

# Matlab-Simulink Implementation of a Memory Polynomial Model for Microwave Power Amplifiers

J.R Cárdenas Valdez<sup>1</sup>, M. Berber Palafox<sup>2</sup>, Christian Gontrand<sup>3</sup>, and J.C. Núñez Pérez<sup>1</sup>

<sup>1</sup> Centro de Investigación y Desarrollo de Tecnología Digital, Instituto Politécnico Nacional ; Av. del Parque 1310, Mesa de Otay, C.P. 22150, Tijuana, Baja California, México

<sup>2</sup> Instituto Tecnológico de Lázaro Cárdenas; Av. Melchor Ocampo No. 2555, Cuarto Sector, Cd. Lázaro Cárdenas Mich. C.P. 60950.

<sup>3</sup> Universite de Lyon, INSA- Lyon, INL, CNRS UMR5270, Villeurbanne, F-69621, France.

**Abstract.** This paper is focused on a model for the power amplifier considering the memory effects in short terms, this work was developed for periodic signals through Matlab software and was implemented in Simulink. This model for the power amplifier implements a Memory-Polynomial models. Memory-polynomials prove to be both accurate and easy to implement and was compared with Ghorbani and Saleh quasi-memoryless models to demonstrate its precision.

**Keywords:** behavioral modeling, memory effects, memory polynomial, periodic signals, power amplifier, RF front-end

## 1 Introduction

An amplifier is a device designed to increase signal power levels. There are mainly two types of amplifiers in Radio Frequency (RF) front end circuits; these are power amplifiers (PA), and low noise amplifiers (LNA). Power amplifiers are mainly present in the transmitters, and are designed to raise the power level of the signal before passing it to the antenna. This power boost is crucial to achieve the desired signal to noise ratio at the receiver, and without which received signals would not be detectable [1]. Nonlinearity is an inherent property of High Power Amplifier (HPA), in wideband applications, HPAs exhibit memory effect as well, which means the current output of an amplifier is stimulated by not only the current input but also previous input. Volterra series are a precise behavior model to describe moderately nonlinear HPAs [2]. However, high computational complexity continues to make methods of this kind rather impractical in some real applications because the number of parameters to be estimated increases exponentially with the degree of nonlinearity and with the memory length of the system.

## 2 Modeling Power Amplifiers with Memory Effects

The Volterra series can be used to describe any nonlinear stable system with fading memory. Memory effects due to the existence of components which store energy, such as inductors and capacitors, impedance of inductors and capacitors is relevant to frequency. PA memory effect is rejected as a non-linear distortion associated with the signal bandwidth and power [8]. However, its main disadvantages are the dramatic increase in the number of parameters with respect to nonlinear order and memory length, which causes drastic increase of complexity in the identification of parameters. As the input signal bandwidth becomes wider, such as in WCDMA (Wideband Code Division Multiple Access), the time span of the power amplifier memory becomes comparable to the time variations of the input signal envelope. A Volterra series is a combination of linear convolution and a nonlinear power series so that it can be used to describe the input/output relationship of a general nonlinear, causal, and time-invariant system with fading memory [6]. The Volterra series in discrete-domain can be represented as (1).

$$y(n) = \sum_k \sum_{l_1} \dots \sum_{l_{2k+1}} h_{2k+1}(l_1, l_2, \dots, l_{2k+1}) \prod_{i=1}^{k+1} x(n - \tau_i) \prod_{i=k+2}^{2k+1} x^*(n - \tau_i) d\tau_{2k+1} \quad (1)$$

where (\*) denote the complex conjugation and  $x(n)$  and  $y(n)$  represents the input and output of the model. It can be observed that the number of coefficients of the Volterra series increases exponentially as the memory length and the nonlinear order increase making it unpractical for modeling power amplifiers in real time applications [4].

