ON THE EFFECTS OF RANDOM TIMING JITTER ON SPECTRUM SENSING ALGORITHMS BASED ON CYCLOSTATIONARITY

Mengüç Öner (Invited Paper)

Department of Electronics Engineering, Isik University, Kumbaba Mevkiı, 34980 Sile, Istanbul, Turkey,
Email: oner@isikun.edu.tr; Phone: +902165287136

ABSTRACT

Cognitive radio is an enabling technology, which is expected to lead to a more efficient utilization of the available spectral resources due to its flexibility and its ability to sense its spectral environment. Recently, spectrum sensing methods based on exploiting the cyclostationary characteristics of communication signals have been drawing interest. In practice, imperfections in the signal generation or reception may affect the cyclic statistics of a signal of interest, leading to a degradation in the performance of cyclostationarity-exploiting spectrum sensing schemes based on an ideal signal model. A typical source of imperfection is random timing jitter, which can occur at the transmitter side, most notably in the form of pulse timing jitter for digitally modulated signals, or at the receiver side in the form of sampling jitter. In this work, we explore the effect of random timing jitter on the second order cyclostationary statistics of wide sense cyclostationary signals. General analytical expressions are derived for the cyclic statistics of signals in the presence of sampling and pulse timing jitter and specific results are provided for cases of practical interest. Subsequently, the effect of the both jitter types on a cyclostationary-based spectrum sensing algorithm is investigated via simulations.

Index Terms— Cognitive radio, detection, spectrum sensing, jitter.

1. INTRODUCTION

Today’s wireless systems are largely characterized by static spectrum allocations, which lead to a largely inefficient use of expensive spectral resources. The emergence of Cognitive Radio (CR) technology is expected to offer an unprecedented flexibility in terms of spectrum usage, paving way to more efficient demand oriented spectrum allocation strategies. Opportunistic spectrum access schemes such as Spectrum Pooling can be regarded as the first steps into this direction [1].

Spectrum sensing (SS) is a key functionality of the CR, providing it with the required environmental awareness for an agile and flexible use of spectral resources. Recently, there has been an increasing interest on spectrum sensing methods based on exploiting the distinct cyclostationary characteristics of communication signals, mainly due to the inherent robustness of such algorithms to noise and interference, and the signal selectivity they offer (see, for example [2], [3], [4],[5]). Virtually all communication signals exhibit cyclostationarity with cycle frequencies related to hidden periodicities underlying the signal, which can be used for this task, whereas some of the recently proposed methods also involve artificially embedding distinct cyclic signatures in signals [6]. SS algorithms based on cyclostationarity usually require an estimate of the second order cyclic statistics of the sampled version of the received signal, i.e. the cyclic autocorrelation function (CAF) or the spectral correlation density (SCD), in order to detect the presence of the cyclic signature of a signal of interest in the frequency environment. In practice, imperfections in the signal generation or reception may affect the cyclic statistics of a signal of interest, leading to a degradation in the performance of cyclostationarity-exploiting SS algorithms based on an ideal signal model. A typical source of imperfection is random timing jitter, which can occur at the transmitter in the form of pulse timing jitter for digitally modulated signals, or at the receiver side in the form of sampling jitter.

In this work, we investigate the effects of random pulse timing jitter and random sampling jitter on SS methods based on cyclostationarity. First, general analytical expressions are provided for the second order cyclic statistics of signals in presence of pulse-timing and sampling jitter and specific results are evaluated for cases of interest. Subsequently, the effect of both jitter types on a SS algorithm based on Dan-dawate’s work in [8] is investigated via simulations.

