Iterative Beamforming and Power Control for MIMO Ad Hoc Networks

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Abstract— We present a distributed joint power control and transmit beamforming selection scheme for multiple antenna wireless ad hoc networks. Under the total network power minimization criterion, a joint iterative beamforming and power control algorithm is proposed to reduce mutual interference at each node. Total network transmit power is minimized while ensuring a constant received signal-to-interference and noise (SINR) at each receiver. First, transmit beamformers are selected from a predefined codebook to minimize the total power in a cooperative fashion. We also study the interference impaired network as a noncooperative beamforming game. By selecting transmit beamforming judiciously and performing power control, convergence of noncooperative beamformer games is guaranteed throughout the iterations. The noncooperative distributed algorithm is compared with centralized and cooperative solutions through simulation results.

Index Terms—MIMO, ad hoc networks, limited feedback beamforming, game theory

I. INTRODUCTION

Using multiple-input multiple-output (MIMO) techniques in communication has attracted an increasing interest. The use of multiple antennas improves the capacity and the spectral efficiency of communication systems [1] [2]. MIMO techniques can be applied to point-to-point links, cellular networks, or ad hoc networks. Point-to-point MIMO links are extensively studied in the literature and the great potential of MIMO in point-to-point communication is shown in [1] [3]. In cellular networks, beamforming algorithms are designed as a means to minimize the total transmit power [4] or to increase capacity using MIMO techniques [5]. In ad hoc networks, without a central controller, beamforming techniques are used to overcome lower system throughput and higher energy consumption. Distributed spatial beamforming algorithms are proposed for ad hoc MIMO networks in [6] where the problem is formulated as a noncooperative game for overall power minimization of the network under a quality-of-service (QoS) constraint.

Linear precoders (eigencoders) and beamformers are well studied for point-to-point MIMO links [7] [8]. For MIMO communication with low overhead between a single transmitter receiver pair, a quantized transmit beamforming codebook was designed using limited feedback beamforming [3]. The concept is based on selecting a codeword in a predetermined codebook that is known to both transmitter and receiver. The transmit beamformer is selected from the predefined codebook to reduce latency in highly mobile and unstable communication networks. When the communication system needs low rate or has bandwidth constraints, feedback overhead in situations like non-reciprocal channels are substantially reduced using the proposed codebook design approach.

In this paper, we are interested in the power minimization problem in ad-hoc networks using distributed algorithms with limited feedback transmit beamformer selection under a QoS constraint which is constant signal-to-interference and noise ratio (SINR) for all receivers. The global solution to this problem is challenging in ad-hoc networks. Optimal power minimization algorithms are difficult to design for beamforming games, due to the lack of a natural ordering of the actions. For example, the transmit beamformer and power of one node pair affects the SINR of other node pairs, and vice versa. Moreover, if the node pairs belong to different regulation entities, the non-cooperative node pairs may only want to minimize their own transmit power rather than the overall transmit power. Therefore, finding the optimal distributed transmit beamformer solution for the power minimization problem is not straightforward. Our contributions in this paper are twofold: first we study an efficient cooperative beamforming algorithm for power minimization problem, and second we design a noncooperative power minimization scheme using beamforming techniques. We compare the performance of the proposed algorithms with the optimal global solution which is found by searching the feasible strategy space.

The rest of this paper is organized as follows. Section II outlines the system model in the paper. The optimization problem and game theoretical interpretation is studied in Section III. The cooperative wireless ad hoc network and non-cooperative counterpart are investigated Section IV. The performance evaluation is provided in Section V. Finally, Section VI concludes this paper.

II. SYSTEM MODEL AND CONCEPTS

In this paper, we consider a wireless ad hoc network shown in Fig. 1. The ad hoc network consists of multiple transmit and receive antenna node pairs. All nodes are assumed to be using the same channel. The interference comes from other
node pairs which operate simultaneously on the same channels. In this ad hoc network model, there are \( N \) node pairs and each node pair \( m \in \{1, 2, \ldots, N\} \) has one transmitter and one receiver. Each node is equipped with \( T \) antennas. The complex symbol stream transmitted is \( b_m \in \mathbb{C} \) with \( E\{|b_m|^2\} = 1 \). Each node has a unit-norm receive/transmit beamformer pair \((w_m, t_m)\) with \( w_m, t_m \in \mathbb{C}^T \).