### 2.1 The Memory-Polynomial Model as special case of the Volterra series

The memory-polynomial model, [3] consists of several delay taps and nonlinear static functions. This model is a truncation of the general Volterra series, which consists of only the diagonal terms in the Volterra kernels. Thus, the number of parameters is significantly reduced compared to general Volterra series, the memory-polynomial model is considered as a subset of the volterra series. The model is shown in Figure 1.

A Memory-polynomial model considering memory effects and nonlinearity is given by the following equation:

$$y(n) = \sum_{q=0}^Q \sum_{k=1}^K a_{2k-1,q} |x(n-q)|^{2(k-1)} x(n-q) \quad (2)$$

Where:

$x(n)$  is the input complex base-band signal.

$y(n)$  is the output complex base-band signal.

$A_{k,q}$  are complex valued parameters.

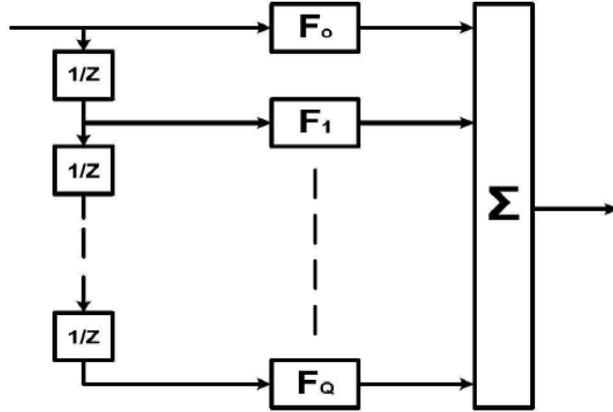


Fig. 1. Memory-Polynomial model

$Q$  is the memory depth.  
 $K$  is the order of the polynomial.

### 2.2 Implementing of the Memory-Polynomial Model

As was expressed in (2), a memory-polynomial system the model can be rewritten as follows:

$$y(n) = \sum_{q=0}^Q F_q(n - q) = F_0(n) + F_1(n - 1) + F_2(n - 2) + \dots + F_Q(n - Q) \quad (3)$$

where  $F_q(n)$  can be expressed as:

$$y(n) = \sum_{k=1}^K a_{2k-1,q} |x(n - q)|^{2(k-1)} x(n) \quad (4)$$

The equation (3) can be defined as block diagram as shown in the Figure 2 and Figure 3.

### 2.3 Implementing of the Memory-Polynomial Model using Matlab Simulink

The Memory-polynomial model can be simulated for any input in Matlab and the gotten parameters can be used for the block diagram developed in Simulink, we present a case demonstration successful treatment of the signal used for this purpose in Matlab and  $y(n)$  gotten of the Memory-Polynomial Model.

To calculate  $y(n)$ , we need the next parameters:

$x$  input of the model.  
 $y$  amplification of the model.

