Optimization of Cooperative Beamforming for SC-FDMA Multi-User Multi-Relay Networks by Tractable D.C. Programming

Ha Hoang Kha, Member, IEEE, Hoang Duong Tuan, Member, IEEE, Ha H. Nguyen, Senior Member, IEEE, and Tung T. Pham

Abstract—This paper addresses the optimal cooperative beamforming design for multi-user multi-relay wireless networks in which the single-carrier frequency division multiple access (SC-FDMA) technique is employed at the terminals. The problem of interest is to find the beamforming weights across relays to maximize the minimum signal-to-interference-plus-noise ratio (SINR) among source users subject to individual power constraints at each relay. Such a beamforming design is shown to be a hard non-convex optimization problem and therefore it is mathematically challenging to find the optimal solution. By exploring its partial convex structures, we recast the design problem as minimization of a d.c. (difference of two convex) objective function subject to convex constraints and develop an effective iterative algorithm of low complexity to solve it. Simulation results show that our optimal cooperative beamforming scheme realizes the inherent diversity order of the relay network and it performs significantly better than the equal-power beamforming weights.

Index Terms—Cooperative beamforming, d.c. programming, frequency-domain equalization, maximin optimization, power allocation, SC-FDMA, wireless relay networks.

I. INTRODUCTION

DESIGNING transmission protocols and techniques for wireless relay networks has drawn phenomenal research interests in recent years. This is attributed to the capability of relaying technology in improving the transmission reliability and coverage range of wireless networks. The most popular relaying protocols studied in the literature are amplify-and-forward (AF) and decode-and-forward (DF). Although the performance advantage of one relaying protocol over the other strongly depends on the network configuration, topology and channel condition, the AF relaying is simpler in terms of signal processing at the relay(s). When the channel state information (CSI) is available at the relay(s), beamforming at each relay or across relays can provide significant performance gains. In fact, relay (or cooperative) beamforming designs have been actively investigated in many studies (e.g., [1]–[8] and references therein).

Most previous works on relay beamforming designs have been carried out for flat fading channels [1], [3], [4], [7], [9]. In [1], [3], the authors focused on the single source-destination transmission with the aid of multiple relays, while in [4], [7], the distributed relay beamforming for multiple source-destination pairs was considered. For frequency-selective fading channels, although orthogonal frequency division multiplexing (OFDM) is a powerful technique to combat frequency selectivity by converting the multi-path channel into a set of parallel frequency-flat channels, its major drawbacks are high peak-to-average power ratio (PAPR) and sensitivity to carrier frequency offset. These drawbacks make OFDM less attractive for applications that require inexpensive transmitters, such as in the uplink of cellular systems. A direct competitor of OFDM is the single-carrier frequency-division multiple access (SC-FDMA) technique, which has been adopted for the uplink transmission in the Third Generation Partnership Project (3GPPP) Long-Term Evolution (LTE) standard [10]–[12]. In essence, SC-FDMA exploits advantages of low-complexity equalization (similar to OFDM) and low PAPR (the same as for any single-carrier transmission) [13], [14].

There have been various optimization approaches to relay beamformings for flat fading channels [15]–[17], where the design problems can be reformulated as convex programs. Obviously, these approaches cannot be extended to SC-FDMA multiple relay beamforming designs since the function of SINR performance metric in SC-FDMA multiple relay systems is highly nonlinear nonconvex with respect to design parameters, as it will be shown in Section III. As far as cooperative beamforming designs for SC-FDMA relay networks are concerned, a beamforming scheme called “filter-and-forward” has been presented in [18], but only for the case of a single source. However, the work in [18] considers continuous single-carrier (SC) transmission and therefore requires time-domain equalization at the destination. The time-domain equalization may not be feasible for future broadband wireless networks as its complexity quickly increases with long channel impulse responses, a consequence of high data rate transmission.

Our previous work in [5] considers beamforming design for the scenario that a single relay helps multiple sources and when block-based SC-FDMA with frequency-domain equalization is employed at both the relay and destination. In [19], the relay selection for multiuser SC-FDMA uplink was considered, but the optimal design of relay beamforming was not involved. More
recently, [8] examines cooperative beamforming design for the same network as in [18] (i.e., single source, multiple relays and frequency-selective fading channels) but when block-based SC transmission and cyclic prefix are used. In particular, the authors in [8] obtain the optimal frequency-domain beamforming coefficients across the relays when the frequency-domain linear equalization is used at the destination. It is important to point out that [8] considers the total transmit power constraint. While such a total relay power constraint leads to a convex optimization problem (hence easier to solve), it does not fully reflect the physical limitation on the transmit power at each individual relay.

This paper makes an important and nontrivial extension from the case of one relay in [5] to the case of multiple relays. In particular, the paper examines wireless relay networks where multiple source nodes communicate with a destination node with the aid of multiple relays. This scenario is practically applicable to the uplink wireless transmission where multiple mobile terminals communicate with the base station, while idle mobile terminals serve as relay nodes [19]. An alternative scenario is deployment of a few fixed relay stations to assist signal transmission to the base station [14]. As far as signal processing at the relays is concerned, each relay performs frequency-domain equalization to mitigate signal distortions and then applies frequency-domain beamforming before forwarding the signal to the destination. The relay beamformers will adjust the amplitudes and phases of signals such that the received signals are most constructively combined at the destination [20]. The optimal beamforming weights can be calculated at the base station as all CSI is available, and then are sent to the relays [5], [17]. The technical problem to be dealt with in this work is how to design the optimal cooperative beamforming across the multiple relays. The problem of cooperative beamforming design is solved under signal-to-interference-plus-noise ratio (SINR) criterion. Specifically, to enable fairness among source users, the (nonsmooth) optimization of maximizing the minimum SINR among source users (i.e., SINR maximin problem) shall be studied. Multiple relays with their individual relay power constraints make the design problem more practically appealing but also pose very challenging issues of nonsmooth and nonconvex optimization for effective solution.

Given the difficulty in locating the global optimal solution of the nonconvex beamforming design problem, our focus is to find an efficient iterative algorithm with low complexity. A main contribution of the paper is to explore partial convex structures of the problem that are useful for optimization purpose. As a result, this highly nonconvex and nonsmooth optimization problem is recast by optimization of a d.c. (difference of two convex) objective function [21] subject to convex constraints. This effective and elegant d.c. representation, an iterative algorithm of sequential convex optimization is developed. The convergence of the proposed iterative algorithm is proved. It shall also be shown that, compared to the equal-power beamforming (in which each relay uses equal-power weights for each subcarrier of each source to share its available power), the proposed method leads to significant performance improvement.

The rest of this paper is organized as follows. Section II introduces the system model and develops signal processing operations at each node. The expression of the SINR, which is used in forming a performance criterion for cooperative beamforming design, is also derived in this section. Section III formulates the beamforming design as a d.c. optimization problem and proposes an iterative algorithm to find the solution. Simulation results are presented in Section IV. Finally, Section V gives concluding remarks.

Notations and Useful Facts. Bold-faced characters denote matrices and column vectors, with upper case used for the former and lower case for the latter. $0_N$, $1_N$ and $I_N$ are $N$-dimensional zero vector, $N$-dimensional vector with all entries equal to one, and the identity matrix of size $N \times N$, respectively. The $(n, m)$th entry of a matrix $X$ is shown by $X(n, m)$, while the $n$th entry of a vector $x$ is $x(n)$. $R^+_N = \{ x \in R^N : x(n) \geq 0, n = 1, 2, \ldots, N \}$. The notations $x^t$, $x|_t$, $x^2|_t$ and $x^r$ are component-wise understood, i.e., for $x \in C^N$ and $y \in C^N$, one has

$$x^t = (x^t(1), \ldots, x^t(N))^T,$$

$$x - (|x(1)|, \ldots, |x(N)|)^T,$$

$$x = \left( \frac{x(1)}{y(1)}, \ldots, \frac{x(N)}{y(N)} \right)^T,$$

$$x^+ = (x^+(1), \ldots, x^+(N)),$$

$$x^{-1} = \left( \frac{1}{x(1)}, \ldots, \frac{1}{x(N)} \right)^T = \frac{1}{x}I_N.$$
The notation $\mathbb{C}^{N \times N}$ denotes the complex-valued diagonal matrix space and $\delta(k)$ is the Kronecker symbol, i.e., $\delta(k) = 1$ if $k = 0$ and $\delta(k) = 0$ otherwise. We use $[\mathbf{x}]_C$ to refer to the circulant matrix with $\mathbf{x}$ as its first column. Note that $[\mathbf{x}]_C(n, n) = \mathbf{x}(1)$. Also a fundamental property for $\mathbf{x} \in \mathbb{C}^N$ is

$$
\mathcal{F}_N [\mathbf{x}]_C \mathcal{F}_N^H = \left[ \sqrt{N} \mathcal{F}_N \mathbf{x} \right]_D, \quad \text{and}
$$

$$
\mathcal{F}_N^H [\mathbf{x}]_D \mathcal{F}_N = \left[ \frac{1}{\sqrt{N}} \mathcal{F}_N^H \mathbf{x} \right]_C.
$$

(1)

The covariance matrix of a random variable $\mathbf{x}$ with mean $\bar{\mathbf{x}}$ is denoted by $\mathbf{R}_x := \mathbb{E}(\{\mathbf{x} - \bar{\mathbf{x}}\}(\mathbf{x} - \bar{\mathbf{x}})^H)$, where $\mathbb{E}\{\cdot\}$ is the expectation operator. Lastly, $\mathbf{x} \sim \mathcal{CN}(\bar{\mathbf{x}}, \mathbf{R}_x)$ means $\mathbf{x}$ is a vector of Gaussian random variables with means $\bar{\mathbf{x}}$ and covariance $\mathbf{R}_x$.

