A Rateless Coding based Multi-Relay Cooperative Transmission Scheme for Cognitive Radio Networks

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Abstract—Existing spectrum management policies have led to significant over-allocation and under-utilization of the licensed spectrum. To overcome this, cognitive radio is proposed for secondary users to share the licensed spectrum without causing harmful interference to primary users. As such, the transmit power of a secondary user is limited even when it accesses the spectrum hole. Therefore, multi-hop transmission is a potential method to deliver the data of secondary users over large distance. In such relay systems, the utilization of rateless codes is suitable for the opportunistic spectrum access of cognitive radio. There has been some work in this area. However, the multi-relay cognitive communication with rateless codes has not been carefully investigated. In this paper, we propose a rateless coding based cooperative transmission scheme for cognitive radio networks, where the average end-to-end throughput is analyzed and optimized. We also propose a block search algorithm to find the optimal number of decoding relays with low complexity. Simulation results show that the optimized relay cognitive cooperative transmission can achieve the maximal throughput.

I. INTRODUCTION

In the traditional fixed allocation scheme, spectrum are regulated and licensed spectrum are not shared. With emerging of new wireless standards and applications, the demand for radio spectrum keeps growing and the amount of unlicensed spectrum becomes scarce. However, actual measurements have shown that most of the licensed spectrum is largely underutilized [1]. Cognitive radio is a promising technique to improve the spectrum utilization which allows a secondary (unlicensed) user (SU) to access the spectrum currently unused by a primary (licensed) user (PU) [2].

In cognitive radio networks, the unused spectrum is sensed and accessed by SUs in an opportunistic way. To avoid harmful interference to PUs, SUs usually operate with low transmit power. As a result, the communication range of SUs is limited. While relay transmission [3], in which one or more relays assist a source in delivering data to a destination, improves the reliability of data transmission and extends communication range. Therefore, inspired by the opportunistic spectrum access of cognitive radio, as well as the performance advantage of wireless relay networks, we consider cognitive relay networks in this paper.

Rateless codes, also known as fountain codes, have recently been shown to be well suited for cognitive radio networks [4]-[6] and wireless relay networks [7]-[11]. Fountain codes are erasure coding schemes which are rateless, in the sense that the source is able to generate infinite number of encoded packets. The rateless property is very attractive in cognitive radio networks because the receiver will be able to correctly decode the message as long as it accumulates enough coded packets in an arbitrary manner. In other words, only how many coded packets have accumulated is the receiver care about, instead of which ones are received. Thus, rateless codes determine the rate “on the fly” by the number of coded packets required for the receiver to achieve correct decoding.

In [4] and [6], rateless codes were used in a single hop transmission for cognitive radio to distribute information over unused spectrum and also to compensate for the loss incurred due to primary user interference. Rateless codes were assumed to be used by PUs, in [5], to enable SU to decode PUs’ message and assist in PUs transmissions; SU used rateless codes to transmit its own messages as well. In [7]-[11], rateless codes were employed in wireless relay networks. Decode-and-forward based single relay networks were considered in [7] and [8]. A distributed space-time coding scheme was used for second hop transmission in [7] and a transmission protocol based on acknowledgement was introduced in [8]. Several multi-relay rateless coded schemes were proposed in [9]-[11]. Both quasi-synchronous and asynchronous schemes were considered in [9]. In [10], the relay with the highest source-relay channel gain was chosen to transmit on the second hop and the source transmitted a new message when there was any one of the relays decoded the message. A rateless coded selection cooperation scheme, which considered the qualities of the source-destination and relay-destination links, was proposed in [11]. The source or any decoding relay having a link to the destination with a gain greater than a preset threshold transmits on the second hop.

To the best of our knowledge, there is little work on cognitive cooperative networks with multiple relays employing rateless codes. In this paper, we propose a rateless coding based cooperative transmission scheme for cognitive radio network, where all the secondary users can transmit only when the PUs are not using the channels. When the channel
is idle, the source transmits until any \( L \) out of \( M \) relays decodes the message. Then the source transmits the next message. The \( L \) decoding relays transmit the same coded packets simultaneously in the idle slot on another channel. The energy is accumulated at the destination to speed up the decode process. We further analyze the average end-to-end throughput of the cognitive relay networks and present a low complexity algorithm to search the optimal \( L \).

