3D Holography: from discretum to continuum

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Overview

1. Is quantum gravity fundamentally holographic?

2. Continuum: dualities for 3D gravity.

3. Regge calculus.

4. Computing the 3D partition function in Regge calculus.

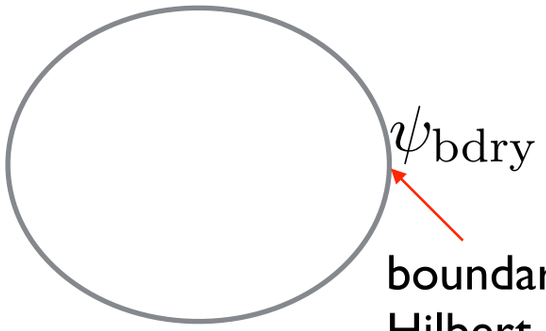
    One loop correction.

    Boundary fluctuations.

Is quantum gravity fundamentally holographic?

- In the last years spin foams have made heavily use of the
  Generalized Boundary Formulation [Oeckl 00's +, Rovelli, et al …]

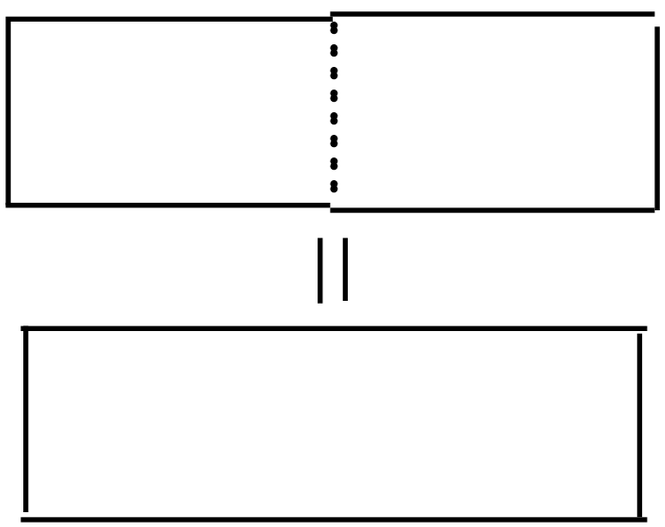
- Encodes dynamics into amplitudes associated to (generalized) space time regions.



$$\mathcal{A}_{\mathrm{Region}}(\psi_{\mathrm{bdry}})$$

-should satisfy (the dualized) Wheeler de Witt equation
-gives no boundary (Hartle-Hawking) wave function

- This looks 'holographic'. However in the standard formulation one has the gluing axiom:

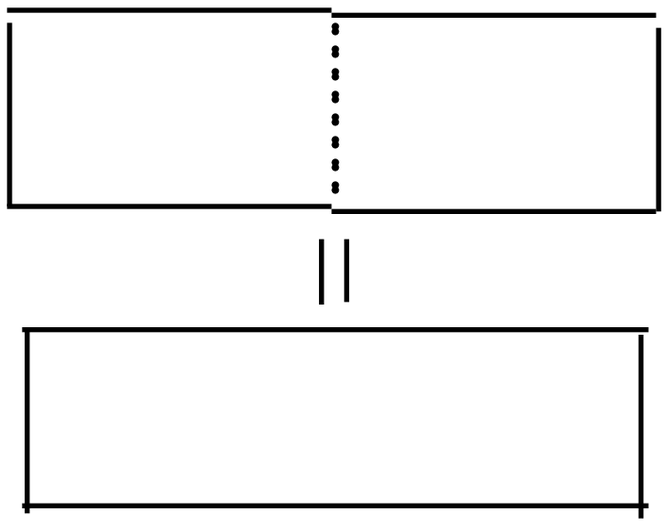


'Gluing axiom':
essential to get more complicated from simpler amplitudes

Amplitude for bigger region
=
glued from amplitudes for smaller regions

This could be / is also called 'locality axiom'.

Gluing / Locality axiom: problems in the discrete



Assumes 'gluing axiom':
needed to get more complicated from simpler amplitudes

Amplitude for bigger region

=

glued from amplitudes for smaller regions

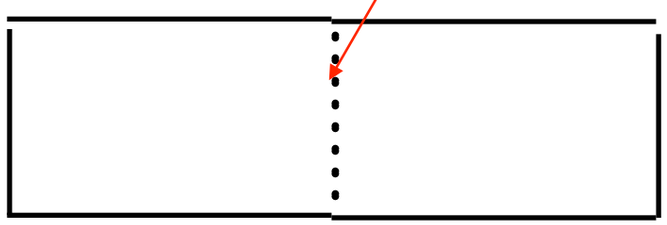
Amplitude for more complicated boundary state

=

glued from amplitudes for less complicated boundary state

However this gluing axiom does not hold in the discrete:
Restricting to discretization we never obtain a full resolution of identity for the continuum Hilbert space.
(This leads eventually to non-local amplitudes if one wants to represent continuum physics.)

Integrating over data associated to discretization misses out on most of the continuum data. Even on the necessary discrete data (depending on boundary discretization).



Gluing axiom / Locality axiom does not hold in the discrete.

[e.g. Bahr, BD, He NJP 11]

The consistent boundary formalism

[BD12, BD14, BD to appear]

We do not assume "gluing axiom" (as fundamental).

However we need a principle to connect amplitudes for less complicated and more complicated boundary data.

