Generalized semi-pre connectedness in intuitionistic fuzzy topological spaces

R. Santhi, D. Jayanthi

Received 25 April 2011; Accepted 11 August 2011

Abstract. In this paper we have introduced the intuitionistic fuzzy generalized semi-pre connected space, intuitionistic fuzzy generalized semi-pre super connected space and intuitionistic fuzzy generalized semi-pre extremally disconnected space. We investigated some of their properties. Also we characterized the intuitionistic fuzzy generalized semi-pre super connected space.

2010 AMS Classification: 54A99, 03E99

Keywords: Intuitionistic fuzzy topology, Intuitionistic fuzzy generalized semi-pre connected space, Intuitionistic fuzzy generalized semi-pre super connected space.

Corresponding Author: D. Jayanthi (jayanthimaths@rediffmail.com)

1. Introduction

Zadeh [10] introduced the notion of fuzzy sets. Fuzzy topological space was introduced by Chang [2]. After that there have been a number of generalizations of this fundamental concept. Atanassov [1] introduced the notion of intuitionistic fuzzy sets. Using the notion of intuitionistic fuzzy sets, Çoker [3] introduced the notion of intuitionistic fuzzy topological space. Connectedness in intuitionistic fuzzy special topological spaces was introduced by Öşçağ and Çoker [7]. Jun and song [4] discussed intuitionistic fuzzy semi-pre opennes and intuitionitic fuzzy semi-pre continuity.

In this paper we have introduced intuitionistic fuzzy generalized semi-pre connected space, intuitionistic fuzzy generalized semi-pre super connected space and intuitionistic fuzzy generalized semi-pre extremally disconnected space. We investigated some of their properties. Also we characterized the intuitionistic fuzzy generalized semi-pre super connected space.
2. Preliminaries

Definition 2.1 ([1]). An intuitionistic fuzzy set (IFS for short) $A$ in $X$ is an object having the form

$$A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$$

where the functions $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set $A$, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by $\text{IFS}(X)$, the set of all intuitionistic fuzzy sets in $X$.

Definition 2.2 ([1]). Let $A$ and $B$ be IFSs of the form $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$ and $B = \{(x, \mu_B(x), \nu_B(x)) : x \in X\}$. Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$;
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$;
(c) $A^c = \{(x, \nu_A(x), \mu_A(x)) : x \in X\}$;
(d) $A \cap B = \{(x, \mu_A(x) \land \mu_B(x), \nu_A(x) \lor \nu_B(x)) : x \in X\}$;
(e) $A \cup B = \{(x, \mu_A(x) \lor \mu_B(x), \nu_A(x) \land \nu_B(x)) : x \in X\}$.

The intuitionistic fuzzy sets $0_\infty = \{(x, 0, 1) : x \in X\}$ and $1_\infty = \{(x, 1, 0) : x \in X\}$ are respectively the empty set and the whole set of $X$.

For the sake of simplicity, we shall use the notation $A = \{(x, \mu_A, \nu_A)\}$ instead of $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$.

Definition 2.3 ([3]). An intuitionistic fuzzy topology (IFT for short) on $X$ is a family $\tau$ of IFSs in $X$ satisfying the following axioms:

(i) $0_\infty, 1_\infty \in \tau$;
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$;
(iii) $\bigcup G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair $(X, \tau)$ is called an intuitionistic fuzzy topological space (IFTS for short) and any IFS in $\tau$ is known as an intuitionistic fuzzy open set (IFOS for short) in $X$. The complement $A^c$ of an IFOS $A$ in IFTS $(X, \tau)$ is called an intuitionistic fuzzy closed set (IFCS for short) in $X$.

Definition 2.4 ([3]). An IFS $A = \{(x, \mu_A, \nu_A)\}$ in an IFTS $(X, \tau)$ is said to be

(i) intuitionistic fuzzy semi-pre closed set (IFSPCS for short) if there exists an intuitionistic fuzzy pre closed set (IFPCS for short) $B$ such that $\text{int}(B) \subseteq A \subseteq B$;

(ii) intuitionistic fuzzy semi-pre open set (IFPOS for short) if there exists an intuitionistic fuzzy pre open set (IFPO for short) $B$ such that $B \subseteq A \subseteq \text{cl}(B)$.

Note that an IFS $A$ is an IFSPCS if and only if $\text{int}((\text{cl}(A))) \subseteq A$ [5].

Definition 2.5 ([5]). Let $A$ be an IFS in an IFTS $(X, \tau)$. Then the semi-pre interior and the semi-pre closure of $A$ are defined as

$$\text{spint}(A) = \cup \{G : G \text{ is an IFPOS in } X \text{ and } G \subseteq A\};$$
$$\text{spcl}(A) = \cap \{K : K \text{ is an IFSPCS in } X \text{ and } A \subseteq K\}.$$

Note that for any IFS $A$ in $(X, \tau)$, we have $\text{spcl}(A^c) = \text{spint}(A)^c$ and $\text{spint}(A^c) = \text{spcl}(A)^c$ [5].
Definition 2.6 ([8]). An IFS \( A \) is an intuitionistic fuzzy generalized closed set (IFGCS for short) if \( \text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is an IFOS. The complement of an IFGCS is called an intuitionistic fuzzy generalized open set (IFGOS for short).