2. PRELIMINARIES

A complex continuous time wide sense cyclostationary (WSCS) process $x(t)$ is characterized by a time varying autocorrelation function (TVAF) $R_x(t,\tau) = E\{x(t)x^*(t-\tau)\}$, which is periodic in time $t$ with a fundamental period $T_f$ and can be represented as a Fourier series:

$$R_x(t,\tau) = \sum_{\alpha} R_x^{\alpha}(\tau)e^{j2\pi\alpha t}$$  \hspace{1cm} (1)
where the sum is taken over integer multiples of fundamental cycle frequency \( \alpha f = 1/T_f \), \( R^\alpha_x (\tau) \) is referred to as the cyclic autocorrelation function, and is nonzero only for \( \alpha = 1/T_f \), i.e. for integer multiples of the fundamental cycle frequency. The SCD \( S^\alpha_x (f) \) is defined as the Fourier transform of \( R^\alpha_x (\tau) \) w.r.t. \( \tau \) and can be seen as a measure of correlation between the spectral components of \( x(t) \) separated in frequency by an amount of \( \alpha \). Clearly, for \( \alpha = 0 \), the SCD is equal to the power spectral density (PSD) of \( x(t) \). Similarly, a complex discrete time WSCS process \( \tilde{x}_n \) has a time varying autocorrelation function \( \tilde{R}_\tilde{x}[n,k] = E\{\tilde{x}_n \tilde{x}_{n-k}^*\} \), periodic in \( n \) with a period of \( N \), which allows a discrete time Fourier series representation:

\[
\tilde{R}_\tilde{x}[n,k] = \sum_\beta \tilde{R}_\tilde{x}^{\beta}[k] e^{2\pi i \beta n}
\]  

(2)

Where the sum is taken over \( N \) integer multiples of the fundamental cycle frequency \( \beta_f = 1/N \). Note that the discrete-time CAF \( \tilde{R}_\tilde{x}^{\beta}[k] \) is periodic in \( \beta \) with a period of 1 and is nonzero only for integer multiples of \( \beta_f \). The discrete time Fourier transform of the discrete-time CAF results in an SCD \( \tilde{S}_\tilde{x}^{\beta}(\omega) \) which is also periodic in \( \omega \) with a period of \( 2\pi \).

Cyclostationarity based signal processing techniques usually require an estimate of the cyclic statistics of the sampled version of the received signal. It is easily shown that a discrete-time random signal \( \tilde{x}_n \) generated by sampling a continuous-time WSCS signal \( x(t) \) with a fundamental cyclostationarity period \( T_f \) is itself WSCS with a fundamental period \( N \), provided that \( T_f \) is an integer multiple of the sampling time \( T_s \), i.e. \( T_f/T_s = N \). In such a case, the CAF and SCD functions of \( \tilde{x}_n = x(nT_s) \) can be expressed in terms of the CAF and SCD functions of the continuous time process \( x(t) \) in the following manner:

\[
\tilde{R}_\tilde{x}^{\beta+\frac{m}{T_s}}(k) = \sum_{p=-\infty}^{\infty} R_x^{\alpha+\frac{m}{T_f}+\frac{p}{T_s}}(kT_s),
\]  

(3)

\[
\tilde{S}_\tilde{x}^{\beta+\frac{m}{T_s}}(\omega) = \frac{1}{T_s} \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} S_x^{\alpha+\frac{m}{T_f}+\frac{p}{T_s}}(\frac{(\omega-q)}{2\pi T_s} + \frac{q}{T_s})
\]  

(4)

(See also [9]). From (4) it is evident that aliasing may take place in both frequency and cycle-frequency domains. However, it can be easily shown that for a WSCS signal bandlimited to the frequency interval [-B,B], aliasing in the \( f \) and \( \alpha \) domains can be prevented by choosing

\[
1/T_s \geq 4B.
\]  

(5)

In such a case we can write:

\[
\tilde{S}_\tilde{x}^{\beta+\frac{m}{T_s}}(\omega) = \frac{1}{T_s} S_x^{\alpha+\frac{p}{T_f}}(\frac{\omega}{2\pi T_s})
\]  

(6)

for \( |\omega| \leq \pi \) and \( |m| \leq N/2 \) (see also [9]).

