Coordinated Regularized Zero-Forcing Precoding for Multicell MISO Systems with Limited Feedback

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Abstract

In this paper, we propose a coordinated regularized zero-forcing (RZF) precoding method for limited feedback multicell multiuser (MU) multiple-input single-output (MISO) systems. We begin by deriving an approximation to the expected signal-to-interference-plus-noise ratio (SINR) for the proposed scheme with perfect channel direction information (CDI) at the base station (BS). We also derive an expected SINR approximation for limited feedback systems with random vector quantization (RVQ) based quantized CDI at the BS. Using the expected interference result for the RVQ based limited feedback CDI, we propose an adaptive feedback bit allocation strategy. The proposed feedback bit allocation minimizes the expected interference at the user by partitioning the total number of bits between the serving and out-of-cell interfering channels. Numerical results show that the proposed coordinated RZF precoding scheme achieves superior average cell-edge spectral efficiency compared to the non-coordinated RZF precoding scheme. Also, the proposed adaptive feedback bit allocation method offers a spectral efficiency gain higher than conventional equal bit allocation and existing coordinated zero-forcing (ZF) schemes.

Keywords

MISO systems, multiuser transmission, RZF precoding, multicell, adaptive feedback bit allocation.

I. INTRODUCTION

MULTIUSER (MU) multiple-input multiple-output (MIMO) achieves the sum-capacity of the broadcast channel with dirty paper coding (DPC) [1]–[3] with perfect channel state information (CSI) at the base station (BS). DPC is a non-linear precoding scheme with high complexity [4]. Research has identified several other more practical sub-optimal precoding schemes which are linear and exhibit low computational complexity [5]–[11], each having its own distinct design and features. Regularized zero-forcing (RZF) [5] is one of these promising precoding techniques which outperforms the conventional zero-forcing (ZF) precoding scheme in the low signal-to-noise ratio (SNR) regime [12] and has proven to be an effective precoding scheme for single-cell communication systems. RZF has also been extensively used in the analysis of 5G technologies such as massive MIMO [13]–[16], where the number of transmit antennas at the BS is scaled up by several orders of magnitude in comparison to conventional MIMO [17].
In cellular communication, the performance of the MU MIMO system is mainly assessed by the average spectral efficiency of the cell and the cell-edge. For single-cell systems, one way to improve the cell-edge performance is by increasing the transmit power at the BS. However, for multicell systems this increases the level of interference to other cells, especially in 4G networks where the frequency reuse factor is one. Despite the gains provided by MIMO technology in the single-cell scenario, in practice users experience inter-cell interference (ICI) and this limits the MIMO gains. Consequently, the spectral efficiency of the cell degrades. Such a loss can be avoided by having coordination between base stations, where information is exchanged among the base stations via a backhaul link to suppress the ICI in the downlink [18]–[23]. An overview of multicell systems is presented in [24]. In multicell MIMO systems, serving and neighboring base stations jointly contribute to reduce ICI. The performance of such techniques is highly dependent on the quality of CSI at the BS.

In codebook-based limited feedback MU MIMO systems [25]–[34], the user feeds back the index of the appropriate codebook entry or codeword to the BS, via a low-rate feedback link. This information is then used to compute the precoding vectors or matrices for the single-stream or multi-stream transmissions, respectively. An overview of limited feedback with multiple antenna wireless communication is presented in [35]. Numerous research works have dealt with multicell MU MIMO systems with perfect CSI and with limited feedback based quantized CSI. As the focus of this study is on limited feedback multicell MU multiple-input single-output (MISO) systems, we briefly give an overview of the work currently proposed in the literature. In [36], a limited feedback strategy for MU MISO multicell systems at high signal-to-noise ratio (SNR) is developed using random vector quantization (RVQ) codebooks [27]. Here, the idea is to design beamforming vectors using a generalized eigenvector approach. The multicell model considered in [36] is based on the Wyner model [37] and feedback bits are allocated adaptively among the serving and interfering channels by minimizing the mean rate loss of the system. An adaptive bit allocation method which maximizes the spectral efficiency is proposed in [38] for limited feedback systems. In [39], an adaptive feedback scheme for limited feedback MISO systems is proposed with a ZF precoding scheme. The adaptive bit allocation is realized by minimizing the expected spectral efficiency loss in multicell systems. The majority of the work discussed above has two common features, in that they all use RVQ codebooks and rely on solving optimization problems to adaptively allocate the bits to the serving and interfering channels.

Alongside other benefits, coordination amongst the base stations can also aid in user scheduling. This
is investigated in [40] for MU MISO, where a scheduling scheme is proposed that mitigates the ICI and also uses an adaptive bit allocation method to minimize the interference with coordinated ZF precoding. Feedback bit allocation schemes for distributed antenna systems that maximize the mean spectral efficiency and minimize the mean spectral efficiency loss are proposed in [41] and [42], respectively. An adaptive bit allocation scheme for joint processing (JP) MISO systems is presented in [43] with limited feedback. Here, base stations not only exchange CSI but also share the data of the users through backhaul links. The work in [44] finds precoders in each cell that maximize a weighted spectral efficiency across the network. It has been shown to achieve twice the spectral efficiency compared to the conventional equal bit allocation scheme in a three cell network. More recently, coordinated beamforming with an adaptive feedback bit allocation method is proposed for MU heterogeneous networks in [45].

Despite the numerous studies on coordinated multicell systems, little attention has been paid to coordinated RZF precoding until massive MIMO became popular. Therefore, in this paper we investigate coordinated RZF precoding but for conventional (small-scale) multicell MU MISO systems. We propose a coordinated RZF precoding strategy, where base stations share out-of-cell interfering CSI to coordinate, such that interference at cell-edge users is suppressed. Thus, we rely on a coordinated precoding strategy for MU MISO systems in order to design the precoding vectors at the BS in each cell to minimize ICI at cell-edge users. For this purpose, we adopt a coordination zone within each cell, as in [39]. The user in this coordination region is defined as a cell-edge user and it feeds back both serving and out-of-cell interfering channels to the serving BS. The serving BS shares the out-of-cell interfering CSI with the relevant interfering base station.

The main reason for sharing interfering channels is that the interfering BS considers these channels while designing precoding vectors to minimize the interference in the network. Although the idea of sharing out-of-cell interfering channels is proposed in [39] using ZF precoding, in this paper, we use RZF precoding which has been shown to be more effective than ZF in the low SNR regime [12]. We also derive expected SINR approximations for the proposed scheme with perfect channel direction information (CDI) and with RVQ codebook CDI at the BS. Furthermore, we develop an adaptive bit allocation scheme that distributes the bits to serving and out-of-cell interfering channels, such that the expected interference is minimized at the user. For this purpose, we use RVQ codebooks for CDI quantization and like many other studies [39], [40], we assume that the perfect knowledge of the channel quality indicator (CQI) is available at the BS. In this paper, we use a regularization parameter derived in [46] for the proposed coordinated RZF scheme.
The main contributions of this paper are summarized below.

- We propose a coordinated RZF precoding scheme for multicell MU MISO systems, where information on interfering channels is shared among the base stations to reduce the interference in the network. As shown through simulations, the proposed scheme achieves higher average cell-edge spectral efficiency compared to the non-coordinated RZF and coordinated ZF schemes.

- Analytical expressions are derived to approximate the expected SINR for the proposed system with perfect CDI and limited feedback RVQ codebook CDI. The expected SINR approximation for RVQ codebooks provides a framework to study the impact of codebook quantization errors on the SINR performance with the coordinated RZF precoding scheme.

- Using the expected interference result for the RVQ codebook, we propose an adaptive bit allocation method for coordinated RZF precoding, where feedback bits are partitioned between the serving and out-of-cell interfering channels at each user, such that the interference is minimized. Through simulations, we show that the proposed adaptive bit allocation method outperforms the existing coordinated ZF based adaptive bit allocation method in [39].

Notation: We use $(\cdot)^H$, $(\cdot)^T$ and $(\cdot)^{-1}$ to denote the conjugate transpose, the transpose and the inverse operations, respectively. $\| \cdot \|$ and $| \cdot |$ stand for vector and scalar norms, respectively. $\mathbb{E}[\cdot]$ denotes statistical expectation. The bold uppercase and lowercase letters are used to represent matrices and vectors, respectively. The lowercase italic letters denote elements of vectors and matrices.