Definition 2.7 ([5]). An IFS \( A \) in an IFTS \( (X, \tau) \) is said to be an intuitionistic fuzzy generalized semi-pre closed set (IFGSPCS for short) if \( \text{spcl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is an IFOS in \( (X, \tau) \).

Every IFCS, IFSPCS is an IFGSPCS but the separate converses may not be true in general [5].

Definition 2.8 ([5]). The complement \( A^c \) of an IFGSPCS \( A \) in an IFTS \( (X, \tau) \) is called an intuitionistic fuzzy generalized semi-pre open set (IFGSPOS for short) in \( X \).

Every IFOS, IFSPOS is an IFGSPOS but the separate converses may not be true in general [5].

Definition 2.9 ([5]). Let \( A \) be an IFS in an IFTS \( (X, \tau) \). Then the generalized semi-pre interior and the generalized semi-pre closure of \( A \) are defined as

\[
\text{gspint}(A) = \bigcup \{ G : G \text{ is an IFGSPOS in } X \text{ and } G \subseteq A \} ;
\]

\[
\text{gspcl}(A) = \bigcap \{ K : K \text{ is an IFGSPCS in } X \text{ and } A \subseteq K \} .
\]

Note that for any IFS \( A \) in an IFTS \( (X, \tau) \), we have \( \text{gspcl}(A^c) = [\text{gspint}(A)]^c \) and \( \text{gspint}(A^c) = [\text{gspcl}(A)]^c \) [5].

Definition 2.10 ([5]). If every IFGSPCS in an IFTS \( (X, \tau) \) is IFSPCS in \( (X, \tau) \), then the space can be called as an intuitionistic fuzzy semi-pre \( T_{1/2} \) space (IFSP \( T_{1/2} \) space for short).

Definition 2.11 ([6]). A mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) is called an intuitionistic fuzzy generalized semi-pre continuous (IFGSP continuous for short) mapping if \( f^{-1}(V) \) is an IFGSPCS in \( (X, \tau) \) for every IFCS \( V \) of \( (Y, \sigma) \).

Definition 2.12 ([6]). A mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) is called an intuitionistic fuzzy generalized semi-pre irresolute (IFGSP irresolute for short) mapping if \( f^{-1}(V) \) is an IFGSPCS in \( (X, \tau) \) for every IFGSPCS \( V \) of \( (Y, \sigma) \).

Definition 2.13 ([8]). Two IFSs \( A \) and \( B \) in \( X \) are said to be q-coincident (\( A q B \) for short) if and only if there exists an element \( x \in X \) such that \( \mu_A(x) > \nu_A(x) \) or \( \nu_A(x) < \mu_B(x) \).

Definition 2.14 ([8]). Two IFSs \( A \) and \( B \) in \( X \) are said to be not q-coincident (\( A q^c B \) for short) if and only if \( A \subseteq B^c \).

Definition 2.15 ([8]). An IFTS \( (X, \tau) \) is said to be an IF \( T_{1/2} \) space if every IFGCS in \( (X, \tau) \) is an IFCS in \( (X, \tau) \).

Definition 2.16 ([9]). An IFTS \( (X, \tau) \) is said to be an intuitionistic fuzzy \( C_5 \)-connected (IF \( C_5 \)-connected for short) space if the only IFSs which are both intuitionistic fuzzy open and intuitionistic fuzzy closed are \( 0_\sim \) and \( 1_\sim \).
Definition 2.17 ([9]). An IFTS \((X, \tau)\) is said to be an intuitionistic fuzzy GO-connected (IFGO-connected for short) space if the only IFSs which are both intuitionistic fuzzy generalized open and intuitionistic fuzzy generalized closed are \(0_{\sim}\) and \(1_{\sim}\).

3. INTUITIONISTIC FUZZY GENERALIZED SEMI-PRE CONNECTED SPACES

In this section we introduce intuitionistic fuzzy generalized semi-pre connected space and intuitionistic fuzzy generalized semi-pre super connected space. We investigate some of their properties. Also we provide a characterization theorem for an intuitionistic fuzzy generalized semi-pre super connected space.

Definition 3.1. An IFTS \((X, \tau)\) is said to be an intuitionistic fuzzy generalized semi-pre connected space if the only IFSs which are both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed are \(0_{\sim}\) and \(1_{\sim}\).

Example 3.2. Let \(X = \{a, b\}\) and \(\tau = \{0_{\sim}, M, 1_{\sim}\}\) be an IFT on \(X\), where \(M = \langle x, (0.5a, 0.6b), (0.5b, 0.4a) \rangle\). Then \((X, \tau)\) is an intuitionistic fuzzy generalized semi-pre connected space.

Theorem 3.3. Every intuitionistic fuzzy generalized semi-pre connected space is an intuitionistic fuzzy C5-connected space but not conversely.