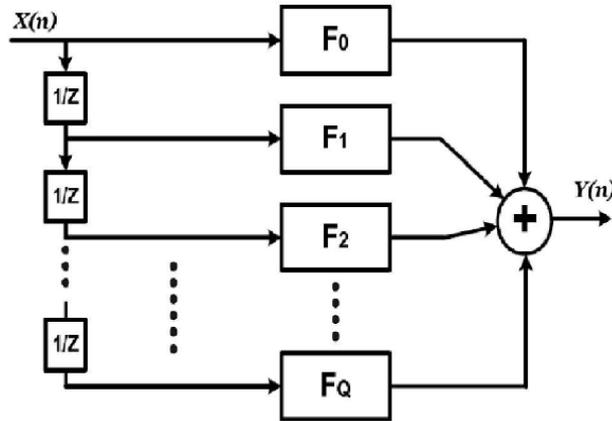


Fig. 2. Implementation of  $F_q(n; q)$  as block diagram

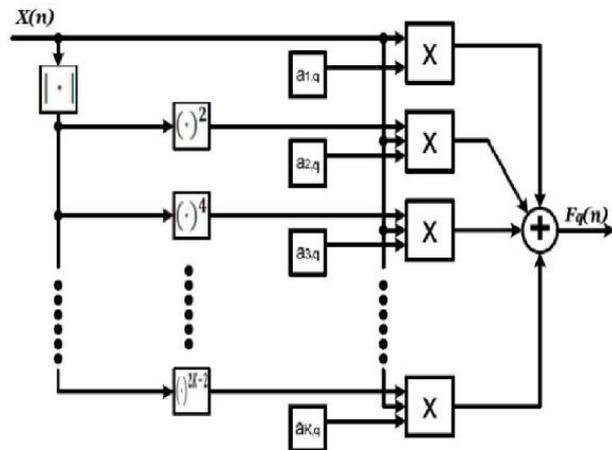
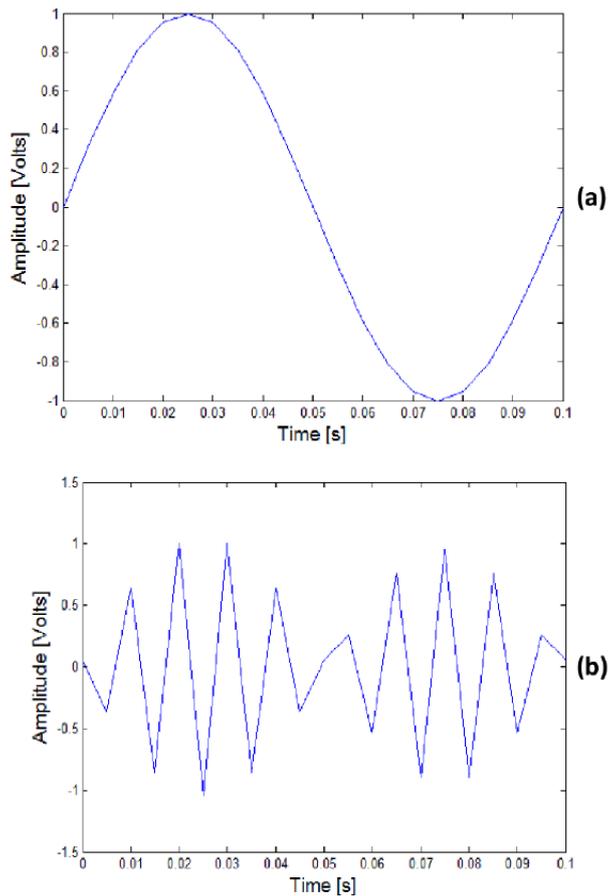


Fig. 3. Implementation of  $F_q(n)$  with parameters

$n$  order of nonlinearity.  
 $m$  order of memory.

We consider  $x$  as the sinewave modulated in amplitude during a time  $t = 0:1$  seconds as shown the Figure 4a,  $y = 5$  amplification of the model,  $n = 1$  and  $m = 0$  for purpose of demonstrate the good performance of the model. Memory effects haven't considered for this case but can modify the parameter. The memory-polynomial model discussed in the previous section was implemented in Simulink. Simulink is a platform for multi-domain simulation and model-based design for dynamic systems. It

provides an interactive graphical environment and customizable set of block libraries, and can be extended for specialized applications [5]. Simulink was chosen because it is easy for implementing



**Fig. 4.** (a) Input  $x(n) = \sin(\mu)$  with  $f = 10Hz$ ; (b) Amplitude modulated signal with  $f_c = 100Hz$

system level models compared to Matlab. Systems implemented in Simulink can be easily modified and upgraded with minimum coding.

Based on the block diagram showed in the Figure (3) using the parameters gotten of a2k1;q is possible to create the same structure using the same sinewave and AM Modulation as can see briefly in the Figure (4b), was inserted a sinewave modulated in amplitude and sampled during 0.1secs, in the Figure (5) is showed that after 0:1 seconds the amplifier is stable and is amplifying  $x(n)$ .

### 3 Implementing Volterra series Model

In comparison with the Volterra series to calculate the parameters  $a_{2k|1;q}$  and the output  $y(n)$ , there is one more internal cycle to generate the output and the parameters so we require more data processing.

