

Testing Lorentz invariance violations in the tritium beta-decay anomaly

J.M. Carmona ^{a,1}, J.L. Cortés ^{b,2}

^a*Dipartimento di Fisica dell'Università and INFN, I-56127 Pisa, Italy*

^b*Departamento de Física Teórica, Universidad de Zaragoza, 50009 Zaragoza, Spain*

Abstract

We consider a Lorentz non-invariant dispersion relation for the neutrino, which would produce unexpected effects with neutrinos of few eV, exactly where the tritium beta-decay anomaly is found. We use this anomaly to put bounds on the violation of Lorentz invariance. We discuss other consequences of this non-invariant dispersion relation in neutrino experiments and high-energy cosmic-ray physics.

Key words: Beta decay; Neutrino; Lorentz invariance; Cosmic rays

1 Introduction

Recent research on the determination of neutrino mass by studying the low-energy beta decay spectrum of tritium has produced a best fit value for m_ν^2 which is significantly negative [1]. This unphysical value is caused by an anomalous excess of electron events at the end of the spectrum, at about 20 eV below the end point. The origin of this anomaly is not known. It seems clear that there is some systematic effect not taken into account, most probably of experimental nature. However, in this letter we want to point that an *apparent* excess of electron events near the end of the spectrum is compatible with a certain deviation of the relativistic dispersion relation for the neutrino. In this way, the tritium experimental results could be used to put bounds on the parameters characterizing this violation of Lorentz symmetry.

¹ E-mail: carmona@df.unipi.it

² E-mail: cortes@posta.unizar.es

The idea that Lorentz invariance is not an exact symmetry, but an approximate one which works extremely well in our low-energy experiments, is not new. In the third section of this letter we will review the main proposals for modifications of the energy-momentum relativistic dispersion relation. There we will argue that a plausible dispersion relation for the neutrino, and only for this particle, is the following one:

$$E_\nu^2 = \mathbf{p}_\nu^2 + m_\nu^2 + 2\lambda|\mathbf{p}_\nu|, \quad (1)$$

where $|\mathbf{p}|$ means the module of the three-momentum \mathbf{p} , m_ν is the neutrino mass, and λ is some mass scale, to be determined afterwards. We will consider $\lambda > 0$ in order to have a positive contribution to the energy squared.

2 Kurie plot

Let us see how the usual Kurie plot of the tritium beta decay is modified by the dispersion relation Eq. (1). The phase-space factor is

$$d\Pi = p_e^2 dp_e p_\nu^2 dp_\nu \delta(Q - E - E_\nu) , \quad (2)$$

where p_e is the momentum of the electron, E is the kinetic energy of the electron ($E_e = m_e + E$) and Q is the energy available to distribute between the neutrino energy and the kinetic energy of the electron. Q is given by

$$M - M' = Q + m_e , \quad (3)$$

M and M' being the masses of the initial and final nuclei. We are neglecting here the kinetic energy of the nucleus, which is typically of order 10^{-1} eV. The Kurie plot is proportional to the function $K(E)$ given by

$$K(E) = \left[\int \delta(Q - E - E_\nu) p_\nu^2 dp_\nu \right]^{1/2} . \quad (4)$$

For the usual relativistic dispersion relations, $E_e^2 = \mathbf{p}_e^2 + m_e^2$ and $E_\nu^2 = \mathbf{p}_\nu^2 + m_\nu^2$, one gets

$$K(E) = \left[(Q - E) \sqrt{(Q - E)^2 - m_\nu^2} \right]^{1/2} , \quad (5)$$

which is a straight line, $(Q - E)$, in the case $m_\nu = 0$.