Proof. Let \((X, \tau)\) be an intuitionistic fuzzy generalized semi-pre connected space. Suppose \((X, \tau)\) is not an intuitionistic fuzzy C5-connected space, then there exists a proper IFS \(A\) which both intuitionistic fuzzy \(g\)-open and intuitionistic fuzzy \(g\)-closed in \((X, \tau)\). That is \(A\) is both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in \((X, \tau)\). This implies that \((X, \tau)\) is not an intuitionistic fuzzy generalized semi-pre connected space. This is a contradiction. Therefore \((X, \tau)\) is an intuitionistic fuzzy C5-connected space. \(\Box\)

Example 3.4. Let \(X = \{a, b\}\) and \(\tau = \{0_{\sim}, M, 1_{\sim}\}\) be an IFT on \(X\), where \(M = \langle x, (0.5a, 0.4b), (0.5b, 0.6a) \rangle\). Then \((X, \tau)\) is an intuitionistic fuzzy C5-connected space but not an intuitionistic fuzzy generalized semi-pre connected space, since the IFS \(M\) in \(\tau\) is both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in \((X, \tau)\).

Theorem 3.5. Every intuitionistic fuzzy generalized semi-pre connected space is an intuitionistic fuzzy GO-connected space but not conversely.

Proof. Let \((X, \tau)\) be an intuitionistic fuzzy generalized semi-pre connected space. Suppose \((X, \tau)\) is not an intuitionistic fuzzy GO-connected space, then there exists a proper IFS \(A\) which both intuitionistic fuzzy \(g\)-open and intuitionistic fuzzy \(g\)-closed in \((X, \tau)\). That is \(A\) is both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in \((X, \tau)\). This implies that \((X, \tau)\) is not an intuitionistic fuzzy generalized semi-pre connected space. This is a contradiction. Therefore \((X, \tau)\) is an intuitionistic fuzzy GO-connected space. \(\Box\)

Example 3.6. In Example 3.4, \((X, \tau)\) is an intuitionistic fuzzy GO-connected space but not an intuitionistic fuzzy generalized semi-pre connected space.
The relation among various types of intuitionistic fuzzy connectedness is given in the following diagram.

```
IFGO-connected space
/  \  /
\  \ \ |
IFGSP connected space
/  \  /
\  \ \ |
IFC5-connected space
```

The reverse implications are not true in general in the above diagram.

**Theorem 3.7.** An IFTS \((X, \tau)\) is an intuitionistic fuzzy generalized semi-pre connected space if and only if there exist no non-zero intuitionistic fuzzy generalized semi-pre open sets \(A\) and \(B\) in \((X, \tau)\) such that \(A = B^c\).

**Proof.** Necessity: Let \(A\) and \(B\) be two intuitionistic fuzzy generalized semi-pre open sets in \((X, \tau)\) such that \(A \neq 0_\sim, B \neq 0_\sim\), and \(A = B^c\). Therefore \(B^c\) is an intuitionistic fuzzy generalized semi-pre closed set. Since \(A \neq 0_\sim, B \neq 1_\sim\), this implies \(B\) is a proper IFS which is both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in \((X, \tau)\). Hence \((X, \tau)\) is not an intuitionistic fuzzy generalized semi-pre connected space. But this is a contradiction to our hypothesis. Thus there exist no non-zero intuitionistic fuzzy semi-pre open sets \(A\) and \(B\) in \((X, \tau)\) such that \(A = B^c\).

Sufficiency: Let \(A\) be both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in \((X, \tau)\) such that \(1_\sim \neq A \neq 0_\sim\). Now let \(B = A^c\). Then \(B\) is an intuitionistic fuzzy generalized semi-pre open set and \(B \neq 1_\sim\). This implies \(B = A^c \neq 0_\sim\), which is a contradiction to our hypothesis. Therefore \((X, \tau)\) is an intuitionistic fuzzy generalized semi-pre connected space. \(\square\)

**Theorem 3.8.** An IFTS \((X, \tau)\) is an intuitionistic fuzzy generalized semi-pre connected space if and only if there exist no non-zero intuitionistic fuzzy generalized semi-pre open sets \(A\) and \(B\) in \((X, \tau)\) such that \(A = B^c, B = (\text{spcl}(A))^c\), and \(A = (\text{spcl}(B))^c\).

**Proof.** Necessity: Assume that there exist IFSs \(A\) and \(B\) such that \(A \neq 0_\sim \neq B, B = A^c, B = (\text{spcl}(A))^c\), and \(A = (\text{spcl}(B))^c\). Since \((\text{spcl}(A))^c\) and \((\text{spcl}(B))^c\) are intuitionistic fuzzy generalized semi-pre open sets in \((X, \tau)\), \(A\) and \(B\) are intuitionistic fuzzy generalized semi-pre open sets in \((X, \tau)\). This implies \((X, \tau)\) is not an intuitionistic fuzzy generalized semi-pre connected space, which is a contradiction. Therefore there exist no non-zero intuitionistic fuzzy generalized semi-pre open sets \(A\) and \(B\) in \((X, \tau)\) such that \(A = B^c, B = (\text{spcl}(A))^c\), and \(A = (\text{spcl}(B))^c\).