3. EFFECT OF THE PULSE TIMING JITTER ON THE CYCLIC STATISTICS

A large class of signal structures used in communications, from direct sequence spread spectrum (DS/SS) to continuous phase modulated (CPM) signals, can be expressed in terms of (or, in case of CPM signals, approximated by) a pulse stream modulated by a complex discrete time wide sense stationary (WSS) or, more generally, wide sense cyclostationary (WSCS) data sequence, both of which result in a continuous time WSCS process. A pulse stream modulated by a discrete-time sequence in presence of random pulse timing jitter can be modeled as:

\[
x(t) = \sum_{n=-\infty}^{\infty} a_n p(t - nT_p - \epsilon_n)
\]  

(7)

with the modulating sequence \( a_n \), assumed to be WSCS with a cyclostationarity period of \( N_a \), the pulse waveform \( p(t) \), the pulse interval \( T_p \) and the stationary jitter sequence \( \epsilon_n \), assumed to be independent of \( a_n \). It is easily shown that \( x(t) \) is WSCS with a fundamental period of \( N_a T_p \). In [7] we have shown that, defining the second order joint characteristic function \( C(\theta, \phi, k) = E \{ e^{-j2\pi\theta n} e^{j2\pi\phi n-k} \} \), and expressing the pulse function \( p(t) \) in terms of its Fourier transform \( p(t) = \int_{-\infty}^{\infty} P(f) e^{j2\pi ft} df \), the SCD of \( x(t) \) can be given as:

\[
S_x^{\alpha=\frac{1}{N_a T_p}}(f) = \frac{1}{T_p} \sum_{k=-N_a}^{N_a} C(f + \frac{l}{N_a T_p}, f, k) R_{\alpha=\frac{1}{N_a}}[k] e^{-j2\pi kT_p f} P(f + \frac{l}{N_a T_p}) P^*(f)
\]  

(8)

Equation (8) is general and apply to arbitrary stationary jitter statistics. Assuming that the jitter sequence is independent identically distributed (i.i.d.), we can write:

\[
C(\theta, \phi, k) = \delta(\theta) \delta(\phi) - \delta(\theta) \cdot \epsilon(\phi) - \delta(\phi) \cdot \epsilon(\theta) - \delta(\theta - \phi),
\]  

(9)

where \( \delta(\theta) = E \{ e^{-j2\pi \theta n} \} \), and \( \delta_p \) is the discrete time impulse function. Thus, the SCD expression simplifies to:

\[
S_x^{\alpha=\frac{1}{N_a T_p}}(f) = \frac{1}{T_p} P(f + \frac{l}{N_a T_p}) P^*(f) \epsilon(f + \frac{l}{N_a T_p})
\]  

\[
\times \epsilon^*(f) R_{\alpha=\frac{1}{N_a}}[0] e^{-j2\pi kT_p f} P(f + \frac{l}{N_a T_p}) P^*(f) R_{\alpha=\frac{1}{N_a}}[0]
\]  

\[
\times \left( \epsilon(f + \frac{l}{N_a T_p}) - \epsilon(f) \right) e^{j2\pi f T_p}
\]  

(10)

It is easily shown that \( |\epsilon(f)| \leq \epsilon(0) = 1 \) for any jitter distribution. Thus, the presence of stationary i.i.d. pulse timing jitter leads to a complex attenuation of the SCD, which is selective both in frequency \( f \) and cycle frequency \( \alpha \), and it adds an additional, previously nonexistent SCD term. The distortion of the SCD increases with increasing \( |\alpha| \). If the data
sequence $a_n$ is wide sense stationary (WSS), zero mean and uncorrelated, the SCD expression reduces to:

$$ S^\alpha_x = \frac{P}{T_p} \frac{\sigma^2_a}{T_p} P(f + \frac{l}{T_p}) P^*(f) \varepsilon (\frac{l}{T_p}) $$

(11)

Where $\sigma^2_a$ is the variance of the data sequence. The effect of the timing jitter for this case can be interpreted as a complex attenuation of the SCD, which is selective only in the cycle frequency $\alpha$. The corresponding expressions for the discrete-time SCD functions, $R_y(\omega)$, of the sampled version of the signal can be easily calculated by substituting (11) and (10) in (6), assuming the pulse waveforms are bandlimited and the sampling is performed according to (5). A more in-depth discussion on the effects of the pulse timing jitter on the second order cyclic statistics can be found in [7].