In this ad hoc network model, there are \( N \) nodes. The additive white Gaussian noise terms are quasi-static.

Each node can select from \( \Theta = \{t_1, t_2, \ldots, t_N\} \) and \( \mathbf{P} = [P_1, P_2, \ldots, P_N] \) denote the selected transmit beamformer and transmit power vectors for \( N \) nodes respectively. The \( T \times T \) interference plus noise covariance matrix at the \( m \)-th receiving node is

\[
\mathbf{R}_m(\Theta_{-m}, \mathbf{P}_{-m}) = \sum_{i \neq m} P_i \mathbf{H}_{m,i} t_i t_i^H \mathbf{H}_{m,i}^H + \sigma^2 \mathbf{I},
\]

where \( \Theta_{-m} \) and \( \mathbf{P}_{-m} \) are the transmit beamformers and powers of nodes other than \( m \).

The normalized receive beamformer at the \( m \)-th receiving node is

\[
w_m = \frac{\hat{w}_m}{||\hat{w}_m||},
\]

where \( \hat{w}_m = \mathbf{R}_m^{-1} \mathbf{H}_{m,m} t_m \). The resulting received SINR at the \( m \)-th receiving node due to the desired transmitter of \( m \)-th node pair is

\[
\Gamma_m = \frac{P_m ||\mathbf{H}_{m,m} t_m||^2}{\sum_{i \neq m} P_i ||\mathbf{H}_{m,i} t_i||^2 + \sigma^2},
\]

where \( ||\mathbf{w}_m||^2 = ||\mathbf{t}_m||^2 = 1 \) for all \( m \).

The proposed distributed algorithms attempt to achieve a target SINR by adjusting transmit powers. To construct a distributed iterative limited feedback beamforming scheme, let us first consider the case when there is only one node pair in the wireless network. The receiver selects the transmit beamformer from the codebook \( \Delta_1 \) as

\[
t_1^* = \arg \max_{t_1 \in \Delta_1} \Gamma_1,
\]

where \( t_1^* \) is the optimal transmit beamformer selection for one node pair. Then, the receiver returns the index of the beamformer for transmit beamformer selection \( t_1^* \) and the received SINR, \( (t_1^*)^H \mathbf{H}_{1,1}^H \mathbf{R}_1^{-1} \mathbf{H}_{1,1} t_1^* \), through a low-rate feedback channel. The transmitter selects the transmit beamformer in order to minimize its own transmission power \( P_1 \), where \( P_1 \) is updated as

\[
P_1 = \gamma_0 \frac{\mathbf{w}_m^H \mathbf{H}_{m,m}^H t_m}{t_1^*^H \mathbf{H}_{1,1}^H \mathbf{R}_1^{-1} \mathbf{H}_{1,1} t_1^*},
\]

where \( \gamma_0 \) is the target SINR value.

Consider now the case where \( N \) node pairs coexist in the wireless network. Note that for each node pair \( m \), the value of the received SINR, i.e. \( \Gamma_m \) is a function of \( (\Theta, \mathbf{P}) \). Therefore, the transmit power of one node pair depends not only on its own transmit beamformer selection, but also the transmit power and transmit beamformer selections of other nodes in the network. Furthermore, in beamforming, if user \( i \neq m \) changes its transmit beamformer \( t_i \) to increase its own SINR \( \Gamma_i \), it can either increase or decrease \( \Gamma_m \), the SINR of link \( m \), depending on the relative positions of the nodes. Therefore, designing an optimal distributed algorithm which converges to a set of beamformers to minimize the overall transmit power while meeting target SINRs for all node pairs is not a straightforward task.
III. OPTIMIZATION PROBLEM AND GAME THEORETICAL INTERPRETATION