Sufficiency: Let \(A\) be both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in \((X, \tau)\) such that \(1_\sim \neq A \neq 0_\sim\). Now by taking \(B = A^c\), we obtain a contradiction to our hypothesis. Hence \((X, \tau)\) is an intuitionistic fuzzy generalized semi-pre connected space. \(\square\)

**Definition 3.9.** An IFTS \((X, \tau)\) is said to be an intuitionistic fuzzy semi-pre \(T_{1/2}^s\) space if every intuitionistic fuzzy generalized semi-pre closed set is an intuitionistic fuzzy closed set in \((X, \tau)\).
Example 3.10. In Example 3.2, the IFTS \((X, \tau)\) is an intuitionistic fuzzy semi-pre \(T_{1/2}^*\) space.

Remark 3.11. Every intuitionistic fuzzy semi-pre \(T_{1/2}^*\) space is an intuitionistic fuzzy semi-pre \(T_{1/2}\) space but not conversely.

Proof. Let \((X, \tau)\) be an intuitionistic fuzzy semi-pre \(T_{1/2}^*\) space. Let \(A\) be an intuitionistic fuzzy generalized semi-pre closed set in \((X, \tau)\). By hypothesis \(A\) is an intuitionistic fuzzy closed set. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy semi-pre closed set, \(A\) is an intuitionistic fuzzy semi-pre closed set in \((X, \tau)\). Hence \((X, \tau)\) is an intuitionistic fuzzy semi-pre \(T_{1/2}\) space.

\(\Box\)

Example 3.12. In Example 3.4, \((X, \tau)\) is an intuitionistic fuzzy generalized semi-pre \(T_{1/2}^*\) space, but not an intuitionistic fuzzy semi-pre \(T_{1/2}^*\) space, since the IFS \(M\) is an intuitionistic fuzzy generalized semi-pre closed set in \((X, \tau)\) but not an intuitionistic fuzzy closed set in \((X, \tau)\), since \(\text{cl}(M) = M^c \neq M\).

Remark 3.13. Every intuitionistic fuzzy semi-pre \(T_{1/2}^*\) space is an intuitionistic fuzzy \(T_{1/2}\) space but not conversely.

Proof. Let \((X, \tau)\) be an intuitionistic fuzzy semi-pre \(T_{1/2}^*\) space. Let \(A\) be an intuitionistic fuzzy generalized closed set in \((X, \tau)\). Since every intuitionistic fuzzy generalized closed set is an intuitionistic fuzzy generalized semi-pre closed set, \(A\) is an intuitionistic fuzzy generalized semi-pre closed set in \((X, \tau)\). By hypothesis, \(A\) is an intuitionistic fuzzy closed set. Hence \((X, \tau)\) is an intuitionistic fuzzy \(T_{1/2}\) space.

\(\Box\)

Example 3.14. In Example 3.4, the IFTS \((X, \tau)\) is an intuitionistic fuzzy \(T_{1/2}\) space, but not an intuitionistic fuzzy semi-pre \(T_{1/2}^*\) space, since the IFS \(M\) is an intuitionistic fuzzy generalized semi-pre closed set in \((X, \tau)\) but not an intuitionistic fuzzy closed set in \((X, \tau)\), since \(\text{cl}(M) = M^c \neq M\).

Theorem 3.15. Let \((X, \tau)\) be an intuitionistic fuzzy semi-pre \(T_{1/2}^*\) space, then the following are equivalent.

(i) \((X, \tau)\) is an intuitionistic fuzzy generalized semi-pre connected space.

(ii) \((X, \tau)\) is an intuitionistic fuzzy GO-connected space.

(iii) \((X, \tau)\) is an intuitionistic fuzzy \(C_5\)-connected space.

Proof. (i)\(\Rightarrow\)(ii) is obvious from Theorem 3.5.

(ii)\(\Rightarrow\)(iii) is obvious from [8].

(iii)\(\Rightarrow\)(i) Let \((X, \tau)\) be an intuitionistic fuzzy \(C_5\)-connected space. Suppose \((X, \tau)\) is not an intuitionistic fuzzy generalized semi-pre connected space, then there exists a proper IFS \(A\) in \((X, \tau)\) which is both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in \((X, \tau)\). But since \((X, \tau)\) is an intuitionistic fuzzy semi-pre \(T_{1/2}^*\) space, \(A\) is both intuitionistic fuzzy open and intuitionistic fuzzy closed in \((X, \tau)\). This implies that \((X, \tau)\) is not an intuitionistic fuzzy \(C_5\)-connected space, which is a contradiction to our hypothesis. Therefore \((X, \tau)\) is intuitionistic fuzzy generalized semi-pre connected space.

\(\Box\)
Theorem 3.16. If \( f : (X, \tau) \to (Y, \sigma) \) is an intuitionistic fuzzy semi-pre continuous surjection and \((X, \tau)\) is an intuitionistic fuzzy generalized semi-pre connected space, then \((Y, \sigma)\) is an intuitionistic fuzzy \(C_5\)-connected space.