II. DOWNLINK SYSTEM MODEL

Consider a multicell MU MISO system with $K$ cells having a single BS each. Each BS has $M$ transmit antennas and it simultaneously serves $L$ single antenna users in the cell. All the $K$ cells are interconnected via backhaul links. The backhaul is assumed to be error free with no delay. In this study, we assume that the total number of users in all the coordinating cells is equal to or less than the number of transmit antennas $M$ at each BS, such that, $KL \leq M$. The channel vector of size $1 \times M$ between the $l^{th}$ user in the $k^{th}$ cell and the serving BS is given by $h_{l,k,k}$. The interfering channel vector between the $l^{th}$ user in the $k^{th}$ cell and the $j^{th}$ interfering BS is denoted by $h_{l,k,j}$, where $j \neq k$. The channel entries $h_{l,k,k}$ and $h_{l,k,j}$ are independent and identically distributed (i.i.d.) complex Gaussian $CN(0, 1)$. The downlink received signal

\footnote{In this paper, we use RZF precoding scheme. We assume that $KL < M$, as at high SNR, RZF precoding is equivalent to ZF precoding, given the fact that $M$ exceeds the total number of receive antennas in the system.}
at the $l$th user in the $k$th cell is given by

$$y_{l,k} = \sqrt{\frac{P_{l,k,k}}{\gamma_k}} h_{l,k,k} s_{l,k} + \sum_{m=1 \atop m \neq l}^{L} \frac{P_{l,k,k}}{\gamma_k} h_{l,k,k} w_{m,k} s_{m,k} + \sum_{j=1 \atop j \neq k}^{K} \frac{P_{l,k,j}}{\gamma_j} h_{l,k,j} \sum_{q=1}^{L} w_{q,j} s_{q,j} + n_{l,k},$$

where $w_{l,k}$ is the non-normalized precoding vector for the $l$th user in the $k$th cell and $\gamma_k$ is the normalization parameter (to be discussed later) for the $k$th cell. $s_{l,k}$ and $n_{l,k}$ denote the data symbol and the noise for the $l$th user in the $k$th cell. The noise is assumed to be an i.i.d. Gaussian random variable with zero mean and variance $N_0$. The data symbols are selected from the same constellation and $\mathbb{E} [ |s_{l,k}|^2 ] = 1$. $P_{l,k,k}$ and $P_{l,k,j}$ are the received powers at the $l$th user in the $k$th cell from serving and interfering base stations, respectively. The received signal powers from the serving BS and interfering BS at the $l$th user in the $k$th cell are modeled by

$$P_{l,k,k} = P_0 \left( \frac{R}{d_{l,k,k}} \right)^a z_{l,k,k}, \quad P_{l,k,j} = P_0 \left( \frac{R}{d_{l,k,j}} \right)^a z_{l,k,j},$$

where $P_0$ is the power received at the distance $R$ in the absence of shadowing, where $R$ is the cell radius and is assumed to be same for all the cells. $a$ is the path loss exponent. The shadowing is modeled as a log-normal random variable, given by $z_{l,k,k} = 10^{(\eta_{l,k,k} \sigma_{SF}/10)}$, for the channel between the $l$th user in the $k$th cell and the $k$th BS, where $\sigma_{SF}$ is the shadowing standard deviation in dB and $\eta_{l,k,k}$ is a zero mean Gaussian random variable with unit variance. The distances $d_{l,k,k}$ and $d_{l,k,j}$ represent the distances from the serving BS and the interfering BS to the $l$th user in the $k$th cell, respectively.

### III. Coordinated RZF precoding and SINR analysis

Each BS uses a regularized zero-forcing (RZF) precoding scheme [5] to compute the precoding vectors for their respective users. In doing so, the serving BS applies RZF not only to the channels of the same cell users but also considers the interfering channels caused by it the users located in the adjacent cell, thus mitigating or suppressing the interference caused by it to those users. All interfering channels and the serving channel at the $l$th user in the $k$th cell are determined using cell-specific pilots and these channels are conveyed to the serving BS. In the next step, all the interfering channels caused by the respective base stations are delivered to them via backhaul links. For example, the $k$th cell BS shares interfering channels
with the $j$th cell BS and in return also receives interfering channels between it and the $j$th cell users, using a backhaul link. The system model for $K = 2$ cells and $L = 2$ cell-edge users is shown in Fig. 1 where interfering channels are represented by red-dashed arrows. The unnormalized RZF precoder $w_{l,k}$, for the $l$th user in the $k$th cell is the $l$th column of $W_k$, given by

$$W_k = H_k^H (H_k H_k^H + \alpha_k I)^{-1},$$

(3)

where $H_k = [X_1^T \ X_2^T \ ... \ X_k^T]^T$ is a $KL \times M$ concatenated matrix, with $X_1 = [h_{1,1,k}^T \ ... \ h_{L,1,k}^T]^T$ and $X_k = [h_{1,k,k}^T \ ... \ h_{L,k,k}^T]^T$. The resulting precoder matrix is normalized by the normalization parameter, such that $\tilde{W}_k = W_k / \sqrt{\gamma_k}$, where $\gamma_k = \|W_k\|_F^2 / M$ to satisfy the total power constraint. The regularization parameter for the $k$th BS is denoted by $\alpha_k$. The choice of regularization parameter, $\alpha_k$, is discussed in Section VI. When perfect CDI and CQI are available at the BSs, the SINR expression for the $l$th user in the $k$th cell using (1) can be written as

$$\text{SINR}_{l,k} = \frac{\frac{P_{l,k,k}}{\gamma_k} |h_{l,k,k} w_{l,k}|^2}{1 + \frac{P_{l,k,k}}{\gamma_k} \sum_{m=1, m\neq k}^{L} |h_{l,k,k} w_{m,k}|^2 + \sum_{j=1, j\neq k}^{K} \frac{P_{l,k,j}}{\gamma_j} \sum_{q=1}^{L} |h_{l,k,j} w_{q,j}|^2}.$$

(4)

4Although the true serving channel at the $l$th user in the $k$th cell is given by $\sqrt{P_{l,k,k}} h_{l,k,k}$, however in this paper, like $[51, 52]$, we do not consider pathloss and shadowing while computing the precoding matrix $W_k$ in (3).

5The CDI and CQI for the channel between the $l$th user in the $k$th cell and the $k$th BS, are defined as, $h_{l,k,k} / \|h_{l,k,k}\|$ and $\|h_{l,k,k}\|$, respectively.
We can take the expectation of the SINR in (4) and approximate it by using the expected SINR approximation in [53], such that

\[
\mathbb{E} [\text{SINR}_{l,k}] \approx \frac{P_{l,k,k}}{\tilde{\gamma}_k} \mathbb{E} \left[ |\mathbf{h}_{l,k,k}\mathbf{w}_{l,k}|^2 \right] + \sum_{j=1}^{K} P_{l,k,j} \tilde{\gamma}_j \sum_{q=1}^{L} \mathbb{E} \left[ |\mathbf{h}_{l,k,j}\mathbf{w}_{q,j}|^2 \right],
\]

(5)

where \( \tilde{\gamma}_k = \mathbb{E} [\gamma_k] \) and \( \tilde{\gamma}_j = \mathbb{E} [\gamma_j] \). We now evaluate (5) by computing the expected signal power and the expected interference power in (5) for the \( KL = M \) case, where the maximum number of users in the network are supported. However, the following analysis can also be applied to the scenarios where \( KL < M \). To compute the expectations, we use the approach presented in [5], and moreover, we evaluate the expectation over the eigenvalues of \( \mathbf{H}_k\mathbf{H}_k^H \), which has not been investigated in [5].

**Expected signal power:** The expected signal power in (5) is

\[
S_{l,k} = \frac{P_{l,k,k}}{\tilde{\gamma}_k} \mathbb{E} \left[ |\mathbf{h}_{l,k,k}\mathbf{w}_{l,k}|^2 \right].
\]

(6)

Using the eigenvalue decomposition, \( \mathbf{H}_k\mathbf{H}_k^H = \mathbf{Q}\Lambda\mathbf{Q}^H \), the expectation in (6), denoted by \( \delta_{l,k} \), is written as [5]

\[
\delta_{l,k} = \mathbb{E} \left[ |\mathbf{h}_{l,k,k}\mathbf{w}_{l,k}|^2 \right] = \mathbb{E} \left[ \left( \sum_{n=1}^{M} \frac{\lambda_n}{\lambda_n + \alpha_k} |q_{l,n}|^2 \right)^2 \right],
\]

(7)

where \( \lambda_n \) is the \( n \)th eigenvalue corresponding to the \( n \)th diagonal entry of \( \Lambda \). The quantity \( q_{l,n} \) denotes the entry of \( \mathbf{Q} \) corresponding to the \( l \)th row and \( n \)th column. Using [5] Appendix A, taking the expectation over the entries of \( \mathbf{Q} \) yields

\[
\delta_{l,k} = \frac{1}{M(M+1)} \left( \mathbb{E}_\lambda \left[ \left( \sum_{n=1}^{M} \frac{\lambda_n}{\lambda_n + \alpha_k} \right)^2 \right] + \mathbb{E}_\lambda \left[ \sum_{n=1}^{M} \left( \frac{\lambda_n}{\lambda_n + \alpha_k} \right)^2 \right] \right). \quad (8)
\]

The value of \( \tilde{\gamma}_k \) is given by

\[
\tilde{\gamma}_k = \frac{1}{M} \mathbb{E} \left[ \|\mathbf{W}_k\|_F^2 \right] = \frac{1}{M} \mathbb{E}_\lambda \left[ \sum_{n=1}^{M} \frac{\lambda_n}{(\lambda_n + \alpha_k)^2} \right], \quad (9)
\]

The expectation of the terms in (8) and (9), with respect to the eigenvalues, are given in Result 1 and Result 2.