If one takes the new dispersion relation for the neutrino, Eq. (1), then the function $K(E)$ becomes (from now on, we will use the notation $m \equiv m_\nu$)

$$K(E) = (Q - E)^{1/2} \left\{ [(Q - E)^2 + \lambda^2 - m^2]^{1/4} - \frac{\lambda}{[(Q - E)^2 + \lambda^2 - m^2]^{1/4}} \right\}. \quad (6)$$

It is easy to check that the end point of this curve is $E = Q - m$, just as in the standard case Eq. (5). On the other hand, for large $Q - E$ values, the curve is very well approximated by a straight line. Considering a point E_l so that $Q - E_l \gg m, \lambda$, we introduce the linear approximation

$$\bar{K}_{E_l}(E) = K(E_l) + \left. \frac{\partial K(E)}{\partial E} \right|_{E=E_l} (E - E_l), \quad (7)$$

i.e., it is the tangent to $K(E)$ at the point $E = E_l$. Expanding $\bar{K}_{E_l}(E)$ in powers of $\lambda/(Q - E_l)$, $m/(Q - E_l)$, we get

$$\bar{K}_{E_l}(E) = (Q - E) - \lambda + \frac{\lambda^2 - m^2}{4(Q - E_l)} + \frac{\lambda^2 - m^2}{4(Q - E_l)^2} (E - E_l) + \dots \quad (8)$$

so that, instead of having a straight line of slope -1 ending at $E = Q$, which is the standard case with $m = \lambda = 0$, we obtain a straight line of slope

$$-1 + \frac{1}{4} \frac{\lambda^2 - m^2}{(Q - E_l)^2} + \dots \quad (9)$$

which ends at the point

$$E \equiv E_0 = Q - \lambda + \frac{1}{2} \frac{\lambda^2 - m^2}{Q - E_l} + \dots \quad (10)$$

On the other hand, one can calculate the exact value of the slope at the end point:

$$\left. \frac{\partial K(E)}{\partial E} \right|_{E=Q-m} = - \left(\frac{m}{\lambda} \right)^{3/2}. \quad (11)$$

Two cases are clearly distinguished: $\lambda > m$ and $\lambda < m$. These are shown in Fig. 1. We see that near the end of the spectrum, the curve $K(E)$ is above the linear approximation $\bar{K}_{E_l}(E)$ when $\lambda > m$, which corresponds to an *apparent*

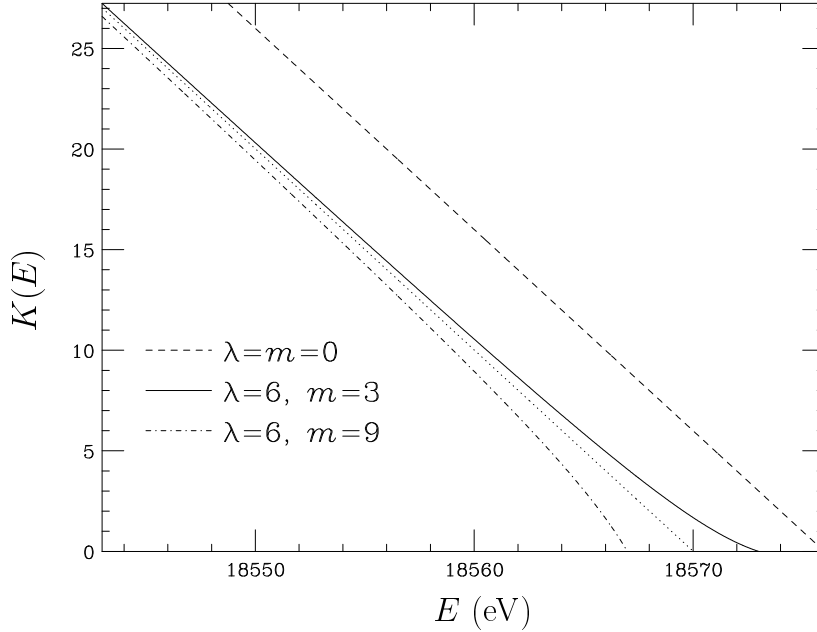


Fig. 1. The function $K(E)$ for the two cases $\lambda > m$ and $\lambda < m$, together with their linear approximation (dotted line). The straight line $Q - E$, corresponding to $\lambda = m = 0$ is also shown. Units in this figure are eV.