The rest of this paper is organized as follows. Section II introduces the system model and the transmission scheme. The average throughput analysis is detailed in section III. Section IV presents and discusses the simulation results. And Section V concludes the whole paper.

II. SYSTEM DESCRIPTIONS

A. System Model

We consider a cognitive relay network, as shown in Figure 1, with one source node, \( S \), one destination node, \( D \), and \( M \) decode-and-forward relay nodes, \( R_1, \ldots, R_M \). Assume that there are two independent primary users, denoted by PU1 and PU2, in the network. Channel 1 and channel 2 are licensed to PU1 and PU2, respectively. All SUs opportunistically utilize the spectrum licensed to the PUs. The source transmits on channel 1 when PU1 is not using it. Since the broadcast nature of the wireless channel, all the relays can receive the signal. The relays decode the message and forward it to the destination on channel 2. As assumed to be FDD capable, the relays can transmit and receive simultaneously. We also assume that the destination cannot receive from the source directly.

Both of the channels are assumed to be time slotted. The occupancies of these channels by different PUs are modeled as independent discrete-time Markov processes with busy \((U_j(t) = 0)\) and idle \((U_j(t) = 1)\) state, respectively. As shown in Figure 2, channel \( j \) transits from state idle to state busy with probability \( q_{Ij} \), and transits from state busy to state idle with probability \( q_{Bj} \), where \( j = 1, 2 \). The stationary distribution of \( j \)-th channel is

\[
p_{Ij} = \Pr(U_j(t) = 1) = q_{Bj}/(q_{Ij} + q_{Bj}), \quad \text{(1)}
\]
\[
p_{Bj} = \Pr(U_j(t) = 0) = q_{Ij}/(q_{Ij} + q_{Bj}). \quad \text{(2)}
\]

The sojourn times of \( j \)-th channel in each state are denoted by \( t_{Ij} \) and \( t_{Bj} \) separately. They follow geometric distribution and their probability mass function is

\[
\Pr(t_{Ij} = n) = q_{Ij}(1 - q_{Ij})^{n-1}, \quad n \geq 1, \quad \text{(3)}
\]
\[
\Pr(t_{Bj} = n) = q_{Bj}(1 - q_{Bj})^{n-1}, \quad n \geq 1. \quad \text{(4)}
\]

The wireless channels between different nodes are assumed to be frequency-flat, block-fading Rayleigh channels. We assume the block length is equal to the duration of one time slot, in other words, the channel gain remains constant during a slot and changes from one slot to the other. The channel gains are independent and exponentially distributed. Let \( \gamma_i \) be the channel gain from the source to the \( i \)-th relay node and \( \lambda_i \) be the channel gain from the \( i \)-th relay node to the destination, with means \( \bar{\gamma}_i \) and \( \bar{\lambda}_i \), respectively.

The secondary users sense the spectrum in each time slot before transmission to avoid the collision with PUs. We assume that rateless coding is employed by SUs in order to combat the effect of loss due to primary users interference and other channel impairments. The source as well as the relays use rateless codes for encoding the message. Each message is composed of \( H \) data packets, which serve as the input packets of the rateless encoder. A coded packet is a binary addition of a random subset of input packets and is transmitted in a time slot. Assume there is a SNR threshold, \( \beta \), at the receiver. In each slot, a coded packet will be dropped if the received SNR is below \( \beta \), which is considered as an erasure, otherwise, it will be received successfully. The erasure probability of \( S-R_i \) link and \( R_i-D \) link are denoted as \( p_{ei}^S \) and \( p_{ei}^R \), respectively.

\[
p_{ei}^S = \Pr\left(\frac{P_S \gamma_i}{w_{Ri}} < \beta\right) = 1 - \exp\left(-\frac{w_{Ri} \beta}{P_S \bar{\gamma}_i}\right), \quad \text{(5)}
\]
\[
p_{ei}^R = \Pr\left(\frac{P_R \lambda_i}{w_D} < \beta\right) = 1 - \exp\left(-\frac{w_D \beta}{P_R \bar{\lambda}_i}\right), \quad \text{(6)}
\]

where \( P_S \) and \( P_R \) are the transmit power of the source and the relay separately, \( w_{Ri} \) and \( w_D \) are the noise power at the receivers.