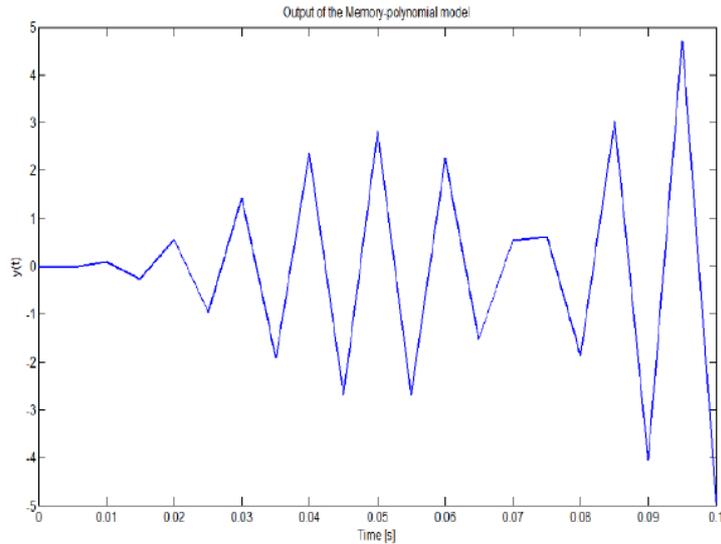


Fig. 5. Exhibits the output signal obtained of the Memory-Polynomial Model

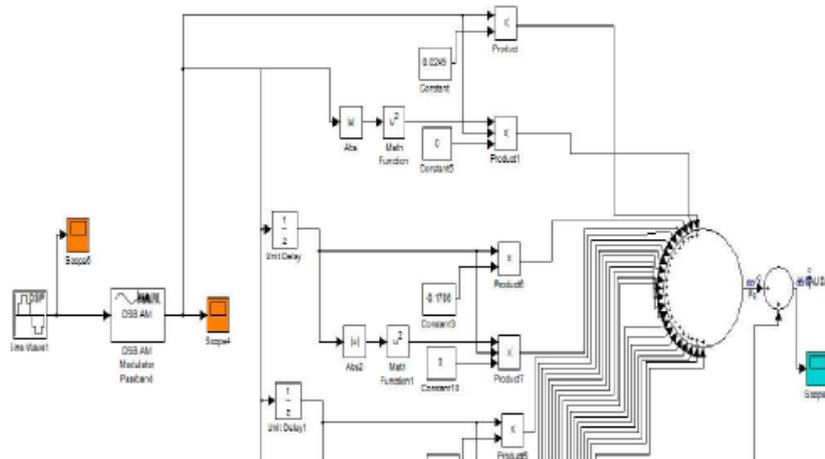


Fig. 6. Overview of Memory-Polynomial model diagram implemented in Simulink

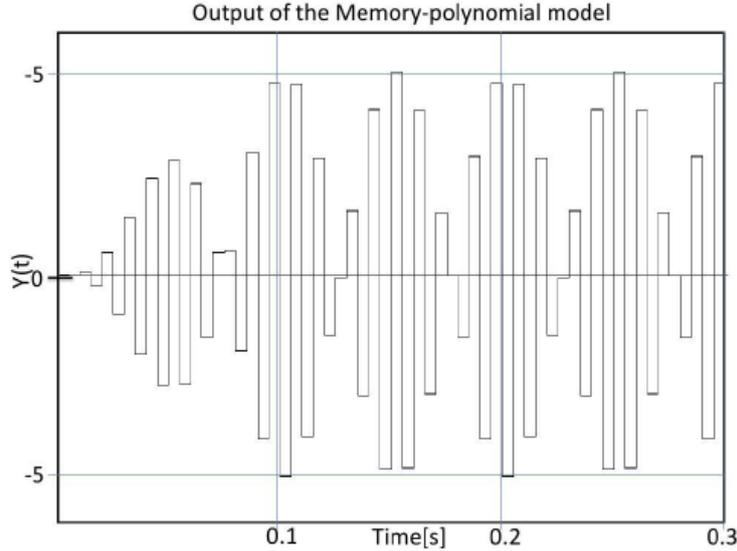


Fig. 7. Output  $y(n)$  of the Memory-polynomial model

#### 4 Comparison of the Memory-Polynomial Model with Quasi-Memoryless Models as Saleh and Ghorbani

Quasi-memoryless models take into account both amplitude and phase distortions. Therefore, they are represented by the amplifier AM/AM as well as AM/PM transfer functions. Static models give reasonable accuracy for applications with a narrow-band frequency spectrum or when memory effects are not important. Quasi-memoryless models have better accuracy for narrow-band applications [1].

##### 4.1 The Saleh model

The Saleh model is a quasi-memoryless model. It uses four parameters to fit the model to measurement data. Its AM-AM and AM-PM conversion functions are described by the following equations

$$g(r(n)) = \frac{\alpha_a r(n)}{1 + \beta_a r(n)^2} \quad (5)$$

$$g(r(n)) = \frac{\alpha_\varphi r(n)^2}{1 + \beta_\varphi r(n)^2} \quad (6)$$

