Interference Alignment for DSL

Sean Huberman and Tho Le-Ngoc
Department of Electrical and Computer Engineering, McGill University, 3480 University Street, Montreal, Quebec, Canada, H3A 2A7
Email: sean.huberman@mail.mcgill.ca; tho.le-ngoc@mcgill.ca

Abstract—Inter-line interference, known as crosstalk, is the most significant source of performance degradation for Digital Subscriber Line (DSL) systems. Vectored DSL is an effective technique for the elimination of crosstalk; however, there are many scenarios in which Vectored DSL is not implementable or practical. Interference alignment is presented as a technique to solve these issues. In particular, the combination of partial vectoring and interference alignment is proposed. The practical advantages of partial vectoring and interference alignment over Vectored DSL are discussed. Finally, simulation results are provided to demonstrate the potential performance gains of the proposed approach.

Index Terms—digital subscriber line, interference alignment, pre-coding, interference cancellation, resource allocation.

I. INTRODUCTION

For Digital Subscriber Line (DSL) systems, inter-line interference, known as crosstalk, is the most significant source of performance degradation. The elimination or mitigation of crosstalk is essential to achieve higher data-rates. To this end, Vectored DSL has emerged as a method for removing crosstalk via pre-coding and/or interference cancellation on a per-subcarrier basis.

While Vectored DSL yields significant data-rate improvements, it also introduces various design and practical challenges [1]. Two significant challenges are related to the computational complexity and geographically separated lines. The Vectored DSL process scales exponentially with the number of users in the system, which imposes computational challenges as the number of users increases. Furthermore, Vectored DSL can only be applied to scenarios in which all the lines are co-located. Hence, for downstream transmission, since all the transmitters are co-located, Vectored DSL makes use of pre-coding. Similarly, for upstream transmission, since all the receivers are co-located, Vectored DSL makes use of interference cancellation. It is not practical to jointly vector lines which are geographically separated at both ends.

Furthermore, even when all lines are co-located, it may not be computationally practical to implement vectoring across all lines when the number of lines is large. Hence, it may be more favorable to independently apply vectoring across sub-groups of lines. This process eliminates intra-sub-group crosstalk; however, inter-sub-group crosstalk still remains. As well, scenarios where multiple DSL Access Multiplexers (DSLAMs) share a binder are also very common in DSL systems. Typically, DSLAMs are not co-located and hence, joint vectoring between different DSLAM lines is not practical.

We refer to the process of applying Vectored DSL to various sub-groups as partial vectoring. In this paper, Interference Alignment (IA) [2] is proposed as a method for effectively removing the effects of the inter-group crosstalk.

IA is a transmission technique which aims to align the interference seen at each receiver in a subspace of the signal space, while ensuring that the direct signal is not in the null space of the interference subspace. Then, each receiver can recover their respective direct signals by projecting their received signal into the null space of the interference.

IA combines various signaling dimensions (e.g., time, frequency, space) and considers them as the number of degrees of freedom. In this paper, the only degrees of freedom present are frequency tones (or sub-carriers). For DSL, unlike typical IA problems, it is common for the number of frequency tones to be much larger than the number of users in the system.

IA is achieved by making use of per-user pre-coders and per-user decoders to align the crosstalk signals seen by each receiver. Intuitively, independent frequency tones are used to jointly encode each user’s signals. Thus, the IA process inherently correlates the signals sent on each frequency tone for each user.

While the Vectored DSL and IA approaches both make use of pre-coding and cancellation, the fundamental difference is that Vectored DSL independently applies pre-coding on each frequency tone, whereas IA uses pre-coding to jointly code over the frequency tones in such a way that the effects can be reversed at the receiver. In particular, both approaches can yield interference-free signals; however, depending on the particular IA scheme, the performance can vary. While IA can ensure interference-free communication, it must sacrifice some degrees of freedom to do so. As such, in general, it is expected that Vectored DSL should outperform IA.

That being said, IA can be applied in a wide-variety of situations where Vectored DSL is infeasible. In particular, IA is feasible in the case of geographically separated DSLAMs, as well as, intra-line-card vectoring. Hence, in such scenarios the combination of Vectored DSL to suppress intra-group crosstalk and IA to suppress inter-group crosstalk is particularly interesting, and is the focus of this paper.

