing and non-line-of-sight propagation environments.

1 Introduction

Mobile location is used to calculate unknown mobile station (MS) position from measurements based on signals transmitted from or received by base stations (BS) of known position. It could support many applications, such as emergency services, roadside assistance, navigation and so on. Industry analysts have viewed location-based services as a multi-billion dollar market waiting just around the corner [1, 2].

The dominant location techniques can be divided into five categories. The first category is the global positioning system (GPS) [3], which provides accurate positioning but fails when satellite signals are blocked, for example in a setting when the MS is in indoors or in urban canyons. The second category estimates the MS location based on received signal strength (RSS) measurements [4, 5]. This approach uses the relationship between the RSS and the distance from the MS to BSs. Unfortunately, the accuracy of this approach is not good enough due to the complex propagation mechanisms. The third category uses the angle-of-arrival (AOA) of a signal from the MS at several BSs [6, 7]. Each AOA estimate can be used to draw a line of bearing (LOB) from the BS to MS; multiple LOBs intersect at the estimated MS location. This technique requires no time synchronisation between stations and
can work with as few as two BSs. However, this method requires a complex antenna system and suffers from non-line-of-sight (NLOS) propagation.

The fourth category measures the time of arrival (TOA) [2, 8] or time difference of arrival (TDOA) [9, 10]. Since electromagnetic waves propagate at the speed of light, each TOA defines a circle centred on the BSs. The MS position then can be determined at intersection of circles. The ability to perform TOA measurements implies full-network synchronisation. This requirement can be relaxed by replacing TOA measurement with TDOA. Each TDOA measurement yields a hyperbola. The solution to TDOA equations is usually obtained by linearising the equations via a Taylor-series expansion, which requires an initial location guess and may suffer from the convergence problem if the initial guess is not accurate enough. Similar to AOA technique, NLOS propagation is a potential disadvantage of TDOA method. The fifth category identifies the MS location by matching the received signal signatures with the entries stored in a database at the network [11, 12]. The location accuracy depends strongly on channel variations and the size of the database, which is time-consuming for construction and update. This ‘fingerprint’ technique is attracting increasing attention for indoor applications, for which database management is easier than in wide outdoor areas.

Each location method has strengths and weaknesses and most of them require multiple BSs to work at the same time. Clearly, adding more curves is expected to yield smaller location errors and therefore the extra information from combining techniques will result in an accuracy advantage even without increasing the number of BSs. Therefore hybrid techniques, such as hybrid RSS/TOA [13], hybrid TOA/TDOA [14], hybrid TOA/AOA [15], hybrid TDOA/AOA [16] and hybrid GPS/wireless network [17], have been suggested in the literature. Because TDOA method is now considered the leading candidate for any future location system, this paper proposes a hybrid signal attenuation difference of arrival (SADOA)/TDOA location scheme. SADOA measurement, derived from the differences of signal attenuations, is the ratio of distances between the MS and BSs. Each SADOA measurement yields a circle on which the mobile may lie. The circles intersect at the estimated mobile position. Since cellular networks provide measurement reports about signal strength received from serving and neighbouring BSs for managing radio resources, for example, the RSSs are reported with a period of 0.48 s from MS to network for GSM systems, the introduction of SADOA measurement increases no additional air interface traffic. By combining SADOA method with TDOA method, this paper shows that the proposed hybrid scheme outperforms the conventional TDOA method without hardware modifications to currently available handsets, whereas the extra computation loading at the network can be relieved by using modern powerful computation machines.
2 Proposed scheme

This section gives the proposed SADOA method, reviews the TDOA method and presents the proposed hybrid SADOA/TDOA scheme.

