Nonholonomic Motion Planning for Mobile Manipulators

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***Abstract*—A nonholonomic motion planner for mobile manipulators moving in cluttered environments is presented. The approach is based on a discontinous feedback law under the infuence of a special potential field. Convergence is shown via Lyapunov's direct method. Utilizing redundancy, the methodology allows the system to perform secondary, configuration dependent, objectives such as singularity avoidance. It introduces an efficient feedback scheme for real time navigation of nonholonomic systems.**

I. Introduction

One of the most severe restrictions to a wider application of robotic manipulators are their limited workspace. To answer the need for increased workspace, mobile manipulators were constructed. The first mobile manipulators operated in space but nowadays their application is spreading to terrestrial and aquatic environments.

Mobile manipulators are systems composed of a robotic arm mounted on a mobile base. This combination gives rise in a new class of robots with remarkable properties. Such systems inherit the dexterity of a robot manipulator and the increased workspace of a mobile robot. Moreover, the merging also results in new properties not encountered in any of the component subsystems, which should be investigated and exploited.

Among the unique properties possessed by such a system, redundancy is one of the most important. Redundancy is created by the increased number of degrees of freedom. This property enables one to use the redundant degrees of freedom to accomplish secondary tasks. Towards redundancy resolution, several methodologies have been presented.

Mobile manipulators have been studied during the last decade and significant work appears in literature [1],[2]. Several papers are related to motion planning for such systems: Desai and Kumar [3] have formulated the problem as an optimal control problem. Pin et. al [4] performed local optimization at velocity level to achieve redundancy resolution using their FSP method. Perrier et. al [5] minimized the total position error using a linearized model. Huang et. al [6] decoupled the motion of the vehicle from that of the manipulator and optimized each one using different criteria.

In this paper the system is kinematically steered between two arbitrary configurations amongst obstacles. Obstacle avoidance, therefore, naturally arises as a secondary objective which is to be achieved by utilization of the system's redundancy. Another important secondary objective could be the avoidance of the manipulator singular configurations.

Due to the kinematic constraints imposed on its base, the mobile manipulator is a nonholonomic system. This complicates considerably the motion planning problem, since no continuous static feedback motion planning scheme can be applied. The motion planning strategy has to be either open-loop, time varying or discontinuous.

Motion planning for nonholonomic systems has traditionally been divided into two stages: path planing and trajectory planning. Path planning consists of defining an admissible continuous sequence of configurations linking the initial position with the goal. Motion planning deals with the time parameterization of the obtained path. In this framework, a motion planning strategy is inevitably open-loop. Alternative methodologies include non smooth feedback and time varying schemes though they usually focus on tracking an existing holonomic path.

Particularly for mobile manipulators, the issue has generally been treated within the framework of optimal control. Optimal control is a powerful tool which is able to incorporate state constraints. It is notoriously known though for its computational requirements that render it impractical in real time applications.

Our approach is based on the use of fast feedback.

The system is kinematically steered in real time using position information to navigate itself at each time step. This is made possible with a discontinuous feedback law and a special kind of potential field functions. The primary merit of such an approach is that it merges path planning and motion planning in a robust and fast feedback scheme. It also allows additional configuration dependent secondary objectives to be simultaneously achieved.

The rest of the paper is organized as follows: Section II presents the basic mathematical tools on which our approach is founded and gives a formal problem statement. Section III briefly discribes the basic mathematical tools used in the paper and presents some stability and convergence properties of the proposed scheme. In section IV several implementation issues are discussed. Section V illustrates the efficiency of the approach through a number of non trivial numerical simulations. The authors conclusions are summarized in section VI.

II. Problem Statement

Consider the autonomous nonlinear system

$$\dot{x} = f(x)$$

where the right hand side can be discontinuous. Specifically, $f : \Re^n \to \Re^n$ is measurable and essentially bounded.