where  $[\alpha_a \alpha_\varphi \beta_a \beta_\varphi]$  are the model's parameters [9].

**4.2 The Ghorbani model**

The Ghorbani model uses eight parameters to fit the model to measurement data, this model is quasi-memoryless, and its AM-AM and AM-PM conversions functions are described by the following equations.

$$g(r(n)) = \frac{x_1 r(n)^{x_2}}{1 + x_3 r(n)^{x_2}} + x_4 r(n) \tag{7}$$

$$f(r(n)) = \frac{y_1 r(n)^{y_2}}{1 + y_3 r(n)^{y_2}} + y_4 r(n) \tag{8}$$

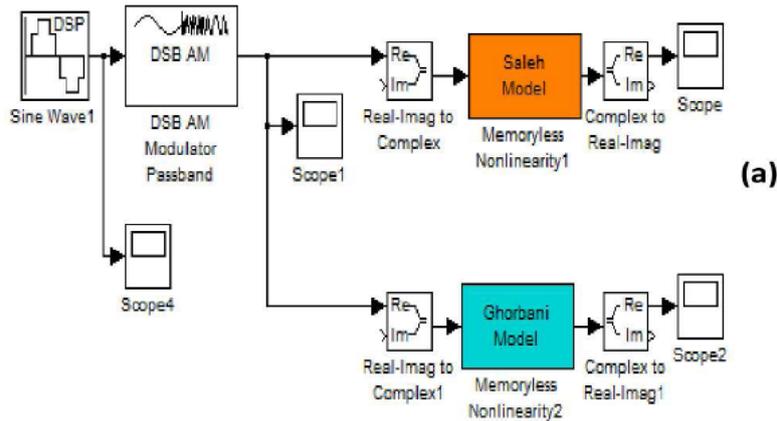
where  $x_1; x_2; x_3; x_4; y_1; y_2; y_3; y_4$  are the models parameters, which are calculated from measurement data by means of curve fitting [10].

**4.3 Implementation of Ghorbani, Saleh and Memory-Polynomial Model**

The Memory-polynomial Model was compared with the quasi-memoryless models as Saleh and Ghorbani was used a sinewave modulated in Amplitude using a  $f_c = 100\text{Hz}$ .

The Saleh Model was adjusted [ $\alpha_a = 2.1587, \alpha_\phi = 4.033, \beta_a = 1.1517, \beta_\phi = 9.1040$ , the parameters  $x_1 = 8.1081, x_2 = 1.5413, x_3 = 6.5202, x_4 = -0.0718, y_1 = 4.6645, y_2 = 2.0965, y_3 = 10.88, y_4 = -0.003$ ..

The Figures 8a and 8B show the amplification gotten of Ghorbani, Saleh and Memory-Polynomial Model.



