Simultaneous and self-referenced amplitude and phase measurement of two frequency combs using multi-heterodyne spectroscopy

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Abstract: We present a new type of multi-heterodyne spectroscopy that does not require separate reference field characterization and use it to demonstrate simultaneous, full-field characterization of two 40-line by 10-GHz optical frequency combs in line-by-line pulse shaping and fiber transmission applications.

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Multi-heterodyne spectroscopy (MHS) can measure the relative amplitude and phase differences between each line of two optical frequency combs (OFC) with slightly different comb spacings ($\Delta f_A$ and $\Delta f_B$) in a single rapid (microseconds) acquisition by transferring the amplitude and phase difference between each comb line to an electrical baseband signal [1–4]. As a coherent measurement technique, MHS is sensitive to both phase and amplitude which makes it ideally suited for spectroscopy, and as a diagnostic for waveform generation. For telecommunications applications, such as coherent wavelength-division multiplexing (WDM) and Nyquist-WDM, knowledge of the comb-line phase is helpful to reduce coherent crosstalk [5].

In MHS, the purposely offset OFC repetition rates cause the two combs to temporally scan through each other, and after coherent detection the result is a field cross-correlation of the two input combs (i.e., $a(t) \ast b^*(t)$ where $a(t)$ and $b(t)$ are the complex envelopes of the two combs, $A$ and $B$). For accurate waveform characterization, an independent measurement of either comb is necessary to unambiguously characterize the other. Due to the difficulty in characterizing the amplitude and phase of broadband OFCs, most demonstrations just assume that the reference OFC is transform limited. As we will show, this can lead to significant measurement errors. In this paper, we extend the MHS technique to enable self referencing to simultaneously characterize two combs from a single interferogram in a simple experimental configuration and without the need to measure, or presume, the characteristics of either field.

Fig. 1 shows the concept and experimental arrangement for simultaneous amplitude and phase measurement of two OFCs. In the time domain, the ‘signal’ comb may be expressed as $a(t) = \sum_{i=0}^{N} A_i \exp(j2\pi \Delta f_A t + f_A)$ and the ‘reference’ comb as $b(t) = \sum_{k=0}^{M} B_k \exp(j2\pi \Delta f_B t + f_B)$ where $N, M$ are the number of lines in each comb, $i, k$ are comb line index numbers, $f_A, f_B$ are the lowest frequency in each comb, and $A_i$ and $B_k$ are a set of complex numbers containing

![Fig. 1: Self-referenced multi-heterodyne spectroscopy concept and experimental arrangement.](image-url)
the amplitude and phase of each comb line (other terms are defined in Fig. 1). The two combs are combined using a 2 × 2 coupler and mixed with balanced coherent detection (balanced detection improves the signal-to-noise ratio when compared to single ended detection). A 50 GS/s digitizer with 50 Ω inputs converts the photocurrent into a voltage and records the signal for 2 µs yielding $v(t) = \sum_{i=0}^{M} A_iB_i^* \exp[j2\pi(\Delta f_A i - \Delta f_B k) + f_A - f_B]$. Each electrical line ($M \times N$ total) contains the phase relationship between one line in comb $A$ and one line of comb $B$ (e.g., $A_1$ and $B_1$, $A_1$ and $B_2$, or $A_2$ and $B_2$, etc.). Most of these beat tones lie outside the electrical bandwidth and are filtered out during detection and never measured. Following Fig. 1, the electrical lines group into sets, $Z_i = \sum_{k=0}^{N-1} A_iB_k^* \exp[j2\pi\Delta f i + j2\pi(f_A - f_B - \Delta f gl)]$ with spacing $\Delta f = \Delta f_A - \Delta f_B$. Here, $i$ is the set number and corresponds to the mixing harmonic (i.e., $k\Delta f_A + \Delta f_B$) between the two combs. The $Z_0$ set appears closest to dc and it corresponds to the beatings between adjacent lines of $A_i$ and $B_i$ (i.e., when $k = i$), and the $Z_{-1}(Z_{+1})$ sets contain beat frequencies between the $A_i$ and $B_{i-1}(B_{i+1})$ and appear around the first harmonic [i.e., $l(\Delta f_A + \Delta f_B)/2$]. To determine the amplitude and phase of each electrical comb line, each line must have a unique frequency which can be ensured by proper selection of start frequencies ($f_A$, $f_B$) and comb line spacings ($\Delta f_A$, $\Delta f_B$). Generally in traditional MHS experiments, $Z_0$ is the only set considered and therefore requires complete knowledge of either $A$ or $B$ to accurately retrieve the amplitude and phase of the unknown comb.