excess of electrons at high energies. Indeed it is only apparent, because the curve lies always below the corresponding curve of a relativistic dispersion relation for a massless neutrino, which is also indicated in the figure. In the $\lambda < m$ case, we get the opposite situation: the effect due to the neutrino mass dominates over the λ term (responsible for the “apparent excess”) and we get a reduction on the number of electrons at high energies.

The tritium anomaly consists in an excess of electron events at high energies, where “excess” here means that the data stay above the straight line which is the linear approximation to the curve at low energies. A possible measure of the anomaly could be given by using the following quantity

$$R_{\Delta} = 4 \frac{\int_{E_0 - \Delta/2}^{E_0} dE K(E)}{\int_{E_0 - \Delta}^{E_0} dE K(E)}, \quad (12)$$

so that $R_{\Delta} > 1$ indicates an apparent excess of electrons at high energies. This quantity could be measured experimentally, and a comparison with the theoretical prediction would put bounds on the values of the parameters λ, m . We will show now the orders of magnitude of these bounds in a hypothetical but realistic example.

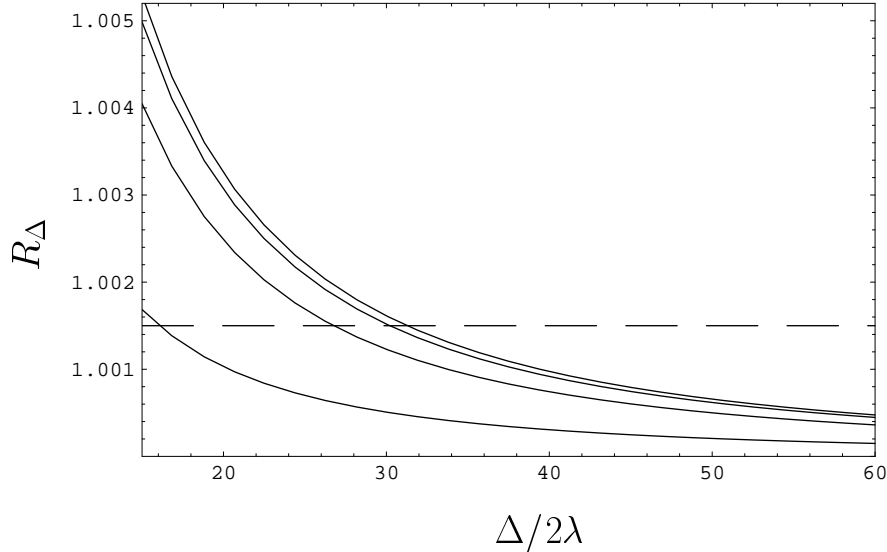


Fig. 2. R_Δ as a function of $\Delta/2\lambda$ for three different values of α : starting from the lower curve, $\alpha = 0.3$ ($\lambda = 1.195m$), $\alpha = 0.75$ ($\lambda = 2m$), $\alpha = 0.9375$ ($\lambda = 4m$) and $\alpha = 1$ ($m/\lambda = 0$). The dashed line $R_\Delta = 1.0015$ is a hypothetical experimental bound on R_Δ (see text).

Introducing the variable $x = (Q - E)/\lambda$, and taking $E_0 \approx Q - \lambda$, R_Δ is rewritten as follows:

$$R_\Delta = 4 \frac{\int_1^{1+\frac{\Delta}{2\lambda}} dx \sqrt{x} [(x^2 + \alpha)^{1/4} - (x^2 + \alpha)^{-1/4}]}{\int_1^{1+\frac{\Delta}{\lambda}} dx \sqrt{x} [(x^2 + \alpha)^{1/4} - (x^2 + \alpha)^{-1/4}]}, \quad (13)$$

where $\alpha \equiv 1 - m^2/\lambda^2$. Fig. 2 shows R_Δ as a function of $\Delta/2\lambda$ for different values of α , that is, for different values of the quotient m/λ .