Definition II.1: (Filipov)[7]: A vector function x is called a solution of $\dot{x} = f(x)$ if x is absolutely continuous and

$$\dot{x} \in K[f](x)$$

where

$$K[f](x) \triangleq \bigcap_{\delta>0} \bigcap_{\mu N=0} \overline{co} f(B(x,\delta) - N)$$

and $\bigcap_{\mu N=0}$ denotes the intersection over all sets of Lebesgue measure zero.
The Filipov set $K[f](x)$ is formed by the images of the vector field of the system on a neighborhood of point x. It is important that we can neglect a set of measure zero where the vector field is not defined.

It is also necessary to define a certain set valued map:

Theorem II.1: [7]: Let $x(\cdot)$ be a Filipov solution to $\dot{x} = f(x)$ and $V : \Re^n \to \Re$ be a Lipschitz and regular function. Then $V(x)$ is absolutely continuous, $\frac{d}{dt}V(x)$ exists almost everywhere and

$$\frac{d}{dt}V(x) \in^{a.e.} \dot{\tilde{V}} \quad \text{where} \quad \dot{\tilde{V}} \triangleq \bigcap_{\xi \in \partial V(x)} \xi^T \begin{pmatrix} K[f](x) \\ 1 \end{pmatrix}.$$

Lyapunov's direct method and LaSalle's invariant principle have been extended to the case where either $f(x)$ is discontinuous, $V(x)$ nonsmooth or both. On these two extended theorems our approach is founded:

Theorem II.2: [7] Let $\dot{x} = f(x,t)$ be essentially locally bounded and $0 \in K[f](0,t)$ in a region $Q \supset \{x \in \Re^n | \|x\| < r\} \times \{t | t_0 \le t < \infty\}$. Also, let $V : \Re^n \times \Re \to \Re$ be a regular function satisfying $V(0,t) = 0$ and $0 < V_1(\|x\|) \le V(x,t) \le V_2(\|x\|)$ for $x \neq 0$ in Q for some V_1, $V_2 \in$ class $\mathcal{K}$. Then,

1. $\dot{\tilde{V}}(x,t) \le 0$ in Q implies $x(t) \equiv 0$ is a uniformly stable solution.
2. If in addition, there exists a class $\mathcal{K}$ function $\omega(\cdot)$ in Q with the property $\dot{\tilde{V}} \le \omega(x) < 0$, then $x(t) \equiv 0$ is uniformly asymptotically stable.

Theorem II.3: [7] Let Ω be a compact set such that every Filipov solution to the autonomous system $\dot{x} = f(x)$, $x(0) = x(t_0)$ starting in Ω is unique and remains in Ω for all $t \ge t_0$. Let $V : \Omega \to \Re$ be a time independent regular function such that $v \le 0$ for all $v \in \dot{\tilde{V}}$. If $\dot{\tilde{V}}$ is the empty set then this is trivially satisfied). Define $S = \{x \in \Omega | 0 \in \dot{\tilde{V}}\}$. Then every trajectory in Ω converges to the largest invariant set, M, in the closure of S.

The problem at hand is to plan the motion of a mobile manipulator between any two arbitrary configurations, coordinating the mobile robot and the attached manipulator to utilize redundancy and achieve secondary objectives. Such objectives could be obstacle and singularity avoidance. Both systems are modeled kinematically. Thus, although the proposed methodology could easily be classified as a control scheme, we would rather refer to it as a motion planning strategy. In such a framework, the mobile base is modeled as a unicycle while the manipulator will be trivially described as a parallel series of integrators. A joint configurations space description of the manipulator can easily accomodate obstacles.

An additional desired feature for the planning scheme is feedback. This would enable the rejection of disturbances and/or noise and the reduction of computational burden which is typical of optimal control methodologies.

