

PACS no.s 04.60.+n, 03.65.-w, 11.10.-z
 Preprint IBR-TH-98-S-023, Dec. 23, 1998

UNIVERSALITY OF THE LORENTZ-POINCARÉ-SANTILLI'S ISOSYMMETRY FOR THE INVARIANT DESCRIPTION OF ALL POSSIBLE SPACETIMES

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Abstract

We review the origin of the physical consistency of the Lorentz- Poincaré symmetry. We outline seemingly catastrophic physical inconsistencies recently identified for noncanonical-nonunitary generalized theories defined on conventional spaces over conventional fields. We review Santilli's isotopic lifting of the Lorentz-Poincaré symmetry, by proving its invariant resolution of said inconsistencies, and universality for the representation of all possible spacetimes with a symmetric metric. The explicit isosymmetry transforms are identified. Particular care is devoted to the recent discovery of the 11-th dimensionality of the conventional Poincaré symmetry and the consequential emergence of an axiomatically consistent grand unification of electroweak and gravitational interactions. The article closes with an outline of the broader geno- and hyper-symmetries and their isodual for the description of single-valued irreversible systems, multivalued irreversible systems and antimatter systems, respectively.

1. Lorentz-Poincaré Symmetry.

Physics is a discipline admitting the reduction of events to primitive symmetries, the most important ones being the symmetries of our spacetime [1], namely, the rotation, boosts, translation and discrete symmetries, hereon called the *Lorentz -Poincaré symmetry* (or the *L-P symmetry* for short) and denoted P(3.1).

We are referring to the most general possible, linear, local-differential and canonical (for classical formulations) or unitary (for operator formulations) symmetries of the Minkowski space $M = M(x, \eta, R)$ with: spacetime coordinates $x = \{x^\mu\} = (x^k, x^4), x^4 = c_o t, \mu = 1, 2, 3, 4, k = 1, 2, 3, c_o$ being the speed of light in vacuum; unit $I = \text{Diag.} (1, 1, 1, 1)$; metric $\eta = \text{Diag.}(1, 1, 1, -1)$; and invariant on the field $R = R(n, +, \times)$ of real numbers n with conventional sum $+$ and associative product \times

$$(x - y)^2 = (x - y)^\mu \times \eta_{\mu\nu} \times (x - y)^\nu =$$

$$= (x-y)^1 \times (x-y)^1 + (x-y)^2 \times (x-y)^2 + (x-y)^3 \times (x-y)^3 - (x-y)^4 \times (x-y)^4 = inv. \quad (1.1)$$

All spacetime theories possessing the Lorentz- Poincaré symmetry have an impeccable axiomatic and physical consistency, as it is the case for relativistic quantum mechanics, special relativity, unified gauge theories of electroweak interactions, and other theories.

These historical successes of the L-P symmetry are due to the *invariant* (rather than covariant) character of the theories, which, in turn, is permitted by their (canonical or) unitary structure on a Hilbert space \mathcal{H} over the field $C(c, +, \times)$ of complex numbers c ,

$$U \times U^\dagger = U^\dagger \times U = I. \quad (1.2)$$

The fundamental Lorentz-Poincaré invariance begins with the invariance under the time evolution of the theories, and implies the numerical invariance of the basic units used for measurements, the preservation in time of Hermiticity-observability, the invariance of the special functions and transforms used in data elaboration, the uniqueness and invariance of the numerical predictions, and other features essential for physical consistency.

In the final analysis, the above mathematical and physical consistency can be traced to the fact that classical or operator Lorentz-Poincaré invariant theories possess a *Lie structure*.

Even though well known, it may be useful for subsequent referrals to recall the basic invariances for unitary theories

$$\begin{aligned} I &\rightarrow U \times I \times U^\dagger = I' = I, \\ A \times B &\rightarrow U \times (A \times B) \times U^\dagger = U \times A \times U^\dagger \times U \times B \times U^\dagger = A' \times B', \\ H \times |\psi \rangle &= E \times |\psi \rangle \rightarrow U \times H \times |\psi \rangle = U \times H \times U^\dagger \times U |\psi \rangle = H' \times |\psi' \rangle = \\ &U \times E \times |\psi \rangle = E' \times |\psi' \rangle, E' = E, \end{aligned} \quad (1.3)$$

THEOREM 1: All theories with a unitary structure on a Hilbert space over the field of complex numbers possess numerically invariant units, products and eigenvalues, thus being suitable to represent physical reality.

2. Inconsistencies of Noncanonical-Nonunitary Generalizations.

This paper will have achieved its first objective if it contributes to stimulate the awareness by the contemporary physica community to come to its senses, and address the rather serious physical inconsistencies of spacetime theories with a noncanonical or nonunitary structure treated via the mathematics of canonical or unitary theories.

Physics is a quantitative science in which, sooner or later, physical truths always emerge. Therefore, silence on these inconsistencies can only damage the authors of papers on noncanonical-nonunitary theories.

The lack of universality of the Poincaré symmetry for the description of the entire universe was identified immediately following its appearance and then confirmed throughout this century. This scientific process lead to the construction of numerous theories representing events in our spacetime, yet violating the Lorentz-Poincaré axioms in favor of broader axioms.

No understanding of the topic of this paper (the isotopies of Lorentz-Poincaré) can be claimed without at least a rudimentary knowledge of the now considerable literature on the indicated inconsistencies.

The first generalization is due to Einstein himself who, immediately following the formulation of the special relativity, identified the impossibility of representing gravitation with the realization of the Lorentz-Poincaré axioms of the time, and formulated the general theory of relativity on Riemannian spaces [2].

While Einstein's studies based on the Lorentz-Poincaré symmetry have passed the test of time and are nowadays more valid than ever, Einstein's theory of gravitation, which departs from said symmetry, has been the subject of endless, still unresolved and actually increasing controversies during this century (see, e.g., representative papers [3] and references quoted therein).

The origin of most of these controversies has been recently identified by Santilli [3f] and can be summarized as follows. The map from the Minkowski metric η to the Riemannian metric $g(x)$ is clearly a *noncanonical* transformation at the classical level and a *nonunitary transformation* at the operator level,

$$\eta \rightarrow g(x) = U(x) \times \eta \times U^\dagger(x), U \times U^\dagger \neq I. \quad (2.1)$$

As a result, *any theory on a curved manifold is structurally noncanonical-nonunitary, beginning with its time evolution.*

Despite an undeniable *mathematical beauty* that has attracted so many scholars throughout this century, a host of rather serious problems of *physical consistency* then follows.

THEOREM 2 [3f]: All theories with a nonunitary structure on a conventional Hilbert space over the field of complex numbers, thus including (but not limiting to) all operator theories of gravity formulated on a curved manifold, possess the following physical inconsistencies:

- 1) *lack of invariant units of space, time, energy, etc., with consequentially impossible applications to real measurements;*
- 2) *lack of preservation of the original Hermiticity in time, with consequential absence of physically acceptable observables;*
- 3) *general violation of causality and probability laws;*
- 4) *lack of invariance of conventional and special functions and transforms used in data elaborations;*
- 5) *lack of uniqueness and invariance of numerical predictions;*
and have other inconsistencies which render them inapplicable to represent physical reality

The proof of these occurrences is elementary. The lack of invariance of the basic units is inherent in the very conception of nonunitary transforms (see later on for details). The lack of preservation in time of Hermiticity-observability is known as *Lopez's lemma* [3g]. The violation of probability laws is an evident consequence of the lack of invariance of the basic units, with consequential violation of causality. Nonunitary transforms do not preserve

elementary functions such as the exponentiation, let alone special functions and transforms. The lack of uniqueness of the numerical predictions is evident from the lack of uniqueness of the value of nonunitary transforms, while the lack of invariance of the numerical predictions is so evident to require no comments.

Even though known, it may have graphical value to review the fundamental noninvariances under nonunitary transforms from which all the physical inconsistencies follow [3f]:

$$I \rightarrow U \times I \times U^\dagger = I' \neq I,$$

$$A \times B \rightarrow U \times (A \times B) \times U^\dagger = U \times A \times U^\dagger \times (U \times U^\dagger)^{-1} \times U \times B \times U^\dagger = A' \times T \times B', T = (U \times U^\dagger)^{-1},$$

$$H \times |\psi \rangle = E \times |\psi \rangle \rightarrow U \times H \times |\psi \rangle = U \times H \times U^\dagger \times (U \times U^\dagger)^{-1} \times U \times |\psi \rangle = H' \times T \times |\psi' \rangle = U \times E \times |\psi \rangle = E' \times |\psi' \rangle, E' \neq E, \quad (2.2)$$

namely, *under nonunitary time evolutions and transforms we have the alteration of the numerical value of all basic units, all product and all eigenvalues.*

Santilli [3f,5l,6c] has identified additional catastrophic inconsistencies which apply to both noncanonical and nonunitary theories. Recall that all physical theories are based on numbers and fields which, in turn, are based on the fundamental (multiplicative) unit. The alteration of the unit by noncanonical-nonunitary transforms then implies the shift to *different numbers and fields*. But noncanonical-nonunitary theories continue to be defined on *conventional* numbers and fields. This implies the collapse of the axiomatic consistency of the entire theory, including the inapplicability of vector and metric spaces, geometries and topologies, algebras, groups and symmetries, etc., with no known exceptions.

Note that the latter arguments rules out the physical value of any *classical* noncanonical theories, again, because they imply the alteration of the basic unit with consequential inapplicability of the carrier spaces used to elaborate the theory.

The above catastrophic physical and axiomatic inconsistencies apply in their entirety to the classical and operator formulation of gravity on curved manifolds. As an example, there is no known physical meaning or consistency in attempting the "experimental verification" of the general relativity at a given time t defined via field equations on a Riemannian space over the fields of real numbers, when the basic unit I is altered at a subsequent time t' , Eq. (2.2a). We then have the consequential lack of physical meaning in preserving the Riemannian space itself because defined on a field no longer applicable at t' . The physical inconsistencies of the operator formulation of gravitation on a curved space are so serious and transparent to require no further comments.

The ultimate origin of the above gloomy scenario investing about one century of studies in gravitation is the very notion of *curvature* itself, because it implies the *breaking of the fundamental Lorentz-Poincaré symmetry* in favor of "covariance" under a broader, often undefined symmetry, with the indicated catastrophic consequences. In fact, the Lorentz-Poincaré invariance and the notion of curvature are mutually exclusive in a transparent and irreconcilable way in their current formulation (see next sections for an alternative).

The limitations of the Lorentz-Poincaré symmetry have also been felt by numerous other scholars besides Einstein, particularly during the recent decades. We here quote: the studies

by Y. S. Kim and others (see [4a] and references quoted therein), which have the important function of extending the applicability of the Lorentz-Poincaré axioms to their ultimate possibilities for the representation of extended particles; the use of broader symmetries in an attempt to reach a grand unification inclusive of the gravitational interactions (see, e.g., [4b]); the broadening of the Lorentz-Poincaré symmetry inherent in contemporary string theories [4c]; and numerous other theories (see other papers in this collection [4d]).