Proof. Let \((X, \tau)\) be an intuitionistic fuzzy generalized semi-pre connected space. Suppose \((Y, \sigma)\) is not an intuitionistic fuzzy \(C_5\)-connected space, then there exists a proper IFS \(A\) which is both intuitionistic fuzzy open and intuitionistic fuzzy closed in \((Y, \sigma)\). Since \(f\) is an intuitionistic fuzzy semi-pre continuous mapping, \(f^{-1}(A)\) is both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in \((X, \tau)\). But this is a contradiction to hypothesis. Hence \((Y, \sigma)\) is an intuitionistic fuzzy \(C_5\)-connected space. \(\square\)

Theorem 3.17. If \( f : (X, \tau) \to (Y, \sigma) \) is an intuitionistic fuzzy semi-pre irresolute surjection and \((X, \tau)\) is an intuitionistic fuzzy generalized semi-pre connected space, then \((Y, \sigma)\) is also an intuitionistic fuzzy generalized semi-pre connected space.

Proof. Suppose \((Y, \sigma)\) is not an intuitionistic fuzzy generalized semi-pre connected space, then there exists a proper IFS \(A\) which is both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in \((Y, \sigma)\). Since \(f\) is an intuitionistic fuzzy semi-pre irresolute mapping, \(f^{-1}(A)\) is both intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed in \((X, \tau)\). But this is a contradiction to hypothesis. Hence \((Y, \sigma)\) is an intuitionistic fuzzy generalized semi-pre connected space. \(\square\)

Definition 3.18. An IFTS \((X, \tau)\) is called intuitionistic fuzzy \(C_5\)-connected between two IFSs \(A\) and \(B\) if there is no intuitionistic fuzzy open set \(E\) in \((X, \tau)\) such that \(A \subseteq E\) and Eq\(^{-1}\)\(B\).

Definition 3.19. An IFTS \((X, \tau)\) is called intuitionistic fuzzy generalized semi-pre connected between two IFSs \(A\) and \(B\) if there is no intuitionistic fuzzy generalized semi-pre open set \(E\) in \((X, \tau)\) such that \(A \subseteq E\) and Eq\(^{-1}\)\(B\).

Example 3.20. Let \(X = \{a, b\}\) and \(\tau = \{0_\infty, M, 1_\infty\}\) be an IFT on \(X\), where \(M = \{x, (0.5_x, 0.3_b), (0.5_x, 0.1_b)\}\). Then \((X, \tau)\) is an intuitionistic fuzzy generalized semi-pre connected between the two IFSs \(A = \{x, (0.5_a, 0.4_b), (0.5_a, 0.3_b)\}\) and \(B = \{x, (0.5_a, 0.2_b), (0.5_a, 0.5_b)\}\).

Theorem 3.21. If an IFTS \((X, \tau)\) is intuitionistic fuzzy generalized semi-pre connected between two IFSs \(A\) and \(B\), then it is intuitionistic fuzzy \(C_5\)-connected between two IFSs \(A\) and \(B\) but the converse may not be true in general.

Proof. Suppose \((X, \tau)\) is not intuitionistic fuzzy \(C_5\)-connected between \(A\) and \(B\), then there exists an intuitionistic fuzzy open set \(E\) in \((X, \tau)\) such that \(A \subseteq E\) and Eq\(^{-1}\)\(B\). Since every intuitionistic fuzzy open set is intuitionistic fuzzy generalized semi-pre open set, there exists an intuitionistic fuzzy generalized semi-pre open set \(E\) in \((X, \tau)\) such that \(A \subseteq E\) and Eq\(^{-1}\)\(B\). This implies \((X, \tau)\) is not intuitionistic fuzzy generalized semi-pre connected between \(A\) and \(B\), a contradiction to our hypothesis. Therefore \((X, \tau)\) is intuitionistic fuzzy \(C_5\)-connected between \(A\) and \(B\). \(\square\)
Example 3.22. Let \( X = \{a, b\} \) and \( \tau = \{0, \dot{\tau}, 1\} \) be an IFT on \( X \), where \( M = \langle x, (0.4_a, 0.3_b), (0.2_a, 0.3_b) \rangle \). Then \( (X, \tau) \) is an intuitionistic fuzzy connected between the IFSs \( A = \langle x, (0.4_a, 0.4_b), (0.6_a, 0.6_b) \rangle \) and \( B = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle \). But \( (X, \tau) \) is not an intuitionistic fuzzy generalized semi-pre connected between \( A \) and \( B \), since the IFS \( E = \langle x, (0.4_a, 0.4_b), (0.5_a, 0.5_b) \rangle \) is an intuitionistic fuzzy generalized semi-pre open set such that \( A \subseteq E \) and \( E \subseteq B^c \).

Theorem 3.23. An IFTS \( (X, \tau) \) is intuitionistic fuzzy generalized semi-pre connected between two IFSs \( A \) and \( B \) if and only if there is no intuitionistic fuzzy generalized semi-pre connected between two IFSs \( A \) and \( B \) such that \( A \subseteq E \) and \( E \subseteq B^c \).