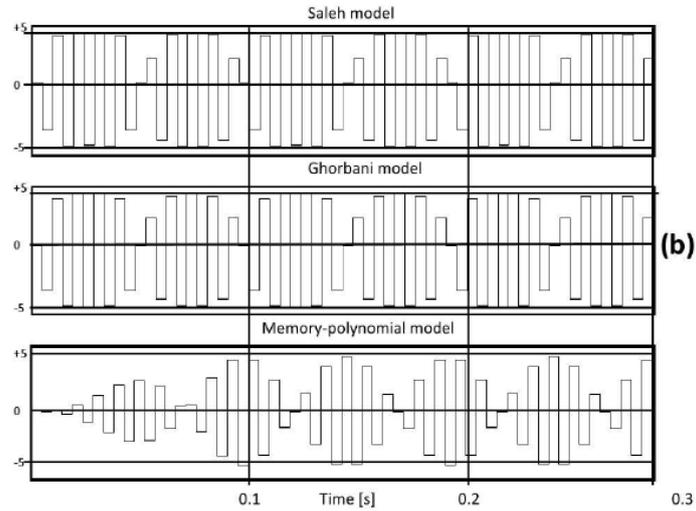


Fig. 8. (a) Overview of Saleh and Ghorbani models, (b) Amplification made by the Saleh, Ghorbani and Memory-polynomial model to a sinewave modulated in Amplitude.

#### 4.4 Comparing Models

An RF Satellite Link (Fig. 9) is simulated. In Figures (10-13) are plotted input, AM signal, memory-polynomial model output, and demodulated output signals are plotted, respectively. This case is for the memory-polynomial model without memory depth.

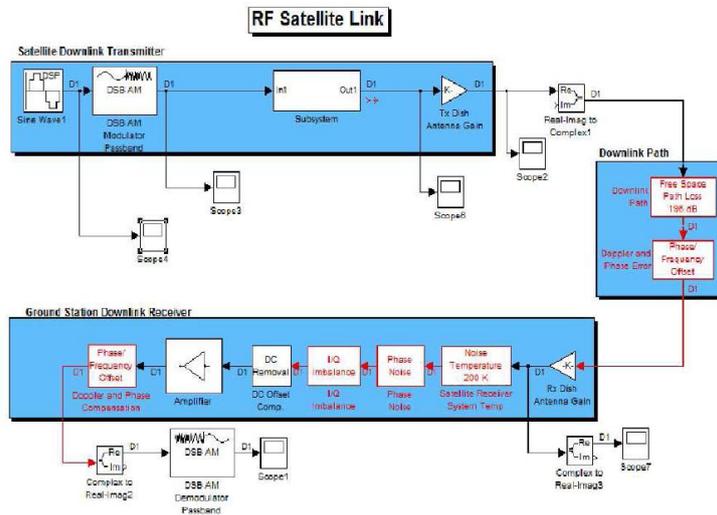
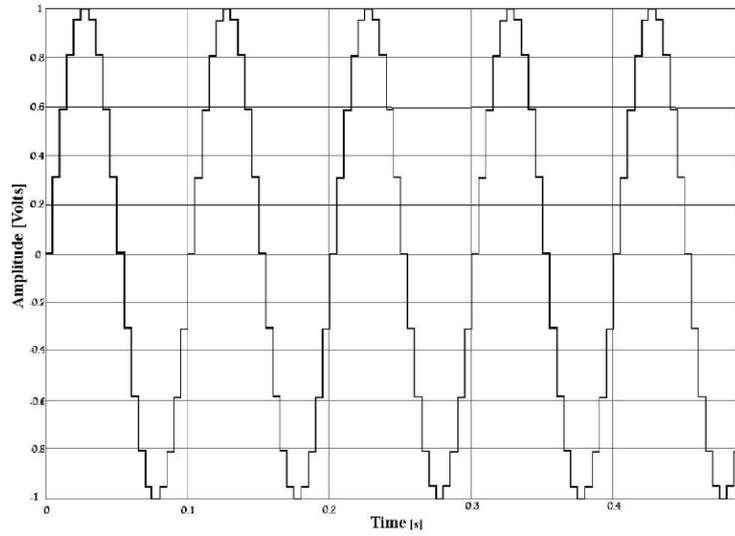
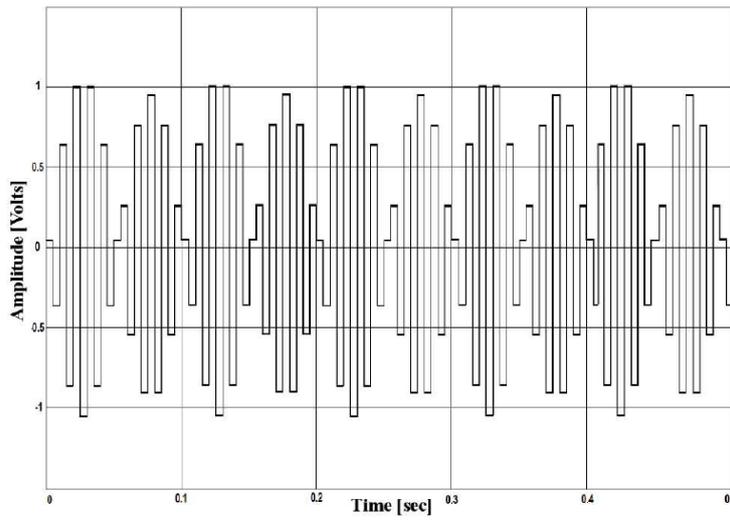