Experimentally, one could put a bound on the excess of electron events, that is, write $R_\Delta < 1 + \epsilon_\Delta$ for a certain value of Δ , which should be chosen in an appropriate way. Δ should be large enough, so that all the anomaly is contained in the region $E_0 - \Delta/2 < E < E_0$, but it should not be too large, because in that case $R_\Delta \rightarrow 1$. Let us do an estimate of orders of magnitude. From Table 1 of Ref. [2], we see that the fit which gives $m_\nu^2 < 0$, signal of the anomaly, is stable and has a good $\chi^2/\text{d.o.f.}$ in a region between 200 and 400 eV before the end point, which is $E_0 \sim 18570$ eV. Let us take $\Delta/2 = 200$ eV. An approximate value of R_Δ comes then from Fig. 2 of Ref. [2], which contains the experimental data for the Kurie plot. R_Δ is approximately given by $4(s+t)/(4s+t)$, where s is the area of a triangle of base and height $\Delta/2$ (the straight line of the Kurie plot has slope -1), and t is the area delimited by the data above the straight line, spread in a range of energies of around 20 eV, and the line itself. A rough estimate of this area gives us a conservative value of $\epsilon_\Delta = 1.5 \cdot 10^{-3}$. For the four curves of Fig. 2, corresponding to four different

values of the quotient m/λ , we obtain bounds for $\Delta/2\lambda$, which, recalling that we have taken $\Delta/2 = 200$ eV, produce bounds for λ which go from $\lambda < 12.5$ eV to $\lambda < 6.5$ eV for the extreme curves. Again, this translates into bounds for the neutrino mass, depending on the value of α . For $\lambda = 2m$, the bound is $m < 3.8$ eV, and for $\lambda = 4m$, we get $m < 1.7$ eV.

We see that the tritium anomaly puts bounds for (λ, m) in a very interesting range (a few eV). Of course, a detailed analysis of the experimental data would give finer bounds to this kind of Lorentz invariance violation. Even more interesting would be a possible (future) experimental bound of the type $R_\Delta > 1 + \epsilon_\Delta$, with $\epsilon_\Delta > 0$ (that is, the confirmation of the anomaly), which would give a lower bound for λ , showing the presence of Lorentz invariance violation effects in the tritium beta decay.

3 Lorentz invariance violations

Special relativity and Lorentz invariance are at the base of our low-energy effective theories. However, it may be possible that these are low-energy symmetries of a larger theory that do not need to be Lorentz invariant. Several attempts have been made to question Lorentz invariance [3] and put bounds on possible violations (see e.g. [4]).

One way to explore the potentially observable effects of departures from exact Lorentz invariance is to consider the consequences in low-energy processes of possible extensions of the Lorentz-invariant particle energy-momentum relation compatible with translational and rotational invariance in a “preferred frame”. A first possibility is given by the Lorentz-violating class of dispersion relations [5]

$$E^2 = \mathbf{p}^2 + m^2 + \mathbf{p}^2 \left(\frac{|\mathbf{p}|}{M} \right)^n, \quad (14)$$

where M is the natural mass scale of the Lorentz non-invariant fundamental theory. An analysis of cosmic ray processes, whose thresholds are drastically changed by the new dispersion relation, puts severe bounds on the scale of the Lorentz violation: M has to be several orders of magnitude larger than the Planck mass scale [6].

