Shades of Gray and Colour Constancy

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Abstract

- Proposed method
  - Max-RGB & Gray-World
    - Instantiations of Minkowski norm
  - Optimal illuminant estimate
    - L_6 norm: Working best overall

Introduction

- Categories of color constancy
  - Representing an image by illuminant invariant descriptors
  - Color constancy methods
    - Physical-based algorithm
    - Statistic-based algorithm
      - Max-RGB, Gray-World, Gray-Edge
    - Gamut constrained algorithm
    - Probability-based algorithm
      - Markov Random Field, Conditional Random Field
    - Learning-based algorithm

- Problem of Max-RGB & Gray-World
  - Two extremes in the Minkoswki family norm
    - Mean(L_1) and Maximum(L_∞)
  - Assuming the optimal illuminant estimate is between L_1 and L_∞

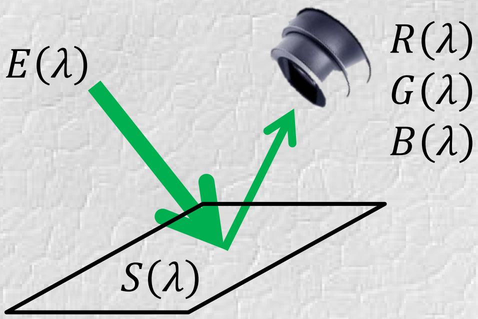
Background

- Modeling a color signal
  - Assuming illuminance $E(\lambda)$ is uniform over a scene
  - A *Lambertian* surface illuminated by a spectral distribution

$$C(\lambda) = E(\lambda)S(\lambda) \quad (1)$$

where $E(\lambda)$: Spectral distribution
$S(\lambda)$: *Lambertian* surface
$C(\lambda)$: Color signal

– Intensity on three sensors (R, G, B)



$$R = \int_{\omega} E(\lambda)S(\lambda)R(\lambda)d\lambda$$

$$G = \int_{\omega} E(\lambda)S(\lambda)G(\lambda)d\lambda$$

$$B = \int_{\omega} E(\lambda)S(\lambda)B(\lambda)d\lambda$$

Sensor response curve
Or
Sensitivity function

– An image represented by three *N*-dimensional vector

- Given image I

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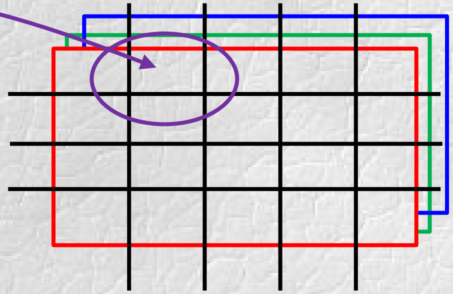
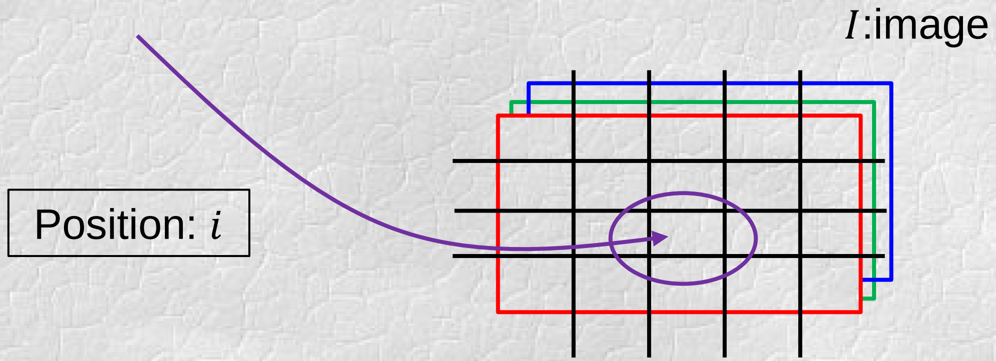


$$\underline{R} = [R_1, R_2, \cdots, R_N]^T$$
$$\underline{G} = [G_1, G_2, \cdots, G_N]^T$$
$$\underline{B} = [B_1, B_2, \cdots, B_N]^T$$

– One pixel intensity over the image

$$R_i = \int_\omega E(\lambda)S_i(\lambda)R(\lambda)d\lambda$$
$$G_i = \int_\omega E(\lambda)S_i(\lambda)G(\lambda)d\lambda \quad (3)$$
$$B_i = \int_\omega E(\lambda)S_i(\lambda)B(\lambda)d\lambda$$