When the receiver obtains any \( N = (1 + \epsilon)H \) coded packets at least, where \( \epsilon \) is the decoding overhead, it can completely recover the message and inform the transmitter via an ACK packet. In practice, the decoding overhead depends on concrete code design. However, it is bounded and not too large [12]. Moreover, we assume there exists a common channel, over which all the ACKs are delivered, among the secondary users.
B. Transmission Scheme

All the secondary users sense the channels at the beginning of each time slot. When the channel is sensed idle, SUs transmit a coded packet on this channel. Otherwise, they remain silent to vacate the spectrum for PUs.

The source has a continuous stream of messages to transmit. Once it starts transmission of a message, the limitless coded packets are broadcast on the channel 1, and the relays accumulate these packets for decoding the message. As soon as any relay has acquired enough coded packets, say \( N \), to reliably decode the message, it immediately informs the source its decoding success via an ACK packet and stores the message in its queue for transmission to the destination. When there are any \( L \) relays decode the message, where \( 1 \leq L \leq M \), the source starts transmitting the next message. Note that, the decoding relays may be different when transmitting different messages.

On the second hop, the \( L \) decoding relays use the same rateless code to encode the message and transmit simultaneously on channel 2 with distributed space-time codes. As assumed that the delay spread is much smaller than a time slot, the destination can accumulate the energy from \( L \) relays. Once the destination decodes the message, it sends an acknowledgement back to all the relays. And if the next message has been decoded by \( L \) relays, it is forwarded to the destination immediately.

Note that \( L \) is a tunable parameter. Raising \( L \) will increase the transmission time of the first hop and decrease that of the second hop, and vice-versus. Thus, the optimal \( L \), which balance the selection diversity of the first hop and the user cooperation diversity of the second hop to maximize the throughput of the cognitive relay networks, is our concern. The optimal \( L \) can be obtained by analyzing the average transmission times of the first hop and the second hop. However, it is not a trivial work to obtain the average transmission times.

III. AVERAGE THROUGHPUT ANALYSIS

We assume that the channel gains of \( S-R \) links are identical distributed for convenience and so are the \( R-D \) links. The analysis is similar for the non-identical links case. We also assume that all nodes have equal transmit power.

A. Transmission time of the first hop

Whether the source can transmit or not is determined by PU1 on channel 1. When the source can transmit, a coded packet will be lost due to the erasure effect of the wireless fading channel. Thus the time required for transmitting a message from the source to a relay is

\[
T_{i}^{SR} = T_{di}^{SR} + T_{Bi}^{SR}, \quad 1 \leq i \leq M
\]  

\( T_{di}^{SR} \) is the transmission time over a wireless fading channel without the effect of PU1 and \( T_{Bi}^{SR} \) is the busy time of PU1 encountered during \( T_{di}^{SR} \). As the source transmits a message, more and more relays decode the message. The time when there are any \( L \) relays decode successfully is defined as the transmission time of the first hop, denoted by \( T_{(L)} \). This is the order statistics of \( T_{i}^{SR} \). In order to obtain \( E[T_{i}^{SR}] \), we give the following lemma.

**Lemma 1:** The average of \( T_{di}^{SR} \) is

\[
E[T_{di}^{SR}] = \frac{N}{1 - p_{c}^{SR}}. \tag{8}
\]

**Proof:** Since the erasure is i.i.d. in each slot, the time for successful transmission of a coded packet is geometric distributed. Thus, \( T_{di}^{SR} \) follows the negative binomial distribution

\[
Pr(T_{di}^{SR} = x) = \frac{(x - 1)}{x - N} \left( 1 - p_{c}^{SR} \right)^{N} \left( p_{c}^{SR} \right)^{x - N}, \quad x \geq N. \tag{9}
\]

The erasure probabilities are the same as the channel gains for different relays are i. i. d.. Therefore, the transmission time, \( T_{di}^{SR} \), for different relays are independent and identical distributed as well. By substituting (9) into

\[
E[T_{di}^{SR}] = \sum_{x=N}^{\infty} x \cdot Pr(T_{di}^{SR} = x), \tag{10}
\]

we get Eqn. (8).

Next, the following lemma presents \( E[T_{Bi}^{SR}] \).