4. EFFECT OF THE SAMPLING JITTER ON THE CYCLIC STATISTICS

Assuming that the timing of the sampling instants are subject to random jitter, the discrete-time process $y_n$ resulting from sampling the continuous time WSCS signal $x(t)$ can be modeled as

$$ y_n = x(nT_s - \epsilon_n) $$

(12)

where $\epsilon_n$ is a discrete-time random process representing the jitter. The TVAF of $y_n$ can be written in terms of $R_x(t, \tau)$:

$$ \tilde{R}_y[n, k] = E\{R_x(nT_s - \epsilon_n, kT_s - \epsilon_n + \epsilon_n - k)\} $$

(13)

The expectation operation is performed over the jitter process. Assuming that the jitter is independent of $x(t)$, after substituting (1) in (13) and expressing the CAF in terms of the SCD, (13) can be reexpressed as

$$ \tilde{R}_y[n, k] = \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} S^\alpha_x = \frac{\pi}{T_f} e^{i2\pi fjT_f} e^{-i2\pi ln/N} $$

$$ \times \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} S^\alpha_x = \frac{\pi}{T_f} e^{i2\pi fjT_f} e^{-i2\pi ln/N} $$

$$ \times E\{e^{-i2\pi f(\epsilon_n - \epsilon_n - k)} e^{-i2\pi \epsilon_n / T_f} \} \} $$

$$ \times C(f + \frac{m}{T_f} + \frac{p}{T_s}, f, e^{i2\pi fjT_f}) $$

(14)

Thus, the CAF of $y_n$ is periodic in $n$ with a period $N$, i.e. $y_n$ is WSCS with the same fundamental cycle frequency $\tilde{x}_n$, provided that the jitter process is stationary. The CAF of $y_n$ can be expressed in terms of the SCD function of $x(t)$ and $C(\theta, \phi, \kappa)$:

$$ \tilde{R}_y[n, k] = \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} S^\alpha_x = \frac{\pi}{T_f} e^{i2\pi fjT_f} e^{-i2\pi ln/N} $$

$$ \times C(f + \frac{m}{T_f} + \frac{p}{T_s}, f, e^{i2\pi fjT_f}) $$

(15)

and the SCD can be calculated by fourier transforming (15) w.r.t. to index $k$. If the jitter sequence is (i.i.d.), the CAF of $y_n$ becomes:

$$ \hat{R}_y = \frac{\pi}{T_f} \hat{R}_y[k] = \left\{ \begin{array}{ll}
\sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} S^\alpha_x = \frac{\pi}{T_f} e^{i2\pi fjT_f} e^{-i2\pi ln/N} \\
\times \varepsilon^*(f) e^{i2\pi ftT_f} df, & k \neq 0 \\
\sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} S^\alpha_x = \frac{\pi}{T_f} e^{i2\pi fjT_f} e^{-i2\pi ln/N} \\
\times \varepsilon(\frac{m}{T_f} + \frac{p}{T_s}) df, & k = 0
\end{array} \right. $$

Finally, fourier transforming the CAF leads to the SCD function:

$$ \hat{S}_y = \frac{\pi}{T_f} \hat{S}_y(\omega) = \frac{1}{T_s} \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} S^\alpha_x = \frac{\pi}{T_f} e^{i2\pi fjT_f} e^{-i2\pi ln/N} $$

$$ \times \varepsilon^*(f) e^{i2\pi ftT_f} df, $$

(16)

Assuming that $x(t)$ is bandlimited and condition (5) is satisfied, we can write:

$$ \hat{S}_y(\omega) = \frac{1}{T_s} \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} S^\alpha_x = \frac{\pi}{T_f} e^{i2\pi fjT_f} e^{-i2\pi ln/N} $$

$$ \times \varepsilon^*(f) e^{i2\pi ftT_f} df $$

$$ + \int_{-\infty}^{\infty} S^\alpha_x = \frac{\pi}{T_f} e^{i2\pi fjT_f} e^{-i2\pi ln/N} $$

$$ \times \varepsilon^*(f) e^{i2\pi ftT_f} df $$

(17)

for $|\omega| \leq \pi$ and $|m| \leq N/2$.