❑ Conventional algorithms

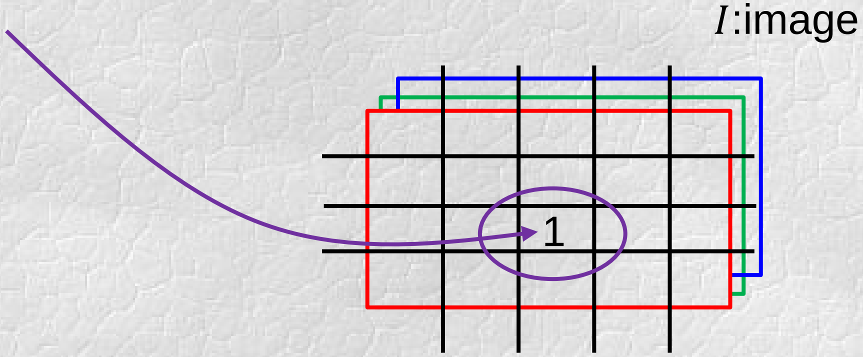
– Max-RGB

- Assuming that at least a white patch exist in an image

$$\max_{i\in\{1,2,\cdots,N\}} R_i = \int_{\omega} E(\lambda)R(\lambda)d\lambda = R_e$$

$$\max_{i\in\{1,2,\cdots,N\}} G_i = \int_{\omega} E(\lambda)G(\lambda)d\lambda = G_e \qquad (7)$$

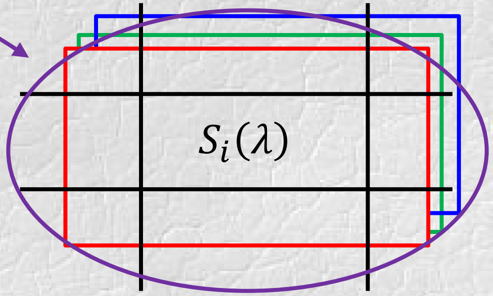
$$\max_{i\in\{1,2,\cdots,N\}} B_i = \int_{\omega} E(\lambda)B(\lambda)d\lambda = B_e$$



– Gray-world

- Assuming that a scene average is grey

$$\mu\left(S(\lambda)\right) = \sum_{i=1}^{N} \frac{S_i(\lambda)}{N} = k$$



$$\mu(\underline{R}) = \int_{\omega} E(\lambda)\left(\sum_{i=1}^{N} \frac{S_i(\lambda)}{N}\right) R(\lambda)d\lambda = kR_e$$

$$\mu(\underline{G}) = \int_{\omega} E(\lambda)\left(\sum_{i=1}^{N} \frac{S_i(\lambda)}{N}\right) G(\lambda)d\lambda = kG_e \qquad (6)$$

$$\mu(\underline{B}) = \int_{\omega} E(\lambda)\left(\sum_{i=1}^{N} \frac{S_i(\lambda)}{N}\right) B(\lambda)d\lambda = kB_e$$

Minkowski family norm

- Minkowski norm
  - Definition of *p*-norm for $\underline{X} = \{X_1, X_2, \cdots, X_N\}^T$

$$\left\|\underline{X}\right\|_p = \left\{\sum_{i=1}^{N} |X_i|^p\right\}^{1/p} \quad (8)$$

  - Example of 2 norm
    - Equal to Euclidean distance

$$\left\|\underline{X}\right\|_2 = \left\{\sum_{i=1}^{N} |X_i|^2\right\}^{1/2} = \sqrt{X_1^2 + X_2^2 + \cdots + X_N^2}$$

❑ Mean of *p*-norm

– Mean of *p*-norm for $\underline{X} = \{X_1, X_2, \cdots, X_N\}^T$

$$\mu_p(\underline{X}) = \frac{\|\underline{X}\|_p}{N^{1/p}} = \sqrt[p]{\frac{X_1^p + X_2^p + \cdots + X_N^p}{N}} \qquad (11)$$

– Property of Minkowski norm

- Triangular inequality: Equation (8) in this paper
- Monotonically increasing sequence

$$\frac{\|\underline{X}\|_p}{N^{1/p}} \leq \frac{\|\underline{X}\|_q}{N^{\frac{1}{q}}}, \qquad if \;\; p \leq q$$

- Infinity norm

$$\|\underline{X}\|_\infty = \max_{0 \leq i \leq N} |X_i|$$

Proposed method

- Expression of Max-RGB & Gray-world with Minkowski norm
  - Max-RGB

$$\begin{bmatrix} R_e \\ G_e \\ B_e \end{bmatrix} = \begin{bmatrix} \mu_\infty(\underline{R}) \\ \mu_\infty(\underline{G}) \\ \mu_\infty(\underline{B}) \end{bmatrix}$$

  - Gray-World

$$\begin{bmatrix} R_e \\ G_e \\ B_e \end{bmatrix} = \begin{bmatrix} \mu_1(\underline{R}) \\ \mu_1(\underline{G}) \\ \mu_1(\underline{B}) \end{bmatrix}$$

