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Economic Fluctuations Without Shocks to Fundamentals; Or, Does the Stock Market Dance to Its Own Music?

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I can calculate the motions of the heavenly bodies, but not the madness of people.

—Sir Isaac Newton

Last October's dramatic 23 percent decline in the U.S. stock market sent shock waves through the economy, policymakers, and economists. Noneconomists and economists alike scurried to find some previously unforeseen new development that might explain the crash. Could the crash have been caused by the sudden appearance of a comet, by a supernova explosion in a distant galaxy, or by a startling change in sunspot activity? Or perhaps it was caused by psychological factors? Until recently, most economists would have pooh-poohed such ideas as crazy.

To an economist (and also to market analysts on Wall Street) it seems natural to look for changes in consumer tastes or technological factors as possible explanations. After all, one would expect that a sudden shift in consumer tastes toward eating out would drive up the stocks of fast-food chains and restaurants or that a new technological development in the computer industry would drive up the stocks of computer firms. (This surely explains why a considerable amount of market research on Wall Street consists of keeping track of technological developments and shifts in consumer trends.) It is not easy, however, to see why there should be any relationship between extraterrestrial happenings and new developments in consumer tastes or technology.

Thus it is that most of the currently popular models of economic fluctuations are based on recurring random shocks to economic *fundamentals.* These fundamentals consist, of course, of consumer tastes and the technological possibilities available to firms. Shocks to consumer tastes affect the demands for various goods, whereas shocks to technology—by affecting costs of production—affect the supplies of various goods. In this way, these shocks give rise to fluctuations in prices and quantities. In the absence of such continued random influences on tastes or technology, the currently popular models would predict that the economy would (in a reasonable amount of time) settle down into a steady state, with no fluctuations whatsoever.[1]

The stock market crash has revived interest in the possibility of explaining fluctuations without such shocks to fundamentals. One clear reason for this renewed interest has been the inability of economists or market analysts to find any new developments in tastes or technology which could explain a crash of that magnitude. The appeal to psychological factors or, in general, random factors unrelated to fundamentals is, however, not new. In 1936, toward the end of the Great Depression, John Maynard Keynes published his classic *General Theory of Employment, Interest, and Money,* in which he attributed business fluctuations not to random shocks to tastes or technology, but to the *animal spirits* of investors. That is, investors may be seized by moods of optimistic or pessimistic expectations which bear no necessary relation to any changes in tastes or technology. Keynes also asserted that such expectations on the part of investors need not necessarily be irrational. The moods of optimism or pessimism can cause investors to either expand or contract investment spending; this, in turn, can lead to either an overall economic expansion or a contraction, thereby justifying the optimistic or pessimistic expectations. Thus, these animal spirits can become self-fulfilling and hence be *rational.*[2] This alternative view of business fluctuations may be described as *nonfundamental, intrinsic,* or *endogenous.*

In this article I explain how economic fluctuations can occur without shocks to fundamentals. This is not to say that taste or technology shocks do not exist or that they are totally unimportant. Instead, the purpose here is to try and understand whether there exist forces intrinsic to an economic system that tend toward instability; whether such instability is bad from the point of view of economic welfare; and, if so, what sorts of policies or institutions may be set in place to avoid such instability and put the economy on a steady course.[3]

To explain these issues, I describe two models that illustrate intrinsic fluctuations and the role of animal spirits. Both models are simplified versions of existing ones that are part of the burgeoning literature on intrinsic fluctuations. Throughout the paper, the emphasis is on explaining how such fluctuations can arise in an environment in which the economic fundamentals consisting of tastes and technology are unchanging over time. Further, in both models, expectations are assumed to be rational. Without this assumption, one can explain anything, given a sufficiently perverse or irrational view of the world. Requiring beliefs to be rational imposes a notion of consistency between beliefs and reality and rules out explanations based on a pathological view of the world.

The first model described is a simple model of stock price determination in which consumers may hold many possible sets of beliefs that may be self-fulfilling and hence rational. Some of these beliefs may even be based on random factors totally unrelated to the objective factors of tastes and technology.[4] Furthermore, some of these beliefs lead the economy to a steady course while many others set the economy on a wildly fluctuating path.[5]

The second model described is a model of frictional unemployment in which production and exchange take place in a decentralized fashion.[6] I show that there may be several stable paths for the economy along which beliefs are self-fulfilling. Among these, some involve high employment and output whereas others involve low employment and output, depending on whether expectations are optimistic or pessimistic. In addition, there are many fluctuating paths corresponding to changing moods of optimism and pessimism. I argue that the low employment and output situation has some resemblance to the widespread lack of confidence and consequent breakdown of market interactions that seem to characterize deep economic depressions.

Can such models explain the qualitative and quantitative properties of economic fluctuations in real economies? Perhaps. But I attempt no such explanations here, since the models described are chosen for their expositional simplicity rather than their ability to explain observed business fluctuations. I believe it is much too early to judge the empirical applicability of these models, for only recently have economists started analyzing such models. Further development and elaboration of such models may prove to be empirically useful, in addition to being theoretically insightful.