Comparing (17) with (6), it is evident that the presence of random i.i.d. sampling jitter leads to a complex attenuation of the SCD, selective both in the frequency parameter $\omega$ and cycle frequency parameter $\beta$, which can be seen in the first summand of (17) and it adds an additional, previously non-existent constant term to the SCD function given in the second summand of (17). The magnitude of the attenuation increases with increasing $|\beta|$, for $|\beta| \leq 1/2$. Obviously, the amount of distortion in the SCD and the CAF functions increases with increasing jitter variance. In the remaining part of this section, the effect of the sampling jitter on the cyclic statistics is investigated for two examples of practical interest.

4.1. SCD of a Digital Pulse Stream With i.i.d. Sampling Jitter

A pulse stream modulated by a WSS discrete-time sequence can be modeled using eq (7) by setting the pulse timing jitter to zero. The SCD of $x(t)$ can be given as:

$$ S^\alpha_x = \frac{\pi}{T_f} S_a(2\pi fT_p) P(f + \frac{m}{T_f}) P^*(f), $$

(18)

where $S_a(\omega)$ is the PSD of $a_n$. Assuming that the sampling is performed such that $T_s = T_p/N$ and the condition (5) is
satisfied, the resulting signal $y_n$ is WSCS with a period $N$. The SCD can be calculated by substituting (18) in (17):

$$
\tilde{S}_y^{\beta-\frac{m}{N}}(\omega) = \frac{S_y(\omega N)}{T_p T_s} P\left(\frac{\omega}{2\pi T_s} + \frac{m}{T_p}\right) \\
\times P^*(\omega + \frac{m}{T_p}) e^{-\frac{\omega^2}{2}} e^{\frac{i\omega m}{T_p}} \\
+ \frac{1}{T_p} \int_{-\infty}^{\infty} S_a(2\pi f T_p) P(f + \frac{m}{T_p}) P^*(f) \\
\times e^{i\left(\frac{m}{T_p} f - \frac{m}{T_p} e^{i\omega f}\right)} df
$$

for $|\omega| \leq \pi$ and $|m| \leq N/2$. Fig.1 shows the normalized estimate of the magnitude of the SCD function for a QPSK modulated pulse stream where a raised cosine pulse waveform with a roll-off factor $\rho = 0.9$ is employed, the sampling time is chosen such that $N = T_p/T_s = 4$ and the modulating data sequence is uncorrelated. Three different cases are displayed: jitter free sampling, Gaussian distributed jitter with variance $\sigma^2 = 0.25 T_s^2$ and $\sigma^2 = 0.5 T_s^2$. Note that even in the presence of sampling jitter with a rather high variance, the attenuation in the main lobes of the SCD surfaces due to jitter is very small, and the magnitude of the constant SCD term is so low that it can only be perceived if logarithmic scale is used. Clearly, this is due to the fact that $\sigma$ is too small compared to the pulse width as a result of oversampling, and for higher harmonics of the cycle frequency, for which the attenuation could have become effective despite the oversampling, the signal does not exhibit any significant amount of spectral correlation to begin with.

