CP-BASED DECISION MAKING FOR SME

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Abstract. Constraint programming (CP) is an emergent software technology for declarative description and effective solution of large combinatorial problems, which has proven to be useful, especially in such areas as integrated production planning. In that context, the CP can be considered as a well-suited framework for the development of decision-making software supporting small and medium size enterprises (SME) in the course of Production Process Planning (PPP). The aim of the paper is to present the CP modelling framework as well as to illustrate its application to decision making in the case of a new production order evaluation. The paper emphasises benefits derived from CP-based Decision Support Systems and focuses on constraint satisfaction driven decision making rather than on optimal solution searching. Copyright © 2005 IFAC

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1. INTRODUCTION

Constraint programming (CP) is a software technology for declarative description and effective solving of large combinatorial problems. Since a constraint can be treated as a logical relationship including several variables, each one taking a value in a given (usually discrete) domain, hence the idea of CP is to solve problems by stating requirements (constraints) that specify the problem at hand, and then finding a solution satisfying all the constraints (Bartak, 1998).

In this context, CP can be considered as a well-suited framework for the development of decision-making software aimed at the support of small and medium size enterprises (SME) in the course of Production Process Planning (PPP). Because of its declarative nature, it is enough for the user to indicate what has to be solved instead how to solve it (Bartak, 2004). That is a reason why different areas of production management can be seen in a unified way, where the descriptive way of decision problems description provides a platform for the integration of a set of distributed task-oriented data bases of a CIM systems, as well as to support a CE-based project e.g., in the course of iterative evaluation of the chain: CAE - CAD/CAM – CAPP.

Solutions obtained through a constraint propagation driven reference engine provide an attractive alternative to costly and time consuming methods of computer simulation as well as mathematical programming. That is because the solutions include the time constraints, and are robust for nonlinear problem-specific constraints.

The only disadvantage follows from the CP language implementation. In general, each language provides different, specific from case to case limitations for constraints representation. The number of backtracking in a searching tree depends on an order the constraints are propagated. So, depending on a problem specification and a language applied the searching process, i.e. the time required to wait for solution, may differ dramatically. It means, that for a given problem and an assumed CP language the best searching strategy has to be looked for. The example presented provides such a case – an intuitive searching strategy looses against another one following different order of constraints propagation.

In order to develop efficient searching strategy, a so-called reference model of possible decompositions of the satisfaction problem is required. However, because of the space limits the relevant considerations are omitted. The aim of the paper is to present a CP modeling framework as well as to illustrate its application to decision making in the case of a new production order evaluation (more precisely, production process planning). Finding an answer to the question whether a given work order
can be accepted for processing in the production system is fundamental, especially in the customer-driven and highly competitive market. In that context, decision making regards the question whether enterprise’s capability allows to satisfy constraints imposed by the production order requirements, i.e., whether its completion time, batch size, and its delivery period satisfy the customer requirements while satisfying constraints imposed by the enterprise configuration that into account available resources, know how, experience, etc. In the case of the response to this question is positive, i.e., there exist a way guaranteeing to complete a production order, the next question addresses the issue of finding the most efficient one.

Examples illustrate the requirements for the prototyping of production planning, as well an implementation of a reference model of constraint satisfaction problem to evaluate the search strategy. It is shown, in particular, that a strategy assuming that a problem of production flow planning can be decomposed into both the production and transportation batch sizing problems is worse than another one that applies not so intuitive decomposition.

The rest of the paper is organized as follows: section 2 describes some issues underlying Production Process Planning (PPP) prototyping, and then provides a problem statement. The CP-based modeling framework aimed at constraint satisfaction-driven enterprise decision-making is presented in section 3. In section 4 an illustrative example of the presented approach is given. In section 5 some conclusions are presented.

2. PPP PROTOTYPING

Integrated information processing requirements for the operational tasks of an industrial enterprise Koenig, 1990; Scheer, 1991) cover mainly the area of PPP, which includes material and capacity requirements planning, cost estimation, master production planning, production and supervisory control.