**Result 1:** When the entries of an \( M \times M \) matrix \( \mathbf{H} \) are i.i.d. \( \mathcal{CN}(0,1) \) random variables, the expected value of \( \sum_{n=1}^{M} \frac{(\lambda_n)^2}{(\lambda_n + \alpha_k)^2} \), where \( \lambda_n \) is the \( n \)th eigenvalue of \( \mathbf{H}\mathbf{H}^H \) (uncorrelated central Wishart matrix),
with respect to \( \lambda_n \ \forall n \), is given by

\[
\mathbf{D}_k^{(t)} = \mathbb{E} \left[ \sum_{n=1}^{M} \left( \frac{\lambda_n}{\lambda_n + \alpha_k} \right)^t \right] \\
= \sum_{i=1}^{M} \sum_{j=0}^{i-1} \sum_{l=0}^{i-1} (-1)^{j+l} \binom{i-1}{i-1-j} \binom{i-1}{i-1-l} \frac{1}{j! l!} \sum_{s=0}^{t+j+l} \binom{t+j+l}{s} (-\alpha_k)^{t+j+l-s} e^{\alpha_k} \int_{\alpha_k}^{\infty} v^{s-2} e^{-v} dv,
\]

where

\[
\int_{\alpha_k}^{\infty} v^{s-2} e^{-v} dv = \begin{cases} 
-\text{Ei}(1, \alpha_k) + \frac{e^{-\alpha_k}}{\alpha_k^t} & \text{for } s = 0 \\
\text{Ei}(1, \alpha_k) & \text{for } s = 1 \\
\Gamma(s - 1, \alpha_k) & \text{for } s \geq 2 
\end{cases}
\]

Proof: See Appendix A.

**Result 2**: When the entries of an \( M \times M \) matrix \( \mathbf{H} \) are i.i.d. \( \mathcal{CN}(0, 1) \) random variables, then the expected value of \( \left( \sum_{n=1}^{M} \frac{\lambda_n}{\lambda_n + \alpha_k} \right)^2 \), where \( \lambda_n \) is the \( n \)th eigenvalue of \( \mathbf{H} \mathbf{H}^H \) (uncorrelated central Wishart matrix), with respect to \( \lambda_n \ \forall n \), is given by

\[
\mathbf{F}_k = \mathbb{E} \left[ \left( \sum_{n=1}^{M} \frac{\lambda_n}{\lambda_n + \alpha_k} \right)^2 \right] = \mathbf{D}_k^{(2)} + \sum_{i=1}^{M} \sum_{j=1, j\neq i}^{M} \\
\left( \sum_{r=0}^{i-1} \sum_{s=0}^{i-1} (-1)^{r+s} \binom{i-1}{i-1-r} \binom{i-1}{i-1-s} \frac{1}{r! s!} \sum_{b=0}^{1+r+s} \binom{1+r+s}{b} (-\alpha_k)^{1+r+s-b} e^{\alpha_k} \int_{\alpha_k}^{\infty} v^{b-1} e^{-v} dv \right)^2 \\
- \left( \sum_{r=0}^{i-1} \sum_{s=0}^{j-1} (-1)^{r+s} \binom{i-1}{i-1-r} \binom{j-1}{j-1-s} \frac{1}{r! s!} \sum_{b=0}^{1+r+s} \binom{1+r+s}{b} (-\alpha_k)^{1+r+s-b} e^{\alpha_k} \int_{\alpha_k}^{\infty} v^{b-1} e^{-v} dv \right)^2,
\]

where

\[
\int_{\alpha_k}^{\infty} v^{b-1} e^{-v} dv = \begin{cases} 
\text{Ei}(1, \alpha_k) & \text{for } b = 0 \\
\Gamma(b, \alpha_k) & \text{for } b \geq 1 
\end{cases}
\]

Proof: See Appendix B.

Using Result 1 and 2, we can write (8) and (9) as

\[
\delta_{l,k} = \frac{\mathbf{F}_k + \mathbf{D}_k^{(2)}}{M (M + 1)}
\]
and
\[
\tilde{\gamma}_k = \frac{D_k^{(1)}}{M}.
\] (15)

Therefore, the expected signal power \( S_{l,k} \) can be written as
\[
S_{l,k} = \frac{P_{l,k,k}}{\tilde{\gamma}_k} \delta_{l,k}.
\] (16)

**Expected interference power:** The expected interference power in (5) is
\[
I_{l,k} = \frac{P_{l,k,k}}{\tilde{\gamma}_k} \sum_{m=1}^{L} \mathbb{E} \left[ |h_{l,k,k}w_{m,k}|^2 \right] + \sum_{j=1}^{K} \frac{P_{l,k,j}}{\tilde{\gamma}_j} \sum_{q=1}^{L} \mathbb{E} \left[ |h_{l,k,j}w_{q,j}|^2 \right].
\] (17)

In order to evaluate (17), we observe that
\[
\psi \triangleq \mathbb{E} \left[ |h_{l,k,k}w_{m,k}|^2 \right] = \mathbb{E} \left[ |h_{l,k,j}w_{q,j}|^2 \right], \forall m, j, q.
\] (18)

Hence,
\[
I_{l,k} = \frac{P_{l,k,k}}{\tilde{\gamma}_k} (L - 1) \psi + \sum_{j=1}^{K} \frac{P_{l,k,j}}{\tilde{\gamma}_j} L \psi.
\] (19)

Now \( \psi \) can be found from \( \psi_{\text{sum}}/(M - 1) \) where
\[
\psi_{\text{sum}} = \sum_{m=1}^{L} \mathbb{E} \left[ |h_{l,k,k}w_{m,k}|^2 \right] + \sum_{j=1}^{K} \sum_{q=1}^{L} \mathbb{E} \left[ |h_{l,k,j}w_{q,j}|^2 \right].
\] (20)

In order to compute \( \psi_{\text{sum}} \), we note that (20) is the expected interference in the absence of the powers and normalization parameters. Hence, (20) is given by the difference between the expected total received power and the expected signal power [5], again neglecting the power and normalization terms. Hence, we can write (20) as
\[
\psi_{\text{sum}} = \xi_k - \delta_{l,k}
\]
\[
= \left[ \frac{D_k^{(2)}}{M} - \frac{F_k + D_k^{(2)}}{M(M + 1)} \right].
\] (21)

where \( \xi_k \) is the expected total received signal (desired and interference) power at the user in the \( k^{\text{th}} \) cell, given by \( \xi_k = \mathbb{E} \left[ ||H_kW_k||_F^2 \right] /M = D_k^{(2)}/M \).

**Expected SINR with perfect CDI:** We can now write the expected SINR in (5) in terms of \( \delta_{l,k}, \psi, \tilde{\gamma}_k \)
\[
\mathbb{E} [\text{SINR}_{l,k}] \approx \frac{P_{l,k,k} \delta_{l,k}}{1 + \frac{P_{l,k,k}}{\gamma_k} (L - 1) \psi + \sum_{j \neq k}^{K} \frac{P_{l,k,j}}{\gamma_j} L \psi}.
\]

Having derived the expected SINR approximation for the perfect CDI case, we now investigate coordinated RZF precoding with limited feedback MU MISO systems in the following section.

IV. LIMITED FEEDBACK WITH RVQ CODEBOOKS

In FDD communication systems, limited feedback techniques are often used to equip the BS with knowledge of the CSI. In doing so, a common codebook is maintained at both the BS and the user, such that the user feedbacks the index of the appropriate codeword to the BS via a low-rate feedback link. In this section, we study the impact of limited feedback with RVQ codebooks [27] on the performance of the coordinated RZF precoding scheme in multicell MU MISO systems.