– Order relationship between Max-RGB and Gray-World

Max-RGB

$$\mu_1(\underline{R}) \le \mu_2(\underline{R}) \le \cdots \le \mu_\infty(\underline{R})$$

$$\mu_1(\underline{G}) \le \mu_2(\underline{G}) \le \cdots \le \mu_\infty(\underline{G})$$

$$\mu_1(\underline{B}) \le \mu_2(\underline{B}) \le \cdots \le \mu_\infty(\underline{B})$$

Gray-World

– Proposed method: (Shade of grey algorithm)

- Assuming that the average of pixels raised to the power of *p* is gray

$$R_{p,i} = \left[\int_\omega E(\lambda)S_i(\lambda)R(\lambda)d\lambda\right]^p = \int_\omega \{E(\lambda)\}^p\{S_i(\lambda)\}^p R(\lambda)d\lambda \quad (15)$$

$$= \int_\omega E_p(\lambda)\sigma_i(\lambda)R(\lambda)d\lambda = R_i^p$$

– Extension of this formula to *R,G,B*

$$R_{p,i} = \left[\int_\omega E(\lambda)S_i(\lambda)R(\lambda)d\lambda\right]^p = \int_\omega E_p(\lambda)\sigma_i(\lambda)R(\lambda)d\lambda = R_i^p$$
$$G_{p,i} = \left[\int_\omega E(\lambda)S_i(\lambda)G(\lambda)d\lambda\right]^p = \int_\omega E_p(\lambda)\sigma_i(\lambda)G(\lambda)d\lambda = G_i^p \qquad (16)$$
$$B_{p,i} = \left[\int_\omega E(\lambda)S_i(\lambda)B(\lambda)d\lambda\right]^p = \int_\omega E_p(\lambda)\sigma_i(\lambda)B(\lambda)d\lambda = B_i^p$$

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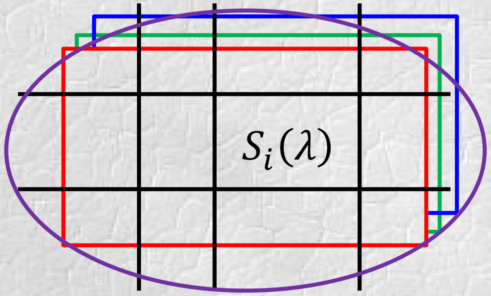

– Shade of grey algorithm

• Assumption

$$\mu_p(S(\lambda)) = \left[\sum_{i=1}^{N}\frac{\{S_i(\lambda)\}^p}{N}\right]^{1/p} = k_p$$

$$\mu_p(\underline{R}_p) = \left[\int_{\omega} E_p(\lambda) \left(\sum_{i=1}^{N} \frac{\{S_i(\lambda)\}^p}{N} \right) R(\lambda) d\lambda \right]^{1/p} = k_p R_e$$

$$\mu_p(\underline{G}_p) = \left[\int_{\omega} E_p(\lambda) \left(\sum_{i=1}^{N} \frac{\{S_i(\lambda)\}^p}{N} \right) G(\lambda) d\lambda \right]^{1/p} = k_p G_e$$

$$\mu_p(\underline{B}_p) = \left[\int_{\omega} E_p(\lambda) \left(\sum_{i=1}^{N} \frac{\{S_i(\lambda)\}^p}{N} \right) B(\lambda) d\lambda \right]^{1/p} = k_p B_e$$

where $\underline{R}_p = \left\{R_1^p, R_2^p, \cdots, R_N^p\right\}^T$

, $\underline{G}_p = \left\{G_1^p, G_2^p, \cdots, G_N^p\right\}^T$

, $\underline{B}_p = \left\{B_1^p, B_2^p, \cdots, B_N^p\right\}^T$

Experimental evaluation

- Evaluation by using angular error
  - Using two databases
    - Data set suggested Barnard et al.
    - One consisting of 321 images of a variety of 32 scenes
    - Another of 220 images of a variety of 22 scenes
    - Both groups taken under 11 coloured illuminant\
    - Comparison measure
      - Angular error: Equation (18) in this paper
      - Distance error in the chromaticity space: Equation (19) in this paper