The problem can be formally stated as follows:

Define $\mathbf{z} \triangleq \begin{bmatrix} x & y & \mathbf{q} \end{bmatrix}^T$ where $\mathbf{q} \triangleq \begin{bmatrix} q_1 & \dots & q_m \end{bmatrix}^T$, and consider the system:

$$\dot{x} = v \cos\theta \tag{1a}$$
$$\dot{y} = v \sin\theta \tag{1b}$$
$$\dot{\theta} = \omega \tag{1c}$$
$$\dot{\mathbf{q}} = \mathbf{u} \tag{1d}$$

Given an obstacle free configuration space, C, a singularity submanifold S, and two arbitrary points $\mathbf{z}_0$, $\mathbf{z}_f$ lying in the same connected component of C, define a feedback scheme $U : \mathbf{q} \mapsto \begin{bmatrix} v & \omega & \mathbf{u} \end{bmatrix}$ such that the resulting trajectory $\mathbf{r} : [0,T] \to C$, satisfies $\mathbf{r}(0) = \mathbf{z}_0$, $\mathbf{r}(T) = \mathbf{z}_f$ and, if possibly, $r(t) \notin S$.

Although the phrase 'if possibly', is not very suited for a formal statement we wish to allow the trajectory to have singular points in the case that no other singularity free admissible trajectory exists.

III. Approach to Solution

Designing a joint path for such a holonomic system as this series of integrators that describe the manipulator, may not be so difficult. The nonholonomic nature of the mobile base, however, poses severe restrictions on the kind of potential motion planning schemes: no smooth, time invariant feedback law would do [8].

Therefore, the adoption of feedback leaves only two options: either nonsmooth [9] or time varying strategies [10]. Both of these have been explored in the literature and significant results have been presented. The authors feel that a nonsmooth, time invariant scheme would suit better, since there are several structural requirements and performance limitations in the latter case.

A. Navigation Functions

For obstacle and singularity avoidance potential fields provide a conceptually appealing option. Navigation functions [11] are a special kind of potential field generating functions that do not have local minima. In this framework a relatively wide class of obstacles can be taken into account. What is more, potential fields can be easily incorporated in a feedback loop and can accomodate configuration constraints such as obstacles and singularities. Singularities can be avoided by maximizing manipulability. Since manipulability is configuration dependent an obstacle function can be built to drive the system away from singular points. The robot can then navigate following the negated gradient of the potential function. The flow of this vector field is guaranteed to reach the destination, whenever the problem admits a solution. In other words, if there is a solution it will always be found. Stability can be established using Lyapunov's direct method.

Navigation functions are positive definite smooth functions, attaining a maximum at the boundary of the free configuration space, and vanishing only at the origin. Their gradient does not vanish except for the origin and perhaps a countable set of isolated points -therefore no local minima are created. Navigation functions serve as natural Lyapunov function candidates.

The mathematical requirements for the existence of navigation functions are not severe: every smooth connected and compact manifold with boundary admits a navigation function [11]. Detailed guidelines for their construction can be found in [11]. Once constructed it can easily be shown that they satisfy the requirements of theorem II.2.

Potential-based motion planning methods assume that the system can be described as a single point in the configuration space. Real robots however are generally multilink mechanisms consisted of rigid bodies. In order to shrink the robot to a point we have to account for its volume and increase the obstacles' volume accordingly [12]. Such a procedure may not be trivial since the shape of the robot is generally nonsmooth and irregular. The issue of determining a constructive procedure for generating grown obstacles for arbitrarily shaped robots such as a mobile manipulator remains under investigation.

B. Motion Planning Scheme

Given a navigation function describing the robot workspace, we propose the following discontinuous feedback law:

$$\begin{aligned} v = -\operatorname{sgn}\left(\frac{\partial V}{\partial x}\cos\theta + \frac{\partial V}{\partial y}\sin\theta\right) \cdot \\ \left\{ k_v \left[\left(\frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2 \right] + k_z\left(x^2+y^2\right) \right\} \end{aligned} \tag{2a}$$

$$\omega = k_\theta(\theta_d - \theta) \cdot \operatorname{sgn}\left(-\frac{\partial V}{\partial \theta}(\theta_d - \theta)\right) \tag{2b}$$

$$u_1 = -\frac{\partial V}{\partial q_1} \tag{2c}$$

$$\vdots$$

$$u_m = -\frac{\partial V}{\partial q_m} \tag{2d}$$

where V is the navigation function, k_v, k_z and k_θ are positive constants and θ_d is defined as:

$$\theta_d \triangleq \arctan2(\mathrm{sgn}(x)\frac{\partial V}{\partial y}, \mathrm{sgn}(x)\frac{\partial V}{\partial x})$$

while the sign function is defined as

$$\mathrm{sgn}(z) \triangleq \begin{cases} 1 & ,z \geq 0 \\ -1 & ,z < 0 \end{cases}$$

Stability for the system can be obtained via Lyapunov's direct method, utilizing features from nonsmooth analysis.