II. SYSTEM MODEL AND SINR DERIVATION

Fig. 1 illustrates a wireless relay network, where $K$ source terminals $\{S_1, \ldots, S_K\}$ communicate with the destination terminal $D$ with the help of $M$ relay terminals $\{R_1, \ldots, R_M\}$. Each terminal is equipped with a single antenna and operates in a half-duplex mode (i.e., it cannot transmit and receive simultaneously). The relays are used to provide spatial diversity in order to improve the transmission reliability and/or extend the network’s coverage range. In the relaying protocol considered (which can be thought of as process-and-forward, a variant of amplify-and-forward) the transmission of one data block requires two communication phases. During the first phase, the source terminals transmit their signals to the destination and relay terminals. In the second phase, relay terminals forward the processed signals to the destination. In the following subsections, the key signal processing operations concerning the sources, relays and destination are described in more details.

A. Transmitted Signals From the Sources

Fig. 2(a) depicts the block diagram of the $k$th source’s transmitter. In the first phase, the $k$th source transmits a block of $Q$ information symbols, represented by a vector $\mathbf{d}_k \in \mathbb{C}^Q$. The data block is firstly transformed into frequency domain by a $Q$-point DFT, where $Q$ is the number of subcarriers assigned to each source. With a total of $K$ sources which can transmit simultaneously, the total number of subcarriers in the system is $N = KQ$. The assignment of every $Q$-point DFT output out of $N$ subcarriers is done by the subcarrier mapping. Although the framework developed in this paper applies to any subcarrier mapping scheme, there are two main types: localized mapping and interleaved mapping. Let matrix $\mathbf{\Theta}_k \in \mathbb{R}^{N \times Q}$ describe the mapping from $Q$ subcarriers of the $k$th source to a subset of $N$ subcarriers. For the localized FDMA,

$$\mathbf{\Theta}_k(n, q) = \delta(n - (k - 1)Q - q),$$

whereas for the interleaved FDMA [23]

$$\mathbf{\Theta}_k(n, q) = \delta(n - (q - 1)K - k).$$

As shown in [5], the interleaved FDMA mapping outperforms the localized mapping as it exploits better the frequency diversity of the channel (as such the interleaved FDMA mapping shall also be used in all simulations in this paper). It is useful to note that the mapping matrix has only one nonzero, unity entry per column. This means that the columns of the mapping matrix are orthogonal, i.e.,

$$\mathbf{\Theta}_k^H \mathbf{\Theta}_j = \delta(k - j)I_Q.$$

Moreover,

$$\mathbf{\Theta}_k^H [\mathbf{d}]_D \mathbf{\Theta}_j = \delta(k - j)[\mathbf{\Theta}_k^H \mathbf{d}]_D, \quad \forall \mathbf{d} \in \mathbb{C}^N.
$$

(2)

After the subcarrier mapping, the resulting signal is transformed back to the time domain by $N$-point IDFT. The discrete-time baseband signal of the $k$th source is given by

$$\mathbf{x}_k = \sqrt{\frac{1}{N_k}} \mathbf{\Phi}_k^H \mathbf{\Theta}_k \mathbf{\Phi}_Q \mathbf{d}_k.$$
where $P_{k}$ is the transmit power of each symbol from the $k$th source.

### B. Signal Processing at the Relays

Let $g_k \in \mathbb{C}^{L_{S,D}}$, $h_{k,m} \in \mathbb{C}^{L_{S,R}}$, and $e_{m} \in \mathbb{C}^{L_{R,D}}$ be the discrete-time baseband channel matrices from the $k$th source to the destination, from the $k$th source to the $m$th relay, and from the $m$th relay to the destination, respectively. Here, $L_{S,D}$, $L_{S,R}$, and $L_{R,D}$ denote the number of taps of the corresponding channels. All these channels are assumed to be Rayleigh block fading. Assume that a cyclic prefix (CP) whose length is greater than or equal to the maximum order of the channels (i.e., $max\{L_{S,D}, L_{S,R}, L_{R,D}\} - 1$) is added to $x_k$ before transmission. With the insertion of the CP at the transmitter and its removal at the receiver, the equivalent channel models can be presented by $N \times N$ circulant channel matrices, namely $G_k := \left[ \begin{array}{c} g_k^T \ b_{L_{S,D}} \end{array} \right]^T$ and $h_{k,m} := \left[ \begin{array}{c} h_{k,m}^T \ b_{L_{S,R}} \end{array} \right]^T$ and $L_{m} := \left[ \begin{array}{c} e_{m}^T \ b_{L_{R,D}} \end{array} \right]^T$. Since all the sources transmit simultaneously, the received signal at the $m$th relay after removing the CP is given by $r_m = \sum_{k=1}^{K} h_{k,m} x_k + n_m$, where $n_m \sim \mathcal{CN}(0, \sigma^2 I_N)$ represents additive white Gaussian noise. The $m$th relay then transforms the received signal $r_m$ into the frequency domain by $N$-point DFT and applies the demapping matrix $\Theta_k$. Next, with available CSI of uplink channels at the relays, frequency-domain equalization (FDE), represented by $\Delta_{k,m} \in \mathbb{C}^{N \times Q}$, is carried out to compensate for the transmission distortion and mitigate interference [24]. The output of the FDE of the $k$th source at $m$th relay is

$$\tilde{r}_{k,m} = \Delta_{k,m} \Theta_k^T \mathcal{F}_N r_m = \Delta_{k,m} \times \left( \sum_{k=1}^{K} \sqrt{P_{S,k}} \Theta_k^T \mathcal{F}_N h_{k,m} \mathcal{F}_N \Theta_k \mathcal{F}_Q d_k + \Theta_k^T \mathcal{F}_N n_m \right).$$

Based on the property in (1), it is noted that $\mathcal{F}_N h_{k,m} \mathcal{F}_N^H = [\sqrt{N} \mathcal{F}_N (h_{k,m}^T b_{L_{S,R}})]^T$. Further, using the property of the subcarrier mapping matrix in (2), the expression for $\tilde{r}_{k,m}$ can be simplified to

$$\tilde{r}_{k,m} = \Delta_{k,m} \left( \sqrt{N} \Theta_k^T \mathcal{F}_N h_{k,m}^T b_{L_{S,R}} d_k + \Theta_k^T \mathcal{F}_N n_m \right).$$

where $x_{k,m}^\dagger := [\sqrt{N} \Theta_k^T \mathcal{F}_N (h_{k,m}^T b_{L_{S,R}})]^{T}$ denotes an effective channel vector related to the channels from sources to relays (hop 1). Using the minimum mean-square error (MMSE) criterion, the frequency-domain equalizer can be found as follows [25]:

$$\Delta_{k,m} = \arg \min_{\Delta_{k,m}} \text{E} \left( \| \sqrt{P_{S,k}} \Theta_k \mathcal{F}_Q d_k - \tilde{r}_{k,m} \|^2 \right)$$

$$= \frac{P_{S,k} \left[ x_{k,m}^\dagger \right] H}{P_{S,k} \left[ x_{k,m}^\dagger \right] H \left[ x_{k,m}^\dagger \right] + \sigma^2 I_Q}$$

where $\beta_{k,m}(q) = \frac{P_{S,k} \left[ x_{k,m}^\dagger \right] (q)}{P_{S,k} \left[ x_{k,m}^\dagger \right] (q)^2 + \sigma^2}$, $q = 1, 2, \ldots, Q$.

