Utility-based joint Power and Rate Control Game with Interference Threshold Elasticity for Cooperative Cognitive Networks

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Abstract—In cognitive networks, the cognitive radio users can share spectrum resource with the primary user while ensuring the normal communication of primary user. In addition, next generation wireless systems needs to provide flexible transmission rate to each terminal. In this paper, a utility-based game-theoretic model is proposed to study the joint power and rate control problem in spectrum underlay fashion in cognitive networks. We adopt a novel utility function based on cognitive network which focuses on social optimal resource allocation through pricing. Further, we show an interference threshold elasticity perspective, which is the key for primary network to maximize its utility by increasing its transmitted power to adjust the tolerable interference constraint. And also, the cognitive network can increase its total throughput capacity. In Stackelberg game, the primary user and the cognitive radio users interact with each other by adjusting their own actions. The cooperative cognitive networks model follows the “best-effort” principle as well as the win-win perspective of primary-cognitive user. Numerical simulations are conducted to demonstrate the performance of the model. The results show that the PU network can get more profit and also the CRU network can increase its total throughput capacity by the adjustment of the interference threshold.

Index Terms—Utility Function, Power & Rate Control, Interference Threshold Elasticity, Cognitive Networks, Stackelberg Game

I. INTRODUCTION

Wireless spectrum suffers from scarcity with the rapid development of communications. FCC studies found that the static frequency allocation policy causes the inefficient usage of spectrum resource [1]. Cognitive Radio (CR) has been viewed as a promising technology to solve this problem [2]. In cognitive networks, the non-authorized users can share the spectrum with the authorized users without affecting normal communication of the authorized users. Here we call authorized user primary user (PU) and call non-authorized user cognitive radio user (CRU). During the sharing process, the CRUs will bring interference to the PU. So power control is vital. Game theory has been used as a powerful tool to study the power control problem. In [3], the non-cooperative game power control model (NPG) was first developed. However, the Nash Equilibrium (NE) may not be optimum. Then, the author imposed a linear pricing function to improve Pareto efficiency [4]. After that, many researchers have designed different utility functions [5-10] such as sigmoid-like utility function to obtain better Quality of Service (QoS) using less power. Next generation wireless systems will provide a wide range of services such as real-time multimedia transmission. Only power control is not enough. Researches [11-14] in the area of joint power and rate control have been proposed. [11] used a two-layered game in which one game determined powers and the other determined rates. Based on [12], [13] also used a single game where powers and rates were computed jointly. [14] focused on GPRS technology and used discrete link adaptation to solve the problem of joint power and rate allocation.

Game theory includes non-cooperative game and cooperative game. The above mentioned articles are almost based on non-cooperative game. Non-cooperative game focuses on individual rationality and optimality. In contrast, cooperative game focuses on group optimality. It has no process for individuals to reach agreement and directly discuss the spectrum resource allocation. In aspect of efficiency and fairness, cooperative game will be more suitable. In this paper, the CRU network is viewed as the public participant. Aiming to the maximum of the total throughput capacity of CRU network, the joint power and rate control utility model with QoS and interference constraints is built. By Lagrange dual layered technology, the model can be divided into the sub-problem of each CRU. Also, we can see that this model combines energy consumption of network with data transmission performance.
Cognitive networks are compound networks consisting of PU network and CRU network. If only considering the CRU network, the power control scheme has no difference with the traditional wireless network power control scheme. Thus, in order to fully utilize the scarce spectrum resource and protect the benefit of PU network, it is very necessary for the PU network as the decision maker to participate in the spectrum sharing process. As opposed to passive spectrum sharing, the PU network has an incentive to allow the CRUs to access its licensed spectrum, that is, the PU network plays an active role in the spectrum sharing process. In [15], the CRU network rewarded the PU network for allowing the CRUs to share its licensed spectrum and penalized them when the total interference became greater than the interference threshold. In [16], the author presented a new pricing function of CRUs based on [15], which made the CRUs achieve their transmission using less powers. In [17], the author introduced an economic-based utility for the PU network as a function of signal-interference-ratio (SIR) and transmitted power, which allowed the CRUs to share the PU network’s spectrum and punished the PU network if its required transmission quality was not satisfied. By analysis, the transmitted power and the interference threshold of the PU network are fixed in the above articles.