Proposition III.1: Under the control law (2), the system (1) converges asymptotically to the origin.

Proof: Consider a smooth navigation function as a Lyapunov candidate. It is positive definite by construction.

Then

$$\dot{\tilde{V}} = \bigcap_{\xi \in \partial V} \xi^T K \begin{bmatrix} v\cos\theta \\ v\sin\theta \\ \omega \\ \mathbf{u} \end{bmatrix}.$$

and since V is smooth,

$$\begin{aligned} \dot{\tilde{V}} &= \nabla V^T K \begin{bmatrix} v\cos\theta \\ v\sin\theta \\ \omega \\ \mathbf{u} \end{bmatrix} \subset \nabla V^T \begin{bmatrix} K[v]\cos\theta \\ K[v]\sin\theta \\ K[\omega] \\ \mathbf{u} \end{bmatrix} \\ &= -K\left[\mathrm{sgn}\left(\tfrac{\partial V}{\partial x}\cos\theta + \tfrac{\partial V}{\partial y}\sin\theta\right)\right]\left(\tfrac{\partial V}{\partial x}\cos\theta + \tfrac{\partial V}{\partial y}\sin\theta\right) \\ &\quad \cdot \left\{ k_v \left[\left(\frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2 \right] + k_z\left(x^2+y^2\right) \right\} \\ &\quad + \tfrac{\partial V}{\partial \theta}(\theta_d - \theta) K\left[\mathrm{sgn}\left(-\tfrac{\partial V}{\partial \theta}(\theta_d-\theta)\right)\right] \\ &\quad - (\frac{\partial V}{\partial q_1})^2 - \cdots - (\frac{\partial V}{\partial q_m})^2 \\ &= -\left|\frac{\partial V}{\partial x}\cos\theta + \frac{\partial V}{\partial y}\sin\theta\right| \cdot \\ &\quad \left\{ k_v \left[\left(\frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2 \right] + k_z\left(x^2+y^2\right) \right\} \\ &\quad - \left|\frac{\partial V}{\partial \theta}(\theta_d - \theta)\right| - (\frac{\partial V}{\partial q_1})^2 - \cdots - (\frac{\partial V}{\partial q_m})^2 \leq 0 \end{aligned}$$

It is $\dot{\tilde{V}} = 0$ only when:

$$\begin{cases} \frac{\partial V}{\partial x}\cos\theta + \frac{\partial V}{\partial y}\sin\theta = 0, \\ \quad \text{or} \\ \frac{\partial V}{\partial x} = \frac{\partial V}{\partial y} = x = y = 0 \end{cases} \text{and} \begin{cases} \frac{\partial V}{\partial \theta} = 0, \\ \quad \text{or} \\ \theta = \theta_d \end{cases} \text{and } \frac{\partial V}{\partial \mathbf{q}} = 0$$

An invariant set inside the region where $\dot{\tilde{V}} = 0$ also requires

$$\theta = \theta_d \quad \text{and} \quad \frac{\partial V}{\partial x} = \frac{\partial V}{\partial y} = x = y = 0$$

However, in that case we have $\theta_d = 0$ so $\theta = 0$ and the invariant set reduces to the origin. ■

A problem here is that the function θ_d is not continuous at 0. This means that in a neigborhood of the origin θ_d could be different from zero, implying that the system would approach the origin with arbitrary orientation and will only correct it after reaching $(x, y) = (0, 0)$. This will be remedied by the use of a special kind of potential field functions, introduced in the following section.

