Technical Section

A real-time cloth draping simulation algorithm using conjugate harmonic functions

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Received 16 January 2006; received in revised form 17 June 2006; accepted 28 September 2006

Abstract

This paper describes a simplified mathematical model and the relevant numerical algorithm that simulates a draped cloth over a virtual human body. The proposed algorithm incorporates an elliptical, or non-consecutive, method to simulate the cloth wrinkles on moving bodies without having to reference the results of past drape simulation time steps. A global–local analysis technique was employed to decompose the drape of a cloth into large-scale deformation and local wrinkles. The large-scale deformation is determined directly by the rotation and translation of body parts to generate a wrinkle-free yet globally deformed shape of the cloth. The local wrinkles are calculated by solving simple elliptical equations based on the orthogonality between conjugate harmonic functions. The large-scale deformation and the local wrinkles are then superposed to simulate the draped cloth. The elliptical equations used to simulate the local wrinkles require no interpolative time frames, even for rapidly moving virtual bodies. Avoiding the incremental approach of time integration used in conventional methods, the proposed method yields markedly enhanced computational efficiency as well as enhanced simulation stability.

Keywords: Interactive cloth simulation; Global–local analysis; Conjugate harmonic functions

1. Introduction

The reality of a graphically created, virtual human body largely depends on the simulated cloth it wears, since a major portion of the body is normally covered with clothing. Recent applications of virtual human bodies have higher demands for rapid and robust cloth simulation capability. A good cloth simulation algorithm should be able to create the cloth wrinkles in as short a CPU time as possible and remain stable for abrupt motions of the virtual body.

Many efficient methods have been developed to simulate clothing on the human body. Weil [1] considered an imitation of a draped cloth using the hyperbolic curve of catenary cables as an elementary function. Agui et al. [2] examined a geometric method to simulate wrinkles at the fold area. The work of Ng and Grimsdale [3] recognized that the folds and the wrinkles, or in their original terms, the fold function and the profile function, are directionally relevant. They also proposed that the fold line could be represented by a sinusoidal function line of equal phase. In the physically based methods proposed by Terzopoulos et al. [4] and Aono [5], wrinkles were considered as a result of elastic deformation of fabric and solved via differential equations concerning the force balance in clothes. Kunii [6] and Taffeier [7] used hybrid geometric and physically based methods in which they adopted trigonometric functions to represent the wrinkles as waves of variable amplitude and wavelength. Volino, Thalmann and others [8–10] considered various forces acting on the cloth to simulate the drape subject to dynamic body motions. Breen et al. [11] employed a particle-based method by using the minimum-energy principle to obtain a state of force equilibrium corresponding to the minimal internal energy between particles. Baraff and Witkin [12] presented an efficient simulation method based on the implicit time-integration scheme to solve the physically based equations. Cordier et al. [13] considered the feedback force from the cloth to the skin to enhance the reality of the simulated cloth.

The drape simulation algorithm proposed in this paper is a hybridization of geometric and physically based methods. A global–local analysis technique was employed to decompose the drape into large-scale deformation and local wrinkles. The large-scale deformation was determined by the rotation and translation of the body parts. Elliptical equations were used for local wrinkles that can be solved without referencing the results of past time steps. The wrinkle equations were solved independently for each keyframe, and the intermediate frames were interpolated to give a smooth evolution in between. Using this interpolative approach, a significant reduction of computation time was achieved over the incremental time-integration methods.

2. Mathematical model

2.1. Geometric observation of cloth draping characteristics

The drape of cloth on a body is governed by internal and external forces. According to Newton’s second law, the force balance on each point of cloth is given as follows:

$$m \frac{d^2 \mathbf{r}}{d t^2} = F_{\text{external}} + F_{\text{internal}},$$  \hspace{1cm} (1)

where \( \mathbf{r} \) is a point on the cloth, \( m \) is the mass, and \( F_{\text{ext}} \) and \( F_{\text{int}} \) are the external and internal forces, respectively. The internal forces include the spring force and the viscous force, among others [4]. The external forces include the body–cloth interaction, the gravitational force and the wind drag. The body–cloth interaction exerted on a contact area is considered to be a concentrated force on a point or a distributed force over a small area. Although Eq. (1) can be considered as a universal equation that describes the overall deformation and small wrinkles of fabric in a unified fashion, solving this equation to obtain a detailed shape of wrinkles can be time-consuming.