From the decision making point of view, PPP can be seen as an integrated framework (based on a distributed data base system) allowing for an efficient interaction among different domains, function, and activities of an enterprise in order to both respond to customer orders and to compete on the market. The main questions regarding the small and medium size enterprise (SME) management are the following: Is the production capacity of the company sufficient to accept a new production order? Is the company able to respond? How to obtain such a response in an on-line mode? What strategy of production order processing is the most efficient one? Of course, answers to the above questions have to be given before entering the order for production. It means the response should be obtained, for example, via a virtual reality environment (Fig.1)

![Virtual reality environment](image1)

Fig.1 Virtual reality environment

However, techniques relying on database resources and virtual reality software are a costly and time-consuming solution offering a potential to exam only few arbitrarily defined versions of work orders. This is due to the combinatorial explosion of possible solutions involving different possible technologies and tool assignments, material handling operations, transportation and storage facilities assignments, production and transportation lot-sizing, scheduling, pricing, and so on.

For illustration, let us consider a production order for a part of female mould (Fig.2). It is assumed that the enterprise considered provides required technologies, such as milling, grinding, drilling, etc., and that for all of them the machine processing cost is known. So, a set of different production and transportation routings as well as relevant resources (machine tools, AGVs, buffers, and so on) assignment should be analyzed.

![Shape analysis](image2)

Fig.2. Shape analysis

\( F_i \) – planes to be machined, \( P_i \) – surfaces, \( O_i \) – holes, \( Z_{ij} \) – fillets
It’s easy to notice that considering only two processes - based on stock material: casting or bar stock - two technological process variants could be considered. Figure 3 illustrates the architecture of the prototype system.

For the purpose of process analysis using the Constructive Solid Geometry approach, a work-piece is defined by elementary volumes (spherical, cylindrical etc.). This set of generic volumes must assure that for every milled part the material to be removed can be decomposed into a union of disjoint delta volumes. The construction of surface in the object needs to be explicitly stated. The framework for production analysis includes the following major modules: image data, a design models database, machine and tools library and cost database.

Assuming that to each partial process two (or more) machines characterized by different exploitation costs and operation times could be assigned, it is easy to notice that the number of variants grows rapidly ($2^n$). The number of variants further increases by adding two materials handling devices and ways of assigning them to the technological process.

The presented way of estimating a potential number of variants of the production processes adopts an imposed order (choice of technology, assignment of tools and devices, etc.). In general case, some mutual local interactions, such as the shape of the designed part, the choice of technology, the production flow architecture (parallel or in series), inter-operating storage system, and, last but not least, simulation should be taken into account.

The tools for coping with PPP prototyping tasks in an on-line mode are of crucial importance. The PPP prototyping belongs to the class of multi-mode problems of production flows scheduling, where finding a feasible solution is NP-complete. Because of real-life constraints, such as requirements imposed by on-line decision-making, to cope with the problem one may consider the use of CP based tools.

3. CP-BASED MODELLING

Decision-making problems occurring in the SMEs mostly concern the acceptance of a new production order. Usually, the first solution, which satisfies the set of constraints, is searched.

3.1 Constraint satisfaction problem

Let us consider the constraint satisfaction problem (CSP) formulated as follows. Given is a finite set of variables $X = \{x_1, x_2, ..., x_n\}$, variable domains $D = \{D_i | D_i = [d_{i1}, d_{i2}, ..., d_{ij}, ..., d_{im}], i = 1..n\}$ and a finite set of constraints $C = \{C_i | i = 1..L\}$ that limit the values of decision variables. Requested is either an admissible solution or an optimal solution following an arbitrarily given goal function.

For the sake of simplicity, let us assume the following notation for the Constraints Satisfaction Problem:

$$CSP = ((X, D), C),$$

where $c \in C$ is a certain predicate $P[x_1,x_2,...,x_n]$ defined on a subset of set $X$. It’s easy to notice that a problem formulated in such a way in natural decomposes into sub-problems, in particular to elementary sub-problems, which are not further decomposed.

To illustrate this, let us consider a CSP $=((X,D), C)$ problem, where $X = \{x_1,x_2,...,x_{12}\}$, $D = \{D_{ij}, D_{ij},...,D_{ij}\}$, $C = \{C_j, C_{ij},...,C_{ij}\}$ , $c_1 := P_1[x_1,x_2,x_3]$, $c_2 := P_2[x_2,x_4,x_5]$, $c_3 := P_3[x_4,x_6]$, $c_4 := P_4[x_7,x_8]$, $c_5 := P_5[x_4,x_7]$, $c_6 := P_6[x_9,x_10]$, $c_7 := P_7[x_8,x_9]$, and $c_8 := P_8[x_11,x_12]$.