IV. Implementation Issues

A. Dipolar Navigation Functions

Although strategy (2) theoretically guarantees convergence to the origin, there are some practical issues. The main idea behind (2) is to align the mobile base with the direction of the navigation function gradient. In practice, the base will follow a nonholonomic trajectory, reach the origin and reorient itself there. This is facilitated by the ability of the unicycle to rotate freely at any fixed position. In order to produce more realistic trajectories we propose the use of a (possibly nonsmooth) potential field the flows of which are tangent to the x axis at the origin. Although this will not avoid the need to rotate in place at locations where $\frac{\partial V}{\partial x} = \frac{\partial V}{\partial y} = 0$, it will cause the mobile base to approach the origin with $\theta \to 0$. We will call this kind of potential field functions *dipolar navigation functions.* The introduction of dipolar navigation functions was motivated by the form of the field produced by a magnetic dipole. A dipolar navigation function, similar to the one used in the simulation examples is shown in Figure 1. The figure is a countour plot of the navigation function on which potential field vectors are marked by arrows.

B. Chattering

Typical to discontinuous control law is the appearance of *chattering.* Chattering is characterized by a high frequency switching in the control signals. In this case, chattering will occur at positions where:

$$\frac{\partial V}{\partial x}\cos\theta + \frac{\partial V}{\partial y}\sin\theta = 0 \qquad \text{or} \qquad \frac{\partial V}{\partial \theta} = 0$$

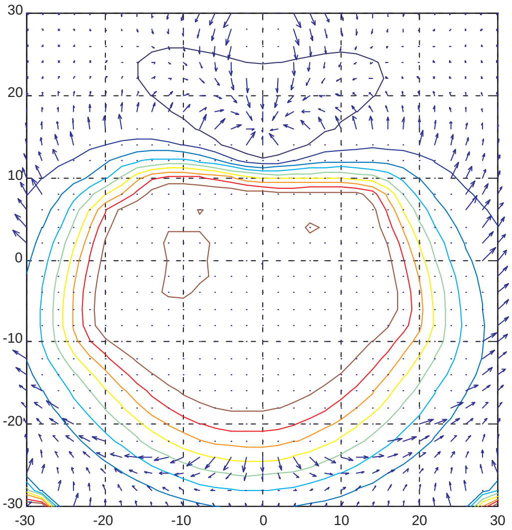

Fig. 1. A dipolar potential field

A common remedy for chattering has been the introduction of a boundary layer along the discontinuity surface to decrease the chattering frequency or perform a smooth transition to the alternating control signals. In this case, however a smooth transition will not do since this would affect the system's convergence properties by creating invariant sets. Chattering can be reduced by applying *hysteretic switching* [13]. We should point out however that the proposed feedback law steers the system away from the first surface of chattering behavior. In fact, it can easily be shown that aligning θ with θ_d makes $\frac{\partial V}{\partial x}\cos\theta + \frac{\partial V}{\partial y}\sin\theta \neq 0$, for every $(x, y, \theta, \mathbf{q}) \neq \mathbf{0}$.

V. Simulations

In the simulation example a two link planar mobile manipulator is considered. The mobile base consists of a planar rectangular region. At its center lies the first rotational manimulator joint. The two manipulator links are of the same size and shape.

The system has five degrees of freedom: three to specify the mobile base position on the plane, (x, y, θ) and two for the manipulator configuration, (q_1, q_2). At the center of the workspace a rectangular obstacle is placed. The destination point is at $(x, y, \theta, q_1, q_2) = (0, 10, 0, 0, \pi/2)$, just above the rectangular obstacle. This configuration gives rise to a relatively difficult motion planning problem, since the destination point is close to the obstacle region and the situation requires careful manoeuvres. At the initial points the angle variables are arbitrarily set to $(\theta, q_1, q_2) = (0, \pi/3, -\pi/4)$. Simulations were conducted with MATLAB5. The figures demonstrate the obstacle avoidance and convergence properties of the proposed scheme.