Proof. Necessity: Let \( (X, \tau) \) be intuitionistic fuzzy generalized semi-pre connected between two IFSs \( A \) and \( B \). Suppose that there exists an intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed set \( E \) in \( (X, \tau) \) such that \( A \subseteq E \subseteq B^c \). Then \( Eq^c B \) and \( A \subseteq E \). This implies \( (X, \tau) \) is not intuitionistic fuzzy generalized semi-pre connected between \( A \) and \( B \), by Definition 3.19. A contradiction to our hypothesis. Therefore there is no intuitionistic fuzzy generalized semi-pre open and intuitionistic fuzzy generalized semi-pre closed set \( E \) in \( (X, \tau) \) such that \( A \subseteq E \subseteq B^c \).

Sufficiency: Suppose that \( (X, \tau) \) is not intuitionistic fuzzy generalized semi-pre connected between \( A \) and \( B \). Then there exists an intuitionistic fuzzy generalized semi-pre open set \( E \) in \( (X, \tau) \) such that \( A \subseteq E \subseteq B^c \). This implies that there is no intuitionistic fuzzy generalized semi-pre open set \( E \) in \( (X, \tau) \) such that \( A \subseteq E \subseteq B^c \). But this is a contradiction to our hypothesis. Hence \( (X, \tau) \) is intuitionistic fuzzy generalized semi-pre connected between \( A \) and \( B \). □

Theorem 3.24. If an IFTS \( (X, \tau) \) is intuitionistic fuzzy generalized semi-pre connected between two IFSs \( A \) and \( B \), \( A \subseteq A_1 \) and \( B \subseteq B_1 \), then \( (X, \tau) \) is intuitionistic fuzzy generalized semi-pre connected between \( A_1 \) and \( B_1 \).

Proof. Suppose that \( (X, \tau) \) is not intuitionistic fuzzy generalized semi-pre connected between \( A_1 \) and \( B_1 \), then by Definition 3.19, there exists an intuitionistic fuzzy generalized semi-pre open set \( E \) in \( (X, \tau) \) such that \( A_1 \subseteq E \) and \( Eq^c B_1 \). This implies \( E \subseteq B_1^c \) and \( A_1 \subseteq E \) implies \( A \subseteq A_1 \subseteq E \). That is \( A \subseteq E \). Now let us prove that \( E \subseteq B^c \), that is let us prove \( Eq^c B \). Suppose that \( Eq B \), then by Definition 2.12, there exists an element \( x \) in \( X \) such that \( \mu_E(x) > \nu_B(x) \) and \( \nu_E(x) < \mu_B(x) \). Therefore \( \mu_E(x) > \nu_B(x) \) and \( \nu_E(x) < \mu_B(x) \), since \( B \subseteq B_1 \), \( Eq B_1 \). But \( E \subseteq B_1 \). That is \( Eq^c B_1 \), which is a contradiction. Therefore \( Eq^c B_1 \). That is \( E \subseteq B^c \). Hence \( (X, \tau) \) is not intuitionistic fuzzy generalized semi-pre connected between \( A \) and \( B \), which is a contradiction to our hypothesis. Thus \( (X, \tau) \) is intuitionistic fuzzy generalized semi-pre connected between \( A_1 \) and \( B_1 \). □

Theorem 3.25. Let \( (X, \tau) \) be an IFTS and \( A \) and \( B \) be IFSs in \( (X, \tau) \). If \( A \cap B \), then \( (X, \tau) \) is intuitionistic fuzzy generalized semi-pre connected between \( A \) and \( B \).

Proof. Suppose \( (X, \tau) \) is not intuitionistic fuzzy generalized semi-pre connected between \( A \) and \( B \). Then there exists an intuitionistic fuzzy generalized semi-pre open set \( E \) in \( (X, \tau) \) such that \( A \subseteq E \) and \( E \subseteq B^c \). This implies that \( A \subseteq B^c \). That is
Remark 3.26. The converse of the above theorem may not be true in general.

Example 3.27. In Example 3.20, (X, τ) is intuitionistic fuzzy generalized semi-pre connected between the IFSs A and B but not q-coincident with B, since μ_A(x) < ν_B(x) and μ_B(x) < ν_A(x).

Definition 3.28. An intuitionistic fuzzy generalized semi-pre open set A is called an intuitionistic fuzzy regular generalized semi-pre open set if A = gspcl(gspint(A)). The complement of an intuitionistic fuzzy regular generalized semi-pre open set is called an intuitionistic fuzzy regular generalized semi-pre closed set.

Definition 3.29. An IFTS (X, τ) is called an intuitionistic fuzzy generalized semi-pre super connected space if there exists no intuitionistic fuzzy regular generalized semi-pre open set in (X, τ).

Theorem 3.30. Let (X, τ) be an IFTS, then the following are equivalent.

(i) (X, τ) is an intuitionistic fuzzy generalized semi-pre super connected space.

(ii) For every non-zero intuitionistic fuzzy regular generalized semi-pre open set A, gspcl(A) = 1_∞.

(iii) For every intuitionistic fuzzy regular generalized semi-pre closed set A with A ≠ 1_∞, gspint(A) = 0_∞.