In this study, a simplified method is suggested to solve for the global deformation and the local wrinkles separately, and the two results are later superposed. In real cloths, inplane cloth deformation depends on the stretch of fabric in tension, and the out-of-plane deformation is caused by transverse buckling. A fabric is, in general, resistant to tensile deformation, while it yields to buckle under transversal loads. Consequently, the tensile stress in a draped cloth aligns the wrinkles in the direction of tension, as seen in Fig. 1. The wrinkle amplitude largely depends on the distance from contact points. Near the point of contact between a cloth and the external object, such as a human body, cloth deformation is constrained by the body, and the wrinkle amplitude is small. The amplitude grows with increasing distance from the contact points. Subject to this point load, the direction of wave-spread can be assumed to be perpendicular to the lines of tensile stress (Fig. 1).

2.2. Decoupling of the in-plane deformation and the out-of-plane corrugation

Based on the geometric observations of cloth deformation due to internal and external forces, simple equations can be derived to describe the direction and the amplitude of the wrinkles. The basic idea of the proposed method is to decouple the global deformation of the fabric; such as global-scale bending or in-plane stretching, and the local-scale wrinkles, so as to handle the two components separately. The solutions to equations for the two different scales are then superposed.

The widely used force-balance equation for global deformation is given in Eq. (1). The equation for local wrinkles, however, calls for additional assumptions. Defining \( \Phi \) as the wrinkle amplitude function that is a

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Fig. 1. Lines of stress and the amplitude of wrinkles subject to a drag force applied on a cloth. Note that the lines of tensile stress and the lines of wave spreading are perpendicular to each other. (a) Simulated wrinkles on a cloth pulled at the corner. (b) Schematic diagram of the draped cloth.
twice-differentiable scalar variable, the stress vector \( \vec{\sigma} \) can be defined as the gradient of \( \Phi \) with a factor \(-k_\Phi\):
\[
\vec{\sigma} = -k_\Phi \nabla \Phi.
\]  
Assuming the stress on a cloth as a conservative force, the divergence of \( \vec{\sigma} \) is zero and hence, the wrinkle amplitude function satisfies the Laplace equation:
\[
-\nabla \cdot \vec{\sigma} = k_\Phi \nabla^2 \Phi = k_\Phi \left( \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} \right) = 0,
\]  
where \( x \) and \( y \) are the coordinates of the body-fitted (local) coordinate system. The wrinkle amplitude \( \Phi \) and the phase angle of the sinusoidal wrinkles \( \Psi \) are assumed to constitute a pair of conjugate harmonic functions. The complex potential \( F \) can be defined as
\[
F(z) = \Phi(x,y) + i\Psi(x,y),
\]
where \( z = x + iy \),
\[z = x + iy,
\]  
where \( z \) is the complex coordinate, with \( x \) and \( y \) are the body-fitted local coordinates.

Since the wrinkle amplitude and the phase angle are assumed to be the conjugate harmonic functions of each other, the Cauchy–Riemann equations [20] are satisfied by multiplying these by the constant factors of \( k_\Phi \) and \( k_\Psi \).

\[
k_\Phi \frac{\partial \Phi}{\partial x} = k_\Psi \frac{\partial \Psi}{\partial y},
\]
\[
k_\Phi \frac{\partial \Phi}{\partial y} = -k_\Psi \frac{\partial \Psi}{\partial x}. \tag{5}
\]

As a result of Eqs. (3) and (5), the phase angle \( \Psi \) is also a solution to the Laplace equation:
\[
k_\Psi \nabla^2 \Psi = k_\Psi \left( \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} \right) = 0. \tag{6}
\]

The two conjugate harmonic functions \( \Phi \) and \( \Psi \) are orthogonal to each other and hence, an orthogonal net can be drawn from the contour lines of the wrinkle amplitude and the phase angle. Using the wrinkle amplitude \( \Phi \) and the phase angle \( \Psi \), the out-of-plane deformation \( \omega \) can be calculated by the following equation:
\[
\omega = \Phi \sin \Psi. \tag{7}
\]