The focus of this paper, however, is interference threshold elasticity. The PU network can maximize its revenue by increasing its transmitted power to adjust the tolerable interference constraint. The word “elasticity” has been presented in some articles. In [18], the author firstly introduced the concept of elasticity into network and refereed elasticity as the applications’ ability to adapt their sending rates according to the available resource. In [19], the author defined a utility-bandwidth elasticity and calculated the corresponding optimal pricing to maximize the total utility. In [20], the author presented the power-interference elasticity perspective and the PU chose the bandwidth and tolerable interference levels for the CRUs. Here the key difference with [20] is that we consider the social optimal resource allocation in the CRU network, and also the PU network will pay for the increment of the transmitted power with respect to an exclusive interference in the PU network utility. So, a dynamic spectrum sharing scheme is designed with a limited interaction between the PU network and the CRU network. Stackelberg game is used in this paper, in which the PU network acts as Stackelberg leader and the CRU network acts as follower. The PU network properly adjusts the interference threshold by the actual requirement of the CRU network, Meanwhile, the CRU network has to adapt its action to the imposed interference constraint.

The remainder of this paper is organized as follows. In Section II, we describe the system model. In Section III, we present the utility functions for both the PU network and the CRU network, and also formulate the cooperative cognitive networks model based on Stackelberg game. In Section IV, we give the primary-cognitive joint power and rate control algorithm. The performance analysis of this model and the conclusions are given in Section V and VI.

II. SYSTEM MODEL

We consider the uplink communication of the single-cell CDMA cognitive networks model with one PU, one PU base station, N CRUs and one CRU base station. Fig.1 illustrates the model where the CRU network is laid over the PU network in spectrum underlay fashion. $h$ and $g$ are the path gain.

![Figure 1. A single-cell CDMA cognitive networks model](Image)

For the i-th CRU, the signal-interference-noise-ratio (SINR) achieved at the CRU base station can be obtained using the following formula:

$$\gamma_i = \frac{W}{r_i} \sum_{j=1}^{N} p_j h_j + \theta$$

where $W$ is the available spread spectrum bandwidth, $\theta = p_i h_i + \sigma^2$ is the interference caused by the PU and ambient noise, $p_i$ and $r_i$ are the transmitted power and the transmission rate of the i-th CRU.

The PU’s target SINR is:

$$\gamma_0 = \frac{p_i h_i}{\overline{Q}_0 + \sigma^2}$$

where $\overline{Q}_0$ is the basic interference threshold.

III. COOPERATIVE COGNITIVE NETWORKS MODEL

A. CRU Network Utility

Considering social welfare, we use the CRU network utility model as follows:

$$\max U_{CRU} = \sum_{i=1}^{N} u_i \quad \text{s.t.} \quad \gamma_i \geq \gamma_i^{th}$$

$$r_i \geq r_i^{min}$$

$$0 < p_i \leq p_i^{max}$$

$$\sum_{i=1}^{N} p_i g_i \leq \overline{Q}_0$$

where $u_i$ is the utility function of the i-th CRU and $u_i = r_i \ln(K \gamma_i)$. $K = e^{(r_i)} / r_i$ and $g(\gamma_i) = [1 - P_e(\gamma_i)]^{\mu}$.
is the frame success rate. So \( u_i \) is the throughput capacity of the \( i \)-th CRU and \( U_{CRU} \) is the total throughput capacity of CRU network.

In addition, the QoS requirements of the \( i \)-th CRU include the SINR threshold \( \gamma_i^{th} \) and the minimal transmission rate \( r_i^{min} \). The transmitted power of the \( i \)-th CRU is bounded by \( (0, p_i^{max}] \). \( I_0 = \sum_{i=1}^{N} p_i g_i \) is the total interference from the CRUs. \( I_0 \) cannot exceed \( Q_0 \). So the maximum of the utility function has to satisfy the above four in equations.

The first three constraint conditions of the utility function can be simplified by [21], and then we can get the simplified model as follows:

\[
\max U_{CRU} = \sum_{i=1}^{N} \mu_i \\
\text{s.t.} \left\{ \begin{array}{l}
\sum_{i=1}^{N} f(r_i) \leq T \\
\sum_{i=1}^{N} p_i g_i \leq Q_0
\end{array} \right.
\] (4)

where \( T = 1 - \frac{f(r_i) \sigma^2}{h p_i^{max}} \) and \( f(r_i) \) is the function of \( r_i \),

\[
f(r_i) = \frac{1}{\frac{\gamma_i^{th}}{r_i} + 1}.
\]

The CRU network utility model is obviously a non-linear optimization problem with two constraint conditions including \( P(p_1, p_2, ..., p_N) \) and \( R(r_1, r_2, ..., r_N) \).

By introducing Lagrange multipliers, the two constraints can be absorbed into the objective function \( U_{CRU}(P, R) \). We have:

\[
Q(P, R, v, \lambda) = \sum_{i=1}^{N} r_i \ln(Kr_i) - v \sum_{i=1}^{N} f(r_i) - T - \lambda \sum_{i=1}^{N} p_i g_i - Q_0
\]

\[
= \sum_{i=1}^{N} (r_i \ln(Kr_i) - v f(r_i) - \lambda p_i g_i) + v T + \lambda Q_0
\]

\[
= \sum_{i=1}^{N} L_i(p_i, r_i, v, \lambda) + v T + \lambda Q_0
\]

(5)

From (5) we can see that the maximum of CRU network utility can be obtained by the maximum of each CRU utility as follows:

\[
(p_i^*, r_i^*) = \max_{p_i, r_i} L_i(p_i, r_i, v, \lambda)
\]

(6)

where \( L_i(p_i, r_i, v, \lambda) = r_i \ln(Kr_i) - v f(r_i) - \lambda p_i g_i \).