4.2. SCD of an OFDM signal With i.i.d. Sampling Jitter

The baseband signal model for an OFDM signal is given as:

$$
x(t) = \sqrt{\frac{1}{N_c}} \sum_{n=-\infty}^{\infty} \sum_{i=0}^{N_c-1} d_{n,i} e^{j2\pi i f_N (t-n T_d)} \\
\times g_n(t-n T_d) e^{-\frac{j2\pi n f_N}{T_d}} e^{j\frac{2\pi n f_N}{T_d} t}
$$

where $d_{n,i}$ is the $n$th information symbol modulated on the $i$th carrier, $N_c$ is the number of carriers, $\Delta_f$ is the carrier separation, and $g_n(t)$ is the rectangular pulse function of length $T_d$, $T_d = T_u + T_g$ is the symbol length, where $T_u = 1/\Delta_f$ is the useful symbol duration and $T_g$ is the length of the guard interval where the symbol is extended cyclically. It is straightforward to show that the OFDM signal is WSCS with a fundamental period of $T_d$. Assuming that the data sequence is zero mean and uncorrelated, the SCD can be calculated as:

$$
S_x^{\alpha-\frac{m}{T_d}}(f) = \frac{A}{T_d} \sum_{i=0}^{N_c-1} G_R(f - i \Delta_f + \frac{N_c-1}{2} \Delta_f + \frac{m}{T_d}) \\
G_R^*(f - i \Delta_f + \frac{N_c-1}{2})
$$

with $A = \sigma^2/\sqrt{N_c}$, $\sigma^2 = E\{d_{n,i} d_{n,i}^*\}$ and $G_R(f) = \sin(\pi f T_d)/\pi f$, the Fourier transform of $g_R(t)$ [2]. Obviously, $G_R(f)$ is not bandlimited, thus (5) cannot be satisfied and the SCD of the sampled OFDM signal in presence of sampling jitter is calculated by substituting (20) in (16). Fig. 2 displays the normalized magnitude estimate of the SCD function of a sampled OFDM signal where $N_c = 12$, $N = T_d/T_s = 20$, $T_g = T_d/5$. Clearly, in contrast to the previous case, the SCD of the signal exhibits a large spectral occupation, even for higher harmonics of the fundamental cycle frequency, and the sampling rate is only slightly higher than the Nyquist rate,
The test statistic for the employed SS algorithm is given as

$$Z_x = N_o \hat{f}_x \hat{\Sigma}_x^{-1} \hat{f}_x^T,$$  \hspace{1cm} (23)$$

thus we expect the cyclic statistics of this signal to be more sensitive to sampling jitter effects than that of the previous example, which is confirmed by Fig. 3 where the SCD magnitude of the signal in presence of Gaussian jitter with a variance $\sigma^2 = 0.25T_s^2$ is displayed. Unlike the previous case, the distortion of the SCD due to the sampling jitter is clearly discernable.

5. DETECTION RESULTS

In the following, the effect of the both jitter types on the detection performance of a cyclostationarity based SS algorithm is investigated via simulations. In this work, we employ a modified version of the time domain constant false alarm rate (CFAR) test for the presence of discrete-time cyclostationarity introduced by Dandawate et al. in [8] for this task.

5.1. The Detection Algorithm

The consistent estimate of the discrete time CAF of the received signal $x_n$ for a given cycle frequency $\beta_0$, is given as:

$$\hat{R}_x^{\beta_0}[k] = \frac{1}{N_o} \sum_{n=0}^{N_o-1} x_n x_{n-k} e^{-j2\pi \beta_0 n} = \hat{R}_x^{\beta_0}[k] + \Delta_x^{\beta_0}[k]$$

(21)

where $N_o$ is the length of the observation window and $\Delta_x^{\beta_0}[k]$ is the estimation error, which vanishes as $N_o \to \infty$. Using (21), the $1 \times 2L$ row vector consisting of cyclic autocorrelation estimates at the cycle frequency $\beta = \beta_0$ is defined as:

$$\hat{f}_x = \left[ Re \{ \hat{R}_x^{\beta_0}(k_1) \}, ..., Re \{ \hat{R}_x^{\beta_0}(k_L) \}, \right.$$

$$\left. Im \{ \hat{R}_x^{\beta_0}(k_1) \}, ..., Im \{ \hat{R}_x^{\beta_0}(k_L) \} \right]$$

(22)