(iv) There exists no intuitionistic fuzzy regular generalized semi-pre open sets A and B in (X, τ) such that A ≠ 0_∞ ≠ B, A ⊆ B^c.

(v) There exists no intuitionistic fuzzy regular generalized semi-pre open sets A and B in (X, τ) such that A ≠ 0_∞ ≠ B, A ≠ B = (gspcl(A))^c, A ≠ (gspcl(B))^c.

(vi) There exists no intuitionistic fuzzy regular generalized semi-pre closed sets A and B in (X, τ) such that A ≠ 1_∞ ≠ B, A ≠ (gspint(A))^c, A ≠ (gspint(B))^c.

Proof. (i)⇒(ii) Assume that there exists an intuitionistic fuzzy regular generalized semi-pre open set A in (X, τ) such that A ≠ 0_∞ and gspcl(A) ≠ 1_∞. Now let B = gspint(gspcl(A))^c. Then B is a proper intuitionistic fuzzy regular generalized semi-pre open set in (X, τ). But this is a contradiction to the fact that (X, τ) is an intuitionistic fuzzy generalized semi-pre super connected space. Therefore gspcl(A) = 1_∞.

(ii)⇒(iii) Let A ≠ 1_∞ be an intuitionistic fuzzy regular generalized semi-pre closed set in (X, τ). If B = A^c, then B is an intuitionistic fuzzy regular generalized semi-pre open set in (X, τ) with B ≠ 0_∞. Hence gspcl(B) = 1_∞. This implies (gspcl(B))^c = 0_∞. That is gspint(B^c) = 0_∞. Hence gspint(A) = 0_∞.

(iii)⇒(iv) Let A and B be two intuitionistic fuzzy regular generalized semi-pre open sets in (X, τ) such that A ≠ 0_∞ ≠ B, A ⊆ B^c. Since B^c is an intuitionistic fuzzy regular generalized semi-pre closed set in (X, τ) and B ≠ 0_∞ implies B^c ≠ 1_∞, B^c = gspcl(gspint(B^c)) and we have gspint(B^c) = 0_∞. But A ⊆ B^c. Therefore 0_∞ ≠ A = gspint(gspcl(A)) ⊆ gspint(gspcl(B^c)) = gspint(gspcl(gspcl(gspint(B^c)))) = gspint(gspcl(gspint(B^c))) = gspint(B^c) = 0_∞. A contradiction arises. Therefore (iv) is true.
(iv)⇒(i) Let $0_\sim \neq A \neq 1_\sim$ be an intuitionistic fuzzy regular generalized semi-pre open set in $(X, \tau)$. If we take $B = (\text{gspcl}(A))^c$, then $B$ is an intuitionistic fuzzy regular generalized semi-pre open set, since $\text{gspint}(\text{gspcl}(B)) = \text{gspint}(\text{gspcl}(\text{gspcl}(A))^c) = \text{gspint}(\text{gspint}(\text{gspcl}(A)))^c = \text{gspint}(\text{gspcl}(A))^c = (\text{gspcl}(A))^c = B$. Also we get $B \neq 1_\sim$, since otherwise, we have $B = 0_\sim$ and this implies $(\text{gspcl}(A))^c = 0_\sim$. This is $\text{gspcl}(A) = 1_\sim$. Hence $A = \text{gspint}(\text{gspcl}(A)) = \text{gspint}(1_\sim) = 1_\sim$. This is $A = 1_\sim$, which is a contradiction. Therefore $B \neq 0_\sim$ and $A \subseteq B^c$. But this is a contradiction to (iv). Therefore $(X, \tau)$ is an intuitionistic fuzzy generalized semi-pre super connected space.

(i)⇒(v) Let $A$ and $B$ be two intuitionistic fuzzy regular generalized semi-pre open sets in $(X, \tau)$ such that $A \neq 0_\sim \neq B$, $B = (\text{gspcl}(A))^c$ and $A = (\text{gspcl}(B))^c$. Now we have $\text{gspint}(\text{gspcl}(A)) = \text{gspint}(B^c) = (\text{gspcl}(B))^c = A$, $A \neq 0_\sim$ and $A \neq 1_\sim$, since if $A = 1_\sim$, then $1_\sim = (\text{gspcl}(B))^c \Rightarrow \text{gspcl}(B) = 0_\sim \Rightarrow B = 0_\sim$. But $B \neq 0_\sim$. Therefore $A \neq 1_\sim \Rightarrow A$ is proper intuitionistic fuzzy regular generalized semi-pre open set in $(X, \tau)$, which is a contradiction to (i). Hence (v) is true.

(v)⇒(i) Let $A$ be an intuitionistic fuzzy regular generalized semi-pre open set in $(X, \tau)$ such that $A = \text{gspint}(\text{gspcl}(A))$ and $0_\sim \neq A \neq 1_\sim$. Now take $B = (\text{gspcl}(A))^c$. In this case we get $B \neq 0_\sim$ and $B$ is intuitionistic fuzzy regular generalized semi-pre open set in $(X, \tau)$, $B = (\text{gspcl}(A))^c$ and $(\text{gspcl}(B))^c = (\text{gspcl}(\text{gspcl}(A)))^c = \text{gspint}(\text{gspcl}(A))^c = \text{gspint}(\text{gspcl}(A)) = A$. But this is a contradiction to (v). Therefore $(X, \tau)$ is an intuitionistic fuzzy generalized semi-pre super connected space.