The goal is to minimize the transmit power of all nodes \( m \in \{1, 2, ..., N\} \) under constant target SINR \( \gamma_0 \). The optimization problem can be defined as,

\[
\min_{\|w_m\| = \|t_m\| = 1, P_{m1} < P_m \leq P_{max}} \sum_{m=1}^{N} P_m \quad (7)
\]

subject to \( \Gamma_m \geq \gamma_0 \), \( m \in \{1, 2, ..., N\} \),

where \( P_{min} \) and \( P_{max} \) are the minimum and maximum transmit powers, respectively. We consider the above problem as a normal form game, which can be mathematically defined by the triplet \( \prod = (N, C, \{U_m\}_{m=1}^{N}) \) where \( \prod \) is a game, \( N = \{1, 2, ..., N\} \) is the finite set of players of the game, \( C = C_1 \times C_2 \times \ldots \times C_N \) represents the set of all available actions for all the players and \( \{U_m\}_{m=1}^{N} : C \rightarrow \mathbb{R} \) is the set of utility functions that the players associate with their strategies. Actions \( c_m \in C_m \) for a player \( m \) are the transmit powers \( P_m \in [P_{min}, P_{max}] \) and the transmit beamformer selections \( t_m \in \Delta_m \).

Players select actions to maximize their utility functions. One of the questions that arise is whether the beamforming selections \( \Theta = [t_{m1}, t_{m2}, ..., t_{mN}] \) and eventually power allocations \( P = [P_1, P_2, ..., P_N] \) will converge to a Nash equilibrium (NE) solution. In the following section, we will discuss scenarios where the node pairs are cooperative and non-cooperative in order to search for the best performance and convergence results.

IV. COOPERATIVE AND NON-COOPERATIVE POWER MINIMIZATION ALGORITHMS USING BEAMFORMING

A. Optimal (Centralized) Solution

In a wireless ad hoc network, the centralized agent can select the transmit beamformers and the corresponding transmit powers to minimize the total transmit power of all transmitting antennas as,

\[
(\Theta^*, P^*) = \arg \min_{\Theta, P} \sum_{m=1}^{N} P_m(\Theta, P_{-m}), \quad (8)
\]

where \( \Theta^* = [t_{m1}^*, t_{m2}^*, ..., t_{mN}^*] \) and \( P^* = [P_1^*, P_2^*, ..., P_N^*] \) are the optimal transmit beamformer and power solutions respectively. The transmit power \( P_m \) of \( m \)-th node pair is defined as

\[
P_m(\Theta, P_{-m}) = \frac{\gamma_0}{t_m^H \mathbf{H}_{mm} \mathbf{H}_{mm}^{-1} \mathbf{H}_{mm} \mathbf{R}_{mm}^{-1} \mathbf{R}_{mm} t_m}, \quad (9)
\]

where \( \mathbf{R}_{mm} \) is a function of \( (\Theta_{-m}, P_{-m}) \) as shown in (2). In order to compute (8), the centralized agent evaluates the total network power for \( T^N \) possible transmit beamforming vector combinations. Finding the centralized transmit beamformer is cumbersome in large-scale wireless ad-hoc network. Next, we will introduce a decentralized power minimization algorithm using cooperative and noncooperative techniques.

B. Cooperative Power Minimization using Beamforming

In this section, we consider the scenarios where all node pairs in wireless network are cooperative. In a cooperative game, nodes in the network are able to coordinate and select the transmit beamformer accordingly. From the system point of view, we want to find beamformer assignments such that the overall power in the whole network is minimized. We analyze a cooperative power minimization algorithm (COPMA) which can converge to the optimal Nash equilibrium (NE) with arbitrarily high probability. This method is analogous to the decentralized negotiation method called adaptive play [9].

In COPMA, each node pair in the network maintains two variables \( P_{current} \) and \( P_{updated} \) which are the total transmit power in the network previously and after the random change of transmit beamformer respectively. The key characteristic of COPMA is the randomness deliberately introduced into the decision making process to avoid reaching a local solution. In COPMA, the choices of players (in our case transmit beamformer selections) lead the system to the optimal NE solution with arbitrarily high probability [9]. The summary of COPMA is provided as follows.

**Initialization:** For each transmitting and receiving pair \( m \in N \), randomly select a transmit beamformer and let \( P_m = P_{max} \).

**Repeat:** Let \( t_m(n) \in \Delta_m \) denote the transmit beamformer of the \( m \)-th node pair in iteration \( n \). Randomly choose a node pair \( q \) to update.