The out-of-plane deformation is accompanied by the shrinkage of the fabric projection area, so that the total area of fabric is conserved through the deformation. This can be compensated for by relocating the in-plane coordinates as follows:
\[
x' = x - \frac{1}{2} \int \left( \frac{d\omega}{dx} \right)^2 \, dx, \quad y' = y - \frac{1}{2} \int \left( \frac{d\omega}{dy} \right)^2 \, dy. \tag{8}
\]

2.3. Boundary conditions

Since the wrinkle amplitude at the contact points is fixed at zero, and the force vectors applied to those points result in a gradient of wave amplitude, the contact points require conjugate boundary conditions:
\[
\Phi = 0,
\]
\[
-k_\Phi \nabla \Phi = \vec{\sigma} = \sigma_i \hat{i} + \sigma_j \hat{j}, \quad \text{at the contact points}, \tag{9}
\]
where \( \hat{i} \) and \( \hat{j} \) are the unit vectors in the orthogonal body-fitted coordinate system.

On the fold lines, the phase angle can be fixed as a constant value [3]:
\[
\Psi = \text{constant}, \quad \text{along the fold lines}. \tag{10}
\]

The force vector \( \vec{\sigma} \) at each contact point can be determined by solving the force equilibrium equation of the cloth and body. At a contact point, there is no out-of-plane deformation of the fabric, thus, the wrinkle amplitude becomes zero. Fig. 2 shows the force equilibrium of a rectangular cloth hanging vertically on the two upper corners, which are the contact points. The force vectors on the contact points, calculated from the force equilibrium, are then used to determine the boundary conditions to solve for wrinkle amplitude. In Fig. 3(a), the boundary conditions and the subsequently calculated wrinkle amplitude are shown for the vertically hanging cloth. The amplitude is zero at the contact points and becomes larger with increasing distance from the contact points. By using the orthogonality principle between the contour lines of conjugate harmonic functions \( \Phi \) and \( \Psi \), the lines of the phase angle can be obtained. Fig. 3(b) shows the contour lines of the phase angle corresponding to the direction of the wrinkles.

In three-dimensional problems, the force vector on a contact point has an additional component: the force in the surface normal direction. The force applied to each contact point in a complex, three-dimensional cloth can be calculated by solving the force balance equation, considering the cloth as shells or tubes seamed together. In many cases, the locations of the primary contact points are given. Shirts usually contact with the shoulders, neck, breasts, elbows and wrists. Pants are belted on the waist and contact with the hips, knees and ankles. These contact points require boundary conditions for the conjugate harmonic equations to be solved. Two typical types of contact force are the stretching force in the in-plane direction, and the normal force in the out-of-plane

Fig. 2. Force equilibrium on a vertical cloth.
direction. A stretching force can be handled in the same manner as described in Fig. 2, by replacing the vertical load with tensile loads applied in the inplane direction. A normal force gives rise to an out-of-plane deformation and the subsequent in-plane forces constituting a balance with the forces on other contact points. These forces can be used as the boundary condition in solving the wrinkle equations.

2.4. Contact force and the boundary conditions

The wrinkle-free shape of cloth simulated for an arbitrary body posture is obtained by conforming the initial cloth form to the body shapes. Each section of the cloth is discretized into vertices and triangular elements for a finite element analysis. The contact points are determined by the distance from each vertex of cloth to the skin of the body. If the distance at a vertex turns out to be smaller than a critical value, it is considered to be a contact point. The forces on the contact points can be calculated by solving the force equilibrium equation. Fig. 4 shows an example of the contact points. The shaded area represents the area of contact, and the thick dashed lines show the folds from bending.

3. Finite element formulation

3.1. Numerical formulation and the solution procedure

A finite element code was developed to simulate the draped cloth. In order to account for the interaction between vertices more efficiently, the control volume approach was incorporated into the conventional finite element formulation [21]. In the body-fitted local coordinate system, the calculation domain is discretized into a finite number of triangular shell elements. Each vertex represents a small area in the polygon made by connecting centroids of the elements and the midpoints of the element borders. The calculation domain is then divided into triangular elements and polygonal control volumes (Fig. 5). The boundary of each polygon constitutes the control surface of the corresponding control volume.