It is important for the contemporary physics community to study, understand and, above all, admit that *all generalized theories with a noncanonical or nonunitary structure, even though possessing an undeniable mathematical beauty, have no known physical application.*

Along these lines, in memoir [6e] of 1996, Santilli clearly states the physical inconsistency of his *Birkhoffian generalization of Hamiltonian mechanics* published in monograph [6g] (by Springer-Verlag in its most prestigious physics series...), precisely because of its noncanonical structure formulated on conventional spaces over conventional fields. The reader should be aware that the Birkhoffian mechanics was proved in the same monograph to be universal for all well behaved, local-differential and nonhamiltonian systems with a generalized Lie-isotopic structure. In the same memoir [6e] Santilli clearly states the additional; physical inconsistency of his broader classical Lie-admissible mechanics of monograph [12f] which is universal for all Newtonian system with a non-Lie, yet algebraically consistent structure. In the same memoir [6e] Santilli presents new invariant classical mechanics of Lie-isotopic and Lie-admissible type we cannot possibly review here for brevity.

Similarly, in memoir [5l] of 1997, Santilli clearly states the physical inconsistency of *all* his generalized operator studies prior to 1997, including all numerous papers written on hadronic mechanics since its proposal of 1978 [6b], including all papers on operator Lie-isotopic and Lie-admissible theories (which are also universal for all possible nonlinear, nonlocal and nonunitary theories with and without an antisymmetric algebras, respectively). In the same memoir [5l] Santilli proposes fully invariant operator, Lie-isotopic and Lie-admissible formulations we shall outline in the next sections).

Regrettably, the same clear statements of physical inconsistencies are lacking at this writing, to our best knowledge, on numerous other generalized theories with a transparent and incontrovertible nonunitary structure, each theory possessing a rather vast literature, such as (see [3f] for complete list and references):

1) Dissipative nuclear models with imaginary potentials, $H = H_o + iV$, and time evolution $idA/dt = A \times H^\dagger - H \times A = [A, H, H^\dagger]$ (these theories lose an "algebra" as commonly understand, in favor of a triple system - as a result of which names such as "proton" and "neutron" lose their physical meaning because of the impossibility to even define spin, mass and other basic characteristics, let alone treat them);

2) Statistical models with external collisions terms with time evolution $id\rho/dt = [\rho, H] + C$ (besides being nonunitary, these theories have no units at all - let alone a noninvariant units - and have no exponentiation at all, under which catastrophic conditions any application to physical reality implies exiting science);

3) q- deformations of the Lie product $A \times B - q \times B \times A$, "q-deformations" of the enveloping associative algebra 'with generalized product $A * B = A \times T \times B$, and other deformations which change the Lie structure while preserving the old mathematics, all being

transparently nonunitary (all these deformations were first introduced by Santilli in his Ph. D. Thesis of 1967 [12a], although this paternity is ignored in the rather vast literature in the field, evidently to the sole detriment of the authors);

4) Certain quantum groups (evidently those with a nonunitary structure);

5) Weinberg's nonlinear theory with nonassociative Lie-admissible envelopes (which lacks any unit, violates Okubo's no quantization theorem prohibiting the use of nonassociative envelopes [3h], and has other serious flaws);

6) All known theories of quantum gravity (the indication of theories in this field with a unitary structure would be appreciated);

7) All known supersymmetric theories (evidently because they broaden the very structure of Lie algebras and groups via the addition of anticommutators, thus resulting in an evident nonunitary structure);

8) all known studies on Kac-Moody superalgebras (also because they depart from Lie's structure with a phase term depending on anticommutators);

9) All known string theories whose nonunitary structure was known since the introduction of the Beta function by Veneziano and Suzuki, and reinforced via supersymmetries in the recent studies (see the specific study [3i]).

Other theories which have a seemingly unitary structure, but depart from other axioms of Lie's theory equally possess serious physical flaws. This is the case, for instance for theories with Hermitean Hamiltonians, yet a structure *nonlinear in the wavefunction* of the type $H(x, p, \psi, \dots) \times |\psi\rangle = E \times |\psi\rangle$ (again, see [3f] for details and references). These theories violate the superposition principle, thus being inapplicable to composite systems; they violate Mackay imprimitivity theorem, thus violating the integrability conditions for the Galilean and Einsteinian relativities; and have other other flaws.

Yet other theories violate the *locality* condition of Lie's theory, e.g., via "integral potentials" in the Hamiltonians. These theories are fundamentally flawed on both mathematical grounds (because the assumption is incompatible with the basic topology) and physical grounds (because nonlocal interactions generally are of contact-zero range type, thus having no potential). As such, these theories deserve no further comment (or attention).

In summary, Santilli has established that *all theories which violate any of the fundamental axioms of linearity, locality and canonicity-unitarity of Lie's theory is physically inconsistent when formulated via the mathematics of quantum mechanics.*

In other cases, the existence of possible inconsistencies requires specific investigations. This is the case of Kim's [4a] theory which replaces the Lorentz-Poincaré *invariance* with a broader *covariance*. These studies are left to the interested readers.

We close this section by indicating that *classical* theories of *antimatter* are generally inconsistent because they only have one channel of quantization for matter and antimatter. As a result, their orator image *does not* yield charge conjugate states, but merely states of particle with the wrong sign of the charge.

The Riemannian treatment of antimatter is afflicted by more catastrophic physical inconsistencies because, in addition to the above inconsistent operator image, they can only represent antimatter via the usual energy-momentum tensors which are notoriously *positive-definite*, thus being in dramatic disagreement with the *negative-definite* energies need for

antiparticles.

These inconsistencies should not be surprising because the biggest unbalance in the physics literature of this century is precisely the treatment of matter at all possible levels, from Newton to quantum field theory, while antimatter is solely treated at the level of second quantization. But antimatter is expected to exist at the macroscopic level, i.e., that of entire galaxies or quasars, thus demanding the restoration of a fully equivalent treatment of matter and antimatter at all levels of study.

By no means all generalized theories of the contemporary physical literature are wrong. In fact, numerous generalized theories constructed on sound foundations have an impeccable axiomatic structure, such as the theories by Ahluwalia [4e], Dvoeglazov [4f], and others.

3. Lorentz-Poincaré-Santilli isosymmetry

By initially working in a rather solitary way, the Italian-American physicist R. M. Santilli [5] has constructed a new realization of the Lorentz-Poincaré axioms which:

1) is "directly universal" for the representation of all infinitely possible, nonlinear, non-local and noncanonical-nonunitary theories in our (3+1)-dimensional spacetime with a well behaved, nowhere singular and symmetric metric (universality), directly in the x-coordinates of the observer without any use of the transformation theory (direct universality);

2) reconstructs the canonicity or unitarity and invariance, on suitably generalized spaces over generalized fields; and

3) resolves the physical inconsistencies indicated in Sect. 2.

Remarkably, Santilli [5] constructed the most general known symmetry of the following most general possible invariant in (3+1)-dimensions with the indicated topological condition on the metric:

$$\begin{aligned} (x-y)^{\hat{2}} &= (x-y)^{\mu} \times \hat{\eta}_{\mu\nu}(x, v, d, \tau, \psi, \dots) \times (x-y)^{\nu} = (x-y)^{\mu} \times \hat{T}_{\mu}^{\rho}(x, v, d, \tau, \psi, \dots) \times \eta_{\rho\nu} \times (x-y)^{\nu} = \\ &= (x-y)^1 \times \hat{T}_{11}(x, v, d, \tau, \psi, \dots) \times (x-y)^1 + (x-y)^2 \times \hat{T}_{22}(x, v, d, \tau, \psi, \dots) \times (x-y)^2 + \\ &+ (x-y)^3 \times \hat{T}_{33}(x, v, d, \tau, \psi, \dots) \times (x-y)^3 - (x-y)^4 \times \hat{T}_{44}(x, v, d, \tau, \psi, \dots) \times (x-y)^4 = inv. \quad (3.1) \end{aligned}$$

where all functions $\hat{T}_{\mu\nu}(= \hat{T}_{\mu}^{\nu})$ are positive definite but otherwise possess an unrestricted, generally *nonlinear, non local and nonhamiltonian* functional dependence on spacetime coordinates x, velocities v, density d, temperature τ , wavefunctions ψ , or any other needed local quantity.

Unexpectedly, Refs. [5] then proved that the universal symmetry of interval (3.1) is locally isomorphic to the symmetry of the *conventional* invariant (1.1), of course, when properly formulated. In fact, Santilli insists in his writings that the symmetry of invariant (3.1) *is not new*, because it is merely a *new realization* of the conventional Lorentz-Poincaré axioms. This implied the reconstruction of the Lorentz-Poincaré symmetry as being *exact* when popularly believed to be broken, as we shall see (e.g., for gravitation).

Santilli [5] then proved the "direct universality" of this symmetry via the explicit construction of the most salient applications.

These results were achieved via the prior construction of a new mathematics, originally proposed in Ref. [6a] under the name of *isomathematics* from the Greek meaning of being

"axiom-preserving", and then developed by various authors [6-8] (see [7n] for a comprehensive literature up to 1984, [5o] for literature up to 1995, and Web Site [7o], Page 18, for a readable outline). The new mathematics is essentially characterized by new numbers, new fields, new spaces, new algebras, etc. called isonumbers, isofields, isospaces, isoalgebras, etc. For this reason the universal symmetry of invariant (3.1) is known as the *Lorentz-Poincaré-Santilli isosymmetry* (also called the *L-P-S isosymmetry* or *Santilli's isopoincaré symmetry* for short), and it is generally denoted $\hat{P}(3.1)$ [6-9].

The main working ideas are essentially the following:

1) the generalization (called *lifting*) of the Minkowski metric η into the most general possible, well behaved, nowhere singular and symmetric metric $\hat{\eta}(x, v, d, \tau, \psi, \dots) = \hat{T}(x, v, d, \tau, \psi, \dots) \times \eta$, where \hat{T} is a 4×4 well behaved, nowhere singular and *positive-definite* (thus diagonalizable) matrix;

2) the joint lifting of the fundamental unit of the Minkowski space, $I = \text{Diag. } (1, 1, 1, 1)$, by the *inverse* of the lifting of the metric, $\hat{I} = 1/\hat{T}$; and

3) the reconstruction of the entire mathematical foundations of Lorentz and Poincaré into a form admitting \hat{I} , rather than I , as the correct left and right unit of the new theory.

The latter condition requires the lifting of the conventional associative product $A \times B$ among generic quantities A, B (numbers, matrices, operators, etc.) into the form $A \hat{\times} B = A \times \hat{T} \times B$, with \hat{T} fixed, for which $\hat{I} \hat{\times} A = A \hat{\times} \hat{I} = A$ for all possible A . In this case (only), \hat{I} is called the *isounit*, and \hat{T} is called the *isotopic element*.