After performing equalization to mitigate signal distortion and interference, the next crucial operation carried out by the relays is beamforming. In essence, the relays need to cooperatively adjust the amplitudes and phases of the transmitted signals based on the available CSI so that the received signals at the destination are most constructively added. Let the beamforming matrix $u_{k,m} \in \mathbb{C}^{Q \times 1}$ scale and rotate the received signal of the $k$th source at the $m$th relay. The transmitted signal at $m$th relay before inserting CP is then given by

$$\tilde{r}_{m} = \mathcal{F}_N^H \sum_{k=1}^{K} \Theta_k [u_{k,m}^T] \mathcal{F}_Q d_k.$$

It is important to determine the transmitted power at the $m$th relay, which is defined as $P_m = \text{E} \left( \| \tilde{r}_{m} \|^2 \right)$. By noting that the signal power does not change by the subcarrier mapping and DFT operation, $P_m$ can be computed as

$$P_m = \text{E} \left( \left\| \sum_{k=1}^{K} [u_{k,m}^T] \mathcal{F}_Q d_k \right\|^2 \right)$$

$$= \sum_{k=1}^{K} \left\| [u_{k,m} \ast x_{k,m} \ast w_{k,m}^\dagger] \right\|^2$$

$$= \sum_{k=1}^{K} \left\| [u_{k,m}]^2 \ast x_{k,m} \ast w_{k,m}^\dagger \right\|^2.$$
where \( \mathbf{v}_2 \sim \mathcal{CN}(0, \sigma^2_2 I_N) \) represents AWGN at the destination in the second phase. After DFT transformation and demapping, the received signal corresponding to the \( k \)th source is
\[
y_{k,2} = \Theta_k^T \mathcal{F}_N z_2
- \sum_{m=1}^{M} \Theta_k^T \mathcal{F}_N L_m \mathcal{F}_N \Theta_j \mathbf{w}_{j,m} \cdot \mathbf{r}_{j,m} + \Theta_k^T \mathcal{F}_N \mathbf{v}_2.
\]
Again, making use of the identity \( \mathcal{F}_N L_m \mathcal{F}_N^H = \sqrt{\mathcal{N}} \mathcal{F}_N (L_m^T 0_I N - L_m^T)^T \) and the orthogonality condition (2), one can write
\[
y_{k,2} = \sqrt{\mathcal{T}_S k} \sum_{m=1}^{M} [b_{k,m} \odot \mathbf{w}_{k,m}] \mathcal{D} \mathcal{F}_Q d_k
+ \sigma^{-1} \sum_{m=1}^{M} [c_{k,m} \odot \mathbf{w}_{k,m}] \mathcal{D} \Theta_k^T \mathcal{F}_N n_m + \Theta_k^T \mathcal{F}_N \mathbf{v}_2,
\]
where \( \mathbf{x}^{(2)}_{k,m} := \sqrt{\mathcal{N}} \Theta_k^T \mathcal{F}_N (L_m^T) \) is an effective channel vector related to the channels from relays to destination (hop 2), and
\[
b_{k,m} := \chi^{(2)}_{k,m} \odot \beta_{k,m} \odot \chi^{(1)}_{k,m},
c_{k,m} := \sigma \chi^{(2)}_{k,m} \odot \beta_{k,m}.
\] (4)

The destination exploits both the received signals from the sources in the first phase and from the relays in the second phase for the final signal detection. It is convenient to rewrite the received signals in the two phases in a vector form as
\[
y_k = \begin{bmatrix} y_{k,1} \\ y_{k,2} \end{bmatrix} - \Psi_k \mathcal{F}_Q d_k + \mathbf{v}_k,
\] (5)
where
\[
\Psi_k = \begin{bmatrix} \sqrt{\mathcal{T}_S k} \mathbf{x}^{(0)}_{k} \\ \sqrt{\mathcal{T}_S k} \sum_{m=1}^{M} [b_{k,m} \odot \mathbf{w}_{k,m}] \end{bmatrix} \in \mathbb{C}^{2Q \times Q}.
\] (6)
and
\[
\mathbf{v}_k = \begin{bmatrix} \Theta_k^T \mathcal{F}_N \mathbf{v}_1 \\ \sigma^{-1} \sum_{m=1}^{M} [c_{k,m} \odot \mathbf{w}_{k,m}] \end{bmatrix} \in \mathbb{C}^{2Q \times Q}.
\]
It is also relevant to determine the covariance matrix of \( \mathbf{v}_k \) to be
\[
\mathbf{R}_\mathbf{v}_k = \begin{bmatrix} \sigma^2 I_Q & 0 \\ 0 & \sum_{m=1}^{M} |c_{k,m} \odot \mathbf{w}_{k,m}|^2 + \sigma^2 I_Q \end{bmatrix} \in \mathbb{C}^{(2Q) \times (2Q)}.
\] (7)

The received signals in the two transmission phases are jointly equalized in the frequency domain. The linear MMSE equalizer, which is performed on the corresponding signal of the \( k \)th source, can be found from
\[
\mathbf{P}_k = \arg \min_{\mathbf{P}_k} \mathbb{E}( \| \mathcal{F}_Q \mathbf{d}_k - \mathbf{P}_k \mathbf{y}_k \|_2^2 )
= \mathbb{E}( \mathcal{F}_Q \mathbf{d}_k \mathbf{y}_k^H ) ( \mathbb{E}( \mathbf{y}_k \mathbf{y}_k^H ) )^{-1} \mathbb{E}( \mathbf{y}_k \mathbf{y}_k^H ) ( \mathbf{y}_k \mathbf{y}_k^H + \mathbf{R}_\mathbf{v}_k )^{-1}.
\] (8)
Furthermore, the corresponding MSE of the \( k \)th source is given by
\[
\text{MSE}_k = \mathbb{E}( \| \mathcal{F}_Q \mathbf{d}_k - \mathbf{P}_k \mathbf{y}_k \|_2^2 )
= \mathbb{E}( \| \mathbf{d}_k - \mathbf{P}_k \mathbf{y}_k \|_2^2 )
= \mathbb{E}( \mathbf{d}_k \mathbf{d}_k^H ) ( \mathbf{y}_k \mathbf{y}_k^H + \mathbf{R}_\mathbf{v}_k )^{-1}.
\] (9)
where, by (6) and (7), one has
\[
\mathbf{R}_k := \mathbb{I}_Q + \Psi_k^H \mathbf{R}_\mathbf{v}_k \Psi_k
= \begin{bmatrix} 1 + \frac{P_{S,k}}{\sigma_1^2} \chi^{(0)}_k \end{bmatrix} \mathbb{I}_Q
+ \frac{P_{S,k}}{\sigma_1^2} \sum_{m=1}^{M} b_{k,m} \odot \mathbf{w}_{k,m}^2
\left( \sum_{m=1}^{M} c_{k,m} \odot \mathbf{w}_{k,m}^2 + \sigma_2^2 I_Q \right) \mathbf{D}
= [\mathbf{r}_k]_D,
\] (10)
with
\[
\mathbf{r}_k := \mathbb{I}_Q + \frac{P_{S,k}}{\sigma_1^2} |\chi^{(0)}_k|^2.
\] (11)
The equalized signals are then transformed into the time domain, denoted by \( \mathbf{d}_k \), for symbol detection. It is given by
\[
\mathbf{d}_k = \mathcal{F}_Q^H \mathbf{P}_k \mathbf{y}_k
= \mathcal{F}_Q^H (\Psi_k^H (\psi_k \mathbf{y}_k^H + \mathbf{R}_\mathbf{v}_k)^{-1} \mathbf{y}_k \mathcal{F}_Q d_k
+ \mathcal{F}_Q^H (\psi_k \mathbf{y}_k^H + \mathbf{R}_\mathbf{v}_k)^{-1} \mathbf{v}_k)
= \mathcal{F}_Q^H (I_Q - \frac{\mathbf{r}_k}{|\mathbf{r}_k|} \mathbf{d}_k
+ \mathcal{F}_Q^H (1 - \frac{1}{|\mathbf{r}_k|}) \mathbf{d}_k
= \mathcal{F}_Q^H (1_Q - \frac{1}{|\mathbf{r}_k|}) \mathbf{d}_k
+ \mathcal{F}_Q^H (\psi_k \mathbf{y}_k^H + \mathbf{R}_\mathbf{v}_k)^{-1} \mathbf{v}_k.
\] (12)
It can be observed from (13) that each component in \( \mathbf{d}_k \), say \( \mathbf{d}_k(q), q = 1, \ldots, Q \), not only depends on the desired component \( \mathbf{d}_k(q) \), but also other components, \( \mathbf{d}_k(q'), q' \neq q \). In addition to the noise term (the second term) in (13), these components constitutes interference. As such, the quality of symbol detection based on the expression of \( \mathbf{d}_k \) in (13) is strongly influenced by the signal-to-interference-plus-noise ratio (SINR). This quantity is computed as follows.
First, from (5), (8) and (12) one has
\[
\mathbf{R}_\mathbf{d}_k := \mathbb{E}( \| \mathcal{F}_Q \mathbf{d}_k - \mathbf{P}_k \mathbf{y}_k \|_2^2 )
= \mathbb{E}( \| \mathcal{F}_Q \mathbf{d}_k \mathbf{y}_k^H \|_2^2 ) ( \mathbb{E}( \mathbf{y}_k \mathbf{y}_k^H ) )^{-1} \mathbb{E}( \mathbf{y}_k \mathbf{y}_k^H ) ( \mathbf{y}_k \mathbf{y}_k^H + \mathbf{R}_\mathbf{v}_k )^{-1}.
\] (14)
where $R_{d_k} = E\{g_k y_k^H\}$. The above together with (9) yields

$$E(\hat{d}_k(q)) = E(\hat{d}_k) = (I_Q - R_{d_k}^{-1}) = Q - \text{MSE}_k,$$

$$E(|\hat{d}_k(q)|^2) = R_{d_k}(q, q) = 1 - \frac{\text{MSE}_k}{Q}.$$  (16)