The draping simulation is performed in the following sequence. First, the wrinkle-free cloth is put on the virtual body and the distance between each vertex and the body surface is calculated. If the distance measures less than a
bodd–cloth interaction (for example, the force named \( C \) and the in-plane forces between vertices, each vertex in the contact point). In addition to the gravitational weight \( W_i \), the vertical arrow denoted as \( R_i \) represents the weight of a small area inside the closed polygon (shown in dashed lines) surrounding the point, as illustrated in Fig. 6. The vertical arrow denoted as \( W_i \) represents the weight of the small polygon surrounding the contact point \( i \). In addition to the gravitational weight and the in-plane forces between vertices, each vertex in the contact area has a normal force that results from the bodd–cloth interaction (for example, the force named \( C_i \) on vertex \( i \) in Fig. 6). The contact forces are found by summing these forces on each vertex and using the force balance in the \( x \)-, \( y \)-, and \( z \)-directions.

The contact force at each vertex in the contact area is then used as an input for Eq. (4) to calculate the distribution of wrinkle amplitudes. The gradient of the wave amplitude is correlated with the phase angle of the wrinkles by Eq. (5). Once the wave amplitude and the phase angle are obtained, Eq. (7) is used to determine the normal deflection of cloth at each vertex. The normal deflection for fixed, in-plane coordinates gives rise to the stretch of fabric. Hence, the in-plane coordinates are moved by Eq. (8), such that the area of each finite element is conserved through the stretch.

It should be noted that the cloth can only transfer tensile loads, not compressions. Therefore, if the force on any link between vertices turns out to be compression, that link needs to be eliminated and the force balance should be recalculated. Using the contact forces on the contact points as boundary conditions, the wrinkle amplitude and the phase angle are calculated by Eqs. (2)–(10).

### 3.2. Addition of random noise

A real draped cloth has numerous random factors that cannot be accounted for in solving the above equations. These factors include defects in the fabric and seams, mislocations and worn-out patches. Paradoxically, these defects contribute to the realism of the cloth on the body. Although these random factors are hardly quantified, a numerical perturbation can be added to enhance the reality of the simulated cloth. An example of the random noise function is

\[
\phi_i^{R,N} = \Phi + \sin\left(\frac{r_i - a_i^\Phi}{b_i^\Phi} \pi\right)[u(r_i - a_i^\phi) - u(r_i - b_i^\phi)],
\]

\[
\Psi_i^{R,N} = \Psi + \sin\left(\frac{r_i - a_i^\Psi}{b_i^\Psi} \pi\right)[u(r_i - a_i^\Psi) - u(r_i - b_i^\Psi)],
\]

\[
\omega_i^{R,N} = \omega_i^{R,N} \sin(\Psi_i^{R,N}) - \Phi \sin \Psi,
\]

\[
\tilde{\omega} = \omega + \sum_{i=1}^{N} \omega_i^{R,N},
\]

where \( r_i \) is the geodesic distance from each contact point and \( u(x) \) is the step function defined as

\[
u(x - a) = \begin{cases} 
0, & x < a, \\
1, & x > a,
\end{cases}
\]

where \( a_i^\phi, b_i^\phi, a_i^\Psi \) and \( b_i^\Psi \) are the randomly generated constants to add noise to the simulated wrinkles. The process of adding noise is optional to the main algorithm.

### 4. Wrinkle simulation results

The draping of a T-shirt on a human body was simulated using the proposed algorithm. The cloth was discretized into triangular finite elements and vertices, and the finite element mesh was put on the virtual body to determine the contact points by calculating the distance from each vertex to the surface of the body. Fig. 7(a) shows the simulation result of the large-scale deformation. The contact points and the fold lines were determined as described in Section 3.1 and used as boundary conditions in solving for the wrinkle amplitude in Eq. (3). The gradient of the wrinkle amplitude was calculated to determine the direction of wrinkles...
corresponding to the constant values of the phase angle. After the wave amplitude and the phase angle were obtained, Eq. (7) was used to determine the normal deflection of each vertex with respect to the surface of the winkle-free cloth. If the deflection of a vertex is positive, it becomes a part of the rise in a wrinkle. Negative deflection constitutes a groove of the wrinkle. The in-plane locations of the vertices were then rearranged using Eq. (8) so that the area of each triangular element is conserved. Since the wrinkle equations are elliptical, the shape of the wrinkles can be calculated directly for the given body posture. A sample result is shown in Fig. 7(b). The total number of triangular finite elements was 6602. The required processing time was 0.72 s on an Intel Pentium IV 1.4 GHz processor.