In turn, the latter liftings imply, for evident reason of consistency, the new *isofields* $\hat{R} = \hat{R}(\hat{n}, \hat{+}, \hat{\times})$ [6b] of *isonumbers* $\hat{n} = n \times \hat{I}$ with *isosum* $\hat{n} \hat{+} \hat{m} = (n + m) \times \hat{I}$, *isoproduct* $\hat{n} \hat{\times} \hat{m} = (n \times m) \times \hat{I}$, *isoquotient* $\hat{A} \hat{/} \hat{B} = (A/B) \times \hat{I}$, and other generalized operations.

Under the above conditions, it is evident that \hat{R} and R are isomorphic, and actually coincide at the abstract level (because \hat{I} and I are topologically identical). Despite this simplicity, the reader should abstain from jumping at conclusion of mathematical triviality to avoid insidious misrepresentations. As an illustration, "two multiplied by two is sixteen" and the number 4 becomes *prime* for isounit $\hat{I} = 4$. This indicates the dependence of number theory from the assumed unit. Following memoir [d] the *Santilli's isonumber theory* has been the subject of comprehensive studies by C. X. Jiang [7g,7m], Kamiya [7h], Trell [7i], and other mathematicians.

By recalling that metric spaces are defined on a given field, the availability of new numbers and fields permitted the construction of the isotopies of the Minkowski space, presented for the first time in Ref. [5a] (see also [5,6]), today called *Minkowski-Santilli isospaces* and denoted $\hat{M} = \hat{M}(\hat{x}, \hat{\eta}, \hat{R})$ with *spacetime isocoordinates* $\hat{x} = x \times \hat{I}$ defined precisely on \hat{R} , and consequential lifting of algebras, groups, geometries, topologies, etc. [5,6,7].

Under the above liftings, i.e.,

$$\eta \rightarrow \hat{N} = (\hat{\eta}_{\mu\nu} \times \hat{I}) = (\hat{T}_{\mu}^{\rho} \times \eta)_{\rho\nu} \times \hat{I}, \hat{T} > 0, I \rightarrow \hat{I} = 1/\hat{T}, A \times B \rightarrow A \hat{\times} B = A \times \hat{T} \times B, \text{ etc.} \quad (3.2)$$

the new isospaces \hat{M} are locally isomorphic to the conventional space M ; the isosymmetry $\hat{P}(3.1)$ is locally isomorphic to the conventional symmetry $P(3.1)$; and *all* properties, axioms and physical laws holding on M over R admit an *identical* image on \hat{M} over \hat{R} . These are

the reasons for the original suggestion of the name *isotopies* [6a] from the Greek meaning of being "axiom-preserving".

In this way, the isorelativistic theories *coincide*, by conception and construction, with conventional relativistic theories at the abstract, realization-free level, by therefore bringing the applicability of the Lorentz-Poincaré symmetry and Einstein special relativity to the unexpected level of universality.

Moreover, Santilli [5] proved that *the conventional Poincaré symmetry is eleven dimensional, and not ten dimensional as believed throughout this century*. This additional unexpected property was proved via the new invariance of the Minkowskian line element [6e],

$$(x^\mu \times \eta_{\mu\nu} \times x^\nu) \times I = [x^\mu \times (\rho^{-2} \times \eta_{\mu\nu}) \times x^\nu] \times (\rho^2 \times I) = (x^\mu \times \hat{\eta}_{\mu\nu} \times x^\nu) \times \hat{I}, \quad (3.3)$$

where ρ is an ordinary parameter, with corresponding novel invariance of the Hilbert product [5j]

$$\langle \phi | \times |\psi \rangle \times I = \langle \phi | \times \rho^{-2} \times |\psi \rangle \times (\rho^2 \times I) = \langle \phi | \hat{\times} |\psi \rangle \times \hat{I}. \quad (3.4)$$

It is evident that Eqs. (3.3) characterizes the isominkowski spaces \hat{M} over \hat{R} in their simplest possible realization, that with isounit characterized by an ordinary parameter, $\hat{I} = \rho^2$. Eqs. (3.4) then characterize the simplest possible realization of the *isohilbert spaces* $\hat{\mathcal{H}}$ defined on the isofield $C(\hat{c}, \hat{+}, \hat{\times})$ of isocomplex numbers $\hat{c} = c \times \hat{I}$, used for the operator formulation of the isosymmetry. It is also evident that the above new symmetries persists at the full isotopic level,

$$(x^\mu \times \hat{\eta}_{\mu\nu} \times x^\nu) \times \hat{I} = [x^\mu \times (\rho^{-2} \times \hat{\eta}_{\mu\nu}) \times x^\nu] \times (\rho^2 \times \hat{I}) = (x^\mu \times \hat{\eta}'_{\mu\nu} \times x^\nu) \times \hat{I}',$$

$$\langle \phi | \hat{\times} |\psi \rangle \times \hat{I} = \langle \phi | \times \rho^{-2} \times \hat{I} \times |\psi \rangle \times (\rho^2 \times \hat{I}) = \langle \phi | \hat{\times}' |\psi \rangle \times \hat{I}'. \quad (3.5)$$

As a result, the *Lorentz-Poincaré-Santilli isosymmetry is also eleven dimensional* (see Sect. 5 for details).

By recalling that any new symmetry of spacetime has far reaching physical implications, Santilli's discovery of a hitherto unknown additional dimension of the fundamental symmetries of our spacetime also has important and novel physical implications outlined below.

The reader should not be surprised that the new symmetry (3.3) has remained unknown since Lorentz-Poincaré-Minkowski's times, and the additional new symmetry (3.4) has remained unknown 'since Hilbert's time. In fact, their identification required the prior discovery of *new numbers*, those *with arbitrary units* [6d].

As a guide to the existing main literature, we here indicate the first construction of the isotopies of: rotational symmetry in Ref. [5b]; Lorentz symmetry in Ref [5a]; SU(2)-spin symmetry in Refs. [5c,5d]; Poincaré symmetry in Ref. [5e]; and spinorial covering of the Poincaré symmetry in Ref. [5f]. In Refs. [5g,5h] Santilli achieved the first axiomatically consistent grand unification of electroweak and gravitational interactions known to this author precisely via the use of the 11-dimensional isopoincaré symmetry; and in Ref. [5i] he presented the isopoincaré invariant *isocosmology*. In memoir [5j] one can find a comprehensive presentation of the underlying isominkowskian geometry and related reformulation of gravity; the operator formulations originated in paper [6b] (of 1978), continued in numerous

publications (see, e.g., Ref. [5k,5-1o,12-14]), and reached maturity in memoir [5l]. Classical realizations of the (isogalilean and) isopoincaré symmetries were studied in detail in monographs [5m,5n], while the operator counterparts were studied in detail in monographs [5o,5p].

Pre-requisites for the above results were the isotopies of Lie's theory in its various branches, the universal enveloping associative algebras (including the Poincaré-Birkhoff-Witt theorem), Lie algebras (including the celebrated Lie first, second and third theorem), Lie's groups, transformation and representation theories. These isotopies were proposed for the first time in Ref. [6a], and then studied in a variety of works (see monograph [6g] for the status of the knowledge as of 1983, and monograph [5o] for the status as of 1995). The emerging theory is today properly called *Lie-Santilli isotheory* and it is the subject of numerous independent studies, such as those by: Tsagas and Sourlas in the papers of Refs. [7] and monograph [7j]; Lohmus, Paal and Sorgsepp in monograph [7k]; Vacaru in papers [7] and monograph [7l]; Kadeisvili in Refs. [8]; and additional authors quoted therein (see the miscellaneous list of papers [9]).

It is evident that we cannot possibly provide a technical treatment in this note of all the above results. To avoid lecture-notes for a two-semester course, we must, therefore, restrict ourselves to only the most essential aspects.

The feature of paramount importance for these introductory lines is the reconstruction on isohilbert spaces $\hat{\mathcal{H}}$ over isofields $\hat{C}(\hat{c}, \hat{+}, \hat{\times})$ of unitarity for all conventionally nonunitary transforms, according to the *isounitariness conditions*

$$\hat{U} \hat{\times} \hat{U}^\dagger = \hat{U}^\dagger \hat{\times} U = \hat{I}, \quad (3.6)$$

In particular, all possible conventionally nonunitary transforms on \mathcal{H} over \mathbb{C} can always be identically rewritten in the isounitary form on $\hat{\mathcal{H}}$ over \hat{C} (first identified in [5l])

$$U \times U^\dagger \neq I, U = \hat{U} \times \hat{T}^{1/2}, \quad (3.7)$$

Once such an isounitary structure is achieved, it remains invariant under all possible, additional isounitary transforms,

$$\begin{aligned} \hat{W} \times \hat{W}^\dagger &= \hat{W}^\dagger \times \hat{W} = \hat{I}, \\ \hat{I} &\rightarrow \hat{W} \hat{\times} \hat{I} \hat{\times} \hat{W}^\dagger = \hat{I}, \\ \hat{A} \hat{\times} \hat{B} &\rightarrow \hat{W} \hat{\times} (\hat{A} \hat{\times} \hat{B}) \hat{\times} \hat{W}^\dagger = \hat{A}' \hat{\times} \hat{B}', \end{aligned}$$

$$\hat{H} \hat{\times} |\hat{\psi}\rangle = E \times |\hat{\psi}\rangle \rightarrow \hat{W} \hat{\times} \hat{H} \hat{\times} |\hat{\psi}\rangle = \hat{H}' \hat{\times} |\hat{\psi}'\rangle = \hat{W} \hat{\times} \hat{E} \hat{\times} |\hat{\psi}\rangle = E \times |\hat{\psi}'\rangle \quad (3.8)$$

Note the invariance of the numerical values of the isounit, isoproduct and the isoeigenvalues, as necessary for physical consistency. Classical noncanonical transforms are similarly turned into *identical* isocanonical versions with resulting invariance not considered here for brevity.

In summary, all nonunitary transforms are rewritten in an *identical* isounitary form which reproduces all the original invariances of conventionally unitary theories, thus resolving the inconsistencies of Sect. 2.

Along the same lines, Santilli reconstructs theories that are nonlinear (in the wavefunction) on \mathcal{H} over \mathbb{C} into identical *isolinear* forms on $\hat{\mathcal{H}}$ over $\hat{\mathbb{C}}$ via the identifications $H(r, p, \psi) \times |\psi \rangle = H_o(r, p)\hat{T}(\psi, \dots) \times |\psi \rangle = H_o(r, p) \hat{\times} |\psi \rangle = E \times |\psi \rangle$, namely, by embedding all nonlinear terms in the isotopic element. This reformulation implies the regaining of the superposition principle, and the resolution of the other inconsistencies.