2.1 SADOA method

Path loss and shadowing (fast fading is ignored because it can be averaged out) attenuate the signal power. Path loss basically increases with the signal travel distance. Shadowing results from differences in levels of clutter along the wave travelling path, causing variations with respect to the nominal value given by path loss models. Shadowing is generally assumed to be a lognormal distributed random process. On the basis of Cost-231 Hata model [18], a generalised form for signal attenuation, A, between the BS and MS, separated by d in a large city, can be modelled as

\[ A(dB) = k_1 + k_2 \log_{10} f + k_3 \log_{10} h_b + 10n \log_{10} d + k_4 \log_{10}(k_5 h_m)^2 + u \]  

(1)

where \( n = (k_6 + k_7 \log_{10} h_b)/10 \) represents the path loss exponent ranging from two to four, \( k_1, k_2, k_3, k_4, k_5, k_6 \) and \( k_7 \) denote different constants in the same clutter type of environment, \( f \) is the carrier frequency, \( h_b \) and \( h_m \) are, respectively, the heights of BS and MS, and \( u \) is a zeromean Gaussian random variable.

Consider a two dimension scenario where a MS whose location is being estimated connects with \( N \) BSs denoted as \( BSi \) and located at \((x_i, y_i)\), \( i = 1, \ldots, N \). Assume that all BSs operate at the same frequency band, have similar height, and are in the same type of clutter environment so that the parameter sets, \( \{ k_1, k_2, k_3, k_4, n \} \), are identical, then the difference between the attenuations of signals from \( BSi \) and \( BSj \) has the form

\[ A_i - A_j = 10 \cdot n \cdot \log_{10}(d_i/d_j) + (u_i - u_j) \]  

(2)

where \( d_i \) and \( d_j \) are the distances between the MS and BSs for \( BSi \) and \( BSj \), respectively, and \( u_i \) and \( u_j \) are the shadowings of the propagation paths. The ratio of \( d_i \) to \( d_j \), symbolized by \( r_{ij} \), can be derived from (2),

\[ r_{ij} = d_i/d_j = 10^{(A_i-A_j)/(10n)+u_{ij}} \]  

(3)

where \( u_{ij}=(u_i-u_j)/(10n) \) is a zero-mean Gaussian random variable. Denote the variance and correlation of shadowings as \( \sigma^2 \) and \( \rho \), respectively, then \( u_{ij} \) has the variance \( 2\sigma^2(1-\rho)/(10n)^2 \). Xia et al. found that the standard deviation ranges from 4.2 to 7.7 dB in suburban/residential environments and from 2.2 to 8.3 dB in urban environments for microcells operating in the 900 MHz frequency band [19]. Saunders found that \( r \) ranges from 0.3 to 0.8 when \( d1 \) is 1 km and \( d2 \) is 2 km [20].

The distance ratio, \( r_{ij}(i \neq j) \), defines a circle along which the mobile may lie,
\[(x - x_{ij})^2 + (y - y_{ij})^2 = R_{ij}^2\]  \hspace{1cm} (4)

Where \[x_{ij} = \frac{(r_j^2 x_j)}{(r_j^2 - 1)}\] , \[y_{ij} = \frac{(r_j^2 y_j - y_i)}{(r_j^2 - 1)}\] and

\[R_{ij}^2 = \left\{1/(r_j^2 - 1)^2 + 1/(r_j^2 - 1)\right\} \cdot [(x_i - j_j)^2 + (y_i - y_j)^2].\]

Note that the circle defined by \(r_{ij}\) is exactly the same as that defined by \(r_{ji}\). Determining the MS location at the mean of the intersections of (4), \(i, j = 1, 2, \ldots, N\), would be the simplest way but has lower accuracy. This paper uses a geometrical approach [21], which generates linear lines of position (LOP) by differencing pairs of circular LOPs and proceeds to solve the MS location using the least-squares algorithm. The linear LOP determined by two circles, centred at \((x_{i1}, y_{i1})\) and \((x_{i2}, y_{i2})\) and with radii of \(R_{11}\) and \(R_{22}\), respectively, is given by

\[(x_{i1} - x_{i2})x + (y_{i1} - y_{i2})y = \frac{R_{11}^2 - R_{22}^2 - (x_{i1}^2 + y_{i1}^2) + (x_{i2}^2 + y_{i2}^2)}{2}\] \hspace{1cm} (5)

For simplicity, setting \(N = 3\) and expressing the set of linear LOPs in matrix form

\[A_x - X_{MS} = B_s\] \hspace{1cm} (6)