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Main Idea: A "discretization" just determines the wave function for a small subset of the degrees of freedom. All other degrees of freedom are put into the simplest state = vacuum state.

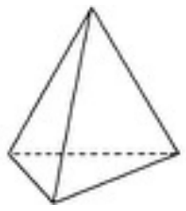
Generalization of inductive Hilbert space techniques used in LQG.

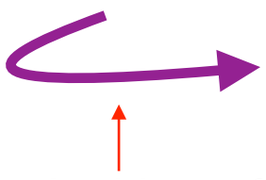
[Ashtekar, Isham, Lewandowski, …]

Main Challenge: Be consistent - observables should not depend on choice of discrete structure, which is used to compute it.

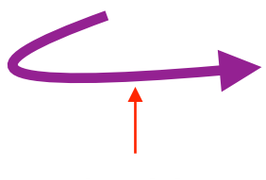
How to express the continuum dynamics [BD NJP 12, BD 14]

Boundary Hilbert space with low complexity wave functions

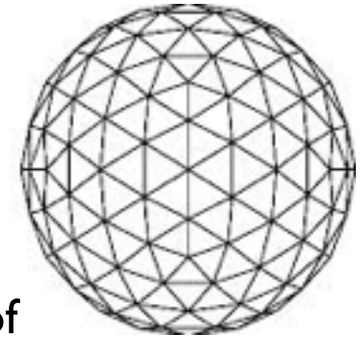


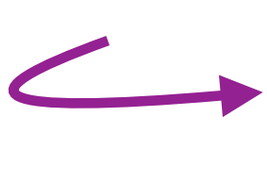


embedding of boundary Hilbert spaces

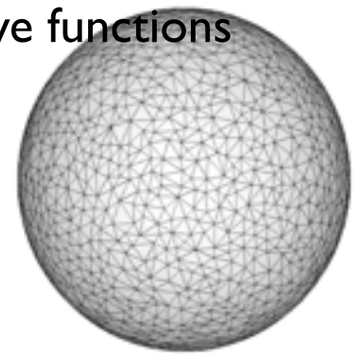


embedding of boundary Hilbert spaces





Boundary Hilbert space with high complexity wave functions



• • •

restricts to

$\mathcal{A}_{vac}^{\text{low com}}(\psi_{\text{low com}})$ ← $\mathcal{A}_{vac}^{\text{med com}}(\psi_{\text{med com}})$ ← $\mathcal{A}_{vac}^{\text{high com}}(\psi_{\text{high com}})$ • • •

(cylindrical) consistency condition

A (complete) family of consistent amplitudes defines a theory* of quantum gravity.

* Corresponds to a complete renormalization trajectory,

with scale given by complexity parameter.

[BD NJP 12, BD 14]

The consistent boundary formalism

- We now only refer to boundary: fundamentally holographic formulation?

- Question: What can we say about boundary ‘dynamics’?

- Even in the case of a topological field theory, where we may have a chance to compute the boundary amplitudes in general?

- Relation to AdS/CFT , Flat/CFT or dS/CFT?

(Further) Motivation

Aim:

Provide a concrete model for holography, that connects non-perturbative and / or discrete approaches to the continuum.

Strategy:

Work with an explicit realization of 3D (quantum) gravity.

Puzzle: [Carlip]

3D gravity is topological. Degrees of freedom can be captured by discretization, in a discretization independent way. How does on obtain a (conformal) field theory from this theory?

Further motivation (and an invitation): [Freidel 08]

Using dualities in non-perturbative approaches? Need to develop a strategy:

Questions:

Can we do finite boundary? Not AdS? Work with (auxiliary) discretization?

Continuum theory:

Thermal flat 3D space

3D gravity - CFT correspondence

- [86 Brown-Henneaux] central extension of asymptotic charge algebra (AdS)
- [Maldacena 97, …] AdS-CFT
- in 3D in particular application to BTZ black holes [Strominger 97, Carlip,…]

- [Giombi-Maloney-Yin 08] one-loop partition function reproduces character of Virasoro algebra

- One-loop partition function is exact.

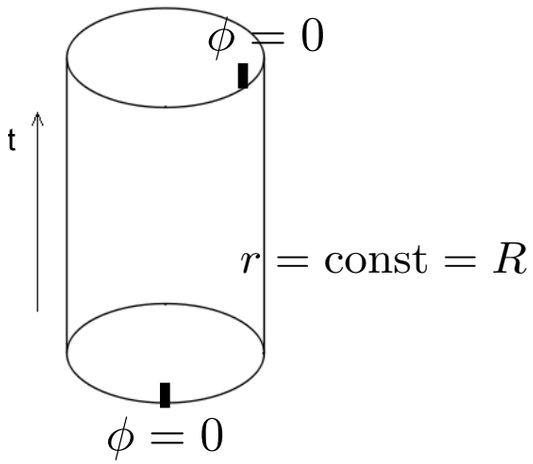
Extension to flat space?