Similarly, isotheries are *nonlocal-integral* (e.g., because admitting volume integrals to represent wave-overlappings). These theory are however reconstructed as local-differential on isospaces over isofields, called *isolocal-isodifferential*, via the embedding of all nonlocal terms in the isotopic element.

In this way, the universal symmetry of invariant (3.1) is the largest possible isolinear, isolocal and isocanonical or isounitary symmetry of isospacetime.

In inspecting the literature on isotopies, the reader should keep in mind that all references prior to memoirs [5], [6e], even though formulated on isospaces over isofields, *are not invariant*. After laborious studies, Santilli identified the origin of the problem where one would expect it the least, in the *ordinary differential calculus* which, contrary to popular beliefs, resulted to be dependent on the fundamental unit of the base field. This point is absent on the vast literature on different calculus through various centuries because for the tacitly assumed trivial unit $I = +1$, we have $d(+1) = 0$, while for more general units with a nontrivial functional dependence, we evidently have $d\hat{I}(x, v, \dots) \neq 0$.

The latter occurrence required a reformulation of the differential calculus into a form, called *isodifferential calculus*, which is compatible with the generalized unit of the base field, first achieved by Santilli in memoir [6e] via the main rules

$$\hat{d}\hat{x}^\mu = \hat{I}_\nu^\mu \times d\hat{x}^\nu, \hat{\partial}/\hat{\partial}\hat{x}^\mu = \hat{T}_\mu^\nu \times \partial/\partial\hat{x}^\nu, \hat{\partial}\hat{x}^\mu/\hat{\partial}\hat{x}^\nu = \hat{\delta}_\nu^\mu = \delta_\nu^\mu \times \hat{I}. \quad (3.9)$$

The above new calculus was then applied in memoir [5j] to the construction of a novel geometry, the *isominkowskian geometry* which resulted to be a symbiotic unification of the Minkowskian features (as reported above), plus the machinery of Riemann (because the isominkowskian metric has an x-dependence), including the *isochristoffel's symbols*, *isocovariant isodifferential*, *isocovariant isoderivative*, etc.,

isocurvature tensor

$$\begin{aligned} \hat{\Gamma}_{\alpha\beta\gamma} &= \frac{\hat{1}}{2} \hat{\times} (\hat{\partial}_\alpha \hat{\eta}_{\beta\gamma} + \hat{\partial}_\gamma \hat{\eta}_{\alpha\beta} - \hat{\partial}_\beta \hat{\eta}_{\alpha\gamma}) \times \hat{I}, \hat{D}\hat{X}^\beta = \hat{d}\hat{X}^\beta + \hat{\Gamma}_{\alpha\gamma}^\beta \hat{\times} \hat{X}^\alpha \hat{\times} \hat{d}\hat{x}^\gamma \\ , \hat{X}_{[\mu}^\beta &= \hat{\partial}_\mu \hat{X}^\beta + \hat{\Gamma}_{\alpha\mu}^\beta \hat{\times} \hat{X}^\alpha, \hat{R}_{\alpha\gamma\delta}^\beta = \hat{\partial}_\beta \hat{\Gamma}_{\alpha\gamma}^\beta - \hat{\partial}_\gamma \hat{\Gamma}_{\alpha\delta}^\beta + \hat{\Gamma}_{p\delta}^\beta \hat{\times} \hat{\Gamma}_{\alpha\gamma}^p - \hat{\Gamma}_{p\gamma}^\beta \hat{\times} \hat{\Gamma}_{\alpha\delta}^p. \end{aligned} \quad (3.10)$$

The isominkowskian geometry then permitted the *identical* formulation of conventional gravitational field equations, such as the Einstein-Hilbert field equations, although now formulated in a space which is *isoflat*, thus resolving the main problems of the conventional formulation outlined in Sect. 2 (see Sect. 4 for details).

By keeping in mind that conventional and isotopic differentials and derivatives coincide at the abstract level, all papers on isotopies prior to 1996 can be easily completed into a fully invariant form via the mere re-interpretation of the symbols "d" and "∂" as being isotopic.

Numerous applications and experimental verifications of the isorelativistic theories have been worked out to date by various authors, among which we indicated:

A) Particle physics: the universality of the isominkowskian geometry for the geometrization of all physical media, whether of low density (such as our atmosphere) or of high density (such as the interior of hadrons and stars) with an excellent fit of experimental data [10a]; the universality of the Lorentz-Poincaré-Santilli isosymmetry for the representation of arbitrary local speeds of light [10b] as established by evidence [11]; the exact-numerical representation of the Minkowskian anomalies in the behavior of the meanlife of unstable hadrons with speed [10c]; the exact-numerical representation of the experimental data from the Bose-Einstein correlation [10d]; the achievement of a true confinement of quarks (with an identically null probability of tunnel effects for free quarks) within a perturbatively convergent theory and conventional SU(3) quantum numbers [10e,10f,10g]; the reconstruction of the *exact parity, Lorentz and Poincaré symmetries* in particle physics when believed to be broken [5p]; and other verifications.

B) Nuclear Physics: the reconstruction of the *exact isospin symmetry* in nuclear physics [5d]; the first exact-numerical representation of *all* total nuclear magnetic moments via the invariant representation of the deformation of shape of the nucleons [10h]; the first exact representation of the synthesis of neutrons as they occur in stars at their beginning, from protons and electrons *only* (thus excluding the yet unavailable remaining baryons with consequential impossibility to use quark theories) [5f]; the prediction that the neutron, a *naturally unstable* particle, can be *stimulated to decay* via suitable resonating mechanisms which are possible for a nonunitary theory although simply inconceivable for quantum mechanics, and consequential prediction of a novel *subnuclear* energy currently under industrial development [10i]; and other verifications.

C) Astrophysics and cosmology: the exact-numerical representation of the large difference in cosmological redshift between quasars and galaxies when physically connected according to gamma spectroscopic evidence (as due to Santilli's isodoppler shift within the huge quasars chromospheres according to which light exits quasars already redshifted to the value of the associated galaxy) [10j]; the first and still the only available numerical representation of the internal quasars redshift and blueshift [10k]; the elimination of the missing mass in the universe [5i]; and other verifications.

D) Superconductivity: the first and only known model of the Cooper pair with an *explicitly attractive force* between the *two identical electrons* of the pair in remarkable agreement with experimental data [10l,10m]; and other verifications.

E) Chemistry: the first known representation of the binding energy, electric and magnetic moments, and other characteristics of the hydrogen, water and other molecules which are *exact to the seventh digit* (quantum chemistry still misses about 2% of the binding energies, with much bigger insufficiencies in electric and magnetic moments, which at times even have the wrong sign) [10n,10o]; several independent experimental verifications of the prediction of a *new chemical species* composed of conventional molecules and atoms under a new magnetic bond originating from the polarization the orbits of the valence electrons (which produce a field about 1,400 times stronger than nuclear magnetic fields) and related new industry of magnetically polarized gases [10p,10q]; and other verifications.

4. Direct Universality of the L-P-S Isosymmetry

The Lorentz-Poincaré-Santilli (L-P-S) isosymmetry is directly universal for closed-isolated systems verifying conventional total conservation laws, with linear and nonlinear, local and nonlocal and potential-Hamiltonian as well as nonpotential-nonhamiltonian internal dynamics, where: 1) the verification of conventional total conservation laws is established by the fact that the generators of the isopoincaré symmetry are conventional (see next section); 2) all linear, local and potential forces are represented via the conventional Hamiltonian; and 3) all "non-non-non" effects are represented with the isounit.

The understanding of isotopic theories requires at least a rudimentary knowledge of the above direct universality, if nothing else, to prevent the alternative use for the same problem of theories with catastrophic physical inconsistencies. The best way to achieve a rapid and intuitive understanding is the geometric way. In turns this is useful to understand the local isomorphism of the conventional and isotopic spacetime symmetries even prior to their treatment in the next section.

As it is well known, the Minkowskian geometry and the rotational-Lorentz-Poincaré symmetry can only characterize perfectly spherical and perfectly rigid shapes $r^2 = x^2 + y^2 + z^2$ which are geometrically represented via the *unit of the Euclidean subspace* $I = \text{Diag.}(1, 1, 1)$. In fact, any shape other than the perfect sphere and any deviation from its perfect rigidity imply the collapse of the pillar of spacetime symmetries, the rotational symmetry.

Santilli [5b] achieves the most general possible, signature preserving (compact) deformation of the sphere while preserving the rotational symmetry as *exact*. Recall that the Euclidean unit represents in a dimensionless form the basic units of length along the three space axes, $I = \text{Diag.}(1\text{cm}^2, 1\text{cm}^2, 1\text{cm}^2)$, where the square is evidently due to quadratic character of the interval. Then, jointly with the lifting of the sphere into the most general possible spheroidal ellipsoids, Santilli lifts the corresponding units by the *inverse* amount,

$$r^2 = x^2 + y^2 + z^2 \rightarrow r^{\hat{2}} = x^2/n_1^2 + y^2/n_2^2 + z^2/n_3^2,$$

$$I = \text{Diag.}(1\text{cm}^2, 1\text{cm}^2, 1\text{cm}^2) \rightarrow \hat{I} = \text{Diag.}(n_1^2\text{cm}^2, n_2^2\text{cm}^2, n_3^2\text{cm}^2), \quad (4.1)$$

It is then easy to see that the deformed sphere is indeed the perfect sphere in *isoeuclidean space* $\hat{E}(\hat{r}, \hat{\delta}, \hat{R})$, $\hat{r} = r \times \hat{I}$, $\hat{\delta} = \text{Diag}(n_1^{-2}, n_2^{-2}, n_3^{-2}) \times \delta$, called the *isosphere* [5]. In fact, each semiaxis is subjected to the lifting $1_k \rightarrow n_k^{-2}$; but the corresponding units are lifted by the inverse amount, $1_k\text{cm}^2 \rightarrow n_k^2\text{cm}^2$. This implies the preservation of the *original numerical value* of the semiaxes in isospace. The latter occurrence is due to the fact that all invariants are elements of the underlying field. As such, they should be written in general $r^2 = (x^2 + y^2 + z^2) \times I = n \times I$, where I is the unit of the field. The preservation of the perfect sphericity under the liftings (4.1) then follows, as established by invariant (3.3). The extension to shapes other than spheroidal ellipsoids is easily achieved via *nondiagonal positive-definite isounits* (see monograph [5p] for brevity).

The understanding of the perfect sphericity of $r^{\hat{2}} = x^2/n_1^2 + y^2/n_2^2 + z^2/n_3^2$ in isospace then permits the understanding of the property that, contrary to all popular beliefs throughout this century, *the rotational symmetry remains indeed perfectly exact for all infinitely possible compact deformations of the sphere*.