**Lemma 2:** The average of \( T_{Bi}^{SR} \) is

\[
E[T_{Bi}^{SR}] = q_{Bi} \frac{N}{1 - p_{c}^{SR}} - 1 + \frac{p_{Bi}}{q_{Bi}}. \tag{11}
\]

**Proof:** During the transmission of a message, the PU1 appears randomly and occupies the channel for a random time, say \( t_{B1} \). Let \( K_{i} \) denotes the number of PU1 appearances during \( T_{di}^{SR} \). Thus, \( T_{Bi}^{SR} \) can be written as \( T_{Bi}^{SR} = \sum_{k=0}^{K_{i}} t_{B1}(k) \). Note that \( K_{i} \) is a random variable correlated with \( T_{di}^{SR} \). Then, the probability mass function of \( K_{i} \) conditioned on \( T_{di}^{SR} \) is

\[
q_{K_{i}}(k) = \Pr(K_{i} = k | T_{di}^{SR} = x) \]

\[
= \begin{cases} p_{11} q_{K_{i}}(k), & k = 0, \\ p_{11} q_{K_{i}}(k) + p_{B1} q_{K}(k-1), & k \geq 1, \end{cases} \tag{12}
\]

where

\[
q_{K_{i}}(k) = \Pr(T_{1}(k+1) \geq T_{di}^{SR} | T_{di}^{SR} = x) - \Pr(T_{1}(k+1) \geq T_{Bi}^{SR} | T_{di}^{SR} = x) + \frac{1}{k} q_{K_{i}}(1 - q_{11})^{x-1-k}, \quad k \geq 0. \tag{13}
\]

In Eqn. (13), \( T_{1}(k) = \sum_{i=1}^{k} t_{1}(i) \), and the probability mass function is

\[
\Pr(T_{1}(k) = t) = \binom{t - 1}{t - k} q_{11}^{k}(1 - q_{11})^{t - k}, \quad t \geq k \geq 1. \tag{14}
\]

The expectation of \( K_{i} \) conditioned on \( T_{di}^{SR} \) is

\[
E[K_{i} | T_{di}^{SR} = x] = \sum_{k=0}^{\infty} k q_{K_{i}}(k) = p_{11} q_{11}(x - 1) + p_{B1} q_{11}(x - 1) + q_{11} x - 1 + p_{B1}. \tag{15}
\]
Let $p_{K_i}(k)$ denote the probability mass function of $K_i$,  

$$p_{K_i}(k) = \Pr(K_i = k) = \sum_{x=N}^{\infty} q_{K_i}(k) \Pr(T_{d_i}^{SR} = x), \quad (16)$$

and the average is  

$$E[K_i] = E[E[K_i|T_{d_i}^{SR}]] = q_{11}(\frac{N}{1 - p_{e}} - 1) + p_{B1}. \quad (17)$$

This is simply because $T_{d_i}^{SR}$ is i.i.d., $K_i$ is i.i.d.

As $T_{d_i}^{SR}$ is the sum of $K_i$ random variables, the probability mass function is given by  

$$\Pr(T_{d_i}^{SR} = t) = p_{K_i}(0) \delta(t) + \sum_{k=1}^{\infty} p_{K_i}(k) \Pr(T_{B1}(k) = t), \quad (18)$$

where $T_{B1}(k) = \sum_{i=1}^{k} t_{B1}(i)$, and the distribution is  

$$\Pr(T_{B1}(k) = t) = \left(\frac{t - 1}{t - k}\right)^{k} q_{B1}^{t - k}, \quad t \geq k \geq 1. \quad (19)$$

According to Wald’s equation, we have  

$$E[T_{d_i}^{SR}] = E[K_i] E[T_{B1}] \quad (20)$$

Thus, the average of $T_{d_i}^{SR}$ can be easily obtained according to Eqn. (20).

Since all the relays are affected by the same PU on receiving a message, the transmission time of different relays is dependent. Therefore, it is not easy to calculate the distribution of $T_{d_i}^{SR}$ from that of $T_{d_i}^{SR}$ directly. However, note that the channel occupancy is the same to all relays, we have the following lemma.

**Lemma 3**: The order of $T_{d_i}^{SR}$ is determined by $T_{d_i}^{SR}$, where $1 \leq i \leq M$.

**Proof**: Here, the lower-case letters denote the observations of random variables. Given the observations of $T_{d_i}^{SR}$, the ordered observations are $t_{d_i}^{SR}(1), \ldots, t_{d_i}^{SR}(M)$. While $k_i$ depend on $t_{d_i}^{SR}$ and channel states $u_{1}(t)$. Since $u_{1}(t)$ is the same to all different relays, $k_i$ is determined by $t_{d_i}^{SR}$ directly. This implies the order of $k_i$ is same to that of $t_{d_i}^{SR}$. Moreover, $t_{d_i}^{SR}$ increase with $k_i$. Therefore, both $T_{d_i}^{SR}$ and $T_{d_i}^{SR}$ have the same order. Since $t_{d_i}^{SR} = t_{d_i}^{SR} + t_{d_i}^{SR}$, the order of $t_{d_i}^{SR}$ is determined by $t_{d_i}^{SR}$.