Two, arbitrarily chosen, admissible decompositions of this problem are shown in Fig.4. Arcs indicate the order of solving the sub-problems (the order of direct preceded sub-problems is unrestricted), and symbol * indicates elementary sub-problems.

3.2. Searching strategy prototyping

Let us introduce the following notation of decomposed subproblems: $CSP_{i,j,k,l}$ means the 1-th decomposition of the i-th problem (where $i = |\{i,k,l\}|$), which is in turn the k-th decomposition of the l-1-th problem, which is the j-th decomposition of the initial CSP problem, $i’$ (i) indicates problem for which its direct decompositions are mutually independent (dependent). Since each sub-problem cor-
responds to a standard sub-problem’s structure, i.e.,
decision variables, domains and constraints, a simpli-
fied notation may be used (see Fig. 5 a).

\[
\begin{align*}
&(({x_1, x_2, \ldots, x_{12}}, \{D_1, D_2, \ldots, D_{12}\}) \{c_1, c_2, \ldots, c_8\}) \\
&(({x_1, x_2, \ldots, x_6}, \{D_1, D_2, \ldots, D_6\}) \{c_1, c_2, c_3\}) \\
&(({x_7, x_8, \ldots, x_{12}}, \{D_7, D_8, \ldots, D_{12}\}) \{c_4, c_6, c_7, c_8\}) \\
&(({x_9, x_{10}}, \{D_9, D_{10}\}) \{c_6\}) \\
&(({x_{11}, x_{12}}, \{D_{11}, D_{12}\}) \{c_8\}) \\
&(({x_7, x_8}, \{D_7, D_8\}) \{c_4\}) \\
&(({x_9, x_{10}}, \{D_9, D_{10}\}) \{c_6\}) \\
\end{align*}
\]

The graphs in Fig. 5 represent CSP decomposition (see Fig. 4) and the admissible search strategies. Figure 5a shows an instance of the CSP problem decomposition tree (In order to simplify the nota-
tion, letters marking each sub-problem were used: A - corresponds to CSP, B - CSP
_1^1, C - CSP
_1^2, E - CSP
_2^1, F - CSP
_2^2, and G -to CSP
_2^3. Fig. 5b shows three admissible, alternative search strategies:

- the order of subproblem resolution,
- subproblems linked by common constraints,
- problem composed of subproblems.

4. ILLUSTRATIVE EXAMPLE

For the purpose of illustrating the application of the presented approach, let us consider a production order characterized by Z (production volume), TZ (production order completion time), and a production system characterized by:

- \( J \) – a number of alternative production routes,
- \( I \) – batch size into which the production volume is divided,
- \( L \) – number of transport batches into which production batch are divided,
- \( K \) – a number of operations along production route,
- \( K+1 \) – a number of transport operations in each production route,
- \( TJ_{j,k} \) – the time of processing per unit for the \( k \)-th operation in the \( j \)-th route production,
- \( TP_{j,k} \) – the length of the \( k \)-th transport operation in the \( j \)-th route production,
- \( H \) – the planning horizon.

An answer to the following question is searched for: whether the production order could be realized in required period and if so, in what possible way? Let us consider the following subtasks of the PPP.

**Production batching**

\[
x_{i,j} \text{ – size of the } i\text{-th batch production, where: } i=1..I \\
\left(\{x_{i,j}\}, \{D_i\}, \{c_i\}\right) \\
D_i : 1..(Z-I+1) \\
c_i : \sum_{i=1}^{I} x_{i,j} = Z
\]

**Production routing**

\[
x_{j,i} \text{ – number of the route in which the } i\text{-th batch will be produced, where: } i=1..I \\
\left(\{x_{j,i}\}, \{D_i\}, \{c_i\}\right)
\]
Production scheduling

\[ x_{3,j,k} \text{ – starting-up time for processing of the } i\text{-th batch on the } k\text{-th workstation along a production route, where: } i=1,..,I, k=1,..,K \]
\[ D_3: 1,..,H \]
\[ c_3: x_{3,j,k} + x_{3,j,k} \cdot TJ_{(x_{3,j}),k} < x_{3,j,k+1} \]

