Probabilistic Communication Complexity of
Boolean Relations

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Abstract

In [KW] it was proved that for every boolean function $f$ there exist a communication complexity game $R_f$ such that the minimal circuit-depth of $f$ exactly equals to the communication complexity of $R_f$. If $f$ is monotone then there also exists a game $R_f^m$ with communication complexity exactly equals to the monotone depth of $f$. It was also proved in [KW] that the communication complexity of $R_{st}$-connectivity$^m$ is $\Omega(\log^2 n)$, or equivalently that the monotone depth of the $st$-connectivity functions is $\Omega(\log^2 n)$.

In this paper we consider the games $R_f$ and $R_f^m$ in a probabilistic model of communication complexity, and prove that the communication complexity of $R_{st}$-connectivity$^m$ is $\Omega(\log^2 n)$ even in the probabilistic case. We also prove that in every $NC^1$ circuit for $s$-$t$ connectivity at least a constant fraction of all input variables must be negated.