The rest of this paper is organized as follows. Section II introduces the system model. Section III introduces the combination of partial vectoring and IA for eliminating crosstalk. Section IV discusses the practical implementation of IA with respect to Vectored DSL. Section V presents some simulation results. Finally, Section VI provides some concluding remarks.
II. SYSTEM MODEL

Consider a DSL network with $M$ sets or groups of users (modems): $N_i$, $i = 1, \ldots, M$, where $|N_i| = N_i$, and the total users in the system is given by the set $N = N_1 \cup \cdots \cup N_M$ with $|N| = N_1 + \cdots + N_M \triangleq N$. The network also uses a set of tones (frequency carriers), $K = \{1, \ldots, K\}$. Let $x^n_k$, $y^n_k$, and $z^n_k$ be the transmitted signal, received signal, and the additive white Gaussian noise for user $n$ on frequency tone $k$. Similarly, let $s^n_k \triangleq \mathcal{E}\{|x^n_k|^2\}/\Delta f$ be the transmit Power Spectral Density (PSD) for user $n$ on frequency tone $k$, where $\mathcal{E}\{\cdot\}$ denotes expected value, and $\Delta f$ denotes the frequency tone spacing. Finally, let $\sigma^n_k \triangleq \mathcal{E}\{|z^n_k|^2\}/\Delta f$.

Without loss of generality, it is assumed that the users are ordered according to their group number ($i.e., N_1, N_2, \ldots, N_M$). Let the full $N \times N$ channel matrix be denoted by $H_k$, where $[H_k]_{n,m} \triangleq h_{n,m}^k$ is the channel gain from the $m$-th user to the $n$-th user on frequency tone $k$. By defining the channel gains from the $j$-th set of users to the $i$-th set of users as the $N_i \times N_j$ matrix $H_{ij}^k$, the full channel matrix can be written as:

$$H_k = \begin{bmatrix} H_{11} & H_{12} & \cdots & H_{1M} \\ H_{21} & H_{22} & \cdots & H_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ H_{M1} & H_{M2} & \cdots & H_{MM} \end{bmatrix}, \quad (1)$$

where $H_{ij}^k, i \neq j$ contains the inter-group crosstalk from group $j$ to group $i$. Note that $H_{ii}^k$ contains both the direct and intra-group-crosstalk channel gains.

Using synchronous discrete multi-tone modulation, transmissions can be modeled independently on each tone $k$ as:

$$y_k = H_k x_k + z_k, \quad (2)$$

where $y_k \triangleq [y^1_k, \ldots, y^N_k]^T$, $x_k$ and $z_k$ are similarly defined.

As well, transmission can be modeled for each user $n$ as:

$$y^n = H^{n,n} x^n + \sum_{m \neq n} H^{n,m} x^m + z^n, \quad (3)$$

where $H^{n,m} = \text{diag}\{h_{n,m}^1, \ldots, h_{n,m}^N\}$, $y^n \triangleq [y^n_1, \ldots, y^n_N]^T$, $x^n$ and $z^n$ are similarly defined. Note that $\text{diag}\{a_1, \ldots, a_K\}$ refers to the diagonal matrix whose diagonal elements are given by the vector $\{a_1, \ldots, a_K\}$.

III. CROSSTALK CANCELLATION

In the following, downstream transmission is assumed; however, the results can be routinely adapted to the upstream transmission case, as well. As discussed, intra-group crosstalk will be dealt with using Vectored DSL, whereas inter-group crosstalk will be dealt with using interference alignment. Vectored DSL is applied on a per-tone basis ($i.e.,$ using transmission equation (2)), while interference alignment is applied on a per-user basis ($i.e.,$ using transmission equation (3)). In particular, Vectored DSL is used to apply pre-coding on the transmitted signal vector of users independently for each frequency tone ($i.e., x_k$). Contrarily, interference alignment is applied using pre-coding and decoding matrices on the transmitted signal vector of frequency tones for each user ($i.e., x^n$). Hence, while both Vectored DSL and interference alignment make use of pre-coding, they do so in fundamentally different ways. Moreover, the pre-coder design for intra-group crosstalk and the pre-coder and decoder design for the inter-group crosstalk can be performed independently.