Another possible extension of the energy-momentum relation, coming from the introduction of a rotationally invariant two-derivative term in the free Lagrangian, is [4]

$$E^2 = \mathbf{p}^2 + m^2 + \epsilon \mathbf{p}^2, \quad (15)$$

where ϵ is a small coefficient which fixes the maximal attainable velocity of each particle ($v^2 = 1 + \epsilon$). Differences among maximal attainable velocities of different particles lead to abrupt effects when the dimensionless ratio $\epsilon \mathbf{p}^2/m^2$ is of order unity. The precise tests of special relativity give very strong constraints on this type of extension [4] ($\epsilon < 10^{-23}$).

A discussion of departures from exact Lorentz invariance in terms of modifications of the energy-momentum relation leads to consider a third possible extension of the dispersion relation with a term linear in $|\mathbf{p}|$ [7],

$$E^2 = \mathbf{p}^2 + m^2 + 2\lambda|\mathbf{p}| . \quad (16)$$

The additional term dominates over the standard kinetic term (\mathbf{p}^2) when $|\mathbf{p}| \lesssim 2\lambda$ and then the nonrelativistic kinematics is drastically changed. Therefore this type of generalized dispersion relation has to be excluded, except just for one case. The neutrino has two characteristic properties: it has a very small mass, and it interacts only weakly. As a result of this combination, we have not any experimental result on its nonrelativistic physics. Therefore, the presence of Lorentz invariance violations affecting the nonrelativistic limit cannot be excluded a priori in the neutrino case.

One possible way to incorporate a departure of Lorentz invariance affecting only to the neutrinos is to assume that the presence of a linear term in the dispersion relation is due to a new interaction which acts as a messenger of the Lorentz non-invariance at high energies. Once again, this is not a weird assumption: the introduction of new interactions is a general practice in the different attempts to explain the smallness of neutrino masses [8].

In conclusion, a dispersion relation of the form (1) can be considered for the neutrino at low energies. We still assume the existence of further Lorentz noninvariant terms, as those contained in Eqs. (14) and (15), for the neutrino as well as for all other particles. In the rest of the letter, we turn our attention to other implications of Eq. (1).

4 Other implications of the new dispersion relation

4.1 Neutrino oscillations

Let us first concentrate on the influence on a characteristic low-energy phenomenon for the neutrino: flavour oscillations. In the case of a Lorentz invari-

ance violation independent of flavour, that is,

$$E_i^2 = \mathbf{p}^2 + m_i^2 + 2\lambda|\mathbf{p}| , \quad (17)$$

one gets the result

$$E_i - E_j = \frac{m_i^2 - m_j^2}{2|\mathbf{p}|} + \dots \quad (18)$$

so that there is not any footprint of Lorentz noninvariance in neutrino oscillations. If we admit a possible dependence of λ on the flavour, we get

$$E_i - E_j = \lambda_i - \lambda_j - \frac{(\lambda_i^2 - \lambda_j^2) - (m_i^2 - m_j^2)}{2|\mathbf{p}|} + \dots \quad (19)$$

and the oscillation probability becomes

$$P(\nu_i \rightarrow \nu_j) \approx \sin^2 2\theta \sin^2 \left(L(\text{Km}) \left[(1.27) \frac{(m_i^2 - m_j^2)(\text{eV}^2)}{E(\text{GeV})} + (3.54)10^9(\lambda_i - \lambda_j)(\text{eV}) \right] \right) , \quad (20)$$

which means an energy independent probability (which goes against the experimental observations in solar neutrino oscillations) unless

$$|\lambda_i - \lambda_j| \leq 10^{-18}(\text{eV}) , \quad (21)$$

where we have used that $L_\odot = 10^8$ Km, and $\lambda_i^2 - \lambda_j^2 \ll m_i^2 - m_j^2$. There will be no footprint of Lorentz invariance violations in the case of atmospheric neutrinos, since $L_{\text{atm}} \sim 10^{-4}L_\odot$, nor in accelerator experiments, with much smaller lengths. The conclusion is that the linear term in $|\mathbf{p}|$ in the generalized dispersion relation (1) has to be flavour-independent.