As proven for any observation of the random variables, the order of $T_{d_i}^{SR}$ is also the order of $T_{d_i}^{SR}$.

According to Lemma 1-3, the average transmission time of the first hop is given by the following theorem.

**Theorem 1**: The average of order statistics, $T_{d_i}^{SR}$, is  

$$E[T_{d_i}^{SR}] = \frac{q_{B1} + q_{11} E[T_{d_i}^{SR}]}{q_{B1}} + \frac{p_{B1} - q_{11}}{q_{B1}}, \quad (21)$$

**Proof**: We calculate the order statistics, $T_{d_i}^{SR}$, at first. Since $T_{d_i}^{SR}$ follow the negative binomial distribution, the average of $T_{d_i}^{SR}$ can be obtained by the following recurrence equation in low complexity [13]

$$E[T_{d_i}^{SR} \leq M] = \sum_{t=0}^{M-1} \sum_{i=0}^{t} \left(\begin{array}{c} M \\ t \end{array}\right) (-1)^t E[T_{d_i}^{SR} \leq M - t + 1]. \quad (22)$$

Finally, according to Lemmas 1 and 2, we have  

$$E[T_{d_i}^{SR}] = \frac{q_{B1} + q_{11} E[T_{d_i}^{SR}]}{q_{B1}} + \frac{p_{B1} - q_{11}}{q_{B1}}. \quad (24)$$

It follows Eqn. (21).

**B. Transmission time of the second hop**

Similar to the analysis of the average transmission time of the first hop, we have the following theorem.

**Theorem 2**: The average of $T_{d_i}^{RD}$ is  

$$E[T_{d_i}^{RD}] = \frac{(q_{B2} + q_{12}) N}{q_{B2}(1 - \frac{p_{RD}}{q_{B2}})} + \frac{p_{RD} - q_{12}}{q_{B2}}. \quad (25)$$

**Proof**: Since $L$ decoding relays transmit the same coded packet simultaneously, the destination accumulates the energy of the relay transmissions. The erasure probability is  

$$p_{eRD} = \Pr\left(\sum_{i=1}^{L} \frac{P_{R} \lambda_i}{\omega} \leq \beta\right) = \Pr\left(\sum_{i=1}^{L} \lambda_i \leq \frac{\omega \beta}{P_{R}}\right) = 1 - \sum_{n=0}^{L-1} \exp\left(-\frac{\omega \beta}{P_{R}}\right) \left(\frac{\omega \beta}{P_{R}}\right)^n / n!. \quad (26)$$

Given $p_{eRD}$, $T_{d_i}^{RD}$ and $T_{d_i}^{RD}$ can be analyzed similar to Lemma 1 and Lemma 2, respectively. Thus, we obtain  

$$E[T_{d_i}^{RD}] = \frac{N}{1 - \frac{p_{RD}}{q_{B2}}} + q_{12} \frac{N}{q_{B2}(1 - \frac{p_{RD}}{q_{B2}})} - 1 + \frac{p_{RD} - q_{12}}{q_{B2}}. \quad (27)$$

It validates Eqn. (25).

**C. Average Throughput**

The average throughput of the cognitive relay network is given by  

$$A = \frac{N}{\max\{E[T_{d_i}^{SR}], E[T_{d_i}^{RD}]\}}. \quad (28)$$

The optimal number of decoding relays, $L^*$, can be found via the following optimization problem  

$$\min_{L} \max_{1 \leq L \leq M} \{E[T_{d_i}^{SR}], E[T_{d_i}^{RD}]\} \quad (29)$$

where $E[T_{d_i}^{SR}]$ is monotonous, and increases with $L$, while $E[T_{d_i}^{RD}]$ monotonously decreases with $L$.

The optimization problem (29) can be analyzed in three cases:
optimal variable. Let large. Thus we propose a low complexity algorithm to find the of the first hop. Since both \( E[T_{SR}^{RD}] \) decreases monotonously, the optimal decoding relay set includes all relays, that is \( L^{*} = M \).