Lagrange multiplier \( v \) is regarded as unit resource pricing factor. \( T \) is the largest resource provided and \( f(r_i) \) is the resource share of CRU \( i \). Lagrange multiplier \( \lambda \) is regarded as unit interference pricing factor.

### B. PU Network Utility

From (2) we can see that PU has to add extra transmitted power \( \Delta p \) to maintain its target SINR when \( I_0 > Q_0 \). To follow the “best-effort” principle, we present a novel PU network utility model as follows:

\[
U_{PU} = Q_0(\overline{Q}_0 - (Q_0 - I_0) - \mu \Delta p_0 (I_0 - \overline{Q}_0)
\]

(7)

where \( Q_0 \) is the maximum interference threshold that PU is willing to tolerate. When \( I_0 \leq Q_0 \), \( \Delta p_0 = 0 \). \( \mu \) is the pricing factor of the extra transmitted power. It means that PU has to pay for the additional transmitted power at the same time it gains more profit. The PU is far from the CRU base station, so the influence of the additional transmitted power for the CRU network is neglected.

We obtain the first-order partial derivative of the model with respect to \( Q_0 \):

\[
\frac{\partial U_{PU}}{\partial Q_0} = -2Q_0 + I_0 + \overline{Q}_0
\]

(8)

When formula (8) is equal to zero, we can get

\[
Q_0^* = \frac{I_0 + \overline{Q}_0}{2}
\]

which maximizes \( U_{PU} \). If \( Q_0 \) cannot be adjusted, that is \( Q_0 = \overline{Q}_0 \), we can get

\[
\Delta Q_0 = Q_0^* - \overline{Q}_0 = \frac{I_0 - \overline{Q}_0}{2}
\]

By the above analysis, we can see that the PU network not only can accommodate more CRUs to share the spectrum resource by properly increasing the transmitted power but also can get more benefit. In a very real sense, the interference threshold elasticity can achieve win-win situation.

### C. Cooperative Model

In cognitive networks, the PU and the CRUs interact with each other by adjusting their own actions. We provide the cooperative cognitive networks model to analyze the behavior as follows:

\[
U_{PU} = u_{pu}(Q_0, I_0)
\]

\[
U_{CRU} = u_{cru}(P, R, T, Q_0, I_0)
\]

(9)

In the above model, the PU network acts as Stackelberg leader and the CRU network acts as follower. The PU network maximizes its utility by adjusting \( Q_0 \) according to the possible interference. The CRU network also maximizes its utility by optimal allocation of spectrum resources including \( P \) and \( R \) according to \( Q_0 \).

### IV. ALGORITHM DESCRIPTION

In the CRU network model, for the optimal unit resource pricing factor \( v^* \), we adopt geometric growth & binary search method to approach it. \( m \) is iterative time, if \( \sum_{i=1}^{N} f(r_i(v(m))) < T \), then \( v(m+1) = v(m)^2 \), and if
For the optimal unit interference pricing factor $\lambda^*$, we adopt sub-gradient iterative algorithm to approach it. The iterative equations of $\lambda$ is expressed as:

$$
\lambda(n+1) = \max\left\{0, \lambda(n) + \alpha(n)\left(\sum_{i=1}^{N} p_i g_i - Q_0\right)\right\}
$$

(10)

where $n$ is the index of iteration, $\alpha(n)$ is the step size factor which effect the convergence rate of the iterative algorithm. We set $\alpha(n) = \frac{\ell}{Q_0}$ to ensure the fast convergence of this algorithm, where $\ell$ is a positive constant.

The optimal values $r_i^*$ and $p_i^*$ of CRU $i$ can be calculated by the best response functions as follows:

$$
r_i^*(v', \lambda') = \arg\max_{p_i \in P, r_i \in R} L_i(p_i, r_i; v', \lambda')
$$

(11)

$$
p_i^*(v', \lambda') = \arg\max_{p_i \in P, r_i \in R} L_i(p_i, r_i, v'; v', \lambda')
$$

(12)

The PU is viewed as the noise source and we can calculate the maximum total transmission rate of CRUs by Shannon formula as follows:

$$
R \leq W \log(1 + \sum_{i=1}^{N} p_i h_i)
$$

(13)

From formula (13) we can see the relation of the maximum total transmission rate and the total transmitted power of CRUs. When each CRU sets its minimum transmission rate according to business requirement, the total transmitted power can be calculated by the total transmission rate.