1) Set \( t_m(n) = t_m(n-1), \forall m \in N \). Calculate \( P_m \) as in (9) \( \forall m \in N \) and record the current total network power in \( P_{current} = \sum_{m=1}^{N} P_m \).

2) To update node pair \( q \), randomly choose a transmit beamformer, \( t_q^{updated} \in \Delta_q \). Then, compute the updated total network power \( P_{updated} = \sum_{m=1}^{N} P_m \) with \( t_q^{updated} \) based on the received power values from all other node pairs \( m \in N \setminus q \).

3) For a smoothing factor \( \tau > 0 \), \( t_q(n) = t_q^{updated} \) for the \( q \)-th node pair with probability

\[
\frac{1}{1 + \exp ((P_{updated} - P_{current})/\tau)} \quad (10)
\]

Until : Predefined number of iteration steps \( \kappa \).

Note that step-3 of the updating rule implies that if \( t_q^{updated} \) yields a better performance, i.e. \( (P_{updated} - P_{current}) < 0 \), the \( q \)-th node pair will change to updated beamformer \( t_q^{updated} \) with high probability. Otherwise, it will keep the current transmit beamformer with high probability. Note also that the tradeoff between COPMA’s performance and convergence speed is controlled by the parameter \( \tau \). Large \( \tau \) represents extensive space search with slow convergence, whereas small \( \tau \) represents restrained space search with fast convergence. The smoothing factor \( \tau \) is selected to be a function of \( n \) such that as \( n \) increases, \( \tau \downarrow 0 \). For example, we choose \( \tau \) inversely proportional to \( n^2 \) in our simulations.

C. Noncooperative Power Minimization using Beamforming

In this section, we want to obtain a distributed noncooperative transmit beamforming scheme in MIMO ad-hoc networks which is guaranteed to converge. The interaction among \( N \) selfish node pairs is defined as Non-cooperative Power Minimization Game (NPMG) where each node pair
attempts to find their own transmit beamformers. In the noncooperative joint iterative limited feedback beamforming and power control game, the $N$ node pairs care about their own utility maximization exclusively, rather than the overall aggregated power.

For NPMG, we use the following utility function for each user for the transmit beamformer and power selection at iteration $n$: 

\[ (t_m(n + 1), P_m(n + 1)) = \arg \max_{t_m, P_m} U_m(P_m, t_m, t_m(n)), \]

\[ U_m(P_m, t_m, t_m(n)) = \log(P_m t_m^H H_{m,m} R_m^{-1} H_{m,m} t_m) - P_m t_m^H H_{m,m} R_m^{-1} H_{m,m} t_m \gamma_0. \]  

(11)

The first term in the above utility function represents the SINR maximizing term and the second term represents a pricing function for power minimization.

The procedure for NPMG can be summarized as following:

**Initialization**: The initial transmit beamformer of each node is selected by maximizing the utility function in (11) when $P_m = P_{\text{max}}$, $\forall m \in N$. Fix the transmit beamformer selections until the end of iterations to prevent unstable transmit beamformer selection oscillations.

**Repeat**: In each iteration $n = 2, \ldots, \kappa$.

1) For each of the node pairs $m \in N$, the $m$-th node pair’s transmit power is

\[ P_m(n + 1) = \arg \max_{P_m} U_m(P_m, t_m, t_m). \]

(12)

Until: Predefined number of iteration steps $\kappa$.

The iterative convergence of NPMG is shown in the next theorem.

**Theorem 1**: Starting from a feasible initial network configuration, NPMG converges to a locally minimum transmitted power solution.