The received signal (before decoding) on frequency tone $k$ can be re-written as follows:

$$y_k = H_k Q_k \tilde{x}_k + z_k, \quad (4)$$

where $Q_k$ is the pre-coder matrix which is used to remove the effects of intra-group crosstalk ($i.e.,$ Vectored DSL), and $\tilde{x}_k$ is the vector of modified transmitted signals for all users on frequency tone $k$, which is used to suppress the effects of inter-group crosstalk ($i.e.,$ interference alignment). The designs of the pre-coder matrix, $Q_k$, and modified transmitted signal vectors, $\tilde{x}_k$, will be described in the following two subsections.

A. Vectored DSL Approach to Intra-Group Crosstalk

It was shown in [3] that for the full vectoring case, near-optimal vectoring performance could be achieved using a diagonalizing pre-coder. Hence, for the partial vectoring case, the pre-coder matrix, $Q_k$, is selected as a block diagonal matrix where each block diagonal element is a diagonalizing pre-coder for the particular group of users.

More specifically, the pre-coder matrix, $Q_k$, is selected as:

$$Q_k = \begin{bmatrix} Q_{11} & 0 & \cdots & 0 \\ 0 & Q_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Q_{MM} \end{bmatrix},$$

where $Q_{ii} = (\beta_k^i)^{-1}(H_{ii}^k)^{-1} \text{diag}(H_{ii}^k)$, $i = 1, \ldots, M$ and $\beta_k^i = \max_{n \in N_i} \|\text{diag}(H_{ii}^k)\|_{row n}$ is selected to ensure compliance with the spectral masks constraints after pre-coding. Note that $\text{diag}\{A\}$ refers to the vector containing the diagonal elements of the matrix $A$. The fact that $\beta_k^i$ ensures that the spectral mask constraint is not violated allows for the separate selection of the Vectored DSL and interference alignment parameters. Mathematically, the effective transmission channel can be re-written as:

$$\tilde{H}_k = \begin{bmatrix} H_{11} & H_{12} & \cdots & H_{1M} \\ H_{21} & H_{22} & \cdots & H_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ H_{M1} & H_{M2} & \cdots & H_{MM} \end{bmatrix},$$

where $H_{ij}^k \triangleq \beta_k^i \text{diag}(H_{ij}^k)$ and $H_{ij}^k, i \neq j$ are the matrices of the modified inter-group crosstalk gains after partial vectoring. For convenience, the $(n, m)$-th element of the modified channel gains matrix is defined as: $[\tilde{H}_k]_{n,m} \triangleq \tilde{h}_{n,m}^k$.

B. Interference Alignment Approach to Inter-Group Crosstalk

Once the intra-group crosstalk has been removed, the remaining performance degradation is caused by inter-group crosstalk. Interference alignment is a technique which is employed to align the interference signals at each receiver such that they are contained within a subspace of signal space. Hence, each receiver can recover their own signal (interference...
free) by projecting the received signal into the null space of the interference subspace. The interference is aligned by processing the per-user vectors (i.e., $x^n$) using pre-coder and decoder matrices. For user $n$, the interference alignment precoders, $V^n$, and decoders, $U^n$, are used to transform the already modified channel gains matrix (i.e., modified using the partial vectoring described in Section III-A).

The effective received signal over all $K$ frequency tones for user $n \in N_i$, $\tilde{y}^n = (U^n)^\dagger y^n$, is given by:

$$
\tilde{y}^n = (U^n)^\dagger H_n V^n x^n + \sum_{m \notin N_i} (U^n)^\dagger \hat{H}^{n,m} V^m x^m + \sum_{m \in N_i, m \neq n} (U^n)^\dagger \hat{H}^{n,m} V^m x^m + \tilde{z}^n,
$$

where $\hat{H}^{n,m} = \text{diag}(\hat{h}^{1,m}, \ldots, \hat{h}^{K,m})$, $V^n$ and $U^n$ are the respective $K \times d_n$ pre-coding and decoding matrices used for interference alignment associated with user $n$, where $d_n$ represents the desired number of degrees of freedom for user $n$. Note that, $\tilde{z}^n = (U^n)^\dagger z^n$. Also note that when $d_n = K/2$, perfect interference alignment will be achieved [2]; however, in practice, it may be costly (in terms of sum-rate) to achieve perfect interference alignment. Hence, one might conjecture that the highest sum-rate will be achieved by maximizing $d_n$ (i.e., $d_n = K$ for all $n$) and simply minimizing the interference leakage while utilizing all degrees of freedom.