Like the perfect CDI case, here we also give the derivation details for the expected SINR approximation with limited feedback. In limited feedback multiple antenna systems, the user quantizes the estimated channel (perfect estimation is assumed in this paper) using a codebook. The quantized channel vector of the \( i^{th} \) user in the \( k^{th} \) cell is denoted by \( \hat{h}_{l,k,k} \). In this paper, we consider an RVQ codebook and \( B_{\text{total}} \) is the total number of feedback bits\(^6\) at the user. Each user quantizes serving and out-of-cell interfering channels, therefore we can write \( B_{\text{total}} = \sum_{i=1}^{K} B_{l,k,i} \). These feedback bits are sent to the BS via a low-rate feedback link. In this case, the perfect concatenated channel matrix for the \( k^{th} \) BS can be modeled as [54], [55]

\[
H_k = \hat{H}_k + E_k,
\]

where \( H_k \sim \mathcal{CN}(0,1) \) and \( E_k = \begin{bmatrix} G_1^T & G_2^T & \ldots & G_k^T & \ldots & G_K^T \end{bmatrix}^T \) is a \( KL \times M \) concatenated quantization error matrix with \( G_1 = \begin{bmatrix} e_{1,1,k}^T & \ldots & e_{L,1,k}^T \end{bmatrix}^T \). The quantization error matrix, \( E_k \), is assumed to have independent Gaussian distributed elements [54], [55] such that the quantization error vector between the \( l^{th} \) user in the \( k^{th} \) cell and the \( k^{th} \) BS, denoted by \( e_{l,k,k} \), is given by \( e_{l,k,k} \sim \mathcal{CN}(0,\sigma^2_{l,k,k}I) \). Using an upper bound on the quantization error for RVQ codebooks in terms of squared chordal distance given in [25], denoting by \( B_{l,k,k} \) the number of bits used to quantize the channel between the \( l^{th} \) user in the \( k^{th} \) cell and the \( k^{th} \) BS, we have, \( \sigma^2_{l,k,k} \leq 2 \frac{B_{l,k,k}}{M} \). In our model, we assume the worst case scenario, where \( \sigma^2_{l,k,k} = 2 \frac{B_{l,k,k}}{M} \). Similarly, the quantized concatenated channel matrix, \( \hat{H}_k = \begin{bmatrix} \hat{G}_1^T & \hat{G}_2^T & \ldots & \hat{G}_k^T & \ldots & \hat{G}_K^T \end{bmatrix}^T \), where \( \hat{H}_k \) is a \( KL \times M \)

\(^6\)We only study the effects of CDI quantization errors as in [32], [33], [39]. The quantization of CQI is out of the scope of this paper.
concatenated quantized channel matrix with $\tilde{G}_1 = \left[ \hat{H}_{1,1,k}^T \ldots \hat{H}_{L,1,k}^T \right]^T$. The entries of $\hat{H}_k$ are assumed to be independent and Gaussian distributed, such that $\hat{h}_{l,k,k} \sim \mathcal{C}\mathcal{N}(0, (1 - \sigma_{l,k,k}^2)I)$. The unnormalized precoding vector of the $l$th user in the $k$th cell, $\hat{w}_{l,k}$, is the $l$th column of the matrix $\hat{W}_k$, given by (for $KL \leq M$)

$$\hat{W}_k = \hat{H}_k^H \left( \hat{H}_k \hat{H}_k^H + \alpha_k I \right)^{-1}, \quad \text{(24)}$$

where $\hat{H}_k = \left[ \bar{X}_1^T \bar{X}_2^T \ldots \bar{X}_k^T \right]$ is a $KL \times M$ concatenated matrix with $\bar{X}_k = \left[ \hat{h}_{1,k,k}^T \ldots \hat{h}_{L,k,k}^T \right]^T$, and $\tilde{h}_{l,k,k} = \hat{h}_{l,k,k}/\sqrt{1 - \sigma_{l,k,k}^2}$ in order to have $\hat{H}_k \sim \mathcal{C}\mathcal{N}(0, I)$. To satisfy the total power constraint, the precoding matrix is normalized by the parameter, $\gamma_k$, such that, $\bar{W}_k = \hat{W}_k/\sqrt{\gamma_k}$, where $\gamma_k = \|\hat{W}_k\|^2/M$.

On the downlink, the received signal for the $l$th user in the $k$th cell is given by

$$\hat{y}_{l,k} = \sqrt{\frac{P_{l,k,k}}{\gamma_k}} (\hat{h}_{l,k,k} + e_{l,k,k}) \hat{w}_{l,k}s_{l,k} + \sqrt{\frac{P_{l,k,k}}{\gamma_k}} \sum_{m=1}^L \left( \hat{h}_{l,k,k} + e_{l,k,k} \right) \hat{w}_{m,k}s_{m,k}$$

$$+ \sum_{j=1}^K \sqrt{\frac{P_{l,k,j}}{\gamma_j}} (\hat{h}_{l,k,j} + e_{l,k,j}) \sum_{q=1}^L \hat{w}_{q,j}s_{q,j} + n_{l,k}. \quad \text{(25)}$$

In the case of RVQ codebooks, the expected SINR at the $l$th user in the $k$th cell, denoted by $\mathbb{E}\left[\text{SINR}_{l,k}\right]$, is approximated by $\mathbb{E}\left[\tilde{\text{SINR}}_{l,k}\right]$,

$$\mathbb{E}\left[\tilde{\text{SINR}}_{l,k}\right] \approx \frac{P_{l,k,k}}{\bar{\gamma}_k} \mathbb{E}\left[ \left| \left( \hat{h}_{l,k,k} + e_{l,k,k} \right) \hat{w}_{l,k} \right|^2 \right]$$

$$1 + \frac{P_{l,k,k}}{\bar{\gamma}_k} \sum_{m=1}^L \mathbb{E}\left[ \left| \left( \hat{h}_{l,k,k} + e_{l,k,k} \right) \hat{w}_{m,k} \right|^2 \right] + \sum_{j=1}^K \frac{P_{l,k,j}}{\bar{\gamma}_j} \sum_{q=1}^L \mathbb{E}\left[ \left| \left( \hat{h}_{l,k,j} + e_{l,k,j} \right) \hat{w}_{q,j} \right|^2 \right], \quad \text{(26)}$$

where $\bar{\gamma}_k = \mathbb{E}[\gamma_k]$ and $\bar{\gamma}_j = \mathbb{E}[\gamma_j]$. As for the perfect CDI case, we compute the various expectation terms in (26) individually, following a similar procedure for the $KL = M$ case.

**Expected signal power:** The expectation of the signal power at the $l$th user in the $k$th cell in (26) is given by

$$S_{l,k}' = \frac{P_{l,k,k}}{\bar{\gamma}_k} \mathbb{E}\left[ \left| \left( \hat{h}_{l,k,k} + e_{l,k,k} \right) \hat{w}_{l,k} \right|^2 \right]. \quad \text{(27)}$$
We can write
\[
\mathbb{E} \left[ \left( \hat{h}_{l,k,k} + e_{l,k,k} \right) \hat{w}_{l,k} \right]^2 = \mathbb{E} \left[ \left( \hat{h}_{l,k,k} \hat{w}_{l,k} + e_{l,k,k} \hat{w}_{l,k} \right) \right] \tag{28}
\]
\[
= \mathbb{E} \left[ \left| \hat{h}_{l,k,k} \hat{w}_{l,k} \right|^2 \right] + \mathbb{E} \left[ \left| e_{l,k,k} \hat{w}_{l,k} \right|^2 \right],
\]
\[
\mathbb{E} \left[ \left| e_{l,k,k} \hat{w}_{l,k} \right|^2 \right] = \mathbb{E} \left[ \left| e_{l,k,k} \right|^2 \right] \mathbb{E} \left[ \left\| \hat{W}_k \right\|_F^2 \right] / M = \left( 2 - \frac{B_{l,k,k}}{M-1} \right) \tilde{\gamma}_k. \tag{30}
\]

Also, we have
\[
\mathbb{E} \left[ \left| e_{l,k,k} \hat{w}_{l,k} \right|^2 \right] = \mathbb{E} \left[ \left| e_{l,k,k} \right|^2 \right] \mathbb{E} \left[ \left\| \hat{W}_k \right\|_F^2 \right] / M = \left( 2 - \frac{B_{l,k,k}}{M-1} \right) \tilde{\gamma}_k. \tag{31}
\]

Therefore, we obtain
\[
\mathbb{E} \left[ \left( \hat{h}_{l,k,k} + e_{l,k,k} \right) \hat{w}_{l,k} \right]^2 = \left( 1 - 2 \frac{-B_{l,k,k}}{M-1} \right) \delta_{l,k} + \left( 2 - \frac{-B_{l,k,k}}{M-1} \right) \tilde{\gamma}_k, \tag{32}
\]
where
\[
\tilde{\gamma}_k = \frac{1}{M} \mathbb{E}_{\lambda_n} \left[ \sum_{n=1}^{M} \frac{\tilde{\lambda}_n}{(\tilde{\lambda}_n + \alpha_k)^2} \right], \tag{33}
\]

Using Result 1 and Result 2, \( \tilde{\gamma}_k \) and \( \delta_{l,k} \) can be written as
\[
\tilde{\gamma}_k = \frac{D_k^{(1)}}{M} \tag{34}
\]
and
\[ \delta_{l,k} = \frac{F_k + D_k^{(2)}}{M(M + 1)}, \]  

and thus the expected signal power in (27) can be expressed as
\[ S'_{l,k} = \frac{P_{l,k,k}}{\gamma_k} \left[ \left( 1 - 2^{-\frac{B_{l,k,k}}{M-1}} \right) \delta_{l,k} + \left( 2^{-\frac{B_{l,k,k}}{M-1}} \right) \tilde{\gamma}_k \right]. \]