However, if we do not ignore the other sets of electrical lines and instead use two adjacent sets, we can simultaneously fully characterize $A$ and $B$. For example, measuring $Z_0$ and $Z_{-1}$ yields $B_i$ and $B_{i-1}$ referenced to the same $A_i$, which provides the phase and amplitude difference between $B_i$ and $B_{i-1}$. Then the entire complex spectrum of $B$ is reconstructed by $B_i = [B_{i-1}Z_{-1,i}/Z_0]^*$, where $i$ refers to the comb index. Similarly, comb $A$ can be reconstructed from the same measurement by $A_{i+1} = A_iZ_{0,i+1}/Z_{-1,i}$. Compared to MHS, self-referenced MHS requires twice the electrical bandwidth. Although not discussed here, measurement of higher order sets can help recover both combs in presence of spectral gaps and provide redundant information to verify retrieval.

Fig. 2 shows results of a self-referenced MHS measurement of two 400-GHz bandwidth OFCs. Comb $A$ and $B$ are both centered near 1550 nm and they have frequency spacings of 10.019 GHz and 10.009 GHz, respectively. Comb $A$ is generated from a single-frequency laser that experiences amplitude and phase modulation from a dual-electrode Mach-Zehnder modulator (DE-MZM), followed by linear pulse compression in standard single mode fiber (SSMF) and spectral broadening in a highly nonlinear fiber. Comb $B$ is generated from a single-frequency laser that experiences amplitude and phase modulation from a DE-MZM followed by two phase modulators and linear compression using SSMF. Fig. 2(a) shows the recorded temporal interference, $v(t)$, of the two combs. The amplitude shape is the convolution of the two combs which occurs every 100 ns [i.e., $1/(\Delta f_A - \Delta f_B)$]. The entire record lasts 4000 ns and contains 40 such measurements (i.e., averaging factor of 40). Fig. 2(b) shows the Fourier transform of $v(t)$ in which five sets of combs fall within the electrical bandwidth (i.e., $Z_{-2}$ through $Z_{+2}$). Extracting the amplitude and phase of the lines in $Z_0$ recovers $A(\omega)B^*(\omega)$, which is the cross-correlation of the two combs [Fig. 2(c,d)]. Note, this extraction algorithm includes correction of phase drift between the two combs, and therefore does not require the two lasers to be phase locked (similar to the DSP in an intradyne receiver). As Fig. 2(e-h) shows, using the proposed retrieval...
algorithm on $Z_0$ and $Z_{-1}$ yields an unambiguous measurement of $A(\omega)$ and $B(\omega)$. Even though $a(t)$ and $b(t)$ appear close to transform limited, the retrieved waveforms are very different when compared to their cross-correlation. This confirms the importance of either completely characterizing the reference, or using the self-referenced MHS technique for accurate waveform measurement. Lastly, independently measured optical spectra (green curves) verify the accurate and simultaneous retrieval of the two spectral intensities.

To further experimentally verify the consistency and relative accuracy of the phase retrieval of the proposed self-referenced MHS technique, we measured $B$ against different $A$ waveforms. Fig. 3 shows self-referenced MHS measurements where $A$ has experienced propagation through 15 km of SSF and then where $A$ is shaped into a binary phase shift keying (BPSK) waveform. For comparison, we also show the $A$ that is shaped assuming a transform-limited reference. Fig. 3(a) shows comb $B$’s retrieved spectral phase for all three cases. The errors between the three measurements are within 0.04 rad, further verifying that $B$ is accurately retrieved independent of the shape of $A$. Fig. 3(b,c) shows the measurement of $A$ after 15 km of SSF where the waveform is spread across the entire 100 ps OFC period due to the fiber’s dispersion. Fig. 3(c) shows the retrieved spectral phase after removing -255 ps/nm of dispersion which matches well with Fig. 2(h).

A potential application for this technique is optical arbitrary waveform generation (i.e., line-by-line pulse shaping) in which accurate measurement of the amplitude and phase is a necessary diagnostic. Fig. 3(d-e) shows a shaped BPSK waveform using a line-by-line waveform shaper to control the amplitude and phase of each comb line using feedback from the self-reference MHS technique. The temporal phase clearly shows $0-\pi$ jumps and flat phase levels and a spectral phase shaping error below 0.1 rad. As a comparison, Fig. 3(f,g) shows the same shaping experiment using feedback from MHS without self referencing (i.e., using only the $Z_0$ term and assuming $B$ is transform limited). In this case, the shaped waveform is incorrect and the spectral phase errors are equal to the spectral phase of $B$. Most importantly, for all three measurements, the spectral phase of $B$ is identical.

In conclusion, we have proposed and demonstrated self-referenced MHS for full optical waveform characterization which improves the accuracy over typical MHS implementations, yet keeps its simple experimental arrangement.

References


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