4.2 Contribution to the energy density of the Universe

The bounds on neutrino masses from their contribution to the energy density of the Universe [9] are not affected by the presence of Lorentz invariance violations, since they only depend on the minimum energy of the neutrino, which is still $E_{\text{min}} = m$ (for $\mathbf{p} = 0$), independently of λ .

4.3 Neutrinos in astrophysics

The spread of arrival times of the neutrinos from SN1987A, coupled with the measured neutrino energies, provides a simple time-of-flight limit on m_{ν_e} .

From Eq. (1) one has for the neutrino velocity

$$\beta = \frac{\partial E}{\partial |\mathbf{p}|} = \frac{1}{\sqrt{1 - \frac{1}{|\mathbf{p}|^2} \frac{\lambda^2 - m^2}{(1 + \lambda/|\mathbf{p}|)^2}}} . \quad (22)$$

Since $|\mathbf{p}| \gg \lambda$, the limit [9] (23 eV) can be taken as an upper limit for $\sqrt{\lambda^2 - m^2}$ which is not far away from the range of parameters suggested by the tritium beta-decay anomaly. Then, neutrinos from supernovas can be a good place to look for footprints of Lorentz invariance violations. In fact, we see another implication from Eq. (22) if $\lambda > m$: neutrinos travel faster than light, so that those carrying more energy will be the latest to arrive, in contrast with the case of a relativistic dispersion relation.

4.4 Consequences in particle and cosmic ray physics

A striking consequence of the presence of the λ -term in the generalized neutrino dispersion relation Eq. (1) is the kinematic prohibition of reactions involving neutrinos at a certain energy. Let us consider ordinary neutron decay: $n \rightarrow p + e^- + \bar{\nu}$. At large momentum, the λ -term, which is proportional to $|\mathbf{p}|$, represents a large contribution to the energy balance, so that the process might be forbidden for neutrons of sufficiently high momentum. This turns out to be the case: it is easy to see that for $\lambda/|\mathbf{p}| \ll 1$ and $|\mathbf{p}| \gg m_n, m_p$, the total energy of the final state is bounded from below,

$$E_p + E_e + E_{\bar{\nu}} > |\mathbf{p}| + \lambda + \frac{(m_p + m_e)^2}{2|\mathbf{p}|} , \quad (23)$$

which implies that *neutrons with momentum*

$$|\mathbf{p}| > \frac{m_n^2 - (m_p + m_e)^2}{2\lambda} = \left[\frac{1.47 \cdot 10^{15}}{2\lambda(\text{eV})} \right] (\text{eV}) \quad (24)$$

are stable particles. A similar conclusion is obtained in the case of pions: the desintegration $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ is forbidden if the pion has a momentum

$$|\mathbf{p}| > \frac{m_\pi^2 - m_\mu^2}{2\lambda} = \left[\frac{8.3 \cdot 10^{15}}{2\lambda(\text{eV})} \right] (\text{eV}) . \quad (25)$$

This means that pions and neutrons with these energies can form part of cosmic rays, since they are stable particles if the neutrino energy-momentum relation is given by Eq. (1). Therefore, cosmic ray physics might be drastically affected at high energies, of order $E \gtrsim 10^{15}$ – 10^{16} eV. In particular, the presence of stable neutrons and pions in cosmic rays at such energies might contribute to avoiding the well-known GZK cutoff [10].

5 Conclusions

It is surprising that a very simple extension of the Lorentz invariant dispersion relation for the neutrino, consistent with all the constraints from neutrino physics, can affect phenomena of such different energy ranges. It could be behind the tritium beta-decay anomaly, and also lead to a drastic change in the composition of cosmic rays beyond 10^{16} eV. It seems worthwhile to explore in more detail all the consequences of this violation of Lorentz invariance and possible models of the Lorentz non-invariant physics at high energies incorporating the extended neutrino dispersion relation considered in this letter as a low-energy remnant.

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