– L_6 norm: Working best overall

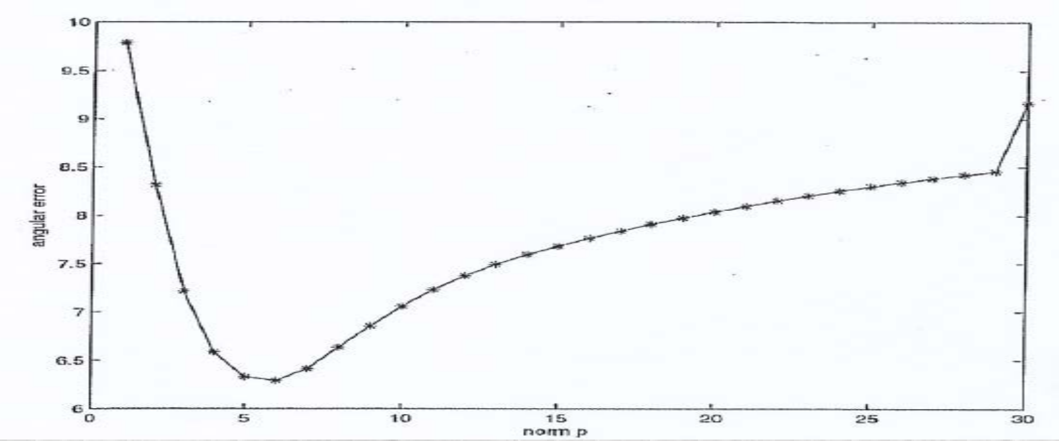

Fig. 2. The figure shows the angular error of the group A images for 30 values of *p*

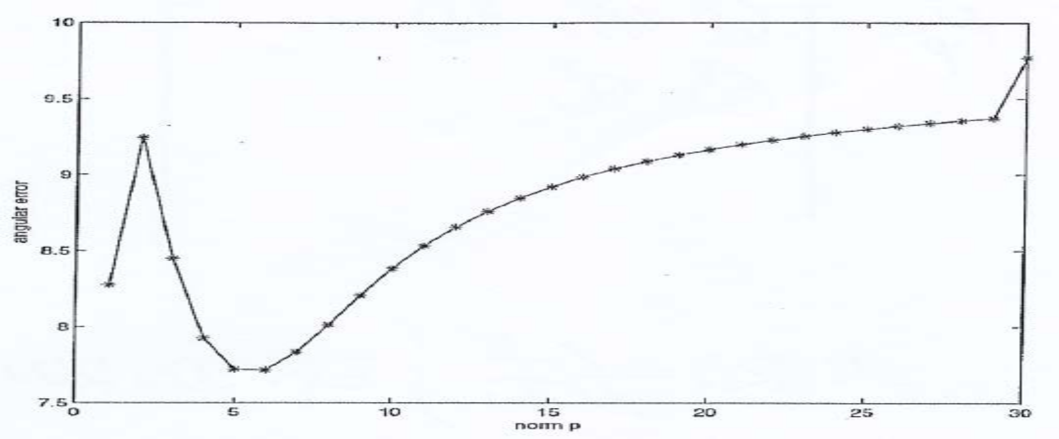

Fig. 3. The figure shows the angular error of the group B images for 30 values of *p*

Table 1. Results for the *p* shade of grey algorithm on two databases considered: the firsts two columns are the mean of angular errors and the lasts two report the distance error in the chromaticities space.

norm	*ang_err*		*dist_err*	
p	group A	group B	group A	group B
1	9.78	8.27	0.0788	0.0624
2	8.32	9.24	0.0640	0.0668
3	7.22	8.45	0.0535	0.0593
4	5.59	7.93	0.0476	0.0549
5	6.33	7.72	0.0448	0.0530
6	6.29	7.71	0.0440	0.0527
7	6.42	7.83	0.0445	0.0533
8	6.64	8.01	0.0456	0.0543
9	6.86	8.21	0.0469	0.0555
10	7.06	8.38	0.0481	0.0566
11	7.23	8.53	0.0492	0.0576
12	7.37	8.66	0.0502	0.0584
13	7.49	8.76	0.0510	0.0590
14	7.59	8.85	0.0517	0.0596
15	7.68	8.92	0.0523	0.0601
16	7.76	8.99	0.0523	0.0605
17	7.84	9.04	0.0534	0.0609
18	7.91	9.09	0.0539	0.0612
19	7.97	9.13	0.0544	0.0615
20	8.04	9.17	0.0549	0.0618
21	8.10	9.20	0.0553	0.0620
22	8.15	9.23	0.0557	0.0622
23	8.20	9.26	0.0561	0.0624
24	8.25	9.28	0.0565	0.0626
25	8.30	9.30	0.0568	0.0627
26	8.34	9.32	0.0571	0.0629
27	8.38	9.34	0.0574	0.0630
28	8.42	9.36	0.0577	0.0631
29	8.46	9.37	0.0580	0.0632
∞	9.16	9.77	0.0630	0.0659

Conclusion

- Shade of grey algorithm
  - Performance
    - L_6 norm: Working best overall
    - Comparable to many advanced colour constancy algorithm for the norm 6 algorithm
    - But, significant computational cost