Although the CPU time of 0.72 s for a single frame cannot be called a real-time rate, it is noted that the simulation could be performed independently for any two postures against any time interval. The CPU time for simulating one frame does not take proportionally long for a sequence of frames, since the intermediate frames can simply be interpolated which is impossible for any incremental time-integration methods. Solving for a few keyframes in a sequence of motion independently, and then engaging the interpolation technique, the proposed method can be used in simulating the cloth at real-time rates. With a polygon counts less than 5000, the real-time simulation can be much easier. This will be illustrated in the following examples.

In real cloths, the wrinkle intensity usually depends on the material properties of the fabric. One body posture can result in different wrinkle intensities for cloths made of different fabrics. Fig. 7(c) shows the variation of wrinkle intensity in fabrics of different $k_F$ values in Eq. (3) and $k_C$ in Eq. (6). The result shows that a larger $k_F$ value results in a smaller wrinkle amplitude. On the other hand, a larger $k_C$ value yields a smaller increase in phase angles, and hence, the space between wrinkles becomes larger.

Fig. 8 illustrates the simulated drape of a sweater on a female body. The simulation procedure is the same as the method used for the T-shirt in Fig. 7. The body was in motion, starting from a standing posture (Fig. 8(a)), then turning around and raising the left arm (Fig. 8(b)). Since the wrinkles are calculated by elliptical equations, the simulation was performed directly for each posture without introducing any incremental time frames. The total number of triangular finite elements was 8814. The required processing time was 0.91 s for the posture in Fig. 8(a), and 1.02 s for the posture in Fig. 8(b) on an Intel Pentium IV 1.4 GHz processor. Fig. 9 shows an example of the instant superposition between a global-scale deformation and local-scale wrinkles in a T-shirt and pants.

Steering away from the incremental approach of time integration used in conventional methods, the proposed method consumes practically no CPU time for the interpolative time frames. Table 1 shows the per-frame simulation time for the proposed method and the conventional methods for similar number of vertices. In the conventional methods, the processing time increases proportionally to the number of interpolative time frames for discontinuous body postures. For example, in the results from Oh et al. [22], the per-frame simulation time was 0.04 s for 1000 vertices and the time increment of simulation was also 0.04. Hence, their method could only be used with up to 1000 vertices to achieve 25 fps for real-time applications.

On the other hand, the proposed method requires computation only for a few keyframe along the path of motion, such as the initial- and the final frames for a given interval. For these keyframes, the proposed method consumes similar CPU time as the conventional methods.
Between these keyframes, the time increments can be obtained by a simple interpolative rule given in Eq. (13), and hence no serious processing time is required for the intermediate time frames.

Consider the case of Fig. 10 in which the leg moves rapidly from the State 1 to State 2. The CPU time required for this swing is for these two states only. Any transient posture between States 1 and 2 can be interpolated by using a weight value

$$o(t) = o_1 + (1 - \varepsilon)o_2,$$

where $o_1$ and $o_1$ are the out-of-plane deformation on the vertices corresponding to the States 1 and 2, respectively.

Once the wrinkle deformation is determined for the two states by the wrinkle simulation equations, the interpolation rule in Eq. (13) can be used for arbitrary instances inbetween without serious time consumption, which is basically the same idea as the tweening/skinning algorithms. Tweening (in-betweening) is a process for generating intermediate frames between two images to give an appearance that the first image evolves smoothly into the second image. Skinning, one of the widely used tweening techniques is to allow a mesh to be deformed based on an underlying transformation matrix set and usually thought of as a skin being deformed by a skeleton [24,25].