It is also interesting to note that the second link is stretched during each manoeuvre to stay as far from the singularity at $q_2 = \pm\pi$ as possible. Only at the end of the manoeuvre does it reach its prescribed final value. This is indicative of the singularity avoidance properties of the proposed scheme. The singularity at $q_2 = 0$ where the rank of the manipulator jacobian is decreased is not taken into account. It is not difficult, however, to do so without removing all set of points with $q_2 = 0$ from the free configuratin space. One can model this singularity as a repulsive 'near-obstacle' region, which is nonetheless reachable in case no other path exist.

Figures 2-4 depict the system motion with different initial conditions.

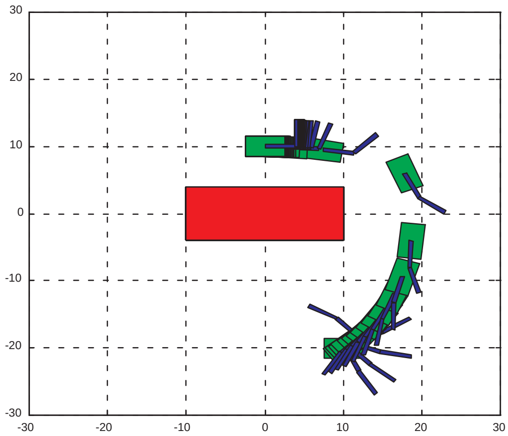

Fig. 2. Initial condition:(x,y)=(10,-20)

VI. Conclusions

In this paper the motion planning problem for nonholonomic mobile manipulator systems is investigated. The proposed motion planning scheme is based on a full state discontinuous feedback law which guarantees convergence the to desired final position and simultaneous obstacle and singularity avoidance. The feedback nature of the scheme makes it particularly useful for real time applications, providing robustness

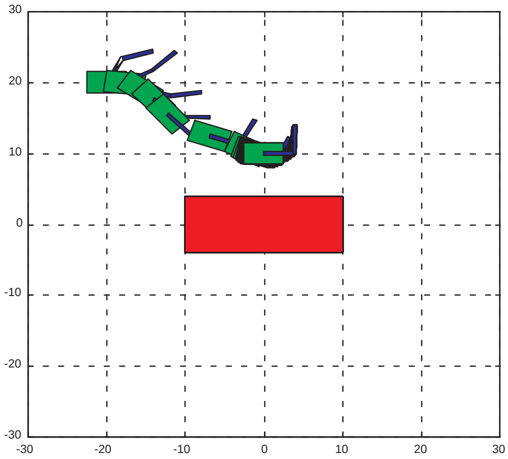

Fig. 3. Initial condition:(x,y)=(20,20)

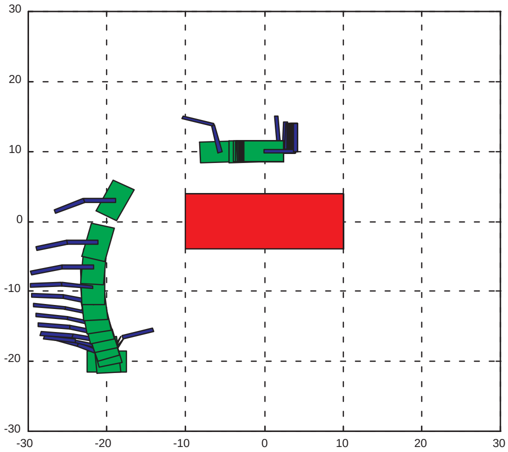

Fig. 4. Initial condition:(x,y)=(-20,-20)

against measurement and modeling errors. Another remarkable novelty is the merging of the path planning and trajectory generation stages of the motion planning problem. Trajectories are generated in real time and solutions, if any, are guaranteed through the use of navigation functions.

One of the intrinsic limitations of the potential-field approach is the requirement for a topological model of the robot's geometry and its environment. At this issue, however, potential-based methods are superior to classical findpath algorithms since the symbolic mathematical representation of the workspace theoretically allows a computationally cheap adaptation of the navigation function parameters as new (or revised) information comes along [11].

Research is continued on the construction of dipolar navigation functions and grown obstacle representation. In the future it is expected that more configuration dependent constraints, such as conditions for avoiding tip-over, would be accomodated.

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