**Expected interference power:** The expected interference in (26) is given by
\[ I'_{l,k} = \frac{P_{l,k,k}}{\gamma_k} \sum_{m=1, m \neq l}^L \mathbb{E} \left[ \left( \mathbf{h}_{l,k,m} + \mathbf{e}_{l,k,m} \right) \mathbf{w}_{m,k} \right]^2 \right] + \sum_{j=1}^K \frac{P_{l,k,j}}{\gamma_j} \sum_{q=1}^L \mathbb{E} \left[ \left( \mathbf{h}_{l,k,j} + \mathbf{e}_{l,k,j} \right) \mathbf{w}_{q,j} \right]^2 \right]. \]

Similar to the perfect CDI case, in order to find the expected interference terms in (37) (without pathloss and shadowing) at the \( l \)th user in the \( k \)th cell, we subtract the expected signal power given in (32) from the total expected received power, giving
\[ \tilde{\psi}_{l,k} = \left( \mathbb{E} \left[ \left( \mathbf{h}_{l,k,k} \tilde{\mathbf{w}}_k \right)^2 \right]_{F} + \mathbb{E} \left[ \left( \mathbf{h}_{l,k,k} \tilde{\mathbf{w}}_k \right)^2 \right]_{F} \right) - \left[ \left( 1 - 2^{-\frac{B_{l,k,k}}{M-1}} \right) \delta_{l,k} + \left( 2^{-\frac{B_{l,k,k}}{M-1}} \right) \tilde{\gamma}_k \right] \]
\[ = \left( \tilde{\gamma}_k M 2^{-\frac{B_{l,k,k}}{M-1}} + \left( 1 - 2^{-\frac{B_{l,k,k}}{M-1}} \right) M \left[ \mathbb{E} \left[ \left( \mathbf{h}_{l,k,k} \tilde{\mathbf{w}}_k \right)^2 \right]_{F} \right] / M - \left( 1 - 2^{-\frac{B_{l,k,k}}{M-1}} \right) \delta_{l,k} + \left( 2^{-\frac{B_{l,k,k}}{M-1}} \right) \tilde{\gamma}_k \right] \]
\[ = \left( \tilde{\gamma}_k M 2^{-\frac{B_{l,k,k}}{M-1}} + \left( 1 - 2^{-\frac{B_{l,k,k}}{M-1}} \right) \xi_k - \left( 1 - 2^{-\frac{B_{l,k,k}}{M-1}} \right) \delta_{l,k} + \left( 2^{-\frac{B_{l,k,k}}{M-1}} \right) \tilde{\gamma}_k \right) \],

where, using Result 1, the value of \( \xi_k \) is
\[ \xi_k = \frac{D_k^{(2)}}{M}. \]

Similarly, the interference at the \( l \)th user in the \( k \)th cell from the \( j \)th cell user, is given by
\[ \tilde{\psi}_{l,j} = \left( \tilde{\gamma}_j M 2^{-\frac{B_{l,j,k}}{M-1}} + \left( 1 - 2^{-\frac{B_{l,j,k}}{M-1}} \right) \xi_j \right) - \left[ \left( 1 - 2^{-\frac{B_{l,j,k}}{M-1}} \right) \delta_{l,j} + \left( 2^{-\frac{B_{l,j,k}}{M-1}} \right) \tilde{\gamma}_j \right] \].

The interference from any single interfering source coming from \( k \)th and \( j \)th cell are given by \( \psi'_{l,k} = \tilde{\psi}_{l,k}/(M - 1) \) and \( \psi'_{l,j} = \tilde{\psi}_{l,j}/(M - 1) \), respectively. Therefore, the expected interference in (37) can be written as
\[ I'_{l,k} = \frac{P_{l,k,k}}{\gamma_k} (L - 1) \psi'_{l,k} + \sum_{j=1}^K \frac{P_{l,k,j}}{\gamma_j} L \psi'_{l,j} \].

**Expected SINR with RVQ:** We can now express the expected SINR approximation in (26) for RVQ
codebooks, by using (36) and (41), to give

\[
E\left[\tilde{\text{SINR}}_{l,k}\right] \approx \frac{P_{l,k,k}}{\bar{\gamma}_k} \left[ \left( 1 - 2 \frac{P_{l,k,k}}{\bar{\gamma}_k} \right) \delta_{l,k} + \left( 2 \frac{P_{l,k,k}}{\bar{\gamma}_k} \right) \tilde{\gamma}_k \right] \frac{1}{1 + \frac{P_{l,k,k}}{\bar{\gamma}_k} (L - 1) \psi'_l,k + \sum_{j=1}^{K} P_{l,k,j} \bar{\gamma}_j L \psi'_l,j}.
\]  

(42)

Comparing (22) and (42), we note that when the number of feedback bits is large, the expected SINR approximation (42) approaches the expected SINR approximation with perfect CDI (22), such that \( E\left[\tilde{\text{SINR}}_{l,k}\right] \rightarrow E[\text{SINR}_{l,k}] \). The results derived in this section are used in developing an adaptive bit allocation for the coordinated RZF scheme. This is investigated in the next section.

V. ADAPTIVE BIT ALLOCATION METHOD

In limited feedback systems, the user assigns bits to the channels (serving and interfering) in each feedback period. Therefore, it is of interest to investigate adaptive bit allocation at the user for each feedback instance. The upper bounds on the quantization errors associated with the error vectors given in (42) can be leveraged to estimate how many bits should be allocated to the serving and interfering channels. This adaptive allocation of the feedback bits is discussed in more detail in this section.

As discussed in Section I, there are numerous studies [39], [40], [42], [43], [48] on adaptive bit allocation for limited feedback systems. Majority of the schemes maximize/minimize a specific performance/distortion metric. While, there are a few studies [52], [56] that consider RZF precoding with adaptive bit allocation for massive MISO systems, adaptive bit allocation with RZF precoding scheme is not well investigated for not so large antenna systems.

We propose an adaptive method to allocate the total number of bits at the user, \( B_{total} = \sum_{i=1}^{K} B_{l,k,i} \), to quantize the serving channel and the out-of-cell interfering channels, by minimizing the mean interference at the user. It is important to note that minimizing the interference at each user in the system reduces overall interference in the multicell network. The mean interference at the \( l^{th} \) user in the \( k^{th} \) cell in (42), is given by

\[
\tilde{I}'_{l,k} = \frac{P_{l,k,k}}{\bar{\gamma}_k} (L - 1) \psi'_l,k + \sum_{j=1}^{K} P_{l,k,j} \bar{\gamma}_j L \psi'_l,j = \frac{P_{l,k,k}}{\bar{\gamma}_k} (L - 1) \tilde{\psi}_{l,k} + \sum_{j=1}^{K} P_{l,k,j} \frac{L}{M - 1} \tilde{\psi}_{l,j}.
\]

(43)
Substituting the values of $\tilde{\psi}_{l,k}$ and $\tilde{\psi}_{l,j}$ from (38) and (40) into (43) and rearranging, gives

$$I'_{l,k} = P_{l,k,k}(L - 1)^2 \left(1 - \frac{B_{l,k,k}}{M - 1}\right)(1 - \Delta_k) + P_{l,k,k}(L - 1)\Delta_k + \sum_{j=1}^{K} P_{l,k,j} L 2 \left(1 - \frac{B_{l,k,j}}{M - 1}\right)(1 - \Delta_j) + \sum_{j=1}^{K} P_{l,k,j} L \Delta_j,$$

(44)

where $\Delta_k = (\xi_k - \delta_{l,k})/(\bar{\gamma}_k(M - 1))$ and $\Delta_j = (\xi_j - \delta_{l,j})/(\bar{\gamma}_j(M - 1))$. We can write (44) as

$$I'_{l,k} = P_{l,k,k}(L - 1)^2 \left(1 - \frac{B_{l,k,k}}{M - 1}\right)(1 - \Delta_k) + P_{l,k,k}(L - 1)\Delta_k + \sum_{j=1}^{K} P_{l,k,j} L 2 \left(1 - \frac{B_{l,k,j}}{M - 1}\right)(1 - \Delta_j) + \sum_{j=1}^{K} P_{l,k,j} L \Delta_j.$$

(45)

It is noted that the expected interference in (45) is composed of two terms, where the first term is the sum of the product of channel gains (with pathloss and shadowing) and codebook quantization errors. On the other hand, the second term in (45) is the sum of weighted channel gains independent of codebook quantization errors. Therefore, in order to solve for the number of bits that minimizes the mean interference at the $l$th user in the $k$th cell, we define an optimization problem, given by

$$\min_{B_{l,k,1}, \ldots, B_{l,k,K}} \sum_{i=1}^{K} \tilde{P}_{l,k,i} 2^{-\frac{B_{l,k,i}}{M-1}}$$

s.t.