Are there any policy implications that emerge from the study of these models? Yes, although these implications are subject to some important qualifications. I show that for each model there exist very simple policies which can eliminate all fluctuations and set the economy on a unique stable course. In addition, for the frictional unemployment model I show that such a policy can move the economy from a state of low employment and output to one of high employment and output in which many people are better off and none is worse off.

A Stock Price Model

In this section I describe and analyze a simple model of stock price determination and then discuss an appropriate stabilization policy.

Consider an environment that is completely stationary and in which there is one unit of a perfectly divisible asset (a *stock*, if you like) which pays a constant and known stream of dividends forever. Consumers can purchase shares in this stock with a view to obtaining dividends and capital gains when the shares are sold. The current stock price depends on the current demand, which in turn depends on the capital gains (or losses) that consumers expect. This, in turn, depends on the price at which the stock can be sold, which again depends on the demand for the stock on the part of future buyers. I show by means of examples how, even in a completely stationary environment, the stock price can be subject to wild gyrations. My exposition is based on the models of Grandmont (1985) and Azariadis (1981).[7]

People, Preferences, and Prices

Suppose that at each date t, numbered 1, 2, 3, . . . , a representative consumer who lives for two periods is born. A consumer born at date t is *young* at t and *old* at $t + 1$. Assume that at date 1, in addition to the young consumer, there is also an old consumer who was born in the previous period. In each period of life, the consumer is endowed with one unit of total time, which may be divided between leisure time and working time. When the consumer is young, each unit of working time results in w_1 units of the consumption good and when old, each unit of working time results in w_2 units of the consumption good. The consumption good is nonstorable and may be either consumed or traded. The old consumer at date 1 is endowed with one unit of a stock which yields a constant dividend stream of d (in units of consumption) each period. The old consumer will, of course, collect the current dividend and then trade the stock for consumption from the young at date 1. The young consumer, in turn, will hold the shares till period 2, then collect the dividend and sell the shares to the new young at date 2. This process then goes on forever.

Let $c_1(t)$ and $c_2(t)$ be the consumptions at date t of the young and the old consumers, respectively, and let $l_1(t)$ and $l_2(t)$ be the amounts of leisure time enjoyed by the young and the old. The young consumer at each date t maximizes lifetime utility, denoted by u and given by

$$(1) \qquad u = U\big(c_1(t)),l_1(t)\big) + V\big(c_2(t+1),l_2(t+1)\big).$$

In equation (1), the functions $U(\cdot)$ and $V(\cdot)$ represent utility derived in the first and second periods of life. Utility in each period of life depends on consumption and the amount of leisure time enjoyed in that period.

The budget constraints faced by the consumer are

$$(2) \qquad c_1(t) = w_1[1-l_1(t)] - p(t)s(t)$$

$$(3) \qquad c_2(t+1) = w_2[1-l_2(t+1)] + [\,p^e(t+1) + d\,]s(t).$$

In equations (2) and (3), $p(t)$ is the stock price at t, $p^e(t+1)$ is the consumer's expectation (held with certainty) of the stock price at $t + 1$, and $s(t)$ is the quantity of shares purchased by the young at t. Equation (2) states that consumption by the young equals the total output produced when young minus the value of shares purchased. Note that $[1-l_1(t)]$ is the amount of time spent working when young, and hence $w_1[1-l_1(t)]$ is the output produced when young. Equation (3) states that consumption by the old equals the total output produced when old plus the dividends on shares held and the proceeds from the sale of shares. The consumer chooses lifetime consumptions, leisure times, and the demand for shares $s(t)$ in order to maximize lifetime utility given by (1).

The determination of the stock price is shown in Figure 1. It is easy to show that the demand for shares depends on $p(t)$ and $p^e(t+1)$ and that demand is downward sloping in the current price $p(t)$. (See the Appendix for a derivation.) Figure 1 depicts a demand curve such that the demand for shares is decreasing in $p(t)$. The position of the demand curve in Figure 1 depends on the expected future price $p^e(t+1)$. The supply of shares is perfectly inelastic at one unit since there is a fixed amount of one unit of the stock available, all of which is supplied by the old inelastically. Thus, the equilibrium condition for shares is given by

$$(4) \qquad s(t) = 1.$$

That is, the equilibrium price $p(t)$ must be such that the demand for shares equals the supply.

Since the position of the demand curve for shares in Figure 1 depends on the consumer's expectation of next period's price, it follows that the current equilibrium price of shares also depends on the price expected to prevail next period. Now assume that the expectations of consumers are *rational;* that is, the price that consumers at t expect will prevail at $t + 1$ is in fact the actual price at $t + 1$. Therefore, we have

$$(5) \qquad p^e(t+1) = p(t+1).$$

It follows that the current equilibrium price $p(t)$ depends on next period's price $p(t+1)$. This relationship is illustrated in Figure 2 for a particular choice of the utility functions $U(\cdot)$ and $V(\cdot)$. These functions have been chosen in such a way as to generate a hump-shaped curve in which the hump occurs to the left of the 45-degree line.