**Proof**: Under fixed transmit beamformers $\Theta$, maximization of the utility function defined in (11) with respect to power is simply a power control algorithm. To find this, we evaluate:

\[ \frac{\partial U_m}{\partial P_m} = \frac{1}{P_m} \frac{t_m^H H_{m,m} R_m^{-1} H_{m,m} t_m}{\gamma_0} = 0, \]

and the maximizing transmit power $P_m$ is given by

\[ P_m = \frac{\gamma_0}{t_m^H H_{m,m} R_m^{-1} H_{m,m} t_m}. \]

(13)

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(14)

Note that $\partial^2 U_m/\partial^2 P_m = -1/P_m^2 < 0, \forall m \in N$. Therefore, $U_m$ is a strictly concave function of transmit power $P_m$ under constant transmit beamformers $\Theta$. Therefore, $U_m$ is a quasiconcave function optimized on a convex set $[P_{\text{min}}, P_{\text{max}}]$. The existence of pure strategy NE point follows directly from the results of game theory [10]. Power control guarantees the convergence of the algorithm to the NE while minimizing the total transmit power iteratively [11].

The above proposed algorithm results in power minimization across the network, while guaranteeing the convergence in a noncooperative manner. We present a detailed performance evaluation of COPMA and NPMG in next section.

V. SIMULATION RESULTS

In this section, we investigate the performance results of centralized optimization, COPMA and NPMG. We assume that there are $N = 4$ and $N = 8$ transmitting and receiving pairs in two different wireless ad-hoc networks. All transmitting and receiving nodes are randomly located in a square of $100m \times 100m$ area. Each transmitter and receiver is separated by a random distance that is uniformly distributed between $10m$ and $20m$. Each entry of the channel matrix $H_{m,i}$, $\forall m, i \in N$ is independent identically distributed according to $CN(0, 1)$. The target SINR is $\gamma_0 = 15$ dB for all the node pairs. We assume that channels do not vary during the iterations and the Grassmannian codebook of [3] is used. The codebook size is selected to be $T = 16$ antennas for all the node pairs. We choose $\tau = 0.01/n^2$ for both $N = 4$ and $N = 8$, where $n$ denotes the iteration number. The noise power is $\sigma^2 = 3.16 \times 10^{-13}$ W (~95 dBm) which corresponds to approximate thermal noise power for a bandwidth of 20 MHz. $P_{\text{max}} = 100$ mW and $P_{\text{min}} = 0$ (no transmission) in our simulations. The maximum number of iterations is $\kappa = 100$.

The global optimum solution is obtained only for $N = 4$ by enumerating all feasible strategies, i.e. $16^4$ profiles, as the performance benchmark. The results are averaged over 100 different configurations.

The change in total transmit power through iterations is shown in Fig. 2. As indicated by the “OPT” curve, the global optimum of minimum power obtained by enumeration approach functions as the lower bound of the overall power for $N = 4$. We observe that COPMA’s performance improves with iterations and settles at the global optimum combination at termination. The non-cooperative players yield inferior performance compared to COPMA in terms of overall power, depicted by NPMG. The inefficiency is due to the single shot update of transmit beamformers at the start of the iterations. We also show the performance results when $N = 8$ pairs coexist on the same figure. In this case, the global centralized solution is not tractable, because the search space includes $16^8$ possible beamforming vector combinations. Therefore, the performances of only COPMA and NPMG are shown in this
case. For $N = 4$ and $N = 8$ users, 99% and 90% of the gain from using COPMA algorithm is realized within the first 3 and 20 iterations respectively, although further improvement results from more iterations. For $N = 4$, the existence of NE in both COPMA and NPMG are corroborated by the convergence of curves in Fig. 2. For $N = 8$, final convergence of COPMA requires more than 100 iterations, however further iterations result in very small decreases in the total transmit power.

![Transmit beamformer indexes versus iteration in COPMA with $N = 4$, $T = 3$ and $\Upsilon = 16$.](image1)

![Transmit powers versus iteration in COPMA with $N = 4$, $T = 3$ and $\Upsilon = 16$.](image2)

VI. CONCLUSION

In this paper, we have considered joint power control and beamforming in MIMO ad-hoc networks under constant QoS requirements. We proposed and compared the performances of iterative cooperative (COPMA) and noncooperative (NPMG) algorithms. In COPMA, users update their beamforming vector to minimize the total transmit power used in the network. COPMA converges to the Nash Equilibrium solution for the network with high probability. In the NPMG algorithm, all the transmit beamformers are updated at the start of the iteration and then power control is performed. Numerical results corroborate the convergence results of NPMG and COPMA. For small problem sizes simulations show that COPMA’s results converge to the global optimum.

REFERENCES