$$\sum_{i=1}^{K} B_{l,k,i} \leq B_{total},$$

(46)

where $\mathbb{Z}^+$ denotes the set of positive integers. This is a convex optimization problem as the objective function is logarithmically convex [39], [40]. Hence, we find the solution in a real space and discretize it to the nearest integer point [39], [40]. Using the Lagrangian function, we define

$$L(B_{l,k,i}, \lambda) = \sum_{i=1}^{K} \tilde{P}_{l,k,i} 2^{-\frac{B_{l,k,i}}{M-1}} + \lambda \left(\sum_{i=1}^{K} B_{l,k,i} - B_{total}\right),$$

(47)

where $\lambda$ denotes a Lagrange multiplier. The first order optimality Karush-Kuhn-Tucker (KKT) conditions
are given by

\[
\frac{\partial L(B_{l,k,i}, \lambda)}{\partial B_{l,k,i}} = -\ln(2)\bar{P}_{l,k,i} - \frac{B_{l,k,i}}{M - 1} + \lambda = 0 \tag{48}
\]

\[
\frac{\partial L(B_{l,k,i}, \lambda)}{\partial \lambda} = \sum_{i=1}^{K} B_{l,k,i} - B_{\text{total}} = 0. \tag{49}
\]

Solving (48) and (49), yields the number of bits required by the \(i\)th user in the \(k\)th cell to quantize the serving and out-of-cell interfering channels, such that the mean interference is minimized. This gives

\[
B_{l,k,k}^* = \min \left\{ B_{\text{total}}, \Re \left( \frac{B_{\text{total}}}{K} + (M - 1) \log_2 \left( \frac{(P_{l,k,k}(L - 1)(1 - \Delta_k))^\frac{K-1}{K}}{\prod_{j=1, j \neq k}^{K} P_{l,k,j} L(1 - \Delta_j)} \right) \right)^+ \right\}, \tag{50}
\]

where \([\cdot]^+ = \max\{0, \cdot\}\). From (50), we can conclude that the bit allocation is a function of serving and interfering channel powers at the user, the number of users, \(L\), the number of cells, \(K\), and the number of transmit antennas at each BS, \(M\). The solution obtained in (50) requires \(L > 1\). Although the case when \(K = 2\) is straightforward, it is important to note that for \(K > 2\) cases, all the interfering channels at the user are given an equal number of remaining bits for quantization, after solving (50), such that

\[
B_{l,k,j}^* = (B_{\text{total}} - B_{l,k,k}^*)/(K - 1), \forall j, \text{ where } j \neq k.
\]

VI. REGULARIZATION PARAMETER ANALYSIS

The optimization of the regularization parameter is considered to be a difficult problem in multicell systems [16]. The regularization parameter is optimized in a single-cell scenario in [5], such that it maximizes SINR. In [46], for single-cell non-homogeneous MU systems, the regularization parameter is defined as

\[
\alpha_k = \frac{1}{L} \sum_{l=1}^{L} 1/P_{l,k,k}, \tag{51}
\]

As the multicell system considered in this paper consists of serving and out-of-cell interfering channels with different link gains, we extend the regularization parameter in [46], such that

\[
\tilde{\alpha}_k = \frac{1}{KL} \sum_{c=1}^{K} \sum_{l=1}^{L} 1/P_{l,k,c}. \tag{52}
\]
In this study, we also consider an optimal regularization parameter, denoted by $\alpha_{k}^{\text{opt}}$, that maximizes the spectral efficiency of the cell \[56\. This optimal regularization parameter is given by

$$
\alpha_{k}^{\text{opt}} = \arg \max_{\alpha_{k}^{\text{opt}} > 0} \sum_{l=1}^{L} \log_{2} \left(1 + \text{SINR}_{l,k}\right). \tag{53}
$$

Finding the optimal regularization parameter, $\alpha_{k}^{\text{opt}}$, that maximizes the spectral efficiency (53) is analytically difficult, therefore we numerically compute $\alpha_{k}^{\text{opt}}$ to evaluate the performance of the proposed coordinated RZF scheme in the next section.

VII. SIMULATION RESULTS

In this section, we present simulation results for the multicell MU MISO system with coordinated RZF precoding. We divide this section into three parts, beginning by explaining the concept of the coordination area in the cell, followed by comparing the expected SINR approximation for perfect CDI and RVQ codebook CDI derived in (22) and (42), respectively, with the corresponding simulated average SINRs. We present average cell-edge spectral efficiency for perfect CDI and limited feedback based RVQ CDI under different transmission scenarios. Next, we present the performance of the coordinated RZF scheme with the adaptive bit allocation method and compare it with the coordinated ZF scheme \[39], for different number of coordinating cells. Finally, we remove the coordination area restriction and present the average cell spectral efficiency comparison between the coordinated RZF scheme and the coordinated ZF scheme \[39]. We assume i.i.d. Rayleigh fading channels. The radius of the single cell is set to $R = 500m$. The standard deviation of the shadowing is 8 dB and the path loss exponent is $a = 3.8$ \[57]\.

A. Coordination area

In this paper, we define the coordination area as an area in the cell where users may experience high interfering power from the adjacent cells and is based on the distance from the BS. We follow the coordination area definition given in \[39], i.e., coordination is needed when the user lies in the region $325m \leq d \leq 500m$, where $d$ represents a distance of the user from the BS. The two- and three-cell coordination areas are illustrated in Fig. 2(a) and Fig. 2(b) respectively. The cell-edge users are uniformly distributed in the coordination area.
B. SINR and spectral efficiency results

In this section, we present SINR and spectral efficiency results for the proposed coordinated RZF scheme. We plot these results against the average received cell-edge SNR, $\rho_0$, obtained by substituting the link distance equal to $R$ in (2) given by $\rho_0 = \mathbb{E} \left[ 10 \log_{10} \left( P_0 10^{(\eta \sigma F / 10) / N_0} \right) \right]$, where the noise power $N_0 = 1$ and $\eta \sim \mathcal{N}(0,1)$. It is important to mention that increasing $\rho_0$ increases the SNR at the users and also increases interference to the users in same or neighboring cells.

Figure 3 shows the average SINR performance of the coordinated RZF scheme with perfect CDI and limited feedback based RVQ CDI. The approximate expected SINRs derived in (22) and (42) are plotted (in the linear scale) in Fig. 3 for perfect CDI and RVQ CDI with various numbers of feedback bits. It is observed that the approximations are tight, however the expected SINR approximation (42) with RVQ codebooks show a small deviation relative to the simulated average SINR at high $\rho_0$ values.

The average cell-edge spectral efficiency for $K = 3$ cells with $L = 2$ cell-edge users is shown in Fig. 4(a) and Fig. 4(b) with $M = 6$ and $M = 8$, respectively. Denoting the cell-edge spectral efficiency by $C$, the average cell-edge spectral efficiency simulations are performed using

$$
\mathbb{E} [C] = \mathbb{E} \left[ \sum_{t=1}^{L} \log_2 (1 + \text{SINR}_{t,k}) \right],
$$

where $\text{SINR}_{t,k}$ is given in (4) and the users are located in the cell-edge area. Therefore we refer to (54) as
Fig. 3: The average SINR for the $l^{th}$ user in the $k^{th}$ cell, with $K = 2$ cells and $L = 2$ users where each BS has $M = 4$ antennas with $B_{l,k,k} = B_{l,k,j} = 20, 15$ and $10$, no shadowing.

the average cell-edge spectral efficiency of the cell. This is plotted against different $\rho_0$ values in Fig. 4. The single cell MU system gives superior average cell-edge spectral efficiency compared to the other scenarios, due to the absence of ICI. However, when ICI is present, the average cell-edge spectral efficiency with the non-coordinated RZF precoding scheme suffers high losses and the performance gap increases compared to the single-cell case, as $\rho_0$ increases. For the proposed coordinated RZF scheme, we consider two cases: 1) only 2 out of 3 cells coordinate and 2) all 3 cells coordinate. In Fig. 4 we consider two regularization parameters for the proposed coordinated RZF case 2: $\alpha_k$ and $\alpha_k^{\text{opt}}$.