Now, using (14) to rewrite (13) as

$$\hat{d}_k - R_{d_k}(1, 1)d_k + |\hat{d}_k - R_{d_k}(1, 1)I_Q|d_k + R_{m_k})^{-1}v_k.$$  (17)

Therefore, the SINR for every symbol $\hat{d}_k(q)$ of the $k$th source is the same and given as

$$\text{SINR}_k = \frac{E(|\hat{d}_k(q)|^2)}{E(|\hat{d}_k(q)|^2 + E(|\hat{d}_k(1, 1)d_k|^2)} - \frac{(R_{d_k}(1, 1))^2}{R_{d_k}(1, 1) - (R_{d_k}(1, 1))^2} = \frac{Q}{\text{MSE}_k} - 1.$$  (18)

The above SINR expression shall be used in the SINR maximin optimization of cooperative beamforming design in the next section. Note also the inverse relationship between $\text{SINR}_k$ and $\text{MSE}_k$, which can be used (see the next section) to show that the beamforming design under SINR maximin criterion is equivalent to the design under the MSE minimax criterion.

### III. Beamforming Design by SINR Maximin Optimization

As discussed before, the optimization problems of multiple source networks in which the objective function is either the total average MSE or total SINR typically lead to unfairness among source users. To maintain a certain fairness level among the sources, improving the performance of the worst-suffering sources should be a priority. In this paper, we focus on cooperative beamforming design whose objective is to maximize the minimum SINR among all sources. In fact, based on the relationship in (18), the following shows that maximizing the minimum SINR is equivalent to minimizing the maximum MSE.

Using (3), the SINR maximin optimization in $\mathbf{w} = (\mathbf{w}_{1,1}, \ldots, \mathbf{w}_{K, M})$ subject to relay power constraints can be expressed by

$$\max_{\mathbf{w}_{k,m} \in \mathbb{C}^M} \min_{k = 1, \ldots, K} \text{SINR}_k(\mathbf{w})$$  (19a)

subject to

$$\sum_{k=1}^K \|w_{k,m}\|^2 \leq P_{R,m}, \quad m = 1, \ldots, M.$$  (19b)

where $P_{R,m}$ is a maximum allowable power at relay $m$. Note that, typically the relays are geographically distributed and such individual power constraints on relays are highly desirable. It follows from (18) that,

$$\max_{\mathbf{w}_{k,m} \in \mathbb{C}^M} \min_{k = 1, \ldots, K} \text{SINR}_k(\mathbf{w})$$

$$= \max_{\mathbf{w}_{k,m} \in \mathbb{C}^M} \min_{k = 1, \ldots, K} \left(\frac{Q}{\text{MSE}_k(\mathbf{w})} - 1\right)$$

$$= \max_{\mathbf{w}_{k,m} \in \mathbb{C}^M} \min_{k = 1, \ldots, K} \left(\frac{Q}{\text{MSE}_k(\mathbf{w})} - 1\right)$$

which clearly shows that (19) is equivalent to the following MSE minimax program:

$$\min_{\mathbf{w}_{k,m} \in \mathbb{C}^M} \max_{k = 1, \ldots, K} \text{MSE}_k(\mathbf{w}) \quad \text{s.t.} \quad (19b).$$  (20)

In the remaining part of this section we present an iterative algorithm to solve minimax (20) by d.c. programming. To this end, first note from (10) that

$$\text{MSE}_k(\mathbf{w}) = \sum_{q=1}^Q f_{k,q}(\mathbf{w})$$

with [see (21) at the bottom of the page], where $b_{k,m}$, $c_{k,m}$, and $a_k$ are defined as in (4) and (11). Obviously, $f_{k,q}(\mathbf{w})$ presents the MSE corresponding to the signal transmitted on subcarrier $q$ of user $k$. For the case of single relay, i.e., $M = 1$, as considered in [5], each $f_{k,q}(\cdot)$ function is a linear fractional function in $|w_{k,1}(q)|^2$ and can be shown to be convex in $|w_{k,1}(q)|^2$. As such minimax (20) is convex optimization, which can be efficiently solved as presented in [5]. In contrast, for the case of multiple sources and multiple relays, i.e., $K > 1$ and $M > 1$, it is quite clear that the functions $f_{k,q}(\cdot)$ are highly nonlinear and nonconvex, which makes minimax (20) a very hard nonsmooth optimization [21].

We now explore partial convex structures of the objective functions $f_{k,q}(\cdot)$ that can be used for effective iterative optimization. Specifically, we aim to represent the optimization problem (20) by an elegant d.c. (difference of two convex functions/sets) program and accordingly develop an efficient iterative procedure to locate its optimal solution. Although each $f_{k,q}(\cdot)$ is a fraction of two convex functions in $|w_{k,1}(q), \ldots, w_{k,M}(q)|^2$, there is no known effective d.c. representation for this class of polynomial fractional functions [21], [26], [27]. Nevertheless, it is readily seen that

$$\sum_{m=1}^M \|b_{k,m}(q)w_{k,m}(q)\|^2 \leq \left(\sum_{m=1}^M |b_{k,m}(q)|w_{k,m}(q)\right)^2$$

$$f_{k,q}(\mathbf{w}) = \sum_{m=1}^M c_{k,m}(q)^2 w_{k,m}(q)^2 + \sigma_k^2$$

$$a_k(q) \left(\sum_{m=1}^M |c_{k,m}(q)|^2 |w_{k,m}(q)|^2 + \sigma_k^2\right) + P_{S,k} \sum_{m=1}^M b_{k,m}(q)w_{k,m}(q)^2.$$  (21)
which implies $f_{k,q}(\mathbf{w}) \geq f_{k,q}(\mathbf{w})$ for $\mathbf{w} := (\mathbf{w}_{11}, \ldots, \mathbf{w}_{K,M})$ and (see the equation at the bottom of the page). The above inequality means that

$$\min_{\mathbf{w}_{k,m}} \max_{k=1,2,\ldots,K} \text{MSE}_{k}(\mathbf{w}) \geq \min_{\mathbf{w}_{k,m}} \max_{k=1,2,\ldots,K} \text{MSE}_{k}(\mathbf{w}) \quad : = \sum_{q=1}^{Q} f_{k,q}(\mathbf{w})$$

and the equality holds if

$$\arg(\mathbf{w}_{k,m}(q)) = \arg(b_{k,m}(q)). \quad (22)$$

i.e., $\mathbf{w}_{k,m}(q)$ are cophased with $b_{k,m}(q)$. Thus, the relay beamforming design can be divided into two steps. First, the optimal phases of $\mathbf{w}_{k,m}(q)$ are pre-determined by (22). With the optimal phases, we have $\text{MSE}_{k}(\mathbf{w}_{k,m}) = \text{MSE}_{k}(\mathbf{w}_{k,m})$ and, therefore, in the second step, the optimal amplitudes of $\mathbf{w}_{k,m}(q)$ are determined by using $\text{MSE}_{k}(\mathbf{w}_{k,m})$. As a consequence, (20) is precisely simplified to

$$\min_{\mathbf{w}_{k,m} \in \mathbb{R}^{Q}_{+}} \max_{k=1,2,\ldots,K} \text{MSE}_{k}(\mathbf{w}_{k,m}) \text{s.t.} (19b). \quad (23)$$