In our method, the cloth can be considered as a skinned mesh. The cloth deformation in the first- and the second frames can be obtained independently by the proposed elliptical equations, and the location of each vertex can easily be evolved from one position to another for a smooth transformation without engaging serious simulation for the intermediate frames. From Table 1, the reduction of per-frame simulation time for the initial- and the final postures may seem rather trivial, but the elimination of intermediate time frames can offer a huge save. This could be made possible only because the cloth deformation in each keyframe can be simulated independently, without referencing the previous time frames.

Another feature of the proposed method is the enhanced stability. A fast-moving human such as a dancer or a sprinter, often swings the arms and the legs 2–3 times per seconds. In conventional methods, the motion of the body should be moderated to avoid the jamming of the cloth and hence a rapid movement of the arms and the legs could hardly be achieved in real-time, especially for high polygon counts. Consider the case of Fig. 10 to solve for a movement between two states, State 1 for the initial posture and State 2 for the final posture for a given interval of motion. In an incremental approach, the CPU time is to be consumed at every time step as well as the initial and final postures, since a large time increment may cause the solution instability problem. Assuming an angular motion of 90° to move the thigh in 1 s, for example, a time increment in the range of 0.05 s can yield only six steps which corresponds to a sequence of angular motions by 15° each. With the time step of 0.1 s, only three frames with 30° of incremental rotation can be processed during the 90° stroke. This could yield a serious intersection problem.

![Fig. 9. The superposition of global-scale deformations and wrinkles in a local-scale. (a) Deformations in a global-scale. (b) The superposition with wrinkles in a local-scale.](image)

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<th>Table 1: Comparison of computational efficiency</th>
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<td>Number of vertices</td>
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<td>Oh et al. [22]</td>
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<td>Our method</td>
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<td>Choi et al. [23]</td>
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<td>Zhou et al. [26]</td>
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<td>Our method</td>
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between the body parts and the cloth, no matter how effective the collision detection algorithm could be.

A study of Choi et al. used a time increment of 0.011 s for a stable cloth simulation for a skirt [23]. Tighter cloths such as jeans and shirts will require more. Zhou et al. [26] extended the immediate buckling model proposed by Choi et al. In their study, stable animations could be obtained at the same time step size of 0.011 s as in the study of Choi et al. Kang [27] animated a streaming flag composed of 2048 triangles for generating plausible appearance of the

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**Fig. 10.** Schematic of the interpolative- and the incremental time integration. (a) Keyframes for the initial posture (State 1) and the final posture (State 2) to be simulated independently by the proposed method. Intermediate frames will be interpolated for smooth evolution. (b) Incremental time integration using incremental frames (CPU time required per each frame). A large time increment may cause the solution instability.

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**Fig. 11.** The continuity of wrinkles against continuous deformations. (a) Discontinuous deformations on a global-scale. (b) The continuous superposition between a global-scale deformation and local-scale wrinkles using an interpolation technique.
cloth. The size of the time step was 0.0033 s. In the conventional manners, therefore, it would be difficult to simulate the cloth deformation in real-time or at interactive rates unless a preliminary data processing is engaged, which confines the movement path [28].

In the proposed method, the stability problem for large time increments can be easily managed as can be seen in Fig. 11. Fig. 11(a) shows the initial- and the final shapes of the global deformation. Fig. 11(b) shows the continuous deformation between the two states using the interpolation rule. In this case, the wrinkle simulation is to be performed for the first step (wrinkle-free) and the last step (largest wrinkle) only. All the intermediate states can be obtained using the interpolation technique in Eq. (13).

5. Conclusion

A visual observation of the cloth deformation characteristic was represented by the mathematical orthogonality between conjugate harmonic functions. Based on this observation, we developed a non-consecutive drape simulation algorithm by hybridizing geometric and physically-based methods to simulate the cloth deformation in real time. The proposed algorithm incorporates a global–local analysis technique, in which the drape of a cloth is decomposed into large-scale deformation and local wrinkles that are later superposed. Since the equation for wrinkles is elliptical, the simulation can be performed without referencing the result of previous time steps. By employing an interpolation technique between two independent frames, the processing time could be markedly reduced, and the numerical algorithm remains stable for abrupt changes in body posture.

References