The test statistic for the employed SS algorithm is given as [8]:

$$Z_x = N_o \hat{f}_x \hat{\Sigma}_x^{-1} \hat{f}_x^T,$$  \hspace{1cm} (23)$$

where $\hat{\Sigma}_x$ is the estimated asymptotic covariance matrix of the cyclic autocorrelation estimation error, which is calculated using fourth order cyclic statistics of the data (see [2], [3] and [8] for details). It can be shown that, if the signal of interest is absent, the distribution of $Z_x$ converges asymptotically to a central $\chi^2$ distribution with $2L$ degrees of freedom, irrespective of the distribution of the input data. Hence, for a given threshold, the false alarm probabilities can be analytically calculated for large enough observation intervals $N_o$, regardless of the particular signal, leading to an asymptotically constant false alarm rate (CFAR) test. If the signal of interest is present, the distribution of the test statistics converges to a normal distribution [8]:

$$\lim_{N_o \to \infty} Z_x \overset{D}{=} \mathcal{N}(N_o \hat{r}_x \hat{\Sigma}_x^{-1} \hat{r}_x^T, 4N_o \hat{r}_x \hat{\Sigma}_x^{-1} \hat{r}_x^T),$$

where $\hat{r}_x$ is the row vector consisting of the actual CAF values of the signal. Thus, any attenuation in the spectral correlation exhibited by the signal of interest is expected to lead to a decrease in the detection performance of the algorithm.

5.2. Simulation Results

In the following simulations, the effect of the pulse timing and sampling jitter on the performance of the detection algorithm in (23) is investigated. In both cases, the probability of detection $P_d$ vs. SNR curves are plotted for a fixed false alarm rate $P_f$ and different values of the jitter variance $\sigma^2$ and the observation interval length $T_o = N_o T_s$. Figure 4 displays the detection performance of the detector for a QPSK modulated pulse stream with a raised cosine pulse shape and a roll off factor $\rho = 1$ in presence of gaussian i.i.d. distributed pulse timing jitter with $\sigma^2 = 0$ (i.e. no jitter), $T_p^2/16$ and $T_p^2/32$ respectively. $P_f = 0.05$ is chosen. In fig.5, on the other hand, the effects of gaussian i.i.d. distributed sampling jitter on the

---

**Fig. 3.** Estimate of $|\hat{S}_d^0(\omega)|$ for an OFDM signal in the presence of Gaussian i.i.d. sampling jitter with $\sigma^2 = 0.25T_s^2$

**Fig. 4.** $P_d$ vs. SNR for a QPSK signal in presence of Gaussian i.i.d. pulse timing jitter, $P_f$ = 0.05
The emergence of previously nonexistent additive terms. For both jitter types, specific SCD results are evaluated for cases of practical interest. It is shown that the presence of i.i.d. jitter in both cases lead to a complex attenuation of the SCD and to the emergence of previously nonexistent additive terms. For both jitter types, specific SCD results are evaluated for cases of practical interest. The detrimental effect of both jitter types on the performance of a SS algorithm based on cyclostationarity is investigated by simulations. Future work will include a thorough statistical analysis of the jitter effects on other existing cyclostationarity based SS algorithms and the design and analysis of algorithms robust to the effects of random jitter.

6. CONCLUSION

In this work, the effects of random sampling- and pulse timing jitter on cyclostationarity based SS methods for cognitive radio systems are investigated. General analytical expressions are provided for the cyclic statistics of signals in presence of both jitter types and specific results are evaluated for cases of practical interest. It is shown that the presence of i.i.d. jitter in both cases lead to a complex attenuation of the SCD and to the emergence of previously nonexistent additive terms. For both jitter types, specific SCD results are evaluated for cases of practical interest. The detrimental effect of both jitter types on the performance of a SS algorithm based on cyclostationarity is investigated by simulations. Future work will include a thorough statistical analysis of the jitter effects on other existing cyclostationarity based SS algorithms and the design and analysis of algorithms robust to the effects of random jitter.

7. REFERENCES