Where \[A_x = \begin{bmatrix} X_{c3} - X_{c2} & Y_{c3} - Y_{c2} \\ X_{c2} - X_{c1} & Y_{c2} - Y_{c1} \end{bmatrix}\] \[X_{MS} = \begin{bmatrix} x \\ y \end{bmatrix}\] and \[B_s = \frac{1}{2} \begin{bmatrix} R_{12}^2 - R_{13}^2 - (x_{c1}^2 + y_{c1}^2) + (x_{c2}^2 + y_{c2}^2) \\ R_{12}^2 - R_{23}^2 - (x_{c1}^2 + y_{c1}^2) + (x_{c3}^2 + y_{c3}^2) \\ R_{13}^2 - R_{23}^2 - (x_{c1}^2 + y_{c1}^2) + (x_{c3}^2 + y_{c3}^2) \end{bmatrix}\]

the least-squares solution is derived from

\[\hat{X}_{MS} = (A_x^T A_x)^{-1} A_x^T B_s\] \hspace{1cm} (7)

To evaluate the performance of SADOA method, Fig. 1 illustrates the hexagonal tested cell surrounded by six neighbouring cells with radius of 500 m. The Cost231-Hata model and a Gaussian random variable were applied for the path loss and shadowing simulations, respectively. One hundred MSs were uniformly distributed in the centre cell. According to the received signals, \(N (N = 3, \ldots, 7)\) BSs with higher received signal powers were used for MS location calculation. The location performance is assessed in terms of the value of the distance error defined as

\[\varepsilon_d = \sqrt{(x_{MS} - \hat{x}_{MS})^2 + (y_{MS} - \hat{y}_{MS})^2}\]

where \((x_{MS}, y_{MS})\) and \((\hat{x}_{MS}, \hat{y}_{MS})\) are the actual and estimated MS locations, respectively. On the basis of 500 independent runs of SADOA location estimation for each MS, Fig. 2 shows the cumulative distribution function (CDF) of location errors in uncorrelated shadowing environments. The location error decreases with the increasing of ability to detect BSs. When only three BSs are available, the SADOA method does not support good performance. However,
with five BSs being available, the 67% of the location errors are below 121 m in the slightly fading environments.

2.2 TDOA method

Assume each BS\(i\) is capable of performing TOA observation, \(t_i\), then TDOA observation is defined as \(T_i = t_i - t_{i-1}, i = 2, \ldots, N\). Expressing TDOA observation as a function of station co-ordinates, a hyperbola has the form

\[
cT_i = \sqrt{(x - x_i)^2 + (y - y_i)^2} - \sqrt{(x - x_{i-1})^2 + (y - y_{i-1})^2}
\]

where \(c\) is the speed of light, \((x_i, y_i)\) and \((x_{i-1}, y_{i-1})\) are the co-ordinates of BS\(i\) and BS\(i-1\), respectively, and \((x, y)\) is the unknown MS position. Consequently, the MS position is determined by solving the intersections of a set of \(N-1\) hyperbolas.

The least-squares estimation is a common technique to solve TDOA equations linearised by using the first two terms of their Taylor series [22, 23]. Denote the initial guess of the MS position as \((x_0, y_0)\), the linearisation of (8) is given by

\[
m_{xi}x + m_{yi}y = cT_i - f_i + m_{xi}x_0 + m_{yi}y_0
\]

Where

\[
m_{xi} = \frac{(x_0 - x_i)}{\sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2}} - \frac{(x_0 - x_{i-1})}{\sqrt{(x_0 - x_{i-1})^2 + (y_0 - y_{i-1})^2}}
\]

\[
m_{yi} = \frac{(y_0 - y_i)}{\sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2}} - \frac{(y_0 - y_{i-1})}{\sqrt{(x_0 - x_{i-1})^2 + (y_0 - y_{i-1})^2}}
\]

\[
f_i = \sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2} - \sqrt{(x_0 - x_{i-1})^2 + (y_0 - y_{i-1})^2}
\]