Transport batching

\[ x_{4,j,l} \text{ – the size of the } l\text{-th transport batch, being a part of the } i\text{-th production batch, where: } i=1,..,I, l=1,..,L \]
\[ D_4: 1,..,Z \]
\[ c_4: \sum_{l=1}^{L} x_{4,j,l} = x_{4,j} \]
\[ c_5: x_{4,j,l} \leq (Z-I-L)+2 \]

Transport scheduling

\[ x_{5,i,l,k} \text{ – the moment the } l\text{-th transport batch of the } i\text{-th production batch start to move to the } k\text{-th workstation in production route, where: } i=1,..,I, l=1,..,L, k=1,..,K+1 \]
\[ D_5: 1,..,H \]
\[ c_6: x_{5,i,l,k} \leq TZ \]
\[ c_7: \sum_{i=1}^{I} x_{5,i,l,k} = x_{5,i,l} \]
\[ c_8: x_{5,i,l,k} \leq (TJ_{(x_{5,i}),k}) + x_{5,i,l} \cdot T \]
\[ c_9: x_{5,i,l,k} \leq x_{5,i,l,k+1} \]

Two, among many other, problem decompositions of the SCP=((\{x_1,..,x_5\}, \{D_1,..,D_5\}),\{c_1,..,c_{11}\}) are shown in Fig.6.

The following goal function is used in order to evaluate the possible search strategies:

\[ f = Z_1^{w_1} \cdot Z_2^{w_2} \cdot \cdots \cdot Z_i^{w_i} \]

where \( w_i \) is the \( i \)-th subproblem index and \( Z_i \) – the \( i \)-th sub-problem computational complexity.

Assuming \( w_1 = 1, w_2 = 1-1,..,w_i = 1 \), where \( I \) – the number of the all sub-problems, the evaluation of particular strategies is summarised in Table 2.
Table 1. Computational complexity of the sub-problems

<table>
<thead>
<tr>
<th>Sub-problem</th>
<th>Decision variables</th>
<th>Computational complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>x₂</td>
<td>8</td>
</tr>
<tr>
<td>C</td>
<td>x₁, x₄</td>
<td>46656</td>
</tr>
<tr>
<td>D</td>
<td>x₃, x₅</td>
<td>10^4</td>
</tr>
<tr>
<td>E</td>
<td>x₁, x₃</td>
<td>6.4 - 10^7</td>
</tr>
<tr>
<td>F</td>
<td>x₅</td>
<td>10^18</td>
</tr>
<tr>
<td>G</td>
<td>x₄</td>
<td>729</td>
</tr>
</tbody>
</table>

Table 2 Strategies computational complexity

<table>
<thead>
<tr>
<th>Strategy</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1.7 - 10⁸⁵</td>
</tr>
<tr>
<td>II</td>
<td>1.1 - 10⁶⁶</td>
</tr>
<tr>
<td>III</td>
<td>7.8 - 10⁵⁵</td>
</tr>
<tr>
<td>IV</td>
<td>1.1 - 10⁶⁶</td>
</tr>
</tbody>
</table>

The best strategy is the strategy II, where subproblems are resolved in the increasing order of their computational complexity, i.e. in the order guaranteeing the lowest amount of backtracking. Fig. 9 illustrates the searching strategy II.

For the presented case, the strategy shown in Fig 6 a) is better. The results are a consequence of a sequence of elementary subproblems consideration. The strategy from Fig 6 a) is characterized (as opposed to strategy in Fig. 6 b) by subproblems solving from least complicated to most. As a result of it, the potential number of backtracks is limited. Fig. 8 illustrates a search solution based on the strategy from Fig. 6 a). The resulting production flow is presented on Gantt’s chart – Fig. 9.

5. CONCLUDING REMARKS

The presented concept of the reference model of CSP decomposition permits to perform the analysis of admissible searching strategies aimed at production flow planning.

The possibilities of verifying the effectiveness of the traditional approach to problems concerning the flow production planning abound. The strategy presented in Fig 4a) belongs to the traditionally applied strategy, which separates the manufacturing problems from the transportation ones. On the other hand, the strategy presented in Fig 4b) belongs to that strategy in which some elementary problems involving transportation are connected to some elementary manufacturing problems. This observation provides a new, based on the concept of CSP decomposition reference model, opportunity to cope with the on-line decision making for SME.

REFERENCES