By comparison, the representation of extended particles by Y. S. Kim [4a] is a particular case of Santilli broader representation [5a,5b]. As indicated earlier, the former can only occur for perfectly spherical and perfectly rigid shapes, while the latter occurs for arbitrarily nonspherical and deformable shapes. Whenever the former is extended to include the latter, the catastrophic physical inconsistencies of Sect. 2 are activated, trivially, because the map from a perfectly spherical to a nonspherical shape is necessarily noncanonical- nonunitary.

The restriction of particle/ charge distributions to be perfectly spherical and perfectly rigid has rather serious physical implications. As an illustration, it prohibits the achievement (indicated in Sect. 3) of an exact representation of nuclear magnetic moments (which require precisely a nonspherical deformation of nucleons), and other applications.

This illustrates the comment of Sect. 2 to the effect that the work of Ref. [4a] and literature quoted therein is invaluable to establish the maximal capability of the conventional realization of the Lorentz-Poincaré axioms, with the clear understanding necessary not to exit science that, by no means, it is the final theory. At any rate, the little groups of Ref. [4a] are contained as a particular case of Refs. [5]; Ref. [4a] departs from the Lorentz-Poincaré teaching of "invariance" in favor of a "covariance, while Refs. [5] restore the "invariance" in its entirety; and, finally, the entire mathematical treatment of Ref. [4a] can be used for the representation of the missing nonspherical and deformable shapes via Santilli's re-interpretation of all symbols as being of isotopic character.

The representation via the Lorentz-Poincaré-Santilli isosymmetry of extended, nonspherical and deformable shapes is only the beginning of its direct universality. The next important applications are the representation of arbitrary speeds of light while preserving on isospace M of the maximal causal speed of M (the speed of light in vacuum), and consequential preservation of the light cone. Contrary to a popular belief throughout this century, this feature establishes that *the Lorentz-Poincaré symmetry is exact for arbitrary speeds of light*.

Recall that Minkowski originally wrote his metric in the form $\eta = \text{Diag.}(1, 1, 1, -c_o^2)$. Therefore, the fourth component of the Minkowski metric represents in a dimensionless form the unit cm^2/sec^2 , and the metric explicitly reads $\eta = \text{Diag.}(1, 1, 1, -1cm^2/sec^2)$. In the isominkowskian space, Santilli [5a] considers: 1) the lifting from c_o^2 to an arbitrary local speed $c^2 = c_o^2/n_4^2(x, v, d, \tau, \psi, \dots)$, where n is the local index of refraction; and 2) the joint lifting the unit by the *inverse* amount. It is then evident that the dual lifting

$$\begin{aligned} \eta &= \text{Diag.}(1, 1, 1, -c_o^2) \rightarrow \hat{\eta} = \text{Diag.}(1, 1, 1, -c_o^2/n_4^2(x, v, d, \tau, \psi, \dots)), \\ I &= \text{Diag.}(1, 1, 1, 1cm^2/sec^2) \rightarrow \hat{I} = \text{Diag.}(1, 1, 1, -n_4^2cm^2/sec^2), \end{aligned} \quad (4.2)$$

implies the preservation of the maximal causal speed c_o on isospaces over isofields (that is, when considered with respect to \hat{I}).

It is evident that, when projected in the ordinary spacetime (that is, when considered with respect to I) the isorelativistic theory represents the local speed $c = c_o/n_4$.

The additional use of the isosphere then yields *Santilli's light isocone* [5] which is the perfect cone in isospace. The abstract identity of the isocone with the conventional cone is such that even the characteristic angles of the two cones coincide (to prevent insidious misrepresentation, one should know that the proof of this occurrence requires the use of

the isotrigonometric and isohyperbolic functions [5p], the use of conventional mathematics in isospace being as fundamental inconsistent as the treatment of conventional theories via isomathematics).

Recall that speeds $c < c_o$ are known since the discovery of the refraction of light, while speeds $c > c_o$ have been experimentally measured in recent times, and can be considered as established for all interior media of high density, such as those in the interior of hadrons or of stars [11].

It then follows that *the isopoincaré symmetry extends the applicability of the conventional Einsteinian axioms, from their sole validity for speeds of light in vacuum, to arbitrary speeds within physical media*. To put it differently, the special relativity becomes "directly universal" when formulated in the form today known as *Santilli's isospecial relativity* [5-10].

A glimpse at the applications may be of some value here. Nowadays the light cone is used for all calculations outside gravitational horizons and in similar conditions. However, in the outside of gravitational horizon we have something dramatically different than the vacuum. In fact we have a region of space filled up of hyperdense chromospheres in which the speed of light, first of all, positively is not that in vacuum and, second it is locally variable. As a result, outside gravitational horizons we have highly deformed "cones". Santilli's light isocone permits a more scientific study of these regions thanks precisely to the admission of local arbitrary speeds.

Note that the traditional hopes of representing light within physical media via photons scattering among molecules (to maintain the speed of light in vacuum c_o) has been discredited by the recent experimental evidence of speeds *bigger* than c_o . At any rate, we are referring here to a purely classical event (the propagation of electromagnetic waves within physical media at speeds $c < c_o$) which, as such, cannot be credibly reduced to *photons in second quantization* without a prior *classical* representation.

By no means the above topics are pure semantic, because they have deep implications in the *numerical values* of physical characteristics throughout the universe.

As an illustration in the macrocosm, the belief of the validity of the conventional light cone everywhere in the universe leads to the current beliefs of the size of the universe (generally derived from the cosmological redshift). However, the admission of the physical reality that the speed of light decreases within astrophysical chromospheres implies the necessary consequence that light exits said chromospheres *already redshifted* (see Santilli's companion paper [10b] for the explicit treatment). The decrease of the currently believed size of the universe is then simply incontrovertible.

As an illustration in the microcosm, the possibility to stimulate the decay of the neutron and related new forms of energy mentioned in Sect. 3, originates precisely from the admission that light travels in the hyperdense ' medium inside hadrons at a speed different than that in vacuum.

Next, it is easy to see that all infinitely possible *Riemannian* metric $g(x)$ are simple *particular cases* of the isometric $\hat{\eta}(x, v, d, \tau, \phi, \dots)$. In fact, Santilli [5j] has introduced the novel *isominkowskian formulation of gravitation and general relativity* based on the *Minkowskian*

factorization of the Riemannian metric

$$g(x) = \hat{T}_{grav.}(x) \times \eta, \hat{I}_{grav.}(x) = 1/\hat{T}_{grav.}, \quad (4.3)$$

and consequential reconstruction of the entire Riemannian formalism into such a form to admit $\hat{I}_{grav.}$ as the correct left and right new unit.

This result was possible thanks to the construction of the novel *isominkowskian geometry* [loc. cit.] as a symbiotic unification of the Minkowskian and the Riemannian geometries indicated in Sect. 3.

A visible illustration is the *isominkowskian formulation of Schwarzschild* [5j]

$$\begin{aligned} \hat{ds}^2 &= \hat{dr}^2 + \hat{r}^2 \hat{\times} (\hat{d}\theta^2 + \hat{\sin}^2 \theta \times \hat{d}\phi^2) - \hat{dt}^2 \times c_o^2, \\ \hat{dr} &= \hat{I}_s \times dr, \hat{dt} = \hat{I}_t \times dt, \hat{I}_s = (1 - 2M/r)^{-1}, \hat{I}_t = 1 - 2M/r, \end{aligned} \quad (4.4)$$

where, as one can see, curvature disappears completely because it is embedded in the differential calculus, thus permitting the regaining of a fully *minkowskian* structure for *Schwarzschild's metric*.

Santilli's isominkowskian formulation of gravity implies considerable structural novelties, such as:

1) The formulation, for the first time to our knowledge, of gravitation under the rigid validity of a *symmetry* (rather than the usual covariance), which results to be isomorphic to the Poincaré symmetry.

2) The abandonment of the conventional curvature in favor of *isoflatness*, that is, flatness in isospace, as transparent in the reformulation (4.4).

3) The unification of the special and general relativities which are now differentiated by the *unit*, rather than by the geometry, while the underlying geometry remains unchanged. Equivalently, we can say that Santilli's isominkowskian representation of gravity extends the direct universality of the *special* relativity to include *gravitation* where nobody looked before, in the unit of the theory.

As shown in Ref. [5j], this reformulation of gravity permits the resolution of at least some of the controversies in gravitation that have raged through this century, such as:

A) The reconstruction on isospaces over isofields of full canonicity or unitarity (isocanonical or isounitary laws). In turn, this permits the regaining for gravitation of invariant basic units of measurements, the preservation of Hermiticity-observability at all times, and resolves the other physical inconsistencies of general relativity indicated in Sec. 2.

B) The compatibility between relativistic and gravitational total conservation laws, which is established via the mere visual inspection that the generators of the isopoincaré and conventional Poincaré symmetries coincide (see next section). It is instructive to compare this geometric- algebraic simplicity with the complexity of the conventional proof of total conservation laws in general relativity.

C) The existence, for the first time to our knowledge, of a consistent *relativistic* limit of Riemann, which is now established via the limit $\hat{I}_{grav.} \rightarrow I$; and other resolutions.

Moreover, the regaining of flat gravity in isospace permitted the achievement of the first grand unification of electroweak and gravitational interactions which is axiomatically

consistent [5g,5h]. In Ref. [5h], p. 324, one can read the viewpoint according to which: "gravitation has always been present in unified gauge theories. It did creep in un-notices because occurring where nobody looked for, in the 'unit' of gauge theories". Electroweak gauge theories can be identically formulated on isospaces. Then, gravity is contained in the unit of the isosymmetries $\hat{U}(2) \times \hat{U}(1)$.

The resulting *iso-grand-unification* (IGU) also identifies the technical reasons for the axiomatic inconsistency of other unified theories. In fact, electromagnetic interactions, as well as electroweak interactions in general, rigorously follow the Poincaré symmetry. Any attempt at adding gravitation without a *symmetry* is then doomed to failure. Via his reformulation of gravitation in such a way to admit a symmetry isomorphic to Poincaré, Santilli has resolved the apparently deepest historical obstacle against a grand unification. This illustrates a reason why all attempts initiated by Einstein and continued by many scholars were doomed to fail from their very foundations. Other resolutions of structural incompatibilities between gauge and gravitational theories are related to the treatment of antimatter and will be discussed later on.

Santilli has also pushed his studies to the formulation of the novel *isocosmology* [5i] which brings the validity of the studies by Lorentz, Poincaré, Einstein, Minkowski, and others to a true "universal" level, that of cosmological character inclusive of gravitation. Some of the rather intriguing implications of the isocosmology are: the elimination of the need for a "missing mass" in the universe because the energy equivalence is now $E = m \times c^2 = c_o^2/n_4^2$, rather than $E = m \times c_o^2$, with an average value of c for galaxies, quasars and the universe in general much bigger than c_o when considering all *interior* gravitational problems; a significant reduction of the currently believed dimension of the universe (indicated earlier in this section); and other intriguing features.