Fig. 4: The average cell-edge spectral efficiency with $K = 3$ cells for $L = 2$ users.
The proposed coordinated RZF case 2 with $\alpha_k^{opt}$ achieves better average cell-edge spectral efficiency compared to the proposed coordinated RZF case 1 (with $\alpha_k$) and the non-coordinated RZF scheme. The proposed coordinated RZF schemes with both cases 1 and 2 outperform the coordinated ZF [39] scheme. However, for $\rho_0 < 2$ dB and $\rho_0 < 3$ dB with case 1 and case 2, respectively, it is noted in Fig. 4(a) that the coordinated RZF schemes with $\alpha_k$ are not effective and the non-coordinated RZF scheme is more effective, as the former leverages the benefit of fewer channels being orthogonalized while precoding, thus leading to a stronger signal power. The same reason is applicable to the proposed coordinated RZF case 1 which yields better performance compared to the proposed coordinated RZF case 2, for $\rho_0 < 8$ dB, with the same number of antennas at the BS (i.e., $M = 6$). In Fig. 4(b) we plot the same cases as in Fig. 4(a) with $M = 8$ antennas at the BS. We observe that for $\rho_0 > -4$ dB, the proposed coordinated RZF schemes using $\alpha_k$ outperforms the non-coordinated RZF scheme. Again, the performance of the proposed RZF case 2 with $\alpha_k^{opt}$ is higher than all the other scenarios. Therefore, by comparing Fig. 4(a) and Fig. 4(b) we note that the coordination gain can be obtained over the non-coordinated RZF method, for range of $\rho_0$ values by either increasing $M$, such that $M > KL$, or by using an effective regularization parameter (here, $\alpha_k^{opt}$).

C. Performance of the proposed adaptive bit allocation scheme

We evaluate the average cell-edge spectral efficiency of the proposed RZF precoding scheme with the adaptive bit allocation scheme discussed in Section V when the numbers of coordinating cells are $K = 2$ and 3. From this point onwards, we use the regularization parameter $\tilde{\alpha}_k$ given in (52).

Fig. 5: The average cell-edge spectral efficiency with $K = 2$ cells and $L = 2$ users where each BS has $M = 4$ antennas with $B_{total} = 8$ ($\sigma_{SF} = 8$ dB).
1) Coordination with 2 cells: The average cell-edge spectral efficiency for the proposed adaptive bit allocation scheme is shown in Fig. 5 with $B_{\text{total}} = 8$, $M = 4$ and $L = 2$. It is compared with the coordinated ZF adaptive bit allocation scheme [39]. The scenario used in the simulation is the $K = 2$ case depicted in Fig. 2(a). It is seen that the proposed scheme improves the average cell-edge spectral efficiency compared to [39]. Even the equal bit partitioning (i.e., 4 bits) to each serving and out-of-cell interfering channel, results in a better average cell-edge spectral efficiency performance than the coordinated ZF adaptive bit allocation scheme [39]. The effect of quantization errors on the average cell-edge spectral efficiency is

Fig. 7: The average proposed adaptive bit allocation for the $l$th user in the $k$th with $\rho_0 = 5$ dB, $K = 2$ cells and $L = 2$ users where each BS has $M = 4$ antennas with $B_{\text{total}} = 10$ (no shadowing).
shown in Fig. 6 with $M = 4$, $L = 2$ and various values of $B_{\text{total}}$. As the total number of bits at the user increases, quantization errors decrease which results in higher spectral efficiency.

The average number of feedback bits allocated to the serving and interfering channels is shown in Fig. 7 with the proposed adaptive bit allocation for $B_{\text{total}} = 10$ bits and $\rho_0 = 5$ dB. It shows the average bit allocation trend with respect to different distance values. The proposed adaptive bit allocation scheme divides bits equally among the serving and interfering channels in the region $395 \text{m} \leq d_{l,k,k} \leq 425 \text{m}$. As the distance increases, the number of bits allocated to the serving channel decreases.

2) Coordination with 3 cells: For $K = 3$ cells, we use the scenario shown in Fig. 2(b) in the simulations. The average cell-edge spectral efficiency is shown in Fig. 8 for $K = 3$ cells using the proposed coordinated RZF with adaptive bit allocation strategy, where $B_{\text{total}} = 9$, $M = 6$ and $L = 2$.

![Fig. 8: The average cell-edge spectral efficiency with $K = 3$ cells and $L = 2$ users where each BS has $M = 6$ antennas with $B_{\text{total}} = 9$.](image)

The trend is similar to Fig. 5 and the proposed adaptive bit allocation strategy yields better average cell-edge spectral efficiency compared to the coordinated ZF based adaptive bit allocation method.

3) Full cell area coordination: Here we assume that there is no restriction on the coordination area, i.e., users always coordinate regardless of their distance in the cell. Figure 9 shows average cell spectral efficiency for the proposed coordinated RZF and the coordinated ZF precoding [39] strategies, at $\rho_0 = 5$ dB. The performance is shown for both adaptive bit allocation and equal bit allocation, with and without shadowing. In all the four cases plotted, the average cell spectral efficiency with the proposed coordinated RZF scheme are higher than the coordinated ZF scheme. Hence, the effectiveness of the proposed coordinated RZF scheme is not only limited to the cell-edge, but it also yields acceptable performance across the entire cell
Fig. 9: The average cell spectral efficiency with $K = 3$ coordinating cells having $L = 2$ users where each BS has $M = 6$ antennas with $B_{\text{total}} = 9$.

VIII. CONCLUSION

We proposed a coordinated RZF precoding strategy for multicell MU MISO systems. We derived expected SINR approximations for both perfect CDI and limited feedback RVQ based CDI. Using the expected interference result for RVQ codebooks, we propose an adaptive feedback bit allocation scheme with the limited feedback RVQ CDI that minimizes the expected interference at the user. Via simulations, we show that the proposed coordinated RZF precoding outperforms the non-coordinated RZF scheme with a suitable choice of the regularization parameter. Also, the proposed adaptive bit allocation scheme yields higher average cell-edge spectral efficiency than the equal bit allocation method and the existing coordinated ZF based adaptive bit allocation method.
APPENDIX A

PROOF OF RESULT 1

Here we provide the derivation details of Result 1. Let \( D_k^{(t)} \) be defined by

\[
D_k^{(t)} = \mathbb{E} \left[ \sum_{i=1}^{M} \frac{(\lambda_n)^t}{(\lambda_n + \alpha_k)^2} \right] = M \mathbb{E} \left[ \frac{1}{M} \sum_{i=1}^{M} \frac{(\lambda_n)^t}{(\lambda_n + \alpha_k)^2} \right] = M \left[ \int_0^\infty \frac{(\lambda)^t}{(\lambda + \alpha_k)^2} f_0(\lambda) \, d\lambda \right], \tag{55}
\]

where \( f_0(\lambda) \) is the pdf of an arbitrary eigenvalue selected from \( \{\lambda_1, \lambda_2, \ldots, \lambda_M\} \). The pdf is given by (for the case \( KL = M \)) \cite{58, 59}

\[
f_0(\lambda) = \frac{1}{M} \sum_{i=1}^{M} e^{-\lambda} L_{i-1}(\lambda)^2, \tag{56}
\]

where \( L_{i-1}(\cdot) \) is a Laguerre polynomial which can be also written as \cite{60}

\[
L_{i-1}(\lambda) = \sum_{j=0}^{i-1} (-1)^j \binom{i-1}{i-1-j} \frac{\lambda^j}{j!}. \tag{57}
\]

Substituting (57) in (56), the pdf of an arbitrary eigenvalue becomes

\[
f_0(\lambda) = \frac{1}{M} \sum_{i=1}^{M} e^{-\lambda} \left( \sum_{j=0}^{i-1} (-1)^j \binom{i-1}{i-1-j} \frac{\lambda^j}{j!} \right)^2. \tag{58}
\]

Substituting (58) in (55) gives

\[
D_k^{(t)} = \int_0^\infty \frac{(\lambda)^t}{(\lambda + \alpha_k)^2} \sum_{i=1}^{M} e^{-\lambda} \left( \sum_{j=0}^{i-1} (-1)^j \binom{i-1}{i-1-j} \frac{\lambda^j}{j!} \right)^2 \, d\lambda
= \sum_{s=0}^{t-1} \sum_{i=1}^{i-1} \sum_{j=0}^{i-1} (-1)^j \binom{i-1}{i-1-j} \binom{i-1}{i-1-l} \frac{1}{j!!} \int_0^\infty \frac{\lambda^{t+j+l} e^{-\lambda}}{(\lambda + \alpha_k)^2} \, d\lambda. \tag{59}
\]

Substituting \( \lambda = v - \alpha_k \), the integral in (59) becomes

\[
\int_0^\infty \frac{\lambda^{t+j+l} e^{-\lambda}}{(\lambda + \alpha_k)^2} \, d\lambda = \sum_{s=0}^{t+j+l} \binom{t+j+l}{s} (-\alpha_k)^{t+j+l-s} e^{\alpha_k} \int_0^\infty v^{s-2} e^{-v} \, dv. \tag{60}
\]
Therefore, we can write (55) as
\[
D(t) = \sum_{i=1}^{M} \sum_{j=0}^{i-1} \sum_{l=0}^{i-1-j} (-1)^{j+l} \left( \frac{i-1}{i-1-j} \right) \frac{1}{j! l!} \sum_{s=0}^{t+j+l-s} \binom{t+j+l-s}{s} (-\alpha_k)^{t+j+l-s} e^{\alpha_k} \int_{\alpha_k}^{\infty} v^{s-2} e^{-v} dv,
\]
where
\[
\int_{\alpha_k}^{\infty} v^{s-2} e^{-v} dv = \begin{cases} 
-\text{Ei}(1, \alpha_k) + \frac{e^{-\alpha_k}}{\alpha_k} & \text{for } s = 0 \\
\text{Ei}(1, \alpha_k) & \text{for } s = 1 \\
\Gamma(s-1, \alpha_k) & \text{for } s \geq 2
\end{cases}
\]
(62)
where \(\text{Ei}(\cdot, \cdot)\) and \(\Gamma(\cdot, \cdot)\) are the generalized exponential integral and incomplete gamma function, respectively.