Before proceeding to solve problem (23), it is worthwhile to discuss the related cooperative beamforming designs as considered in [5] and [8] but under the total relay power constraint. One can write

$$f_{k,q}(\mathbf{w}) := f_{k,q}(\mathbf{w}_{k,1}(q), \ldots, \mathbf{w}_{k,M}(q))$$

and then use Lemma 1 in the Appendix to recognize that the sensitivity functions as shown in (24) at the bottom of the page, are explicitly defined, convex and decreasing in the sub-carrier powers $\rho_{k,q}$. Here, $\rho_{k,q}$ is a total power which all relays assign to the $q$th subcarrier of the $k$th source. Then, the objective function in minimax program (20) is convex in $\rho_{k,q}$ as well. Under the total relay power constraint expressed by

$$\sum_{k=1}^{K} \sum_{q=1}^{Q} \rho_{k,q} = P_{R}, \quad (25)$$

the minimax program

$$\left\{ \begin{array}{l}
\min_{\mathbf{w}_{k,m} \in \mathbb{R}^{Q}_{+}} \max_{k=1,2,\ldots,K} \text{MSE}_{k}(\mathbf{w}_{k,m}) \text{s.t.} (25) \\
\min_{\rho_{k,q} > 0} \max_{k=1,2,\ldots,K} \sum_{q=1}^{Q} g_{k,q}(\rho_{k,q}) \text{s.t.} (25) 
\end{array} \right\} \quad (26)$$

is thus convex. Furthermore, for the particular case of $K = 1$, this program is separated convex and its optimal solution can be located quickly through the so-called iterative bisection procedure (IBP) [28] as done in [8]. Another more effective and direct approach is to find an approximated solution by a water-filling procedure as in [29]. For $K > 1$, the convex minimax program (26) can be efficiently solved by the so-called water-discharging scheme [5]. However, the individual relay power constraints (19b) for the minimax program in (23) involve the optimal solutions $\mathbf{w}_{k,m}(q)$ for attaining the optimal values $g_{k,q}(\rho_{k,q})$ in (24), which are highly nonlinear and nonconvex in $\rho_{k,q}$ (according to Lemma 1 in the Appendix). In other words, the tractable convex constraints (19b) in $\mathbf{w}_{k,m}$ become highly nonlinear and nonconvex in the new variables $\rho_{k,q}$. Consequently, the minimax program (23) in decision variables $\mathbf{w}_{k,m}$, which is in fact a minimization of nonconvex objective function subject to convex constraints, becomes a minimization of the convex objective function $\max_{k=1,2,\ldots,K} \sum_{q=1}^{Q} g_{k,q}(\rho_{k,q})$ subject to highly nonconvex constraints in decision variables $\rho_{k,q}$. The latter is much harder to be solved than the former [21]. The sensitivity function approach is obviously not suitable.

We now return to problem (23) and follow a different approach to solve it. First, write

$$f_{k,q}(\mathbf{w}) = \frac{1}{a_{k}(q)} - \tilde{f}_{k,q}(\mathbf{w})$$

and then use Lemma 1 in the Appendix to recognize that the sensitivity functions as shown in (24) at the bottom of the page, are explicitly defined, convex and decreasing in the sub-carrier powers $\rho_{k,q}$. Here, $\rho_{k,q}$ is a total power which all relays assign to the $q$th subcarrier of the $k$th source. Then, the objective function

$$f_{k,q}(\mathbf{w}) := \frac{1}{a_{k}(q)} \sum_{m=1}^{M} c_{k,m}(q) \mathbf{w}_{k,m}(q)^{2} + \sigma_{k}^{2}$$

$$= a_{k}(q) \left( \sum_{m=1}^{M} |c_{k,m}(q)|^{2} \mathbf{w}_{k,m}(q)^{2} + \sigma_{k}^{2} \right) + P_{S,k} \left( \sum_{m=1}^{M} |b_{k,m}(q)| \mathbf{w}_{k,m}(q) \right)^{2}$$

$$g_{k,q}(\rho_{k,q}) := \min_{\mathbf{w}_{k,1}(q), \ldots, \mathbf{w}_{k,M}(q)} \left\{ \begin{array}{l}
\tilde{f}_{k,q}(\mathbf{w}_{k,1}(q)), \ldots, |\mathbf{w}_{k,M}(q)| : \sum_{m=1}^{M} |\mathbf{w}_{k,m}(q)|^{2} X_{k,m}(q) = \rho_{k,q} \\
k = 1, 2, \ldots, K; q = 1, 2, \ldots, Q
\end{array} \right\} \quad (24)$$
with (see equation at the bottom of the page). It is also relevant to note that

\[
\min_{\mathbf{w}_{m}, m \in R_+^{KQ}} \max_{k = 1, 2, \ldots, K} \left[ \sum_{q=1}^{Q} \tilde{f}_{k,q}(\mathbf{w}) \right] = - \max_{\mathbf{w}_{m}, m \in R_+^{KQ}} \min_{k = 1, 2, \ldots, K} \left[ \sum_{q=1}^{Q} \tilde{f}_{k,q}(\mathbf{w}) - \sum_{q=1}^{Q} \frac{1}{a_k(q)} \right].
\]

It then follows that the optimization problem (23) is equivalent to the following

\[
\max_{\mathbf{w}_{m}, m \in R_+^{KQ}} \min_{k = 1, 2, \ldots, K} \left[ \sum_{q=1}^{Q} \tilde{f}_{k,q}(\mathbf{w}) - \sum_{q=1}^{Q} \frac{1}{a_k(q)} \right] \quad \text{s.t. (19b)}. \tag{27}
\]

The first partial convexity recognition is summarized in the following theorem.

**Theorem 1:** Problem (27) is equivalent to the following optimization over convex constraints involving additional variables \( \mathbf{x} = (x_{k,1}, \ldots, x_{K,Q}) \in R_+^{KQ} \): See (28a) and (28b) at the bottom of the page.

**Proof:** It follows from the fact that the equality sign for (28b) must hold at the optimality of (28), i.e., (28) is actually the following optimization problem, shown in the equation at the bottom of the page, which is the same as (27).

Next, we give the following theorem as it can be used to obtain d.c. representation [21] for the objective function in (28a).

**Theorem 2:** Each function

\[
\varphi_{k,q}(\mathbf{w}, x_{k,q}) := \left( \sum_{m=1}^{M} |b_{k,m}(q)| w_{k,m}(q) \right)^2 \quad \text{s.t. (28b)}.
\]

is convex in its variables \((w_{k,1}(q)), \ldots, w_{k,M}(q), x_{k,q}) \in R_+^{KQ+1} \).

**Proof:** Introduce the function \( \phi(t, x) = \frac{t^2}{x} \) in \( R_+^{Q^2} \). Its Hessian is

\[
\nabla^2 \phi(t, x) := \begin{bmatrix}
\frac{\partial^2 \phi}{\partial t^2} & \frac{\partial^2 \phi}{\partial t \partial x} \\
\frac{\partial^2 \phi}{\partial x \partial t} & \frac{\partial^2 \phi}{\partial x^2}
\end{bmatrix} = \frac{1}{x} \begin{bmatrix}
-1 & -1 \\
-1 & -1
\end{bmatrix} \succeq 0
\]

for all \((t, x) \in R_+^{Q^2} \). Therefore \( \phi(t, x) \) is convex in its variable \((t, x) \in R_+^{Q^2} \), i.e.,

\[
\phi(\alpha(t, x) + (1 - \alpha)(t^1, x^1)) \leq \alpha \phi(t, x) + (1 - \alpha) \phi(t^1, x^1) \quad \forall \alpha \in [0, 1],
\]

\(\forall (t, x, (t^1, x^1)) \in R_+^{Q^2} \).

\[
\tilde{f}_{k,q}(\mathbf{w}) = \frac{P_{S,k} \left( \sum_{m=1}^{M} |b_{k,m}(q)| w_{k,m}(q) \right)^2}{\sum_{m=1}^{M} |c_{k,m}(q)|^2 w_{k,m}(q)^2 + \sigma_2^2 + a_k(q) P_{S,k} \left( \sum_{m=1}^{M} |b_{k,m}(q)| w_{k,m}(q) \right)^2}, \tag{28a}
\]

\[
\max_{\mathbf{w}} \min_{k = 1, 2, \ldots, K} \left[ \sum_{q=1}^{Q} \frac{P_{S,k} \left( \sum_{m=1}^{M} |b_{k,m}(q)| w_{k,m}(q) \right)^2}{x_{k,q}} - \sum_{q=1}^{Q} \frac{1}{a_k(q)} \right] \quad \text{s.t. (19b)}, \tag{28b}
\]