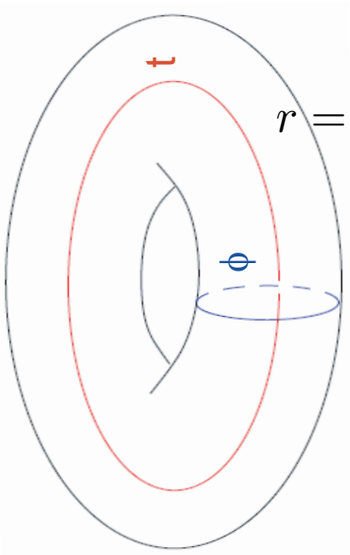
- [06 Barnich-Compere, … ,Barnich et al] central extension of asymptotic charge algebra for flat case (BMS group at null infinity)

- [15 Barnich-Gonzalez-Maloney-Oblak] one-loop partition function reproduces character of BMS group [15 Oblak]

→ review of continuum computation of one-loop partition function

Thermal spinning flat space





[Pictures: Giombi]

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 = dt^2 + dr^2 + r^2 d\phi^2$$

periodic identification:

$$(t, r, \phi) \sim (t + \beta, r, \phi + \alpha)$$

inverse temperature and angular potential:
moduli parameter for the (boundary) torus

Want to study partition function:

$$Z(\beta, \alpha) = \int \mathcal{D}g \, \exp(-\frac{1}{\hbar} S_E)$$

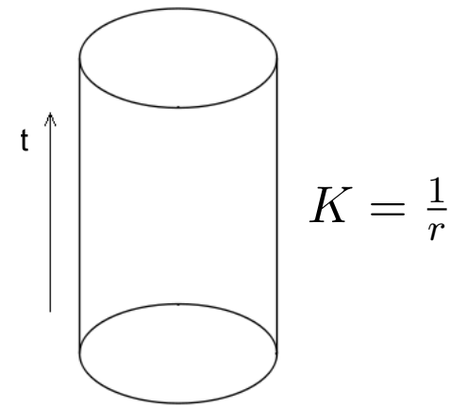
This is expected to reproduce:

$$Z_{\text{Vira}} = \text{Tr} e^{-\beta H - i\alpha J}$$

Boundary Hamiltonian and angular momentum

Classical limit: Action evaluated on solution

$$
\begin{aligned}
S &= -\frac{1}{16\pi G}\int_{\mathcal{M}} d^3x\sqrt{g}R - \frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^2x\sqrt{h}K \\
&\underset{\text{sol}}{=} -\frac{1}{8\pi G}\mathrm{Ar}(\text{torus})_{|r=1} = -\frac{\beta}{4G} \qquad \text{no } \alpha \text{ dependence}
\end{aligned}
$$



There is a factor of 1/2 difference to the result one ‘wishes’ to obtain.
This is due to a different boundary term one (can) adopt for a boundary at infinity.
[14 Detournay et al]

As for now we do choose a finite boundary we stick with the standard boundary term.
Later comments on the alternative boundary term.

One loop correction

[AdS: 08 Giombi-Maloney-Yin]

[15 Barnich-Gonzalez-Maloney-Oblak]

- although topological theory, one loop contribution might be non-vanishing, as graviton loops might not be completely canceled by ghost loops

$$S^{(1)} = -\frac{1}{2}\ln\det\Delta^{(2)} + \ln\det\Delta^{(1)} - \frac{1}{2}\ln\det\Delta^{(0)}$$

Laplacians for spin 2, spin 1 and spin 0

$$-\ln\det(\Delta - m^2) \quad = \quad \int_0^\infty \frac{dt}{t}\int d^3x\, K(t,x,x)$$

- evaluate determinants by heat kernel approach (regulate with mass term)
- use method of images to get heat kernel on quotient space from heat kernels on R^3 (summation over images of the action of the group, gives the sum over k below)

Summing the three contributions together, and taking mass to zero:

$$e^{S^{(1)}} = \prod_{k=2}^{\infty}\frac{1}{|1-q^k|^2} \qquad \text{with} \qquad q = e^{i\alpha}$$

Depends only on angular potential.

Remarks

- the calculation does not make use of the topological nature of the theory

- Can we change that?
- Understand better contributing degrees of freedom and structure of the result? (Why k >1?)

- finite boundaries?

- address more directly the puzzle: how to get field theory (on boundary) from topological theory?

Regge calculus

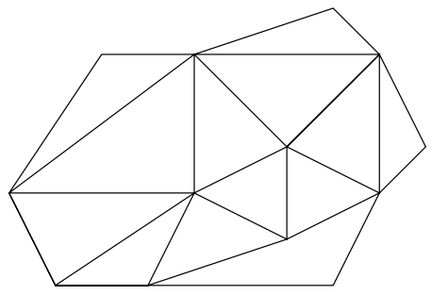
(at least in 3D) semi-classical limit of spin foams

Linearized Regge calculus:

[Roceck, Williams 81]
[BD, Freidel, Speziale 07 : in the context of GBF and graviton propagator]

Regge calculus [61 Regge]

classical version of the Ponzano Regge model [68 Ponzano, Regge]

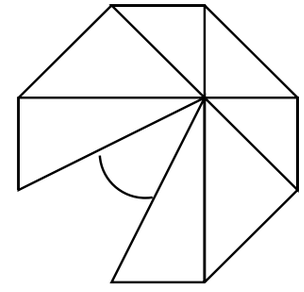


- discretization of general relativity
- use an (arbitrary) triangulation
- assign length variables to edges of the triangulation: defines (piecewise flat) geometry

- dynamics defined by the (here 3D) Regge action:

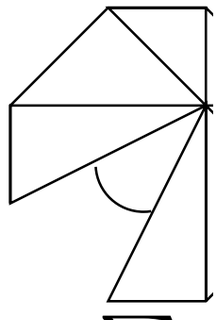
$$S_{\text{Regge}} = -\sum_{\text{bulk edge}} l_{\text{edge}}\epsilon_{\text{edge}} - \sum_{\text{bdry edge}} l_{\text{edge}}\omega_{\text{edge}}$$

$$S_{\text{Eucl}} = -\frac{1}{2}\int d^3x\sqrt{g}R - \int d^2x\sqrt{h}K$$



$$\epsilon_{\text{edge}} = 2\pi - \sum_{\text{tetrahedra}} \theta^{\text{tetrahedron}}_{\text{edge}}$$

dihedral angle



boundary

$$\omega_{\text{edge}} = \pi - \sum_{\text{tetrahedra}} \theta^{\text{tetrahedron}}_{\text{edge}}$$

Regge calculus: Hamilton-Jacobi functions

$$S_{\mathrm{Regge}} = -\sum_{\text{bulk edge}} l_{\mathrm{edge}}\epsilon_{\mathrm{edge}} - \sum_{\text{bdry edge}} l_{\mathrm{edge}}\omega_{\mathrm{edge}}$$

Schlaefli identity for tetrahedron:

$$\sum_{e} l_{\mathrm{edge}}\delta\theta^{\mathrm{tetrahedron}}_{\mathrm{edge}} = 0$$

continuum analogue:

contraction of the variation of Ricci tensor gives total divergence

→ Variation of action:

$$\delta S_{\mathrm{Regge}} = -\epsilon_{\mathrm{edge}}\,\delta l_{\mathrm{edge}} \stackrel{!}{=} 0$$

EOM:

3D gravity is flat.

$$S_{\mathrm{Regge}_{|\mathrm{sol}}} = -\sum_{\text{bdry edge}} l_{\mathrm{edge}}\omega_{\mathrm{edge}}$$

Hamilton-Jacobi function (of continuum GR) for piecewise flat boundary data.

Independent of choice of bulk triangulation.

(Together with local form of action shows topological nature of the theory.)

Quantum Regge calculus

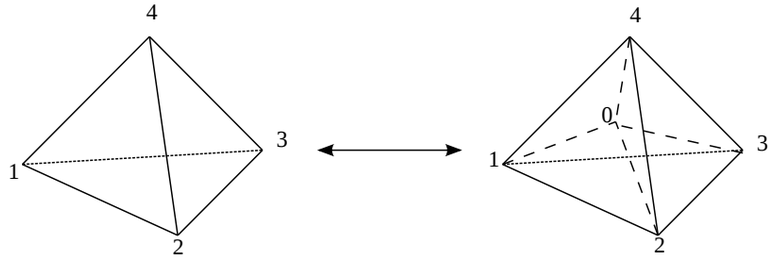
[reviews by Hamber, Williams]

$$Z_{\mathrm{Regge}} = \int \mathcal{D}\mu(l)\ \exp(-\frac{1}{\hbar} S_{\mathrm{Regge}}(l))$$

- 'heated' debate on the correct measure

[Lund-Regge, Hamber-Williams, Menotti et al, …]

- [11 BD, Steinhaus]: Measure should be chosen such so that (at least the linearized) theory is independent of choice of (bulk) triangulation.

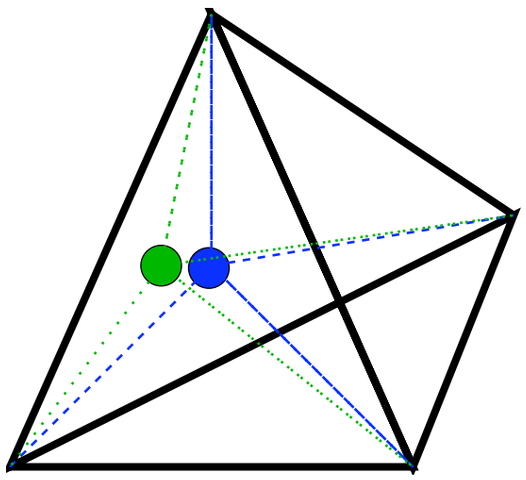


- [11 BD, Steinhaus]: Key point:
  The (3D) Hessian for Pachner move configurations (local changes of the triangulation) factorizes over tetrahedra and edges.
  This allows to construct a triangulation invariant measure!

$$\mu(l) = \frac{\prod_e \frac{l_e}{\sqrt{12\pi}}}{\prod_\tau \sqrt{V_\tau}}$$

- function of background (flat) solution
- matches exactly asymptotics of Ponzano Regge amplitude for one tetrahedron

Diffeomorphism symmetry in Regge calculus



3D Regge EOM impose flatness. Given a flat solution with bulk vertices, the position of any of these vertices can be changed as long as the geometry remains flat. Thus one expects three gauge modes per vertex.
In the linearized theory this results in three null modes per vertex.

[Roceck-Williams 81]

This 'vertex translation' symmetry is the discrete remnant of the continuum diffeomorphism symmetry and is responsible for the topological nature of the theory.

[Bahr, BD 09]: This diffeomorphism symmetry is broken for 4D Regge solutions with curvature. This has severe repercussions (e.g. triangulation dependence) and necessitates a continuum limit.