APPENDIX B

PROOF OF RESULT 2

Let, \(F_k\) be defined as
\[
F_k = \mathbb{E} \left[ \left( \sum_{n=1}^{M} \frac{\lambda_n}{\lambda_n + \alpha_k} \right)^2 \right] = \mathbb{E} \left[ \sum_{n=1}^{M} \left( \frac{\lambda_n}{\lambda_n + \alpha_k} \right)^2 \right] + \mathbb{E} \left[ \sum_{a=1}^{M} \sum_{b=1, b \neq a}^{M} \frac{\lambda_a}{\lambda_a + \alpha_k} \frac{\lambda_b}{\lambda_b + \alpha_k} \right] = M \mathbb{E} \left[ \frac{1}{M} \sum_{n=1}^{M} \left( \frac{\lambda_n}{\lambda_n + \alpha_k} \right)^2 \right] + M(M-1) \mathbb{E} \left[ \frac{1}{M(M-1)} \sum_{a=1}^{M} \sum_{b=1, b \neq a}^{M} \frac{\lambda_a}{\lambda_a + \alpha_k} \frac{\lambda_b}{\lambda_b + \alpha_k} \right],
\]
(63)
where \(\lambda_a\) and \(\lambda_b\) are two distinct arbitrary eigenvalues. Using Result 1 given in Appendix A to solve the first term in (63) and denoting \(f_0(\lambda_a, \lambda_b)\) as the joint pdf of two distinct arbitrary eigenvalues, we can write (63) as
\[
F_k = D_k^{(2)} + M(M-1) \int_0^{\infty} \int_0^{\infty} \frac{\lambda_a}{\lambda_a + \alpha_k} \frac{\lambda_b}{\lambda_b + \alpha_k} f_0(\lambda_a, \lambda_b) d\lambda_a d\lambda_b.
\]
(64)
The joint pdf of two distinct arbitrary eigenvalues (for the case \(KL = M\)) is given by 
\[
f_0(\lambda_a, \lambda_b) = \frac{1}{M(M-1)} \sum_{i=1}^{M} \sum_{j=1, j \neq i}^{M} e^{-(\lambda_a + \lambda_b)} Z(\lambda_a, \lambda_b).
\]
(65)
where
\[ Z(\lambda_a, \lambda_b) = \mathbf{L}_{i-1} (\lambda_a) \mathbf{L}_{j-1} (\lambda_b)^2 - \mathbf{L}_{i-1} (\lambda_a) \mathbf{L}_{j-1} (\lambda_a) \mathbf{L}_{i-1} (\lambda_b) \mathbf{L}_{j-1} (\lambda_b). \] (66)

So now we can write (64) as
\[ F_k = D_k^{(2)} + \sum_{i=1}^{M} \sum_{j=1, j \neq i}^{M} \int_0^\infty \int_0^\infty \frac{\lambda_a}{\lambda_a + \alpha_k} \frac{\lambda_a}{\alpha_k + \lambda_b + \alpha_k} e^{-(\lambda_a+\lambda_b)} Z(\lambda_a, \lambda_b) d\lambda_a d\lambda_b. \] (67)

Expanding Y in (67) gives
\[ Y = \int_0^\infty \int_0^\infty \frac{e^{-\lambda a}}{\lambda_a + \alpha_k} \frac{e^{-\lambda b}}{\lambda_b + \alpha_k} (\mathbf{L}_{i-1} (\lambda_a)^2 \mathbf{L}_{j-1} (\lambda_b)^2) d\lambda_a d\lambda_b 
- \int_0^\infty \int_0^\infty \frac{e^{-\lambda a}}{\lambda_a + \alpha_k} \frac{e^{-\lambda b}}{\lambda_b + \alpha_k} (\mathbf{L}_{i-1} (\lambda_a) \mathbf{L}_{j-1} (\lambda_a) \mathbf{L}_{i-1} (\lambda_b) \mathbf{L}_{j-1} (\lambda_b)) d\lambda_a d\lambda_b 
\] (68)

As the double integrals in (68) are of the same function but with different variables, we can also write Y as
\[ Y = \left( \int_0^\infty \frac{e^{-\lambda}}{\lambda + \alpha_k} \mathbf{L}_{i-1} (\lambda)^2 d\lambda \right)^2 - \left( \int_0^\infty \frac{e^{-\lambda}}{\lambda + \alpha_k} \mathbf{L}_{i-1} (\lambda) \mathbf{L}_{j-1} (\lambda) d\lambda \right)^2 
= \left( \int_0^\infty \frac{e^{-\lambda}}{\lambda + \alpha_k} \left[ \sum_{r=0}^{i-1} (-1)^r \left( \frac{i-1}{i-1-r} \frac{\lambda^r}{r!} \right) \right] d\lambda \right)^2 
- \left( \int_0^\infty \frac{e^{-\lambda}}{\lambda + \alpha_k} \sum_{r=0}^{i-1} (-1)^r \left( \frac{i-1}{i-1-r} \frac{\lambda^r}{r!} \right) \sum_{s=0}^{i-1} (-1)^s \left( \frac{i-1}{i-1-s} \frac{\lambda^s}{s!} \right) d\lambda \right)^2 
= \left( \sum_{r=0}^{i-1} \sum_{s=0}^{i-1} (-1)^{r+s} \left( \frac{i-1}{i-1-r} \frac{\lambda^r}{r!} \right) \left( \frac{i-1}{i-1-s} \frac{\lambda^s}{s!} \right) \frac{1}{(\lambda + \alpha_k)^2} \int_0^\infty \frac{\lambda^{1+r+s}}{\lambda + \alpha_k} e^{-\lambda} d\lambda \right)^2 
- \left( \sum_{r=0}^{i-1} \sum_{s=0}^{j-1} (-1)^{r+s} \left( \frac{i-1}{i-1-r} \frac{\lambda^r}{r!} \right) \left( \frac{j-1}{j-1-s} \frac{\lambda^s}{s!} \right) \frac{1}{(\lambda + \alpha_k)^2} \int_0^\infty \frac{\lambda^{1+r+s}}{\lambda + \alpha_k} e^{-\lambda} d\lambda \right)^2. \] (69)
Solving the integrals in (69) by substituting \( v = \lambda + \alpha_k \) and then substituting the resulting expression in (67), we can write (67) as

\[
F_k = E \left[ \left( \sum_{n=1}^{M} \frac{\lambda_n}{\lambda_n + \alpha_k} \right)^2 \right] = D_k^{(2)} + \sum_{i=1}^{M} \sum_{j=1, j \neq i}^{M} \left( \sum_{r=0}^{i-1} \sum_{s=0}^{j-1} (-1)^{r+s} \left( \begin{array}{c} i-1 \r s \\ i-1-r \end{array} \right) \left( \begin{array}{c} j-1 \r s \\ i-1-s \end{array} \right) \frac{1}{r! s!} \sum_{b=0}^{1+r+s} \left( 1 + r + s - b \right) \left( -\alpha_k \right)^{1+r+s} - b e^{\alpha_k} \int_{\alpha_k}^{\infty} v^{b-1} e^{-v} d\nu \right)^2
\]

\[
- \left( \sum_{r=0}^{i-1} \sum_{s=0}^{j-1} (-1)^{r+s} \left( \begin{array}{c} i-1 \r s \\ i-1-r \end{array} \right) \left( \begin{array}{c} j-1 \r s \\ j-1-s \end{array} \right) \frac{1}{r! s!} \sum_{b=0}^{1+r+s} \left( 1 + r + s - b \right) \left( -\alpha_k \right)^{1+r+s} - b e^{\alpha_k} \int_{\alpha_k}^{\infty} v^{b-1} e^{-v} d\nu \right)^2,
\]

where

\[
\int_{\alpha_k}^{\infty} v^{b-1} e^{-v} d\nu = \begin{cases} 
\text{Ei}(1, \alpha_k) & \text{for } b = 0 \\
\Gamma(b, \alpha_k) & \text{for } b \geq 1.
\end{cases}
\]

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