Based on (5), the modified transmitted signal for user $n$ on frequency tone $k$ (i.e., $\tilde{x}^n_k$ from Equation (4)) is given by:

$$
\tilde{x}^n_k = [V^n x^n]_k = \sum_{i=1}^{K} [V^n]_{k,i} x^n_i.
$$

The received signal vector at receiver $n$, $y^n$, is processed using the interference suppression rotation (or decoder) matrix, $U^n$. For perfect interference alignment, the matrices $V^n$ and $U^n$ are selected such that all the interference generated to receiver $n$ lies in the null space of $U^n$. Hence, the following conditions must be satisfied for interference alignment to occur at receiver $n \in N_i$:

$$
(U^n)^\dagger \hat{H}^{n,m} V^m = 0_{d_n \times d_n}, \quad \forall m \notin N_i,
$$

$$
\text{rank}((U^n)^\dagger \hat{H}^{n,n} V^n) = d_n.
$$

Iterative algorithms were derived in [4]–[6] to optimize the selection of $U^n$ and $V^n$ by alternating between fixing one matrix and optimizing the other on a per-user basis. Following the approach of [4], the objective function for user $n \in N_i$ is selected as the total interference leakage, denoted by $TL^n$:

$$
TL^n = \text{Tr} \left[ \sum_{m \notin N_i} \mathcal{E} \left( ((U^n)^\dagger \hat{H}^{n,m} V^m x^m) ((U^n)^\dagger \hat{H}^{n,m} V^m x^m)^\dagger \right) \right],
$$

where $\text{Tr}[A]$ denotes the trace of matrix $A$. $TL^n$ represents the total variance of the interference in the system (i.e., the total leakage interference). It is assumed that $\mathcal{E}(x_i^n x_j^n) = 0$ for $i \neq j$ and hence, $\mathcal{E}(x^m(x^n)^\dagger) = \text{diag}\{s_1^n, \ldots, s_{d_n}^n\}$. Then, it can be shown that:

$$
TL^n = \text{Tr} \left[ (U^n)^\dagger \left( \sum_{m \notin N_i} \hat{H}^{n,m} V^m S^m (V^m)^\dagger (\hat{H}^{n,m})^\dagger \right) U^n \right],
$$

where $S^m = \text{diag}\{s_1^m, \ldots, s_{d_n}^m\}$. The optimization problem for user $n$ with fixed $V^n$ and $S^m$ can be written as:

$$
\min_{U^n, S^m} TL^n
$$

subject to: $\langle U^n \rangle U^n = I_{d_n \times d_n}$. (6)

Expanding on what is shown in [4], the solution to this optimization problem for fixed values of $S^m$ for user $n \in N_i$ is given by the matrix, $U^n$, composed of the eigenvectors corresponding to the $d_n$ smallest eigenvalues of the matrix $\sum_{m \notin N_i} \hat{H}^{n,m} V^m S^m (V^m)^\dagger (\hat{H}^{n,m})^\dagger$. Furthermore, when $U^n$ is fixed, the optimization problem for $V^n$ can be formulated similarly to optimization problem (6). Moreover, the optimal solution for user $n \in N_i$ will be given by the matrix, $V^n$, composed of the eigenvectors corresponding to the $d_n$ smallest eigenvalues of the matrix $\sum_{m \notin N_i} \hat{H}^{n,m} U^n S^m (U^n)^\dagger \hat{H}^{n,m}$. An iterative algorithm which alternates between optimizing $U^n$ and $V^n$ is proposed in [4]. Optimization problem (6) differs from the optimization problem in [4] in that it also involves the optimization of the transmit PSD, whereas [4] assumes a flat transmit PSD. Due to the fact that direct DSL channel gains are decreasing functions with respect to frequency, the most interference-free sum-rate gains will occur at low frequencies. As such, a non-flat PSD is crucial for achieving strong performance gains. In order to simplify the procedure, a flat transmit PSD is assumed when optimizing the values of $U^n$ and $V^n$ and then the values of $S^m$ are computed using typical Dynamic Spectrum Management (DSM) techniques. The $U^n$ and $V^n$ update algorithms are summarized in Algorithm 1. It is shown in [4] that the iterative algorithm to optimize the pre-coder and decoder matrices will converge and hence, Algorithm 1 will converge. The full algorithm is summarized in Algorithm 2.