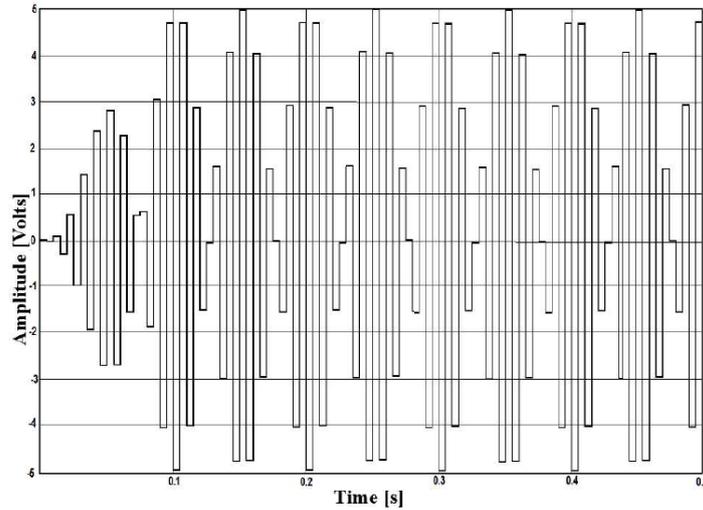
Fig. 9. RF Satellite Link Schematic.



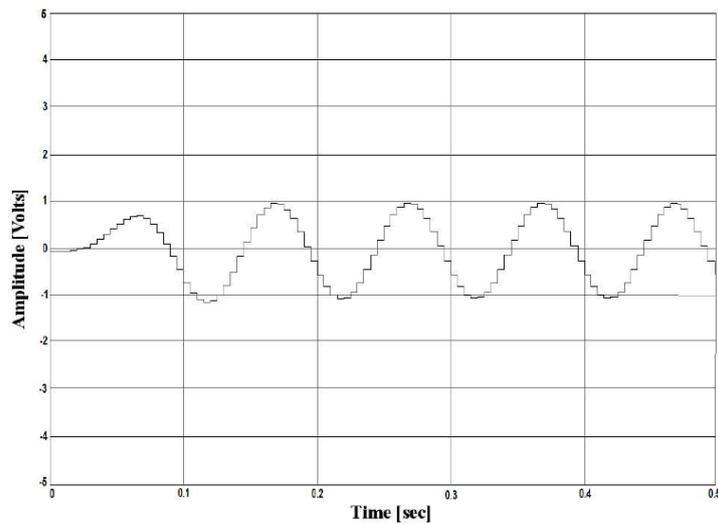
**Fig. 10.** Input Signal.