By no means this exhausts all the applications of the isominkowskian geometry, isopoincaré symmetry and isospecial relativity. The next application is the study of relativistic and gravitational *interior* problems at large, e.g., the formulation of the *Schwarzschild solution for interior problems with local speed $c = c_o/n_4(x, v, d, \tau, \dots)$* [5j].

In particular, *gravitational horizons (singularities) result to be the zeros of the time (space) component of the isounit*, as one can verify from structures (4.4). This is not a mere mathematical curiosity. Gravitational collapse is one of the most complex physical events in the universe, with the consequentially most complex possible dependence of the metric on all conceivable local quantities. In particular, as typical of interior trajectories (such as those of missiles in atmosphere), we must expect in the gravitational collapse an *arbitrary dependence of the metric in the velocities* (which is simply impossible for Riemann), nonlocal- integral effects due to total wave-overlappings of a large number of wavepackets in a small region of space (which effects are precluded by the topology of Riemann), and interactions which violate the integrability conditions for the existence of a Lagrangian (the *conditions of variational selfadjointness* [6g]) which is also beyond any dream of representation via Riemann. In short, the assumption of the Riemannian geometry as being exact for gravitational collapse in general and for the study of gravitational singularities in particular, is so questionable to imply exiting science.

All the above interior features are directly represented by Santilli's isominkowskian ge-

ometry, thus permitting, for the first time, more realistic studies of interior gravitational problems in general, gravitational collapse in particular, and related topic, such as whether or not the universe started from a "big bang".

The direct universality of the isorelativistic formalism is also established by the fact that it admits as a particular case all infinitely possible *Galilean*-types of space and time. They are evidently admitted under the particular Kronecker structure of the isounit

$$\hat{I} = \{\hat{I}_{Space}\} \times \hat{I}_{time}. \quad (4.5)$$

with consequential factorization of the isopoincaré symmetry into the *isogalilean symmetry* [5m.5n]. The latter aspects are not considered here for brevity.

5. Explicit form of the L-P-S Isotransforms.

In this section we outline the operator version of the L-P-S isosymmetry $\hat{P}(3.1)$, with particular reference to the explicit form of the isosymmetry transformations (called *isotransforms*).

In inspecting this section the reader should keep in mind that it provides a *direct operator theory of gravity* called *operator isogravity* [5j,5l] under the sole restriction of the isominkowskian metric $\eta(x, v, d,]\tau, \psi, \dots)$ to be the conventional Riemannian metric $g(x)$. The resulting new theory coincides at the abstract level with the conventional relativistic quantum mechanics, thus preserving *all* its properties. This occurrence is sufficient, alone, to establish the axiomatic consistency of operator isogravity beyond scientific doubt. Such a 'consistency should then be compared, for scientific objectivity, with the catastrophic physical inconsistencies of quantum gravity outlined in Sect. 2.

The clear understanding is that the operator isorelativistic theory outlined below has applications much beyond that of the mere operator formulation of gravity (Sect. 4).

The isosymmetry $\hat{P}(3.1)$ is characterized by the *conventional* ten generators and parameters of $P(3.1)$, only lifted into their corresponding forms on isospaces over isofields, *plus* the 11-th generator \mathcal{S} for the new symmetries (3.3), (3.4) with parameter ρ

$$X = \{X_k\} = \{M_{\mu\nu} = x_\mu \times p_\nu - x_\nu \times p_\mu, p_\alpha, \mathcal{S}\} \rightarrow \mathcal{X} = \{\mathcal{M}_{\mu\nu} = \hat{\mathfrak{S}}_\mu \hat{\mathfrak{A}} \hat{\mathfrak{A}}_\nu - \hat{\mathfrak{S}}_\nu \hat{\mathfrak{A}} \hat{\mathfrak{A}}_\mu, \hat{\mathfrak{A}}_\alpha, \hat{\mathcal{S}}\},$$

$$w = \{w_k\} = \{(\theta, v), a, \rho\} \in R \rightarrow \hat{w} = w \times \hat{I} \in \hat{R}(\hat{n}, \hat{+}, \hat{\times}), \mu, \nu = 1, 2, 3, 4; k = 1, 2, \dots, 11. \quad (5.1)$$

The isotopies preserve the original connectivity properties of the Lorentz group $L(3.1)$ [5-8]. The $\hat{P}(3.1)$ isosymmetry is then given by

$$\hat{P}(3.1) = [\hat{L}(3.1) \hat{\times} \hat{T}(3.1)] \times \hat{\mathcal{S}} \quad (5.2)$$

, where $\hat{L}(3.1)$ is the *isolorentz group* [5a], $\hat{T}(3.1)$ is the group of *isotranslations* [5e], and $\hat{\mathcal{S}}$ is the one-dimensional group of symmetries (3.3), (3.4), which is evidently in the center of the isogroup. Note that the latter essentially acts as the isotopies of the conventional "scalar extensions" of Lie's symmetries, as familiar for the Galilei's (but not for the Poincaré) symmetry. Santilli has identified their origin in the change of the unit of the underlying field and found its explicit symmetry transforms.

The *isoexponentiation* characterized by the *Poincaré–Birkhoff–Witt–Santilli theorem* [6a,6g,7g,8] of the underlying enveloping isoassociative algebra $\hat{\mathcal{A}}(\hat{P}(3.1))$

$$\hat{e}^A = \hat{I} + A/1! + A \hat{\times} A/2! + \dots = (e^{A \times \hat{T}}) \times \hat{I} \quad (5.3)$$

permits to write the connected component of the L-P-S isosymmetry $\hat{P}_o(3.1) = \hat{S}O(3.1) \hat{\times} \hat{T}(3.1)$ in the form

$$\hat{P}_o(3.1) : \hat{A}(\hat{w}) = \Pi_{k=1,\dots,10} \hat{e}^{i \times X \times w} = (\Pi_k e^{i \times X \times \hat{T} \times w}) \times \hat{I} = \tilde{A}(x, v, d, \tau, \psi, \dots) \times \hat{I}. \quad (5.4)$$

Note the appearance of the isotopic element $\hat{T}(x, v, d, \tau, \psi, \dots)$ in the *exponent* of the group structure. This illustrates the nontriviality of the Lie-Santilli isothory and, in particular, its *nonlinear, nonlocal and nonunitary* characters in its projection on conventional spaces over conventional fields. Intriguingly, the isopoincaré symmetry recovers linearity, locality and unitarity on \hat{M} over \hat{R} .

Conventional linear transforms on M *violate* isolinearity on \hat{M} and must then be replaced with the *isotransforms*

$$\hat{x}' = \hat{A}(\hat{w}) \hat{\times} \hat{x} = \hat{A}(\hat{w} \cdot \hat{x}, \hat{v}, \dots) \times \hat{T}(x) \times \hat{x} = \tilde{A}(w, x, v, \dots) \times \hat{x}. \quad (5.5)$$

The preservation of the original ten dimensions is ensured by the *isotopic Baker–Campbell–Hausdorff Theorem* [6a]. Structure (5.4) then forms a connected *Lie–Santilli isogroup* with laws

$$\hat{A}(\hat{w}) \hat{\times} \hat{A}(\hat{w}') = \hat{A}(\hat{w}') \hat{\times} \hat{A}(\hat{w}) = \hat{A}(\hat{w} + \hat{w}'), \hat{A}(\hat{w}) \hat{\times} \hat{A}(-\hat{w}) = \hat{A}(0) = \hat{I} = \hat{T}^{-1}. \quad (5.6)$$

The use isodifferential calculus on \hat{M} then yields the L-P-S isoalgebra $\mathbf{p}(\hat{\mathbf{3}}.1)$ [5]

$$\begin{aligned} [\hat{M}_{\mu\nu}, \hat{M}_{\alpha\beta}] &= i \times (\hat{\eta}_{\nu\alpha} \times \hat{M}_{\mu\beta} - \hat{\eta}_{\mu\alpha} \times \hat{M}_{\nu\beta} - \hat{\eta}_{\nu\beta} \times \hat{M}_{\mu\alpha} + \hat{\eta}_{\mu\beta} \times \hat{M}_{\alpha\nu}), \\ [\hat{M}_{\mu\nu}, \hat{p}_\alpha] &= i \times (\hat{\eta}_{\mu\alpha} \times \hat{p}_\nu - \hat{\eta}_{\nu\alpha} \times \hat{p}_\mu), \\ [\hat{p}_\alpha, \hat{p}_\beta] &= [\hat{M}_{\mu\nu}, \hat{\mathcal{S}}] = [\hat{p}_\mu, \hat{\mathcal{S}}] = 0 \\ \hat{p}_k \hat{\times} |\hat{\psi}\rangle &= i \hat{\partial}_k |\hat{\psi}\rangle, [A, B] = A \times \hat{T} \times B - B \times \hat{T} \times A \end{aligned} \quad (5.7)$$

where $[A, B]$ is the *Lie–Santilli isoproduct* (first proposed in [6b]), which does indeed satisfy the Lie axioms in isospace, as one can verify.

Note for the particular case $\hat{\eta} = g(x)$ the appearance of the *Riemannian metric as the 'structure functions'*. Note also that the *momentum components isocommute* (while they are notoriously non-commutative for quantum gravity). This confirms the achievement of the isoflat representation of gravity indicated in Sect. 4, which is seemingly mandatory to achieve a consistent grand unification of gravity with other interactions.

The local isomorphism $\hat{\mathbf{p}}(\mathbf{3}.1) \approx \mathbf{p}(\hat{\mathbf{3}}.1)$ is ensured by the positive-definiteness of \hat{T} . In fact, the use of the generators in the form $\hat{M}_\nu^\mu = \hat{x}^\mu \hat{\times} \hat{p}_\nu - \hat{x}^\nu \hat{\times} \hat{p}_\mu$ yields the *conventional* structure constants under a *generalized* Lie product, as one can verify. The above local isomorphism is sufficient, per se, to guarantee the axiomatic consistency of the L-P-S isosymmetry and all applications in which it is exact, including operator isogravity.