\[
\tilde{a}_k^2(q) \left( \sum_{m=1}^{M} |c_{k,m}(q)|^2 w_{k,m}(q)^2 + \sigma_2^2 + a_k(q) P_{S,k} \left( \sum_{m=1}^{M} |b_{k,m}(q)| w_{k,m}(q) \right)^2 \right) \leq x_{k,q}, \quad q = 1, \ldots, Q; k = 1, \ldots, K.
\]
Therefore,
\[
\varphi_{k,q}(\alpha (\mathbf{w}, k, q^*): \mathbf{z}) + (1 - \alpha) (\mathbf{w}^{(1)}: \mathbf{z}^{(1)})
\]
\[
= \phi \left( \alpha \left( \sum_{m=1}^{M} \mathbf{b}_{k,m}(q) \| \mathbf{w}_{k,m}(q) \| \mathbf{x}_{k,q} \right) + (1 - \alpha) \left( \sum_{m=1}^{M} \mathbf{b}_{k,m}(q) \| \mathbf{w}_{k,m}(q) \| \mathbf{x}_{k,q}^{(1)} \right) \right)
\]
\[
\leq \alpha \phi \left( \sum_{m=1}^{M} \mathbf{b}_{k,m}(q) \| \mathbf{w}_{k,m}(q) \| \mathbf{x}_{k,q} \right)
\]
\[
+ (1 - \alpha) \phi \left( \sum_{m=1}^{M} \mathbf{b}_{k,m}(q) \| \mathbf{w}_{k,m}(q) \| \mathbf{x}_{k,q}^{(1)} \right)
\]
\[
= \alpha \varphi_{k,q}(\mathbf{w}, \mathbf{x}_{k,q}) + (1 - \alpha) \varphi(\mathbf{w}^{(1)}, \mathbf{x}_{k,q}^{(1)})
\]

for all \( \alpha \in [0, 1] \) and \((\mathbf{w}, \mathbf{x}_{k,q}), (\mathbf{w}^{(1)}, \mathbf{x}_{k,q}^{(1)})\), which shows that \( \varphi_{k,q}(\cdot) \) is convex.

Using Theorem 2, one has the following d.c. representation for the objective function in (28a):

\[
\min_{k=1, \ldots, K} \left[ \sum_{q=1}^{Q} P_{S,k} \left( \sum_{m=1}^{M} \mathbf{b}_{k,m}(q) \| \mathbf{w}_{k,m}(q) \| \mathbf{x}_{k,q} \right)^{2} - \sum_{q=1}^{Q} \frac{1}{\mathbf{a}_{k}(q)} \right]
\]
\[
= \min_{k=1, \ldots, K} \left[ \sum_{q=1}^{Q} P_{S,k} \varphi_{k,q}(\mathbf{w}, \mathbf{x}_{k,q}) - \sum_{q=1}^{Q} \frac{1}{\mathbf{a}_{k}(q)} \right]. \tag{30}
\]

As indicated, the objective function in (30) is expressed as a difference of the following two functions:

\[
f_{1}(\mathbf{w}, \mathbf{x}) := \sum_{k=1}^{K} \sum_{q=1}^{Q} P_{S,k} \varphi_{k,q}(\mathbf{w}, \mathbf{x}_{k,q}) - \sum_{q=1}^{Q} \frac{1}{\mathbf{a}_{k}(q)}
\]
\[
f_{2}(\mathbf{w}, \mathbf{x}) := \max_{j=1,2 \ldots, K} \sum_{k=1 \neq k}^{K} \sum_{q=1}^{Q} P_{S,k} \varphi_{k,q}(\mathbf{w}, \mathbf{x}_{k,q}) - \sum_{q=1}^{Q} \frac{1}{\mathbf{a}_{k}(q)}.
\]

These functions are convex as they are a sum and a maximum of convex functions, respectively \[21\]. Now, the optimization problem (28) can be rewritten as a minimization of a d.c. objective function over convex sets:

\[
\min_{\mathbf{w}, \mathbf{x}} \left[ f_{2}(\mathbf{w}, \mathbf{x}) - f_{1}(\mathbf{w}, \mathbf{x}) \right] \text{ s.t. } (19b), (28b). \tag{31}
\]

Although (31) belongs to the realm of d.c. optimization \[21\], its nonconvexity rank expressed by the dimension of the actual variable of \( f_{1}(\cdot) \)\[[30]\] is too large for a meaningful application of global optimization algorithms \[21\], which perform a global search in the whole \( KQ(M + 1) \)-dimensional space of the actual variables \((\mathbf{w}, \mathbf{x})\) of \( f_{1}(\cdot) \). In fact, a crucial prerequisite in global optimization is to have an effective d.c. representation in (31) such that the actual variable dimension of \( f_{1}(\cdot) \) is low. It is unlikely that such a d.c. representation for (28) is possible. In contrast, motivated by the theory and results in \[31\]–\[33\], an alternative approach is to seek a d.c. representation, which may involve the whole set of variables in function \( f_{1}(\cdot) \) but nevertheless leads to an easily-implemented iterative procedure of local search in nature but targeting at the approximation of the global optimal solution in reality. Specifically, our recent work \[34\] shows that our approach indeed brings a quick global optimal solution for very large dimension problems of wireless network's power control, for which any application of global optimization algorithms as done in \[2\] predictably fails.

With the d.c. representation in (31) for the highly nonconvex optimization problem (28), the d.c. iterations to obtain the solutions of (28) are simple (but far from trivial) and described in the following. Suppose \((\mathbf{w}^{(\infty)}, \mathbf{x}^{(\infty)})\) is feasible to (28). Since \( f_{1}(\cdot) \) is clearly smooth (continuously differentiable), it is known from convex analysis \[21\] that

\[
f_{1}(\mathbf{w}, \mathbf{x}) \geq f_{1}(\mathbf{w}^{(\infty)}, \mathbf{x}^{(\infty)})
\]
\[
+ \langle \nabla f_{1}(\mathbf{w}^{(\infty)}, \mathbf{x}^{(\infty)}), (\mathbf{w} - \mathbf{w}^{(\infty)}, \mathbf{x} - \mathbf{x}^{(\infty)}) \rangle
\]
\[
- f_{1}(\mathbf{w}^{(\infty)}, \mathbf{x}^{(\infty)}) \forall (\mathbf{w}, \mathbf{x}).
\]

Thus, a convex majorant of the objective function \( f_{2}(\cdot) - f_{1}(\cdot) \) is

\[
f^{(\infty)}(\mathbf{w}, \mathbf{x}) := f_{1}(\mathbf{w}, \mathbf{x}) - f_{1}(\mathbf{w}^{(\infty)}, \mathbf{x}^{(\infty)})
\]
\[
- \langle \nabla f_{1}(\mathbf{w}^{(\infty)}, \mathbf{x}^{(\infty)}), (\mathbf{w} - \mathbf{w}^{(\infty)}, \mathbf{x} - \mathbf{x}^{(\infty)}) \rangle.
\]

Because \( f^{(\infty)}(\mathbf{w}, \mathbf{x}) > f_{2}(\mathbf{w}, \mathbf{x}) - f_{1}(\mathbf{w}, \mathbf{x}) \forall (\mathbf{w}, \mathbf{x}) \), the following convex program provides a global upper bound (robust) minimization for d.c. program (28):

\[
\min_{\mathbf{w}, \mathbf{x}} f^{(\infty)}(\mathbf{w}, \mathbf{x}) \text{ s.t. } (19b), (28b). \tag{32}
\]

Moreover, the optimal solution \((\mathbf{w}^{(\infty + 1)}, \mathbf{x}^{(\infty + 1)})\) of (32) is a better feasible solution of (28) than the feasible \((\mathbf{w}^{(\infty + 1)}, \mathbf{x}^{(\infty + 1)})\) of (28). This is because

\[
f_{2}(\mathbf{w}^{(\infty + 1)}, \mathbf{x}^{(\infty + 1)}) - f_{1}(\mathbf{w}^{(\infty + 1)}, \mathbf{x}^{(\infty + 1)}) \leq f^{(\infty)}(\mathbf{w}^{(\infty + 1)}, \mathbf{x}^{(\infty + 1)}) - f^{(\infty)}(\mathbf{w}^{(\infty)}, \mathbf{x}^{(\infty)}).
\]

It is also obvious from the above analysis that if initialized by any feasible solution \((\mathbf{w}^{(0)}, \mathbf{x}^{(0)})\) of (28), the sequence \((\mathbf{w}^{(\infty)}, \mathbf{x}^{(\infty)})\) of improved solutions of convex program (32), is bounded and thus compact, i.e., it must converge to a cluster \((\mathbf{w}^{(\infty)}, \mathbf{x}^{(\infty)})\), which is feasible to (28). A straightforward gradient calculation for function \( f_{1}(\cdot) \) at \((\mathbf{w}^{\infty}, \mathbf{x}^{\infty})\) concretizes (32) as shown in (33) at the bottom of the next page, where \( \mathbf{B}_{k,l}(i,j) = b_{k,l}(q) b_{k,l}(q), i = 1, \ldots, M; j = 1, \ldots, M. \)

The implementation of the d.c. iterations just described is sketched in Algorithm 1. Note that the optimization problem (33) can be easily rewritten as a semi-definite program (SDP).
Therefore, problem (33) can be efficiently solved by available SDP solvers of the polynomial complexity (for example, the SeDuMi program in [35]).