The solid torus partition function a la Regge

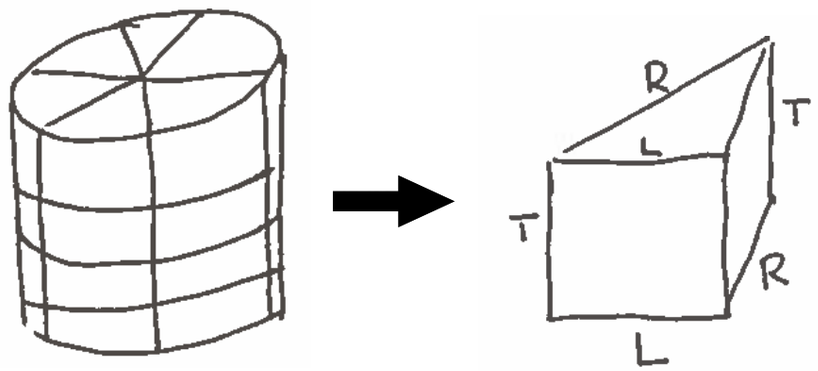
Questions to answer

- Can we take a boundary at finite radius?

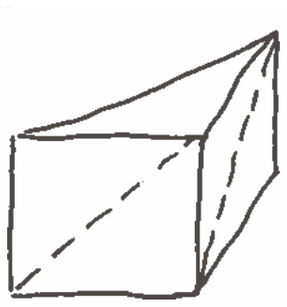
- Do we need to take the continuum limit on the boundary?
  In angular and in time direction?

Set-up

- Choose a triangulation of the solid torus. Allow a very fine triangulation on the boundary but choose bulk triangulation as simple as you can think of.



splits into three tetrahedra



$$V_1 = V_2 = V_3 =: V$$

To get flatness (around time-like axis):

$$a := \frac{L^2}{2R^2} \stackrel{!}{=} 1 - \cos\frac{2\pi}{N}$$

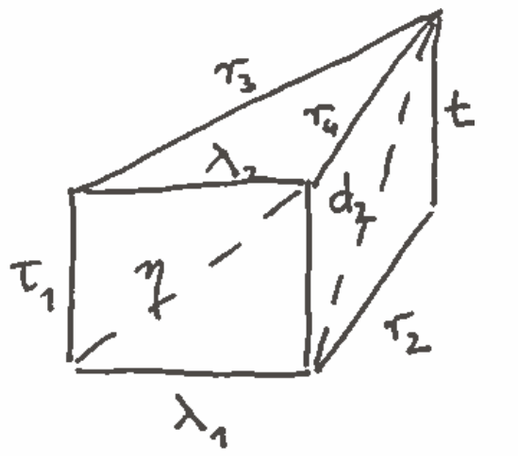
number of prisms in a time slice

- Evaluation of classical Regge action on solution is straightforward and works with simplest triangulation (even just using three tetrahedra).
  Reproduces continuum result. No continuum limit (on boundary) necessary.

Hessian of the Regge action I

- The one-loop correction is determined by (determinant of) the Hessian of the Regge action

$$H_{ee'} = \frac{\partial S_{\mathrm{Regge}}}{\partial l_e \partial l_{e'}} = \sum_{\mathrm{tetra}} \frac{\partial \theta_e^{\mathrm{tetra}}}{\partial l_{e'}}$$



$$H_{ee'} = \frac{l_e l_{e'}}{12V} M_{ee'}$$

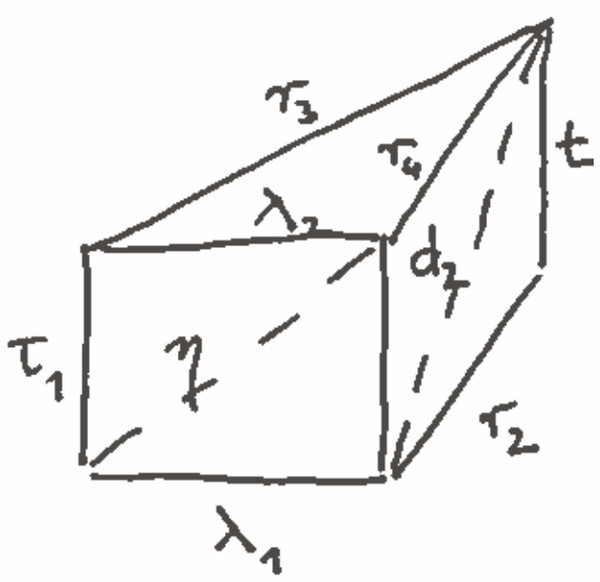
dimension free, only depends on $a = \frac{L^2}{2R^2}$

factors (partially) cancelled by measure term

[similar scaling behaviour found in BD, Freidel, Speziale 07 for cuboids]

- Only ratio L/R matters, this is fixed by the number N of prism in one time slice.
- Two null vectors corresponding to overall scaling symmetry and scaling in time direction only.