**Algorithm 1: $U^n$ and $V^n$ Updates**

Given $S^m$;
Set $V^n \in \mathcal{C}^{K \times d_n}, \forall n$ as a random orthogonal matrix;

repeat
for $n = 1, \ldots, N$
do
Step 1:
Set pre-coder as: $V^n$, canceller as: $U^n$; Fix $V^n$;
Set $U^n$ corresponding to the eigenvectors of $\sum_{m \notin N_i} \hat{H}^{n,m} V^m S^m (V^m)^\dagger (\hat{H}^{n,m})^\dagger$;
Step 2:
Set pre-coder as: $U^n$, canceller as: $V^n$, Fix $U^n$;
Set $V^n$ corresponding to the eigenvectors of $\sum_{m \notin N_i} \hat{H}^{n,m} (U^n)^\dagger S^m (U^n)^\dagger \hat{H}^{n,m}$;
until $V^n, U^n$ converge $\forall n$;

The interference alignment sum-rate is given by:

$$
R = \sum_{n=1}^{N} f_n \log_2 \left( |A_n + \frac{1}{\Gamma} (\Sigma + \sum_{m \neq n} C_{m,n})^{-1} C_{n,n}| \right),
$$
Algorithm 2: Interference alignment algorithm

Initialize \( S^n \) to a flat PSD. ;
Update \( V^n \) and \( U^n \) \( \forall \ n \) using Algorithm 1 ;
Compute \( S^n \) \( \forall \ n \) using typical DSM techniques ;
Re-update \( V^n \) and \( U^n \) \( \forall \ n \) using Algorithm 1 ;
Re-compute \( S^n \) \( \forall \ n \) using typical DSM techniques ;

where \( \Sigma^n = (U^n)^\dagger \text{diag}\{\sigma^n_1, \ldots, \sigma^n_{d_n}\} U^n \), \( f_s \) is the DMT symbol rate, \( \Gamma \) is the signal-to-noise ratio gap, and the \( d_n \times d_n \) covariance matrices are given by:

\[
C_{n,m} = (U^n)^\dagger H_{n,m} V^m S^m(V^m)^\dagger(H_{n,m})^\dagger U^n. 
\]

IV. Practical Implementation

Two significant factors in the implementation of Vectored DSL and/or interference alignment are their feasibility and computational complexity. The complexity of Vectored DSL is well-known: it grows linearly with the number of frequency tones, and exponentially with the number of users (i.e., \( O(KN^2) \)). Conversely, the complexity of interference alignment grows linearly with the number of users and exponentially with the number of degrees of freedom (an upper bound on the number of degrees of freedom is the number of frequency tones, i.e., \( O(NK^2) \)); however, the scalability of interference alignment can reduce this complexity.

More specifically, full interference alignment can be achieved by applying interference alignment on various subsets of frequency tones (e.g., the interference can be aligned over 100 tones by separately aligning the interference over the first 50 tones and the second 50 tones). Hence, by creating multiple interference alignment sub-problems, the resulting complexity will be linear in the number of users and the number of tone-groups and only exponential in the number of tones per tone-group. Let \( N_s \) be the number of tone-groups and let \( \Delta K \) be maximum number of frequency tones per tone-group. Hence, the computational complexity of the interference alignment approach can be reduced to \( O(N N_s (\Delta K)^2) \). This process can significantly reduce the computational complexity of the interference alignment approach when \( \Delta K \ll K \).