3) \( \min_{L} E[T_{SR}^{RD}] \leq \max_{L} E[T_{SR}^{RD}] \) and \( \max_{L} E[T_{SR}^{RD}] > \min_{L} E[T_{SR}^{RD}] \): In this case, there is a point where \( E[T_{SR}^{RD}] \) and \( E[T_{SR}^{RD}] \) meet and the optimal \( L \) is between 1 and \( M \). The brute-force search for \( L^{*} \) is time exhaustive when \( M \) is large. Thus we propose a low complexity algorithm to find the optimal variable. Let \( f(L) \) denote \( \max \{ E[T_{SR}^{RD}], E[T_{SR}^{RD}] \} \).

Algorithm 1 Block Search Algorithm

1. The initial searching block \( W \) is set to \( M \).
2. The number of blocks, \( n \), is required to be \( 4 \leq n \leq M \).
3. \( \text{while } 2W/n > n - 1 \) do
4. Divide the searching block \( W \) into \( (n-1) \) equal blocks
5. Calculate \( f([W/n]), \ldots, f([(n-1)W/n]) \)
6. Find the minimum and denote as \( f[mW/n] \)
7. Reduce \( W \) to \( [(m-1)W/n, (m+1)W/n] \)
8. end while

The block search algorithm can find the optimal \( L \) with complexity \( O(n \log_{2} M) \) and the simulation results validate the feasibility of the algorithm.

IV. SIMULATION RESULTS

In this section, we present the simulation results of the cognitive relay network. For simplicity, we set \( N = 50, \beta = 0.3, P_{S} = P_{R} = 1, M = 5 \). In all of the figures, markers are used to plot the simulation results and lines plot the theoretical results.

Figure 3 shows the average transmission times of the first hop and the second hop, where the channel states of different channels have the same statistics \( q_{11} = q_{12} = 1/50, q_{21} = q_{22} = 1/50 \). Note that the average sojourn time of state idle and state busy are equal to \( N \). The transmission times of the first hop are order statistics. For example, when \( L = 1 \), the transmission time is the decode time of the relay who decodes a message first. Therefore, it is clear that the transmission times of the first hop increase with \( L \) monotonously. The decoding relays cooperate to transmit on the second hop to obtain the user cooperation diversity gain. The larger the number of cooperative relays, the smaller the erasure probability, furthermore, the smaller the transmission time of the second hop, as shown in Figure 3.

In Figure 4 we show the performance of the cognitive relay network under different channel occupancy statistics. In Case 1, the idle probability of channel 1 is smaller than that of channel 2 according to \( q_{11} = q_{22} = 1/40, q_{12} = q_{21} = 1/60 \). As a result, the first hop is the bottleneck. Thus the optimal \( L \) equals one. This implies the source needs to transmit a new message as soon as possible. While Case 2 is opposite to Case 1 \( q_{11} = q_{22} = 1/60, q_{12} = q_{21} = 1/40 \). The channel 1 more likely than channel 2 to being idle. Therefore, the transmission time of the second hop bounds the system throughput. In order to increase the throughput, we hope more relays cooperate on the second hop, which indicates \( L^{*} = M \). However, the backlog of decoding relays will approach infinity. In Case 3, both the PUs have the same statistics according to \( q_{11} = q_{22} = 1/50, q_{12} = q_{21} = 1/50 \). The optimal number of decoding relays is determined by the tradeoff between the selection diversity of the first hop and the user cooperation diversity of the second hop. As shown in Figure 4, when there are 2 relays cooperate on the second hop, the maximum throughput is achieved.

By comparing the average end-to-end throughput and optimal number of decoding relays in different cases, we can draw a conclusion from Figure 4 that the optimal \( L \) will not exceed three according to \( M = 5 \) because the performance
improvement is negligible when three more relays cooperate on the second hop.

V. CONCLUSION

A rateless coding based cooperative transmission scheme was introduced for cognitive relay networks. We presented the average end-to-end throughput by using order statistics. Block search algorithm with low complexity was proposed in order to find the optimal number of decoding relays. We found that the transmission time of the first hop increases with $L$ while the transmission time of the second hop decreases with $L$. The optimal number of decoding relays, which achieves the maximal average throughput, was found to be less than the total number of potential relays and validated by the simulation results.

REFERENCES