The *isocasimir invariants* of $\hat{\mathbf{p}}(\mathbf{3.1})$ are the simple isotopic images of the conventional ones

$$\begin{aligned} C^o &= \hat{I} = [\hat{T}(x, v, d, \tau, \psi, \dots)]^{-1}, \\ C^{(2)} &= \hat{p}^{\hat{2}} = \hat{p}_\mu \hat{\times} \hat{p}^\mu = \hat{\eta}^{\mu\nu} \times \hat{p}_\mu \hat{\times} \hat{p}_\nu, \\ C^{(4)} &= \hat{W}_\mu \hat{\times} \hat{W}^\mu, \hat{W}_\mu = \epsilon_{\mu\alpha\beta\pi} \hat{M}^{\alpha\beta} \hat{\times} \hat{p}^\pi \end{aligned} \quad (5.8)$$

. From them, one can construct any needed *isorelativistic equation*, such as the *Dirac-Santilli isoequation* [5f]

$$\begin{aligned} (\hat{\gamma}^\mu \hat{\times} \hat{p}_\mu + \hat{i} \hat{\times} \hat{m}) \hat{\times} | > &= [\hat{\eta}_{\mu\nu}(x, v, \dots) \times \hat{\gamma}^\mu \times \hat{T} \times \hat{p}^\nu - i \times m \times \hat{I}] \times \hat{T} \times | > = 0, \\ \{\hat{\gamma}^\mu, \hat{\gamma}^\nu\} &= \hat{\gamma}^\mu \times \hat{T} \times \hat{\gamma}^\nu + \hat{\gamma}^\nu \times \hat{T} \times \hat{\gamma}^\mu = 2 \times \hat{\eta}^{\mu\nu}, \hat{\gamma}^\mu = \hat{T}_{\mu\mu}^{1/2} \times \gamma^\mu \times \hat{I} \text{ (no sum)}, \end{aligned} \quad (5.9)$$

where γ^μ are the conventional gammas and $\hat{\gamma}^\mu$ are the *isogamma matrices*.

Note that, again for the particular case $\eta(x, v, d, \dots) = g(x)$, the *anti-isocommutators of the isogamma matrices yield twice the Riemannian metric*, thus confirming the representation of gravitation in the *structure* of Dirac's equation.

As an illustration, we have the *Dirac-Schwarzschild isoequation* characterized by $\hat{\gamma}_k = (1 - 2M/r)^{-1/2} \times \gamma_k \times \hat{I}$ and $\hat{\gamma}_4 = (1 - 2M/r)^{1/2} \times \gamma^4 \times \hat{I}$. Similarly one can construct the isogravitational version of all other equations of relativistic quantum mechanics.

These equations are not a mere mathematical curiosity because they establish the compatibility of operator isogravity with experimental data in view of the much smaller value of gravitational over electromagnetic, weak and strong interactions. The isotopic unification of the special and general relativities is, therefore, compatible with experimental evidence at both classical and operator levels.

The explicit form of the isosymmetry transformations are by:

1) Isorotations [5b], which can be computed from isoexponentiations (5.4) resulting in the explicit form in the (x,y)-plane (were we ignore hereon the factorization of \hat{I} for simplicity)

$$\begin{aligned} x' &= x \times \cos(\hat{T}_{11}^{\frac{1}{2}} \times \hat{T}_{22}^{\frac{1}{2}} \times \theta_3) - y \times \hat{T}_{11}^{-\frac{1}{2}} \times \hat{T}_{22}^{\frac{1}{2}} \times \sin(\hat{T}_{11}^{\frac{1}{2}} \times \hat{T}_{22}^{\frac{1}{2}} \times \theta_3), \\ y' &= x \times \hat{T}_{11}^{\frac{1}{2}} \times \hat{T}_{22}^{-\frac{1}{2}} \times \sin(\hat{T}_{11}^{\frac{1}{2}} \times \hat{T}_{22}^{\frac{1}{2}} \times \theta_3) + y \times \cos(\hat{T}_{11}^{\frac{1}{2}} \times \hat{T}_{22}^{\frac{1}{2}} \times \theta_3), \end{aligned} \quad (5.10)$$

(see [5p] for general isorotations in all there Euler angles). Isorotations (5.10) leave invariant all ellipsoidal deformations of the sphere indicated in Sect. 4, as the reader is encouraged to verify. The local isomorphism between $\hat{O}(3)$ and $O(3)$ then confirms the perfect sphericity of ellipsoids on isospace (the isosphere).

Note that the space components of all gravitational theories characterize an isosphere when reformulated on isoeuclidean spaces over isofields.

2) Isolorentz transformations [5a], which are characterized by the isorotations and the *isoboosts*, e.g., in the (3,4)-plane

$$x^{3'} = x^3 \times \sinh(\hat{T}_{33}^{\frac{1}{2}} \times \hat{T}_{44}^{\frac{1}{2}} \times v) - x^4 \times \hat{T}_{33}^{-\frac{1}{2}} \times \hat{T}_{44}^{\frac{1}{2}} \times \cosh(\hat{T}_{33}^{\frac{1}{2}} \times \hat{T}_{44}^{\frac{1}{2}} \times v) =$$

$$\begin{aligned}
& \tilde{\gamma} \times (x^3 - \hat{T}_{33}^{-\frac{1}{2}} \times \hat{T}_{44}^{\frac{1}{2}} \times \hat{\beta} \times x^4) \\
x^{4'} &= -x^3 \times \hat{T}_{33} \times c_0^{-1} \times \hat{T}_{44}^{-\frac{1}{2}} \times \sinh(\hat{T}_{33}^{\frac{1}{2}} \times \hat{T}_{44} \times v) + x^4 \times \cosh(\hat{T}_{33}^{\frac{1}{2}} \times \hat{T}_{44}^{\frac{1}{2}} \times v) = \\
& \tilde{\gamma} \times (x^4 - \hat{T}_{33}^{\frac{1}{2}} \times \hat{T}_{44}^{-\frac{1}{2}} \times \tilde{\beta} \times x^3), \\
\tilde{\beta} &= v_k \times \hat{T}_{44}^{\frac{1}{2}}/c_0 \times \hat{T}_{44}^{\frac{1}{2}}, \tilde{\gamma} = (1 - \tilde{\beta}^2)^{-\frac{1}{2}}
\end{aligned} \tag{5.11}$$

Note that the above isotransforms are formally similar to the Lorentz transforms, as expected from their isotopic character.

Note also that all (3+1)-dimensional Riemannian models of gravity, when subjected to their isominkowskian reformulation, characterize precisely light isocones on \hat{M} over \hat{R} . All possible gravitational models are therefore unified into one single primitive geometric notion.

3) Isotranslations [5e], which can be written

$$\begin{aligned}
x' &= (\hat{e}^{i \times \hat{p} \times a}) \hat{\times} \hat{x} = [x + a \times A(x, v, d, \dots)] \times \hat{I}, \hat{p}' = (\hat{e}^{i \times \hat{p} \times a}) \hat{\times} \hat{p} = \hat{p}, \\
A_\mu &= \hat{T}_{\mu\mu}^{1/2} + a^\alpha \times [\hat{T}_{\mu\mu}^{1/2}, \hat{p}_\alpha]/1! + \dots
\end{aligned} \tag{5.12}$$

and they are also nonlinear, as expected.

4) Isoinversions [5e], which are given by

$$\hat{\pi} \hat{\times} x = \pi \times x = (-r, x^4), \hat{\tau} \hat{\times} x = \tau \times x = (r, -x^4) \tag{5.13}$$

where $\hat{\pi} = \pi \times \hat{I}$, $\hat{\tau} = \tau \times \hat{I}$, and π , τ are the conventional inversion operators.

Despite such a simplicity, the physical implications of the isoinversions are nontrivial because of *the possibility of reconstructing as exact discrete symmetries when believed to be broken*, which can be achieved by embedding all symmetry breaking terms in the isounit [5p].

One should be aware that the reconstruction of exact spacetime and internal symmetries is a rather general property of the Lie–Santilli isotheory, thus holding also for continuous symmetries. In fact, contrary to popular beliefs, this section shows that *the Lorentz and Poincaré symmetries are exact for gravitation*.

5) Isoselfscalar transforms [5h], which are characterized by invariances (3.3)-(3.4), i.e.,

$$\hat{I} \rightarrow \hat{I}' = \rho^2 \times \hat{I}, \hat{\eta} \rightarrow \hat{\eta}' = \rho^{-2} \times \hat{\eta}, \tag{5.14}$$

where ρ is the parameter characterizing the novel 11-th dimension.

The implications of the 11-th invariance of spacetime is now clear: it permits the achievement of a consistent grand unification of gravitation and electroweak interactions according to a mechanism essentially equivalent to the unification of electromagnetic and weak interactions, the generalization of the parameter ρ into the positive-definite function $\hat{T}_{grav.}(x)$ and the rule

$$\begin{aligned}
(x^\mu \times \eta_{\mu\nu} \times x^\nu) \times I &= \{x^\mu \times [\hat{T}_{grav.}(x)_\mu^\rho \times \eta_{\rho\nu}] \times x^\nu\} \times [\hat{T}_{grav.}^{-1}(x) \times I] = \\
& [x^\mu \times g_{grav.\mu\nu}(x) \times x^\nu] \times \hat{I}_{grav.}(x).
\end{aligned} \tag{5.15}$$

where the equality holds for the two sides computed in their respective spaces and fields.

By looking in retrospect, we can say that the apparent reason why a grand unification was not achieved during this century until recently was the absence of one dimension in the basic symmetry of spacetime [5g,5h].

The above results can be summarized with the following:

THEOREM 3 (Direct universality of the L-S-P isosymmetry [5]): The 11-dimensional, Lorentz-Poincaré-Santilli isosymmetry on isominkowski spaces over real isofields with common, 4×4 -dimensional, positive-definite isounits constitutes the largest possible isolinear, isolocal and isocanonical-isounitary invariance of isoseparation (3.1), thus being directly universal for all possible, Galilean, relativistic or gravitational, interior and exterior spacetime theories with a well behaved symmetric metric.

It should be stressed that for any arbitrarily given diagonal metric $\hat{\eta} = \hat{T} \times \eta$ there is nothing to compute because one merely plots the $\hat{T}_{\mu\mu}$ terms in the above given isotransforms. The invariance of interval (3.1) is then assured by Theorem 3. The (2 + 2)-de Sitter or other diagonal cases can be derived from the above theorem via mere changes of signature or dimension of the isounit. The case of nondiagonal metrics will be considered in the next section.

6. Santilli's geno- and hyper-symmetries and their isoduals.

The limitations of fundamental theories can be best identified in the writings of their originators (rather than of their followers). For instance, Lorentz was the first to identify the insufficiency of his historical symmetry for the speeds of light of the physical reality and conducted the first search for a possible broader symmetry for speeds $c < c_o$ [11a] (while his followers proclaimed for the rest of this century the "universal constancy of the speed of light").

Receptive to the and other historical teachings, Santilli has identified the major limitation of his isotopies as being *their inability to permit axiomatically correct studies on irreversibility*. In fact, being "axiom-preserving", the isotopies preserve the original inability by the L-P axioms to describe irreversibility.

For these reasons, Santilli conducted his studies via the broader *genomathematics* initiated in his Ph.D. thesis back in 1967, [12a] and then studied in various works [5a,5b,5c, 5l,6e,12]. Invariance was achieved for the first time in memoir [12e] of 1997. This is a broader mathematics possessing a *Lie-admissible* (rather than Lie-isotopic) structure (a generally nonassociative algebra U with abstract product ab is said to be Lie-admissible when the attached algebra U^- , which is the same vector space as U equipped with the product $[a, b] = ab - ba$, is Lie-isotopic).