Algorithm 1: Iterative Algorithm for Relay Beamforming

**Initialization**: Set $\kappa = 0$. Choose $(\mathbf{w}^{(0)}, \mathbf{z}^{(0)})$ and calculate $\text{MSE}_{\max}^{(0)} = \max_{k=1, \ldots, \kappa} \text{MSE}^{(0)}(\mathbf{w}^{(0)})$.

**repeat**

— At the $\kappa$th iteration, solve the convex optimization problem (33) to obtain the solution $(\mathbf{w}_{\text{opt}}, \mathbf{z}_{\text{opt}})$.

— Set $\kappa = \kappa + 1$, $(\mathbf{w}_{\kappa}^{(\kappa)}), (\mathbf{z}_{\kappa}^{(\kappa)}) = (\mathbf{w}_{\text{opt}}, \mathbf{z}_{\text{opt}})$ and calculate $\text{MSE}_{\text{max}}^{(\kappa)} = \max_{k=1, \ldots, \kappa} \text{MSE}_{k}(\mathbf{w}^{(\kappa)})$.

**until** $|\text{MSE}_{\text{max}}^{(\kappa)} - \text{MSE}_{\text{max}}^{(\kappa-1)}| \leq \epsilon$.

As conventionally assumed for relay networks (see, for example, [5], [8], [17], and references therein) the destination collects all CSI and calculates the optimal beamforming weights. Then, the optimal parameters are conveyed to the corresponding relays using feedback channels. In this scenario, the CSI between sources and the destination, and between the relays and the destination can be obtained by channel estimation using training sequences. Additionally, the relays send the CSI between the sources and the relays to the destination. In this paper, we assume that perfect CSI is available at the destination.

IV. ILLUSTRATIVE RESULTS

In this section, simulation results are provided to illustrate the performance of the proposed cooperative beamforming method in multi-source, multi-relay SC-FDMA wireless relay networks. Similar to [5], [19], we consider SC-FDMA networks supporting $K = 8$ source users. The number of subcarriers per source user is $Q = 8$ and hence the total number of subcarriers is $N = 64$. The channels are assumed to be independent Rayleigh fading with $L_{\text{SD}} = L_{\text{SR}} = L_{\text{RD}} = 7$ channel taps. Exponential power delay profiles [36], [37] for channels from the sources to the destination, the sources to the relays and relays to the destination are given, respectively, by

\[
\begin{align*}
E\{g_k(\ell)\}^2 &= \sum_{n=-\infty}^{\infty} \frac{B_{\text{SD}}}{2 \pi \rho_{\text{SD}}^2} \exp(-n^2) \exp(-\ell), \quad \ell = 0, \ldots, L_{\text{SD}} - 1, \\
E\{h_{k,m}(\ell)\}^2 &= \sum_{n=-\infty}^{\infty} \frac{B_{\text{SR}}}{2 \pi \rho_{\text{SR}}^2} \exp(-n^2) \exp(-\ell), \quad \ell = 0, \ldots, L_{\text{SR}} - 1, \text{and} \\
E\{t_{k,m}(\ell)\}^2 &= \sum_{n=-\infty}^{\infty} \frac{B_{\text{RD}}}{2 \pi \rho_{\text{RD}}^2} \exp(-n^2) \exp(-\ell), \quad \ell = 0, \ldots, L_{\text{RD}} - 1,
\end{align*}
\]

where $\rho_{\text{SD}}$ and $\rho_{\text{SR}}$ present the path loss of channels from sources to the destination and from sources to relays, respectively. For the sake of convenience and fairness in comparison, it is assumed that the maximum allowable powers at relays are the same and equal to $P_{\text{R,m}} = \frac{P_S}{M}, m = 1, \ldots, M$, where $P_S$ is the total power of all relays. All the sources use quadrature phase-shift keying (QPSK) modulation and the interleaved subcarrier mapping is employed. The noise powers at the relays and destination are normalized to unity, i.e., $\sigma^2 = \sigma_{\text{SD}}^2 = \sigma_{\text{SR}}^2 = 1$. We set the transmit power at all the sources to be $P_S = 14$ dB in normalized units, while the (total) relay power shall be varied relatively to the source power. We always chose an equal power allocation as an initial point in our iterative algorithm and we set $\epsilon = 10^{-4}$.

**Example 1**: This example evaluates the performance of the proposed cooperative beamforming and compare it with equal-power beamforming (EPB) given by

\[
w_{k,m}(q) = \sqrt{\frac{P_{\text{R,m}}}{\sum_{q'=1}^{M} x_{k,m}(q')^2}}.
\]

Such a comparison is reasonable as there exists no other beamforming design for multi-source, multi-relay SC-FDMA wireless relay networks. The number of relays is set to be $M = 4$. To take the channel path loss into consideration, we set $\eta_{\text{S,k}} = 2^{(k-4)}$ for $k > 4$ and $\eta_{\text{S,m}} = 2$ for $m < 2$ while $\eta_{\text{S,m}} = 2$ for $m > 2$ and $\eta_{\text{S,m}} = 1$ for $m = 2$. The performance of our design and the equal-power method are shown in Fig. 3, where the results are averaged over 200 random channel realizations. It can be observed from Fig. 3 that an average of the minimum SINR among source users achieved by our proposed beamforming is better than that of the EPB method, namely by more than 5 dB, while the averages of the maximum SINRs are about the same by the two methods. Such results also lead to a higher average SINR by our method than the EPB method. It can also be seen that the fairness among all source users is much better in our design.

As the bit error rate (BER) is the most important metric in digital communications, Fig. 4 plots the BER performance curves, averaged over all sources, obtained by the proposed and EPB methods. As can be seen, our proposed method offers significantly better BER performance as compared to the EPB. In fact, the proposed beamforming achieves a higher diversity order than the EPB.

**Example 2**: This example investigates the performance of multi-source, multi-relay SC-FDMA networks with different numbers of relays. Specifically, we consider the cases with $M = 1$, $M = 2$, and $M = 4$ relays. The path loss coefficient $\eta_{\text{S,k}}$ is the same as in Example 1 while $\eta_{\text{R,m}} = 1$ for all relays. Fig. 5 compares the SINR, whereas Fig. 6 plots the BER performance curves. It can be observed from Fig. 5 that for the same total

\[
\min_{\mathbf{w}, \mathbf{z}} \left\{ f_1(\mathbf{w}, \mathbf{z}) - f_1(\mathbf{w}(\kappa), \mathbf{z}(\kappa)) - \sum_{k=1}^{K} \sum_{q=1}^{Q} 2P_{\text{S,k}} \left( B_{k,q} \left| \mathbf{w}_{k}^{(\kappa)}(q) \right|^{2} \right)^2 \left( x_{k,q}^{(\kappa)} - x_{k,q}^{(\kappa)} \right)^2 \right\} 
\]

s.t. (19b), (28b)

(33)
Fig. 3. Average SINR versus the normalized total relay power \( \frac{P_r}{P_s} \). Solid lines are for the proposed method and dashed lines are for equal power beamforming.

Fig. 4. Average BER versus the normalized total relay power \( \frac{P_r}{P_s} \) for \( M = 4 \) relays.

relay power, the minimum SINR values improve as the number of relays increases. For example, with the normalized total relay power of \( \frac{P_r}{P_s} = 6 \) dB the single-relay network offers SINR=6.61 dB, while the network with 4 relays provides SINR=11.12 dB. In addition, the SINR gap between the average maximum SINR and the average minimum SINR among source users becomes smaller as the relay power increases. This means that the fairness among sources is improved.

As expected, similar observations can be made with respect to the BER performance illustrated in Fig. 6. For example, with \( \frac{P_r}{P_s} = 10 \) dB, the network with \( M = 1 \) relay offers the BER of about \( 1.12 \times 10^{-2} \), whereas for the network with \( M = 4 \) relays a BER level of \( 1.13 \times 10^{-5} \) is achieved. It is obvious from Fig. 6 that with our proposed cooperative beamforming a higher diversity is attained with a larger number of relays, an intuitively satisfying result.