Expressing the set of linearised equations in matrix form,

\[
A_iX_{MS} = B_i
\]

where

\[
A_i = \begin{bmatrix} m_{x2} & m_{y2} \\ m_{x3} & m_{y3} \\ \vdots & \vdots \\ m_{xN} & m_{yN} \end{bmatrix} \quad \text{and} \quad B_i = \begin{bmatrix} cT_2 - f_2 + m_{x2}x_0 + m_{y2}y_0 \\ cT_3 - f_3 + m_{x3}x_0 + m_{y3}y_0 \\ \vdots \\ cT_N - f_N + m_{xN}x_0 + m_{yN}y_0 \end{bmatrix}
\]

the least-squares solution is derived from \(\hat{X}_s = (A_i^T A_i)^{-1} A_i^T B_i\). To obtain the location estimate with higher accuracy, the least-squares estimation is performed iteratively by linearising hyperbolas about the point \(\hat{X}_{MS}\) fed back from previous estimate into the
new estimated until \( \hat{X}_{MS} \approx [x_0, y_0]^T \).

Fig. 1 Hexagonal cell geometry.

Fig. 2 CDF of location errors using SADOA method with different abilities to detect BSs in shadowing environments.

NLOS propagation has been identified as one of the primary factor that limits the accuracy of TDOA method [24, 25]. The same simulation scenario as the previous one was used to evaluate the performance of TDOA method in NLOS propagation environments. Conventional TDOA method sues serving BS position as the initial guess, but its performance is generally not good enough. In this work, the initial guess
is set as the centre of the polygon formed by the BSs connecting to the MS [26]. Fig. 3 reveals the CDF of location errors in 6 dB shadowing and different NLOS propagation environments, where the noises representing the errors of TDOA observations are zero-mean Gaussian random variables with standard deviations of 300, 200 and 100 m. The location error noticeably decreases with the decreasing of NLOS propagation. With five BSs being available, the 67% of the location errors are 73 m in slightly NLOS propagation environments.

Fig. 3 CDF of location errors using TDOA method in different NLOS propagation environments.

2.3 Hybrid SADOA/TDOA

The proposed hybrid SADOA/TDOA location scheme linearises circles and hyperbolas defined by the SADOA and TDOA measurements, respectively, and iteratively performs the least-squares estimation until the solution converges to a minimum. The linear LOP derived from the linearization of (4) using Taylor-series expansion about the point \((x_0, y_0)\) has the form

\[
n_{ij} x + n_{ij} y = \frac{1}{2} (D_{ij}^2 - g_{ij}) + n_{ij} x_0 + n_{ij} y_0
\]

(11)

where \(g_{ij} = (x_0, x_{ij})^2 + (y_0, y_{ij})^2\), \(n_{ij} = x_0 - x_{ij}\) and \(n_{ij} = y_0 - y_{ij}\). For simplicity, setting \(N=3\) and expressing the set of linearised equations in matrix form,

\[
AX_S = B
\]

(12)

where
\[
A = \begin{bmatrix}
m_{12} & m_{12} \\
m_{23} & m_{23} \\
n_{12} & n_{12} \\
n_{23} & n_{23}
\end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix}
cT_2 - f_2 + m_{12}x_0 + m_{12}y_0 \\
cT_3 - f_3 + m_{13}x_0 + m_{13}y_0 \\
\frac{D_{12}^2 - g_{12}}{2} + n_{12}x_0 + n_{12}y_0 \\
\frac{D_{13}^2 - g_{13}}{2} + n_{13}x_0 + n_{13}y_0 \\
\frac{D_{23}^2 - g_{23}}{2} + n_{23}x_0 + n_{23}y_0
\end{bmatrix}
\]

the least-squares solution is derived from \( \hat{\mathbf{X}}_{MS} = (A^T A)^{-1} A^T \mathbf{B} \). Similar to TDOA method, the least-squares estimation is performed iteratively by feeding back \( \hat{\mathbf{X}}_{MS} \) into new estimate until \( \hat{\mathbf{X}}_{MS} \approx [x_0 \quad y_0]^T \).