We cannot possibly provide a technical review of the covering genotopic formalism to avoid another two-semester volume of lecture notes. However, this presentation would be insufficient in our view (if not potentially misleading) without at least the main idea of the genotheories.

In essence, while other scholars searched for *departures* from the Lie axioms, Santilli

devoted his research life to *preserve* the ' same axioms and search instead for broader *realizations*. This approach was eventually rewarded because, while other generalizations outside Lie have the catastrophic physical inconsistencies of Sect. 2, the preservation of the abstract Lie axioms permitted their resolutions while achieving a structurally broader theory.

The main idea of the genoties is best presented in Ref. [12d] and consists in the identification in Lie groups and algebras the following *abstract bimodular structure*

$$\begin{aligned}
A(w) &= U \times A(0) \times U^\dagger = e^{iX \times w} \times A(0) \times e^{-iw \times X} = e_{>}^{iX > w} > A(0) < e_{<}^{-iw < X} \\
idA/dw &= A \times X - X \times A = A < X - X > A = (A, X), \\
e_{>}^{iX > w} &= [e_{<}^{-iw < X}]^\dagger
\end{aligned} \tag{6.1}$$

characterized by: I) a modular associative action to the right $>$; II) a modular-associative axioms to the left $<$; and III) an inter-relation between the two actions generally given by Hermitean conjugation.

The genotopic/Lie-admissible formulations then introduce a *realization* of these axioms more general than that by the isotopic formulations given by the mere *relaxation of the symmetric character of the isounit*.

This yields *two different generalized mathematics*, one for the ordered product to the right $>$ (representing motion forward in time), and one for the ordered product to the left $<$ (representing motion backward in time), with *two genounits, two genoproducts*, etc.,

$$\begin{aligned}
\hat{I}^> &= 1\hat{S}, A > B = A \times \hat{S} \times B, \hat{I}^> > A = A > \hat{I}^> = A, \\
<\hat{I} &= 1/\hat{P}, A < B = A \times \hat{R} \times B, <\hat{I} < A = A << \hat{I} = A \\
A &= A^\dagger, B = B^\dagger, \hat{R} = \hat{S}^\dagger
\end{aligned} \tag{6.2}$$

The above elements must then be completed, for necessary reasons of consistency, with the forward and backward genofields, genospaces, genodifferential calculus, genogeometries, etc. [6e,12e]. The explicit Lie-admissible realization of Lie's axioms I, II and III then reads (at a fixed value of the parameter w , thus without its ordering)

$$\begin{aligned}
A(w) &= e_{>}^{iX > w} > A(0) < e_{<}^{-iw < X} = [e^{iX \times S \times w} \times \hat{I}^>] \times S \times A(0) \times R \times [<\hat{I} \times e_{<}^{-iw \times R \times X}] \\
id\hat{A}/\hat{d}w &= (A, X) = A < X - X > A = A \times \hat{R} \times X - X \times \hat{S} \times A, \\
X &= X^\dagger, \hat{R} = \hat{S}^\dagger
\end{aligned} \tag{6.3}$$

As one can see, the above structures permit an axiomatic treatment of irreversibility. In fact the formulation is *structurally irreversible* in the sense that it is *irreversible* for all possible conventional, reversible Hamiltonians. This is precisely what needed for a serious study of irreversibility because all action-at-a-distance interactions are well known to be reversible while physical reality is irreversible.

The observation (and admission) of this physical reality is sufficient, alone, to establish that *irreversibility should be represented with anything except the Hamiltonian*. Santilli represents irreversibility with nonhermitean, thus irreversible, generalized units. The selection

of the units is evidently preferable over an other possible choices because it assures the invariance of the representation.

In memoirs [51,6e,12e] Santilli identifies the Lie-admissible structure of the historical Hamilton equations (those with external terms); introduces a new invariant genohamiltonian mechanics; identifies the new genoquantization; and works out the invariant Lie-admissible operator theory. These studies have permitted the reduction of the irreversibility of our macroscopic physical reality to the most elementary levels of nature, such as an electron in the core of a star considered as external.

Mutatis mutandae, the belief that an electron in the core of a star is a reversible system or, worse, that it can be described by quantum mechanics, implies the exiting of science (because it implies the belief of the perpetual motion within a hyperdense physical medium because of the usual conservation laws of the theory). Rather than adapting physical reality to [re-existing theories, Santilli has constructed a theory that can represent physical reality in an invariant way.

Note that the theory is manifestly open-nonconservative because $idH/dt = (H, H) = H \times (R - S) \times H \neq 0$. Yet, the notion of *genohermiticity* on $\hat{\mathcal{H}}^>$ over $\hat{C}^>$ coincides with conventional Hermiticity. Therefore, the Lie-admissible theory provides the only operator representation of open systems known to this author in which the *nonconserved Hamiltonian and other quantities are Hermitean, thus observable*. In other treatments of nonconservative systems the Hamiltonian is generally *nonhermitean* and, therefore, *not observable*.

Intriguingly, Ref. [12e] proves that the product $A < B - B > A = A \times \hat{R} \times B - B \times \hat{S} \times A, A \neq B$, is manifestly non-Lie on conventional spaces over conventional fields, yet it becomes fully antisymmetry and Lie when formulates on the bimodule of the respective envelopes to the left and to the right, $\{<\hat{\mathcal{A}}, \hat{\mathcal{A}}>\}$ (explicitly, the numerical values of $A \times B$ computed with respect to I is the same as that of $A > B = A \times \hat{S} \times B$ when computed with respect to $\hat{I}^> = 1/\hat{S}$).

The same quoted contributions on genotopies identified the limitations of the formulations themselves as being *single-valued* (e.g., a Hamiltonian has only *one* genoeigenvalue per each direction of time). Illert and Santilli [13a] provided evidence of the need for *multi-valued methods in biological structures*.

In fact, mathematical treatments complemented with computer visualization establish that the *shape* of sea shells can be described via the conventional single-valued three-dimensional Euclidean space according to the empirical perception of our three Eustachian tubes. However, the same space is basically insufficient to represent *the growth in time* of sea shells. In fact, computer visualization show that, under the exact imposition of the Euclidean axioms, sea shells first grow in time in a distorted way and then crack.

Ref. [13a] then showed that the minimally consistent representation of sea shells growth requires *six dimensions*. But sea shells exist in our environment and can be observed via our *three-dimensional* perception. The solution proposed by Santilli [13b] is that via his *multi-valued hypermathematics* essentially characterized by the relaxation of the single-valued character of the genounits (while preserving their nonsymmetric character as a necessary condition to represent irreversible events).

We have in this way the ordered *hyperunits* and *hyperproducts* [6e,13b]

$$\begin{aligned}
\hat{I}^{\triangleright} &= \{\hat{I}_1^{\triangleright}, \hat{I}_2^{\triangleright}, \hat{I}_3^{\triangleright}, \dots\} = 1/\hat{S}, \\
A > B &= \{A \times \hat{S}_1 \times B, A \times \hat{S}_2 \times B, A \times \hat{S}_3 \times B, \dots\}, \hat{I}^{\triangleright} > A = A > \hat{I}^{\triangleright} = A, \\
<\hat{I} &= \{<\hat{I}_1, <\hat{I}_2, <\hat{I}_3, \dots\} = 1/\hat{S}, \\
A < B &= \{A \times \hat{R}_1 \times B, A \times \hat{R}_2 \times B, A \times \hat{R}_3 \times B, \dots\} <\hat{I} < A = A << \hat{I} = A \\
A &= A^\dagger, B = B^\dagger, \hat{R} = \hat{S}^\dagger
\end{aligned} \tag{6.4}$$

All aspects of the dual Lie-admissible formalism admit a unique, and significant extension to the above hyperstructures (for their expression via *weak equalities and operations* one may consult Ref. [13c]).

The belief in the existence of a "final theory for everything" can only occur to feverish minds because so dissonant with the complexity of our reality. Despite their remarkable generality, hyperformulations too cannot describe the entire universe. In fact, as indicated in Sect. 2, all available classical theories (including conventional, isotopic, genotopic and hyperstructural theories) cannot consistently represent *antimatter* at the *classical* level (see the end of Sect. 2).

After several years of research, Santilli resolved the unbalance between matter and antimatter in the physics of this century by introducing (for the first time in Ref. [5b] of 1985), the map, called *isoduality*, for an arbitrary quantity A with underlying spaces and fields

$$A(x, v, \psi, \dots) \rightarrow A^d = -A^\dagger(-x^\dagger, -v^\dagger, -\psi^\dagger, \dots) \tag{6.5}$$

The above map is mathematically nontrivial, e.g., because it implies the first construction on records of *numbers with negative units and norm* [6d]. Physically the map is also nontrivial because it implies an isodual image of our universe which coexists with our own, yet it is physically distinct.

We have in this way the *isodual conventional, isotopic, genotopic and hyperstructural mathematics* [14], which he then applied to the construction of a new *isodual theory of antimatter*. In particular, at the operator level, *isoduality is equivalent to charge conjugation*. The main property here is that charge conjugation is only applicable at the level of *second quantization*, while *isoduality holds for all levels of study, from Newton to second quantization*, thus resolving the historical unbalance of this century indicated at the end of Sect. 2.

The reader can see the inevitability of the isodual treatment of antimatter by noting that *the fundamental novel invariants (3.3) and (3.4) also hold for negative-definite units*. This guarantees that all properties and physical laws of the conventional invariants also apply to antimatter under isoduality, the main difference being that the treatments of matter and antimatter are anti-isomorphic to each others, as they should be.

The most general formulation of the theories presented in this paper is the *isoselfdual hypercosmology* [5i], in which the "universe": has a multi-valued structure perceived by our Eustachian tubes as a single-valued three-dimensional structure; is defined to include

biological structures (as it should be); is open-irreversible; admits equal amounts of matter and antimatter (in its limit formulation verifying Lies axioms III of Eq. (6.1)); and *possesses all identically null total characteristics of time, energy, linear and angular momentum, etc.*

In closing, the reader should be aware that isotopic, genotopic and hyperstructural formulations and their isoduals can be constructed in their entirety via simple nonunitary transforms of conventional theories, provided that they are applied to the *totality* of the original mathematics. For brevity, we refer the reader to [5l,10b,12e].

The latter methods are easily applicable for the explicit construction of the iso-, geno- and hyper-liftings of the Lorentz-Poincaré symmetry, including the case of nondiagonal-nonsymmetric metrics.

7. Concluding remark

The question raised by the studies reviewed in this paper is: why use generalized theories with limited representational capabilities and catastrophic physical inconsistencies, rather when we have available axiomatically consistent, invariant and universal formulations ?

Acknowledgments.

The author has no words to thank Prof. R. M. Santilli for invaluable criticisms in the writing of this paper as well as the use of his computer files.

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