As discussed in Section I, interference alignment is practical in some scenarios where Vectored DSL is not (e.g., linecard vectoring or geographically separated DSLAMs). Another significant factor with respect to the feasibility of interference alignment is its invariance to users entering or leaving the system. In particular, once the interference is aligned for all users in the system (i.e., active and inactive users), regardless of whether or not users enter or leave the system, the interference will remain aligned. Conversely, for Vectored DSL, the pre-coders are required to be re-designed every time a user enters or exits the system. Hence, the interference alignment solution only needs to be re-computed whenever the channel gains change. Due to the slow-time-varying nature of the DSL channel, the interference alignment solution will not need to be updated frequently, relative to the Vectored DSL case.

V. Simulation Results

The test case presented in this section represents a Fiber-To-The-Node (FTTN) downstream scenario. It consists of 24 total users organized into three groups of eight users. The line lengths of first group, \( L_1, \ldots, L_8 \), were uniformly distributed between 300 to 800 m. The second group of users were offset by 250 m from the first group of users and their line lengths, \( L_9, \ldots, L_{16} \), were uniformly distributed between 250 to 550 m. Finally, the third group of users were offset by 450 m from the first group of users and their line lengths, \( L_{17}, \ldots, L_{24} \), were uniformly distributed between 150 to 350 m, as shown in Fig. 1.

![Fig. 1. 24-user FTTC test case.](image)

The test case used the ANSI model [7] and assumed that 26-gauge (0.4 mm) lines were used. The target symbol error probability was set at \( 10^{-7} \). The coding gain and noise margin were set to 3 dB and 6 dB, respectively. The frequency tone spacing was set to \( \Delta_f = 4.3125 \text{ kHz} \), and the DMT symbol rate was set to \( f_s = 4 \text{ kHz} \). PSD masks were applied using VDSL Profile 17a band plan [8], which consists of three downstream bands (0.276 - 3.75 MHz, 5.2 - 8.5 MHz, and 12 - 17.664 MHz). A maximum transmit power of 11.5 dBm was applied to each modem.

As discussed earlier, the number of degrees of freedom for each user was selected as the number of frequency tones in order to achieve maximum sum-rate (i.e., \( d_n = K \forall n \)). As discussed in Section IV, interference alignment can be achieved by independently aligning the interference on subsets of frequency tones. Let \( \Delta K > 1 \) be the maximum number of frequency tones per subset. Hence, the number of subsets will be \( N_s = \lceil K/\Delta K \rceil \), where the final subset may have less than \( \Delta K \) frequency tones. The value of \( \Delta K \) was varied from 2 to 24. Note that when \( \Delta K > 1 \) is smaller, the runtime per-subset will be shorter but the total number of subsets will be larger; however, since each subset can be optimized (minimizing leakage interference) independently, the solutions can be computed using parallel processing if multiple CPUs are available, and combined afterwards. Also note that \( \Delta K > 1 \), since if \( \Delta K = 1 \), the pre-coder and decoder matrices become scalar values and therefore the minimization of leakage interference corresponds to a trivial solution where \( |U^n| = |V^n| = 1 \forall n \), due to the unitary constraint (i.e., the no interference alignment case).

Table I shows the performance of partial vectoring combined with the interference alignment approach for varied values of \( \Delta K \) as compared to partial vectoring and Vectored DSL, relative to the channel capacity. Two methods for selecting the frequency tones for each sub-problem are compared.
More specifically, a sequential approach (i.e., Fig. 2-(a)) and a spaced approach (i.e., Fig. 2-(b)) are compared. The sequential approach forms subsets using adjacent frequency tones, while the spaced approach maximizes the separation between frequency tones within each subset. The total leakage interference is also provided.

Fig. 2. Frequency tone selection methods: (a) sequential, (b) spaced.

Recall that for such scenarios, full vectoring is not practically implementable; however, mathematically, full vectoring can be used as a reference point in order to judge the effectiveness of the interference alignment-based approach. The DSM algorithm used is that of Distributed Spectrum Balancing (DSB) [9]. The results of Table I show that partial vectoring combined with interference alignment has the potential to increase the performance of DSL systems when applying full Vectored DSL is not practical. Note that the results of Table I show that minimizing the leakage interference is not equivalent to maximizing the sum-rate. In particular, Table I shows that the maximum sum-rates correspond to the largest interference leakage values. More importantly, it can be seen that the frequency tone grouping selection method and subset size have significant impacts on the achieved sum-rate.