3 Location estimation simulation

Owing to the lack of TDOA measurement data, this paper uses ray tracing technique for propagation simulations. A software package, SignalProw by EDX Engineering, Solid line (A–B) represents a trajectory the mobile moving along, encircled crosses designate the BSs with omni-direction antennae and polygons stand for buildings including a set of planning tools for wireless communication systems, was used to facilitate the simulations. Fig. 4 illustrates the simulation environment, which involves 17 BSs with an average BS separation of 500 m, an omni-direction antennae, and a mean BS height of 36.9 m. To assess the impact of NLOS propagation on TDOA observation, Fig. 5 shows the statistics of distance errors of TDOA observations, where the mean value is 40 m and standard deviation is 240 m. Meanwhile, the Cost231-Hata model was applied for the path loss simulations, and uncorrelated zero-mean Gaussian random variables were assumed for shadowing simulations, where the standard deviation of shadowing was set to 5.55 dB on the basis of measurement reported in [27]. According to Cost231-Hata model, the path loss exponent was set to \( 3.46((44.9 - 6.55 \log_{10} h_b)/10, \) where \( h_b = 36.9 \).
Fig. 4 Simulation environment.

Fig. 5 Statistics of distance errors of TDOA observations. Mean value is 40 m and standard deviation is 240 m.
Fig. 6 CDFs of location errors with three BSs in range of the MS. Fig. 6 reveals the comparisons of location performances with three BSs used per location estimation. The performance is inadequate by using TDOA method with initial guesses set as serving, which is labelled as ‘TDOA (IG:Serving BS)’. With
initial guess set as the centre of polygon formed by BSs connecting to the MS, which is labelled as ‘TDOA(IG:Polygon Centre)’, the MS can be located within 237 m with a confidence of 67%. By using the proposed hybrid scheme, the 67% of the location errors are below 206.4 m. Fig. 7 shows the comparisons of location performances with five BSs used per location estimation. Again, TDOA(IG:Serving BS) does not support good accuracy. The performances of SADOA and TDOA(IG:Polygon Centre) methods are similar by locating the MS within 200 m with a confidence of 67%. By using the proposed hybrid scheme, the 67% of the location errors are below 137.9 m. Table 1 summarises the statistics of the location simulations. Generally, the location errors are reduced with the increasing of BS number. The proposed hybrid scheme performs significantly better than that of the conventional TDOA method by about 50% improvement.

<table>
<thead>
<tr>
<th>Number of base stations</th>
<th>3BSs</th>
<th>4BSs</th>
<th>5BSs</th>
<th>6BSs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional TDOA Method</td>
<td>Median</td>
<td>388.7</td>
<td>280.7</td>
<td>214.8</td>
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<tr>
<td></td>
<td>67%</td>
<td>596.7</td>
<td>414.5</td>
<td>331.4</td>
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<td></td>
<td>95%</td>
<td>2039.4</td>
<td>1293.7</td>
<td>1058.0</td>
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<tr>
<td>Proposed hybrid method</td>
<td>Median</td>
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<td>125.3</td>
<td>107.7</td>
</tr>
<tr>
<td></td>
<td>67%</td>
<td>206.4</td>
<td>161.0</td>
<td>137.9</td>
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<td></td>
<td>95%</td>
<td>362.7</td>
<td>282.1</td>
<td>241.6</td>
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<tr>
<td>Performance improvement at the 67th percentile</td>
<td>65%</td>
<td>61%</td>
<td>55%</td>
<td>48%</td>
</tr>
</tbody>
</table>

Rows 67% and 95% denote the 67th and 95th percentile, respectively

4 Conclusions

TDOA method is considered the leading candidate for any future location system. The SADOA method needs no perfect path loss modelling, reduces shadowing impact on location and can be applied to an existing system without hardware modifications. This paper has proposed a hybrid TDOA/SADOA location scheme for wireless communication systems. The proposed hybrid scheme supports good location performance. Simulations demonstrate that the proposed scheme performs significantly better than that of the conventional TDOA method by about 50% improvement.
5 References