We assume that $\bar{P} = [\bar{p}_1, \bar{p}_2, ..., \bar{p}_N]$ is the interference power vector of CRUs received by PU base station. Because $\bar{p}_i = p_i g_i$, we can get:

$$
\bar{P} = \frac{\bar{p}_1 h_1}{g_1} + \frac{\bar{p}_2 h_2}{g_2} + ... + \frac{\bar{p}_N h_N}{g_N}
$$

(14)

According to the cooperative cognitive networks model, the primary-cognitive joint power and rate control algorithm is formally stated below:

1. Set $Q_0$ by PU.
2. Initialization. $t = 0$ , $r_i^{\textrm{min}} \leq r_i(0) \leq r_i^{\textrm{max}}$ , $0 < p_i(0) \leq p_i^{\textrm{max}}$ .
3. Compute a sequence of power and rate vectors as follows for $t = t + 1$ .
   - $r_i(t) = \arg\max_{r_i \in R} L_i(P, R)$ and set rate as $r_i(t) = \min(r_i(t), r_i^{\textrm{max}})$
   - $p_i(t) = \arg\max_{p_i \in P} L_i(P, R)$ and set power as $p_i(t) = \min(p_i(t), p_i^{\textrm{max}})$
4. Update Lagrange multipliers $\nu$ and $\lambda$ .
5. Continue (3) and (4) until $|p_i(t+1) - p_i(t)| \leq \varepsilon$ , in which $\varepsilon$ is a minimal value.
6. When the new CRU enters the cognitive networks, the PU will renew the interference threshold $Q_0$ .
7. Repeat (2) (3) (4) and (5).

V. SIMULATION RESULTS

We consider a single-cell CDMA system with 1 km radius. There are 6 CRUs located at different distances from the CRU base station. The PU is 200m distance from the PU base station. The system parameters are considered as follows: $W = 10^6$ Hz , $\sigma^2 = 5 \times 10^{-15}$ W , $p_i^{\textrm{max}} = 0.5$ W , $10^2 \leq r_i^{\textrm{max}} \leq 10^4$ bps , $\gamma_i^d = 12$ , $h = A / d^4$ with $A=0.097$ , $\bar{Q}_0 = 10^{-12}$ , $K = 0.21886$ .

![Figure 2](image)

Figure 2. The convergence curve of unit interference pricing factor $\lambda$.

Fig. 2 shows the convergence process of unit interference pricing factor $\lambda$. $\Delta \kappa = \sum_{i=1}^{N} p_i g_i - \bar{Q}_0$ denotes the difference of the actual total interference and the basic interference threshold. When $\Delta \kappa > 0$ , $\lambda$ increases and CRUs have to reduce their transmitted powers. From Fig. 2 we can see that $\lambda$ will achieve the convergence value when the iteration times is 8.
VI. CONCLUSIONS

This paper presents a cooperative model of cognitive networks in spectrum underlay fashion. In this model, a joint power and rate allocation scheme based on CRU network is used. This CRU network utility is bounded by feasible constraint conditions which ensure the PU and the CRUs’ QoS requirements. In this paper, Lagrange duality optimization theory has been used to solve the nonlinear model with constraints. This CRU network utility is provably convergent to the globally optimal pair of powers and rates. Further, this paper introduces the concept of interference threshold elasticity into the cognitive networks. Instead of the fixed interference threshold, the PU allows more CRUs to access its licensed spectrum and maximizes its utility by moderately adding the transmitted power to increase the interference threshold. In the cooperative model, the PU and the CRUs have dynamic primary-cognitive network interaction to achieve win-win situation. Numerical simulations have been performed to illustrate the proposed cooperative game. Simulation results have shown that the PU network can get more profit and also

Fig. 5 shows the relation of PU utility and different interference thresholds. From Fig. 5 we can see that $U_{PU}$ increases with $I_0$ increasing when $I_0 \leq Q_0$. When $I_0 > Q_0$, $U_{PU}$ has a cost associated to the extra transmitted power $\Delta p_0$. In a certain range, $U_{PU}$ remains increasing, but the growth rate gradually slows. When $Q_0 = 5 \times 10^{-11}$, $U_{PU}$ reaches the maximal value. When the cost is out of range the PU is willing to tolerate, $U_{PU}$ begins to decrease. Compared to $Q_0$, the maximum interference threshold $Q_0$ increases $5 \times 10^{-11}$.

Fig. 6 shows the effect of interference threshold elasticity for the throughput capacity of the CRU network. From Fig. 6 we can see that the increase in $Q_0$ can improve the CRU network throughput capacity which can accommodate more CRUs to share spectrum resource.
the CRU network can increase its total throughput capacity by adjusting the interference threshold.

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REFERENCES