Hessian of the Regge action II



$$H_{ee'} = \frac{l_e l_{e'}}{12V} M_{ee'} \qquad a = \frac{L^2}{2R^2}$$

$$M^{\text{prism}} = \begin{pmatrix}
2(-1+a) & 2 & 0 & 0 & 2(1-a) & -2 & 2a & 0 & 0 & 0 & 0 & 0 \\
2 & 2(-1+a) & 0 & 0 & -2 & 2(1-a) & 2a & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -2a & 0 & 2a & 0 & 0 & 2(1-a) & 0 & 0 & 2 & -2 \\
0 & 0 & 0 & -2a & 0 & 2a & 0 & -2 & -2a & -2 & 0 & 2 \\
2(1-a) & -2 & 2a & 0 & -2 & 2 & 0 & 0 & -2 & 0 & -2 & 2 \\
-2 & 2(1-a) & 0 & 2a & 2 & -2 & 0 & 0 & 2 & 2 & 0 & -2 \\
2a & 2a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 \\
0 & 0 & 2(1-a) & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & -2a & -2 & 2 & 0 & 0 & -2 & -1 & 0 & 2 \\
0 & 0 & 0 & -2 & 0 & 2 & -2 & 0 & -1 & -1 & 0 & 1 \\
0 & 0 & 2 & 0 & -2 & 0 & 0 & 1 & 0 & 0 & -1 & 1 \\
0 & 0 & -2 & 2 & 2 & -2 & 0 & 0 & 2 & 1 & 1 & -2
\end{pmatrix}
\begin{matrix}
r_1 \\ r_2 \\ r_3 \\ r_4 \\ d_1 \\ d_2 \\ t \\ \tau_1 \\ \tau_2 \\ \lambda_1 \\ \lambda_2 \\ \eta
\end{matrix}$$

Hessian of the Regge action III

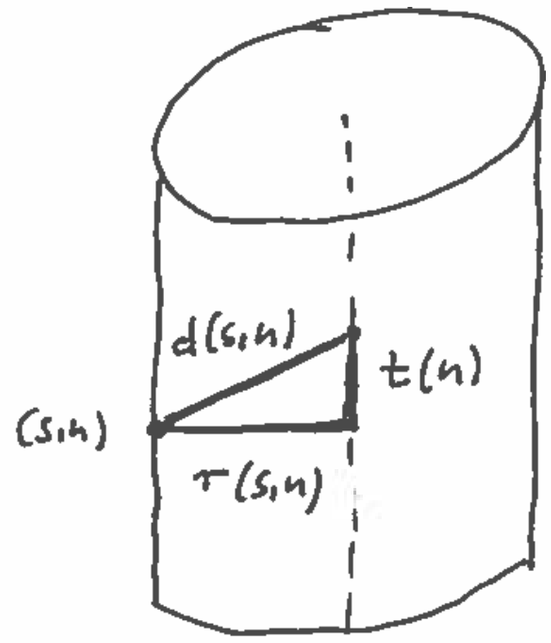
Hessian for the full triangulation

- re-sort (bulk) variables by associating them to vertices (s=angular step, n=time step) on the bdry
- variable transformation to absorb scaling factors

- Fourier trafo

$$x(k,\nu) = \frac{1}{\sqrt{N N_T}} \sum_n e^{-i\frac{2\pi}{N_T}(\nu - \frac{\alpha}{2\pi}k)\cdot n} e^{-i\frac{2\pi}{N}k\cdot s}\, x(s,n)$$

taking care of twisted periodification in time



$$v := \nu - \frac{\alpha}{2\pi}k$$ shifted time frequency

$$S^{(2)} = \frac{1}{2}\sum_{e,e'} x_e H_{ee'} x_{e'} = \frac{1}{2}\sum_{k,\nu} (\hat{x}(k,\nu))^{\mathrm{t}} \cdot \tilde{M}(k,\nu) \cdot (\hat{x}(-k,-\nu))$$

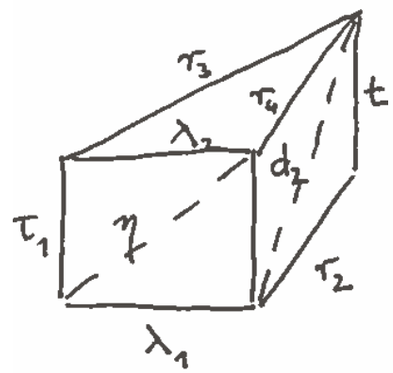
original fluctuation variables

rescaled and Fourier transformed variables

Fourier transform leads to $\delta(k+k')\delta(\nu+\nu')$

Hessian of the Regge action IV

$$\tilde{M}(k,\nu) \quad = \quad \begin{array}{c} t \\ \\ \\ \\ \cdots \end{array} \begin{array}{c} \begin{array}{ccc} t & r & d \end{array} \\ \begin{pmatrix} 0 & -2a\sqrt{N}\delta_{k,0} & 0 & \cdots \\ -2a\sqrt{N}\delta_{k,0} & \Delta_k & -\Delta_k + 2a(1-\omega_v) & \cdots \\ 0 & -\Delta_k + 2a(1-\omega_v^{-1}) & \Delta_k & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix} \end{array}$$



Lattice Laplacian in angular direction $\Delta_k = 2 - \omega_k - \omega_k^{-1}$

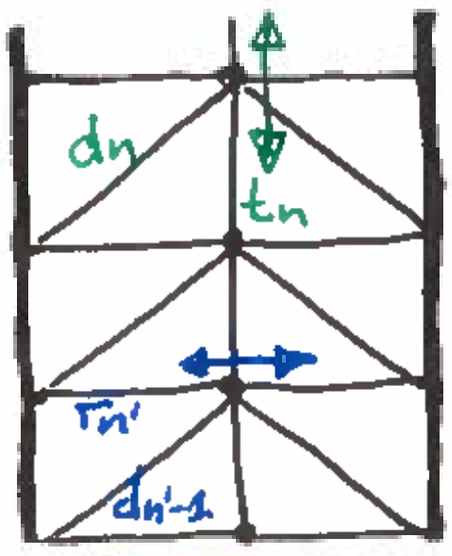
$\omega_k = \exp(i\frac{2\pi}{N}k)$

$\omega_v = \exp(i\frac{2\pi}{N_T}v)$

$2a = \Delta_{k=1}$

- Rather weak coupling between time slices.
- Need time coupling to get dependence on angular twist variable
- (so should not take limit of vanishing $2a$ here)

Null vectors: diffeomorphism symmetry



Null vectors (for each time step) correspond to vertex displacements:

- involving d(k=0) and t variables: displacement of bulk vertex along axis
- involving r(k=+1), d(k=+1) variables: displacement of bulk vertex in radial direction
- involving r(k=-1), d(k=-1) variables: displacement of bulk vertex in (some other) radial direction

Only modes k=0,+1,-1 affected by gauge symmetry
(3 null vectors per bulk vertex)

We can integrate out the d-variables for k>0.