**Fig. 11.** Signal modulated in Amplitude.



**Fig. 12.** Output signal of the Memory-polynomial Model.



**Fig. 13.** Demodulated Received Signal.

It can be concluded from Fig. (10 - 13) that the implemented model fits the amplifier behavior very well. In comparison with models of memory (Saleh and Ghorbani), the polynomial model has a signal delay of  $108 \text{ degrees} = 0.6\frac{1}{4} \text{ rad}$ .

This is because the amplification of the AM modulated signal starts, all states start at zero amplifier and is up to the second cycle of the periodic signal when the amplifier is stable and in all states of the amplifier is part of the input signal.

In reality the output of the power amplifier depends on previous inputs as well as the current input of the amplifier.

## Conclusion

The Volterra series is the most general model and is the most accurate one but the number of parameters needed increases dramatically.

Quasi-memoryless models have better accuracy for narrowband applications, for higher frequency applications some times aren't enough Quasi-memory less models and it is better to consider the memory effects for such issues.

The Memory-Polynomial Model considers only odd-order nonlinear terms, because the even-order terms are usually outside of the operational bandwidth of the signal and can be easily filtered out.

This model was proved in a RF Satellite Link simulated in Matlab and confirmed in Simulink, was showed good performance of this model as a truncation of the fully Volterra series.

## References

- [1]. Eyad Arabi, Sadiq Ali: Thesis: Behavioral Modeling of RF front end devices in Simulink. Chalmers University of Technology Gteborg, Sweden. pp. 7 (2008).
- [2]. Xiaofang Wu, Jianghong Shi, Huihuang Chen: On the Numerical Stability of RF Power Amplifiers Digital Predistortion. Proceedings of the 15th Asia-Pacific Conference on Communications pp. 1 (2009).
- [3]. Hyunchul Ku, and J. Stevenson Kenney: Behavioral Modeling of Nonlinear RF Power Amplifiers Considering Memory Effects IEEE Trans. on Microwave Theory and Techniques, Vol. 51, No 12, December 2003.
- [4]. Lei Ding: Digital Predistortion of Power Amplifiers for Wireless Applications. School of Electrical and Computer Engineering Georgia Institute of Technology, March 2004. pp 11.3
- [5]. Simulink: Simulation and Model-Based Design. On line: <http://www.mathworks.com/products/simulink>. Access date: September 2011.
- [6]. Anding Zhu, Jos C. Pedro and Thomas J. Brazil. Dynamic Deviation Reduction-Based Volterra Behavioral Modeling of RF Power Amplifiers. December 2006.
- [7]. Qiang Luo, M. Pirola, V. Camarchia, R. Quaglia, R. Tinivella, Shu Shen, G. Ghione. Article: FPGA implementation of adaptive baseband predistortion for FET-based wireless power amplifiers Politecnico di Torino, Torino, Italy (2009).
- [8]. Hanxin Zhou, Guojin Wan and Limin Chen. Article: A Nonlinear Memory Power Amplifier Behavior Modeling and Identification Based on Memory Polynomial Model in Soft-defined Shortwave Transmitter. Department of Electronic Information Engineering, Nanchang University Nanchang, 330031, China 2010.
- [9]. Saleh, A.A.M., .Frequency-independent and frequency-dependent nonlinear models of TWT amplifiers,. IEEE Trans. Communications, vol. COM-29, pp.1715-1720, November 1981.
- [10]. Ghorbani, A. and M. Sheikhan, .The Effect of Solid State Power Amplifiers (SSPAs) Nonlinearities on MPSK and M-QAM Signal Transmission,. Sixth Int.l Conference on Digital Processing of Signals in Comm., 1991, pp. 193-197.