(v)⇒(vi) Let $A$ and $B$ be two intuitionistic fuzzy regular generalized semi-pre closed sets in $(X, \tau)$ such that $A \neq 1_\sim \neq B$, $B = (\text{gspint}(A))^c$ and $A = (\text{gspint}(B))^c$. Taking $C = A^c$ and $D = B^c$, $C$ and $D$ become intuitionistic fuzzy regular generalized semi-pre open sets in $(X, \tau)$ with $C \neq 0_\sim \neq D$, $D = (\text{gspcl}(C))^c$ and $C = (\text{gspcl}(D))^c$, which is a contradiction to (v). Hence (vi) is true.

(vi)⇒(v) can be easily proved by the similar way as in (v)⇒(vi).

□

Definition 3.31. An IFTS $(X, \tau)$ is said to be an intuitionistic fuzzy generalized semi-pre extremally disconnected if the generalized semi-pre closure of every intuitionistic fuzzy generalized semi-pre open set in $(X, \tau)$ is an intuitionistic fuzzy generalized semi-pre open set.

Theorem 3.32. Let $(X, \tau)$ be an intuitionistic fuzzy generalized semi-pre $T_{1/2}$ space, then the following are equivalent.

(i) $(X, \tau)$ is an intuitionistic fuzzy generalized semi-pre extremally disconnected space.

(ii) For each intuitionistic fuzzy generalized semi-pre closed set $A$, $\text{gspint}(A)$ is an intuitionistic fuzzy generalized semi-pre closed set.

(iii) For each intuitionistic fuzzy generalized semi-pre open set $A$, $\text{gspcl}(A) = (\text{gspcl}(\text{gspcl}(A)))^c$.

(iv) For each intuitionistic fuzzy generalized semi-pre open sets $A$ and $B$ with $\text{gspcl}(A) = B^c$, $\text{gspcl}(A) = (\text{gspcl}(B))^c$.

Proof. (i)⇒(ii) Let $A$ be any intuitionistic fuzzy generalized semi-pre closed set. Then $A^c$ is an intuitionistic fuzzy generalized semi-pre open set. So (i) implies that $\text{gspcl}(A^c) = (\text{gspint}(A))^c$ is an intuitionistic fuzzy generalized semi-pre open set. Therefore $\text{gspcl}(A)$ is an intuitionistic fuzzy generalized semi-pre closed set in $(X, \tau)$. 

252
(ii)⇒(iii) Let \( A \) be an intuitionistic fuzzy generalized semi-pre open set. Then we have \( \text{gspcl}(\text{gspcl}(A))^c = \text{gspcl}(\text{gspint}(A^c)) \). Therefore \( (\text{gspcl}(\text{gspcl}(A))^c)^c = (\text{gspcl}(\text{gspint}(A^c)))^c \). Since \( A \) is an intuitionistic fuzzy generalized semi-pre open set, \( A^c \) is an intuitionistic fuzzy generalized semi-pre closed set. So by (ii) \( \text{gspint}(A^c) \) is an intuitionistic fuzzy generalized semi-pre closed set. That is \( \text{gspcl}(\text{gspint}(A^c)) = \text{gspint}(A^c) \). Hence \( (\text{gspcl}(\text{gspint}(A^c)))^c = (\text{gspint}(A^c))^c = \text{gspcl}(A) \).

(iii)⇒(iv) Let \( A \) and \( B \) be any two intuitionistic fuzzy generalized semi-pre open sets in \( (X, \tau) \) such that \( \text{gspcl}(A) = B^c \). (iii) implies \( \text{gspcl}(A) = (\text{gspcl}(\text{gspcl}(A))^c)^c = (\text{gspcl}(B))^c \).

(iv)⇒(i) Let \( A \) be any intuitionistic fuzzy generalized semi-pre open set in \( (X, \tau) \). Put \( B = (\text{gspcl}(A))^c \). Then \( \text{gspcl}(A) = B^c \). Hence by (iv), \( \text{gspcl}(A) = (\text{gspcl}(B))^c \).

Since \( \text{gspcl}(B) \) is an intuitionistic fuzzy generalized semi-pre closed set as the space is an intuitionistic fuzzy semi-pre \( T_{1/2} \) space, it follows that \( \text{gspcl}(A) \) is an intuitionistic fuzzy generalized semi-pre open set. This implies that \( (X, \tau) \) is an intuitionistic fuzzy generalized semi-pre extremally disconnected space.

\[ \square \]

References


R. Santhi \( \text{(santhifuzzy@yahoo.co.in)} \)
Department of Mathematics, NGM College, Pollachi, Tamil Nadu, India

D. Jayanthi \( \text{(jayanthimaths@rediffmail.com)} \)
Department of Mathematics, NGM College, Pollachi, Tamil Nadu, India