After integrating out d-variables

lattice Laplacian in time direction with angular twist

$$(\tilde{M}'(l,\nu))_{rr} \quad = \quad 2a\,\Delta_v(1-\frac{2a}{\Delta_k})$$

- vanishes for k=+1 and k=-1 modes as $\Delta(k) = 2 - 2\cos(\frac{2\pi}{N}k)$ and $2a = 2 - 2\cos(\frac{2\pi}{N})$

- For computation of determinant we need:

$$\prod_{\nu=0}^{N_T-1}\left(2-\omega_v-\omega_v^{-1}\right) \quad = \quad 2-2\cos(\alpha k)$$

vanishes for $\alpha = 0$

independent of number of time steps

- higher order in a terms:

$$\prod_{k=2}^{N_A-2}\left(1-\frac{2a}{\Delta_k}\right) \quad = \quad \frac{1}{4-2a}$$

One loop correction

Finally: (for N odd)

from previous integration

$$\frac{1}{\sqrt{\det(M_{\rm red})}} = \frac{1}{N} \prod_{2\le k\le (N-1)/2} \frac{1}{2a}\left(1-\frac{2a}{\Delta(k)}\right)^{-1} \frac{1}{(1-e^{ik\alpha})(1-e^{-ik\alpha})}$$

$$= \frac{1}{N}\sqrt{4-2a} \prod_{2\le k\le (N-1)/2} \frac{1}{2a} \frac{1}{(1-e^{ik\alpha})(1-e^{-ik\alpha})}$$

$$= \frac{2}{N}(1+\mathcal{O}(a)) \prod_{2\le k\le (N-1)/2} \frac{1}{2a} \frac{1}{(1-e^{ik\alpha})(1-e^{-ik\alpha})}$$

factors get absorbed into measure term

Features

$$e^{S^{(1)}} = \prod_{k=2}^{\infty} \frac{1}{|1-q^k|^2} \qquad \text{with} \qquad q = e^{i\alpha}$$

- much simpler than continuum calculation:
  gauge fixing only for k=0,+/-1 modes, no (further) regularization necessary, at finite boundary

- identifies contributing degrees of freedom and explains structure of the result

- need to take continuum limit in angular direction:
  for getting correction term small and for getting the product over all modes

- result for alpha-dependent part is independent of number of time slices!

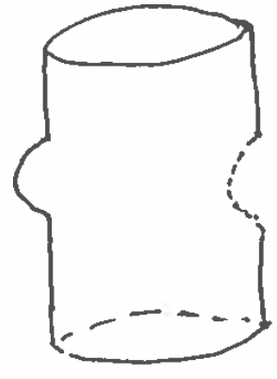
[free particle in a 'perfect discretization':
Bahr, BD, Steinhaus 11]

- Can we see the contributing degrees of freedom in the effective action for the boundary fluctuations?

Boundary Fluctuations: what to expect

$$S_{\text{eff}} = \frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^2x\sqrt{h}K$$

bumps= conformal mode



- functional of the 2D boundary geometry (non-uniqueness due to 'inside vs outside bumps')

[related to Freidel 08]

- invariant under boundary diffeomorphism: should only depend on conformal metric mode
- This is harder to evaluate than it seems (using metric variables)!

[Skenderis, Solodukhin 99: for adS one should obtain the effective action by integrating out Liouville field, heavily use of Fefferham Graham expansion]

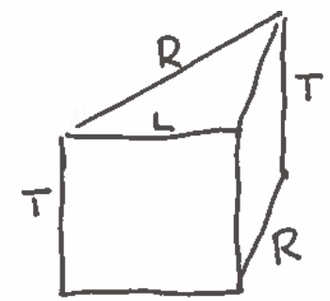
[Carlip 05: for adS: Liouville field arises because it describes position of boundary (using Fefferham-Graham expansion) and describes breaking of 3D diffeomorphism symmetry giving rise to entropy]

[Riello (unpublished notes): for flat case, can consider an auxiliary field which describes position of boundary from the central axis]

- Suggest that our r variable is related to (Liouville?) auxiliary field.
  However to show this directly is involved due to (broken) diffeo invariance in angular direction.

