Erratum

Erratum to: “Ruled surfaces with timelike rulings”  

Emin Kasap

Faculty of Arts and Sciences, Department of Mathematics, Ondokuz Mayis University, 55139 Samsun, Turkey

Abstract


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2.4. Maximal timelike ruled surfaces

A classical result of Catalan states that the only ruled minimal surface in Euclidean 3-space $E^3$ are the plane and the helicoid. As a continuation of the results obtained by Woestijne [3] on maximal timelike ruled surface, we introduced a new technique different of that used in his study to give a classification of maximal timelike ruled surfaces.

The mean curvature $H$ of the surfaces in Lorentz space is given by $H = \frac{1}{2} h_{\alpha\beta}g^{\alpha\beta}$ where $g_{\alpha\beta}$ are the components of the contravariant metric tensor. It is easy to see that, the mean curvature of the ruled surfaces (I) is given by

$$H = \frac{1}{2} \left[ \frac{2\lambda_1 \langle \hat{L}, \hat{L} \rangle \ell_1 + \langle \hat{\alpha} \wedge \hat{L} + v\hat{L}', \wedge \hat{L}, \epsilon \hat{e}_2 + v\hat{e}_n \rangle}{(-1 - 2v \langle \hat{\alpha}, \hat{L} \rangle - v^2 \langle \hat{L}, \hat{L} \rangle - \ell_1^2 )^{3/2}} \right].$$  \hspace{1cm} (2.12)

In [3], we have proved that for a maximal timelike ruled surface, the generator must be coincident with the principal normal field of its base curve. Then, the ruled surface (I) may be a maximal if $\hat{L}(s) = \hat{e}_2$ i.e. $\ell_1 = \ell_3 = 0$ and $\ell_2 = 1$. In this case, one can show that, the ruled surface (I) is a timelike ruled surface. From (1.2), (2.1) and (2.12), one can see that the timelike ruled surface (I) is maximal if and only if

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E-mail address: kasape@omu.edu.tr

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\[ H = \frac{1}{2} v \frac{\tau' + v(\tau' \kappa - \kappa' \tau)}{(-1 - 2v\kappa - v^2(\kappa^2 + \tau^2))^{3/2}} = 0 \]

or equivalently \( \tau' = 0 \) and \( \kappa \tau' - \tau \kappa' = 0 \), \( \forall v \in \mathbb{R} \).

This is valid if \( \kappa \) and \( \tau \) are constants.

From (2.2), the striction curve of the maximal timelike ruled surface with timelike rulings is given by

\[ \beta(s) = \bar{z}(s) + \left( \frac{\kappa}{\kappa^2 + \tau^2} \right) \bar{e}_2(s). \]

From (1.2) it follows that \( \beta = \bar{z}(s) \) is a spacelike curve if \( |\kappa| > |\tau| \) or \( |\tau| > |\kappa| \). From these inequalities it is easy to see that then, the parametrization of maximal timelike ruled surfaces with timelike rulings is given as

**Class I:** \( |\kappa| = |\tau| \neq 0 \)

\[ \phi(s, v) = \left( \frac{\kappa}{\kappa^2 + \tau^2} + v \right) sh \sqrt{\kappa^2 + \tau^2}, \left( \frac{\kappa}{\kappa^2 + \tau^2} + v \right) ch \sqrt{\kappa^2 + \tau^2}, \frac{\tau}{\sqrt{\kappa^2 + \tau^2}} \right). \]

**Class II:** \( |\kappa| = |\tau| = 0 \) Since \( \bar{L}(s) = \bar{e}_2 \), and from (1.2) one can see that \( \bar{L}''(s) = 0 \) i.e., \( \bar{z}''(s) = 0 \). That is, \( \bar{z}(s) \) is a polynomial of degree 3. Then, we have

\[ \phi(s, v) = \left( \sum_{i=1}^{3} a_i s^i + v \sum_{i=2}^{3} i(i - 1) a_i s^{i-2} \right). \]

Thus, we have the proof of the following theorem:

**Theorem 7.** Every maximal timelike ruled surface with timelike rulings, whose ruling is not null, is a congruent to a part of one of the following surfaces:

(i) Time-like plane.

(ii) Helicoid of third kind \( (|\kappa| = |\tau| \neq 0) \).

(iii) Conjugate of Enneper’s surface of second kind \( (|\kappa| = |\tau| = 0) \).

The types of maximal timelike ruled surfaces (ii) and (iii) of this theorem are plotted as examples which are shown in Figs. 1 and 2.

**Fig. 1.** Helicoid of third kind.
Example. For the ruled surface

\[ \varphi(s, v) = \left( \sqrt{2} \cos s - 2\sqrt{2}v \sin s, s + 3v, \sqrt{2} \sin s + 2\sqrt{2}v \cos s \right) \]

it is easy to see that \( \xi(s) = (\sqrt{2} \cos s, s, \sqrt{2} \sin s) \) and \( \bar{L}(s) = (-2\sqrt{2} \sin s, 3, 2\sqrt{2} \cos s) \) are the base curve (spacelike) and the generator (timelike). The striction curve is the base curve. This surface is a timelike ruled surface for which \( \lambda = 1/2 \) (Fig. 3).

References