The results indicated that when using sequential frequency tone groupings, the interference alignment approach provided sum-rate increases over the partial vectoring approach for almost all values of $\Delta K$ tested. On the other hand, when using spaced frequency tone groupings with $\Delta K = 2$, the interference alignment approach provided better results than partial vectoring; however, for all other values of $\Delta K$ tested, interference alignment provided a worse sum-rate than the partial vectoring approach.

In general, as $\Delta K$ increased, the spaced frequency tone selection method provided a lower sum-rate. This is due to the nature of the DSL channel (e.g., Fig. 3). In particular, DSL channels are characterized as having strong direct channel gains at low frequencies and weak direct channel gains at high frequencies. Conversely, DSL channels are characterized as having weak crosstalk channel gains at low frequencies and strong crosstalk channel gains at high frequencies. As such, the majority of sum-rate gains for DSL systems is achieved at low frequencies. Using spaced frequency tone groupings, as $\Delta K$ increases, the subsets begin to group more high frequency tones with low frequency tones. As a result, in order to minimize the leakage interference, the interference is rotated from the higher frequencies to the lower frequencies. The resulting effective channel has reduced interference at high frequencies, but has greater interference at low frequencies, which reduces the potential sum-rate gains at low frequencies. Hence, the spaced frequency tone grouping approach leads to lower leakage interference than the sequential tone grouping approach and as such, the overall sum-rate decreases.

![Typical DSL Channel](image)

The results indicate that the interference alignment approach has the potential to provide performance gains over the partial vectoring approach. The results also indicate that selecting the pre-coding and decoding matrices based on minimizing leakage interference does not always correspond to a better sum-rate. Hence, a more improved selection method for the pre-coding and decoding matrices is required to fully utilize the potential benefits of the interference alignment approach.

### Table I

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>$\Delta K$</th>
<th>% of Capacity Leakage Interference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Vectoring</td>
<td>–</td>
<td>94.9</td>
</tr>
<tr>
<td>Partial Vectoring</td>
<td>–</td>
<td>53.5</td>
</tr>
<tr>
<td>Partial</td>
<td>2</td>
<td>60.7</td>
</tr>
<tr>
<td>Vectoring +</td>
<td>5</td>
<td>41.6</td>
</tr>
<tr>
<td>Interference</td>
<td>12</td>
<td>60.5</td>
</tr>
<tr>
<td>Alignment</td>
<td>17</td>
<td>50.1</td>
</tr>
<tr>
<td>(Sequential)</td>
<td>24</td>
<td>60.0</td>
</tr>
<tr>
<td>Partial</td>
<td>2</td>
<td>61.0</td>
</tr>
<tr>
<td>Vectoring +</td>
<td>5</td>
<td>46.4</td>
</tr>
<tr>
<td>Interference</td>
<td>12</td>
<td>53.0</td>
</tr>
<tr>
<td>Alignment</td>
<td>17</td>
<td>44.1</td>
</tr>
<tr>
<td>(Spaced)</td>
<td>24</td>
<td>42.3</td>
</tr>
</tbody>
</table>

### VI. Concluding Remarks

A joint partial vectoring and interference alignment solution was proposed as an alternative solution for cases where Vectored DSL is impractical. It was shown that the partial vectoring pre-coders and the interference alignment pre-coders and decoders can be solved for independently. The structure of the partial vectoring pre-coders was provided. The classic distributed interference alignment algorithm was used to optimize the interference alignment pre-coder and decoder matrices. A discussion on the practical advantages of the proposed approach with respect to Vectored DSL was provided. It was shown that the proposed approach could be decomposed into a series of independent sub-problems, which can be solved independently using parallel processing to significantly reduce the computation complexity. The simulation results show that the proposed approach has the potential to improve the sum-rates in scenarios where full Vectored DSL is not implementable. Finally, a more sophisticated selection method
for the pre-coding and decoding matrices is required for the full potential of the interference alignment based approach to be realized.

VII. ACKNOWLEDGEMENTS

The authors would like to thank Prof. Ignacio Santamaria for his interesting discussions on interference alignment.

REFERENCES