Boundary Fluctuations: what we get

- the effective action can be 'easily computed': encoded in 3x3 matrix for the three types of boundary edges
- diffeomorphism symmetry in time direction is exactly realized, even on the discrete level
- diffeomorphism symmetry in angular direction is more complicated to recover:
- boundary continuum limit $T = \hat{T}\varepsilon\,, \quad L = \hat{L}\varepsilon$



$$\rightarrow \quad \omega_k = \exp\left(i\frac{\hat{L}\epsilon}{R}k\right) = \exp(i\epsilon K)\,, \qquad \omega_v = \exp\left(i\frac{2\pi\hat{T}\epsilon}{\beta}v\right) = \exp(i\epsilon V)\,, \qquad a = 1-\cos(\epsilon K)$$

- expand in epsilon, and truncate at lowest (zeroth) order
- the resulting matrix still breaks angular diffeomorphisms
- matrix entries are rational function in k,v: terms of homogeneity degree 0 and 2
- take only the degree 2 terms: these dominate as k,v = 0,1, 2, …
- the resulting matrix has now two null modes, corresponding to spatial and time diffeos's

Boundary Fluctuations: what we get

$$\omega_k \;=\; \exp\left(i\frac{\hat{A}\epsilon}{R}k\right) \;=\; \exp(i\epsilon K)\,, \qquad\qquad \omega_v \;=\; \exp\left(i\frac{2\pi\hat{T}\epsilon}{\beta}v\right) \;=\; \exp(i\epsilon V)\,,$$

- after this limiting procedure there is just one eigenvalue for the normal metric deformation mode:

$$\text{Eigenvalue} \;=\; -\frac{R^2}{\hat{L}^2}\frac{K^2+KV+V^2}{V^2}$$

- consider discretized and linearized Liouville theory and compare the resulting effective action

$$S = \int d^2x\sqrt{g}(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + {}^{2\mathrm{D}}R\phi)$$

$$\downarrow$$

$$S_{\text{eff}} \sim -\frac{K^2+KV+V^2}{\hat{T}^2K^2+\hat{L}^2V^2}$$

$$S' = \int d^2x\sqrt{g}(\frac{1}{2}g^{00}\partial_0\phi\partial_0\phi + \gamma\ {}^{2\mathrm{D}}R\phi)$$

$$\downarrow$$

$$S'_{\text{eff}} \sim -\gamma^2\frac{K^2+KV+V^2}{\hat{L}^2V^2}$$

Matches our result.

Remarks

Include cosmological constant

- The calculation can be easily generalized to dS or AdS background.
  [wip Bonzom, BD, Mizera, Riello]
- One has to use Regge calculus with homogeneously curved tetrahedra, only this will give triangulation independence. [Bahr, BD 09a, Bahr, BD 09b]

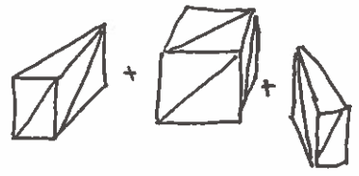
Full non-perturbative version?

- The full (non-linear) quantum version of Regge calculus is given by the Ponzano Regge (spin foam) model. [Ponzano, Regge 68]
  Need to consider BF-theory with boundary. [Bonzom, Smerlak, unpublished notes]
- Using (3D) spherical tetrahedra corresponds to the Turaev Viro model.
- Using (3D) hyperbolic tetrahedra: analytical continuation of Turaev Viro (results)?
  [eg Geiller, Noui 13]

Conclusions

- in 3D discrete models provide elegant method to evaluate partition functions
- provides an explicit model for holography (based on topological field theory)
- provides toy model to understand continuum limit in background independent theories in consistent boundary formulation [BD 12,14]
- confirm [Carlip 05] interpretation: auxiliary field (radial edges) gives rise to entropy
- regaining angular diffeomorphism symmetry is involved:
  Use different building blocks with one side being curved? [similar to Bahr, BD 09b]

Many further directions to explore



- introduce subdivision in radial direction:
  explicit model for holographic renormalization [Skenderis] / MERA-AdS [Vidal, Swingle, …]

- (BTZ) black holes: identify degrees of freedom responsible for entropy (at the horizon) [Carlip]

- applications to dS?

- Applications to 4D: need to take full continuum limit.
- By comparing to corresponding continuum computation, fix the measure for 4D Regge calculus. [14 BD, Kaminski, Steinhaus: Discretization independence implies non-locality in 4D gravity]

- Hamiltonian analysis of charges in (Ponzano-) Regge realization:
  on finite boundaries [recent paper for AdS continuum: Andrade, Marolf]

[canonical Regge calculus: and Hamiltonian constraints: 09 Bahr, BD, 12, 13 BD, Hoehn;
constraint algebra of 3D constraints in the discrete: 13 BD, Bonzom;
quantum Hamiltonians and algebra of quantum Hamiltonians: 09 Bonzom, Freidel; 14 BD, Esterlis]

Alternative boundary term

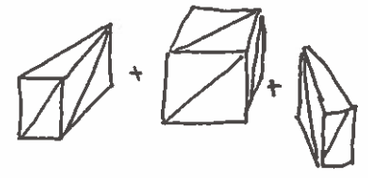
$$S = -\frac{1}{16\pi G}\int_{\mathcal{M}} d^3x\sqrt{g}R - \frac{1}{2}\frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^2x\sqrt{h}K$$

[14 Detournay, Grumiller, Scholler, Simon]

- derive an alternative variational principle for infinite boundary

- for finite boundary this does however change the equations of motion

- in particular for Regge calculus:
  -the triangulation we use turns out to be inconsistent (for finite R) with the alternative boundary (due to induced deficit angle at horizontal edges)
  -moreover it gives not the background action one wishes for (due to induced deficit angle at time axis)



- can use alternative triangulation: glue the current triangulation with an outer ring built from prisms and cuboids

- need furthermore insert a ring of wedges (prisms and pyramides alternating) to be consistent at finite radius R

This issue does not arise in adS where the counter boundary term is just a functional of the boundary metric.