Finally, it is appropriate to comment on the computational complexity of the proposed iterative algorithm. In general, the complexity of any iterative algorithm is governed by its convergence and the computational complexity of each iteration. Each iteration of Algorithm 1 solves the convex optimization problem (33), shown at the bottom of the previous page, with complexity \( O(N^3(M + 1)^3) \)[38]. Moreover, the proposed algorithm converges to the optimal solution in the small number of iterations, as shown in Fig. 7. The average iteration numbers and the computational complexity per iteration are provided by in Table I for different numbers of relays. Table I reveals that the proposed iterative algorithm converges in a few iterations independently of the number of relays.
TABLE I

<table>
<thead>
<tr>
<th>Number of relays</th>
<th>Iteration number</th>
<th>Computational complexity per iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>M = 2</td>
<td>22,483</td>
<td>( \mathcal{O}(27N^3) )</td>
</tr>
<tr>
<td>M = 4</td>
<td>22,805</td>
<td>( \mathcal{O}(125N^3) )</td>
</tr>
</tbody>
</table>

V. CONCLUSION

This paper has solved the cooperative beamforming design for multi-user multi-relay wireless relay networks where the terminals exploit single-carrier frequency division multiple access (SC-FDMA) techniques. Appropriate signal processing operations were developed at each terminal and the expression of the SINR at the destination for each source user was obtained. The challenging nonconvex optimization problem of beamforming based on the SINR maximin criterion was tackled by recasting it as a d.c. optimization problem. A low-complexity iterative algorithm in which convex programming is efficiently solved in each iteration was presented to obtain the solution. Simulation results show that using the proposed cooperative beamforming across relays offer a significantly-improved system performance, both in terms of the average SINR and BER, as compared to the equal-power beamforming.

APPENDIX

Lemma 1: For \( \mathbf{a} \in \mathbb{R}^M_+ \), \( \mathbf{b} \in \text{int}(\mathbb{R}^M_+) \) and \( \rho > 0 \), \( \sigma > 0 \) the following equalities hold true

\[
\max_{\mathbf{x} \in \mathbb{R}^N_+} \frac{\langle \mathbf{a}, \mathbf{x} \rangle^2}{\langle \mathbf{b}, \mathbf{x} \rangle^2} = \langle \mathbf{a}^2, \mathbf{b}^{-1} \rangle. 
\]

\[
\arg \max_{\mathbf{x} \in \mathbb{R}^N_+} \frac{\langle \mathbf{a}, \mathbf{x} \rangle^2}{\langle \mathbf{b}, \mathbf{x} \rangle^2} = \left\{ \frac{\alpha \mathbf{a}}{\mathbf{b}}, \alpha > 0 \right\}, \quad (34)
\]

\[
\max_{\mathbf{x} \in \mathbb{R}^N_+} \frac{\langle \mathbf{a}, \mathbf{x} \rangle^2}{\langle \mathbf{b}, \mathbf{x} \rangle^2 + \sigma} = \left\{ \mathbf{a}^2, \left( \mathbf{b} + \sigma \mathbf{c}/\rho \right)^{-1} \right\}, \quad (35)
\]

\[
\arg \max_{\mathbf{x} \in \mathbb{R}^N_+} \frac{\langle \mathbf{a}, \mathbf{x} \rangle^2}{\langle \mathbf{b}, \mathbf{x} \rangle^2 + \sigma} = - \langle \mathbf{c}, \left( \mathbf{b} + \sigma \mathbf{c}/\rho \right)^{-1} \rangle \mathbf{c} \mathbf{a}^2 \left( \mathbf{b} + \sigma \mathbf{c}/\rho \right)^{-2} \left( \mathbf{b} + \sigma \mathbf{c}/\rho \right)^{-1} \mathbf{c} \mathbf{a} \mathbf{x}^2, \quad (36)
\]

The sensitivity function \( \langle \mathbf{a}^2, \frac{\mathbf{b} + \sigma \mathbf{c}}{\rho} \rangle^{-1} \) is concave and increasing in the parameter \( \rho \).

Proof: First, (34) is a straightforward consequence of the Cauchy-Schwarz inequality:

\[
\frac{\langle \mathbf{a}, \mathbf{x} \rangle}{\langle \mathbf{b}, \mathbf{x} \rangle^2} = \frac{\langle \mathbf{a} \odot \mathbf{b}^{-1/2}, \mathbf{b}^{1/2} \odot \mathbf{x} \rangle^2}{\| \mathbf{b}^{1/2} \odot \mathbf{x} \|^2} \leq \left| \mathbf{a} \odot \mathbf{b}^{1/2} \right|^2 = \langle \mathbf{a}^2, \mathbf{b}^{-1} \rangle
\]

where the equality holds if and only if \( \mathbf{a} \odot \mathbf{b}^{1/2} = \mathbf{b}^{1/2} \odot \mathbf{x} \), \( \alpha > 0 \). Next, by rewriting \( \sigma = \langle \mathbf{c}, \mathbf{x} \mathbf{c} \rangle \), one has

\[
\max_{\mathbf{x} \in \mathbb{R}^N_+} \frac{\langle \mathbf{a}, \mathbf{x} \rangle^2}{\langle \mathbf{b}, \mathbf{x} \rangle^2 + \sigma} = \max_{\mathbf{x} \in \mathbb{R}^N_+} \frac{\langle \mathbf{a}, \mathbf{x} \rangle^2}{\langle \mathbf{b} + \sigma \mathbf{c}/\rho \mathbf{c}, \mathbf{x} \rangle^2}. \quad (37)
\]

The equalities in (35) and (36) are obtained by applying the following equalities hold true

\[
\langle \mathbf{a}^2, \frac{\mathbf{b} + \sigma \mathbf{c}}{\rho} \rangle^{-1} = \sum_{m=1}^{M} \frac{\mathbf{a}^2(m)}{\mathbf{b}(m) + \sigma \mathbf{c}(m)/\rho},
\]

Finally,

\[
\langle \mathbf{a}^2, \frac{\mathbf{b} + \sigma \mathbf{c}}{\rho} \rangle^{-1} = \sum_{m=1}^{M} \frac{\mathbf{a}^2(m)}{\mathbf{b}(m) + \sigma \mathbf{c}(m)/\rho},
\]

which is obviously concave and increasing in \( \rho \) because each function \( \langle \mathbf{a}^2, \frac{\mathbf{b} + \sigma \mathbf{c}}{\rho} \rangle^{-1} \) is concave and increasing in \( \rho \).

REFERENCES


Hoang Duong Tuan (M’94) was born in Hanoi, Vietnam. He received the Diploma and Ph.D. degree in applied mathematics from Odessa State University, Ukraine, in 1987 and 1991, respectively.

From 1991 to 1994, he was a Researcher with the Optimization and Systems Division, Vietnam National Center for Science and Technologies. He was an Assistant Professor with the Department of Electronic-Mechanical Engineering, Nagoya University, Japan, from 1994 to 1999, and an Associate Professor with the Department of Electrical and Computer Engineering, Toyota Technological Institute, Nagoya, from 2003 to 2011. Currently, he is a Professor with the Centre for Health Technologies, Faculty of Engineering and Information Technology, University of Technology, Sydney. His research interests include theoretical developments and applications of optimization based methods in many areas of control, signal processing, communication, and bioinformatics.

Ha H. Nguyen (M’01–SM’05) received the B.Eng. degree in electrical engineering from Hanoi University of Technology, Hanoi, Vietnam, in 1995, the M.Eng. degree in electrical engineering from the Asian Institute of Technology, Bangkok, Thailand, in 1997, and the Ph.D. degree in electrical engineering from the University of Manitoba, Winnipeg, MB, Canada, in 2001.

In 2001, he joined the Department of Electrical Engineering, University of Saskatchewan, Saskatoon, SK, Canada, where he is currently a Full Professor. He holds adjunct appointments with the Department of Electrical and Computer Engineering, University of Manitoba, and TRLabs, Saskatoon. From October 2007 to June 2008, he was a Senior Visiting Fellow with the School of Electrical Engineering and Telecommunications, University of New South Wales, Sydney, NSW, Australia. He is a coauthor, with E. Shvedyk, of the textbook A First Course in Digital Communications (Cambridge, U.K.: Cambridge University Press). His research interests include digital communications, spread spectrum systems, and error-control coding.

Dr. Nguyen is a Registered Member of the Association of Professional Engineers and Geoscientists of Saskatchewan. From 2007 to 2011, he was an Associate Editor for the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS. He currently serves as an Associate Editor for the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY and the IEEE WIRELESS COMMUNICATIONS LETTERS. He was a Cochair for the Multiple Antenna Systems and Space–Time Processing Track, IEEE Vehicular Technology Conferences (Fall 2010, Ottawa, ON, Canada, and Fall 2012, Quebec City, QC, Canada).

Ha H. Nguyen

Tung T. Pham received the Ph.D. degree in electrical engineering from the University of Saskatchewan, Saskatoon, Canada, in 2011.

He is a Postdoctoral Fellow with the Department of Electrical and Computer Engineering, University of Saskatchewan. His current research interests include wireless communications and signal processing.

KHA et al.: SC-FDMA MULTI-USER MULTI-RELAY NETWORKS 479