It is important to understand the reason for the particular hump-shaped curve (with the hump occurring to the left of the 45-degree line) shown in Figure 2, since this shape is the source of fluctuations to be described. This shape arises due to the conflict between the *substitution effect* and the *wealth effect* of a change in $p(t+1)$ on the demand for shares. These effects may be explained as follows. An increase in $p(t+1)$ increases the rate of return on the stock, thereby making saving for future consumption more attractive. This induces the consumer to reduce current consumption and increase saving, and therefore increases the demand for shares. This is the substitution effect. However, an increase in $p(t+1)$ also increases the value of savings in the form of shares and therefore increases wealth. This perceived increase in wealth causes the consumer to increase current (as well as future) consumption. The increase in current consumption reduces the demand for shares. This is the wealth effect. Consequently, the substitution effect and the wealth effect of an increase in $p(t+1)$ have opposite effects on the demand curve for shares. At low values of $p(t+1)$ the substitution effect dominates the wealth effect; as a result, an increase in $p(t+1)$ increases the demand for shares. Thus, the demand curve in Figure 1 shifts to the right, thereby increasing the current equilibrium price $p(t)$. At high values of $p(t+1)$ the wealth effect dominates the substitution effect; as a result, an increase in $p(t+1)$ reduces the demand for shares. Therefore, the demand curve in Figure 1 shifts to the left, thereby lowering the current equilibrium price $p(t)$. This conflict between the two effects is the reason for the hump-shaped relationship between $p(t)$ and $p(t+1)$ depicted in Figure 2—a relationship which yields a variety of possibilities for fluctuations.

Since Figure 2 gives a relationship between the stock price today and the stock price tomorrow, it is possible to calculate some equilibrium time paths for the stock price for various parameter values. The way to do this is also illustrated in Figure 2. Start with some price $p(1)$ at date 1. Then find a price $p(2)$ such that the point $\big(p(1),p(2)\big)$ is on the hump-shaped curve. Then use the 45-degree line to transpose $p(2)$ to the vertical axis and find a price $p(3)$ such that the point $\big(p(2),p(3)\big)$ is on the curve. By proceeding this way, we can construct a time path for the stock price. This time path constitutes a perfect foresight equilibrium path because each pair of prices $\big(p(t),p(t+1)\big)$ has the property (by construction) that $p(t)$ is the equilibrium price at t, given that consumers expect the price at t + 1 to be $p(t+1)$.

Once we have an equilibrium time path for the stock price, we can also calculate time paths for the real interest rate and total output by making use of the following relationships. The real interest rate $r(t)$ from t to t + 1 is given by

(6) $$r(t) = [\,p(t+1) - p(t) + d\,]/p(t).$$

There is a simple linear relationship between total output $y(t)$ and the stock price $p(t)$ for the chosen utility functions $U(\cdot)$ and $V(\cdot)$; that is,

(7) $$y(t) = a + bp(t).$$

Equation (7) is derived in the Appendix.

Illustrations of Intrinsic Fluctuations

In what follows, I illustrate the variety of fluctuations that can be generated by the model. Each illustration corresponds to a different choice of utility functions.

At this point it is worth emphasizing that each economy illustrated is completely stationary in terms of its characteristics over time. Each generation looks exactly the same as any other in terms of its tastes, endowments, and productivities. That is, the fundamentals of each economy are constant over time. In spite of this constancy in the fundamentals, we will see that it is possible for the stock price, real interest rate, and output to exhibit pretty wild behavior.

□ *Periodic and Bizarre Paths*

The model can generate a variety of periodic time paths. In Figures 3 and 4 we see that there is indeed a constant time path that can be generated for the stock price. This price, denoted p^*, corresponds to the intersection in Figure 3 of the 45-degree line and the hump-shaped curve between $p(t)$ and $p(t+1)$. If all consumers expect that the price next period will be p^*, then it will be p^* today and hence forever. From equations (6) and (7), it follows that the interest rate and output will also be constant over time in this example. However, Figures 3 and 4 also show how another time path for the stock price can be generated, along which it follows an up-and-down cyclical path that repeats every two periods. Therefore, equations (6) and (7) imply that along this alternative path, the interest rate and output will also exhibit a similar pattern. In Figures 5 and 6 we see the generation of a four-period cycle in stock prices (and hence also in the interest rate and output). Figures 7 and 8 show how a three-period cycle can be generated.

The model can also generate some bizarre time paths. Figure 9 depicts a pretty bizarre time path for the stock price in which it is hard to discern any strictly periodic pattern. Figure 10 shows a pattern that is hard to distinguish from a time path that might be generated due to the presence of random shocks, even though such shocks have been explicitly ruled out in constructing these illustrations.

Although we have shown only one or two of the possible time paths of the stock price for each example, there are in fact many possible time paths for each set of parameter values. For instance, the example that gives rise to the four-period cycle of Figure 6 can also give rise to a two-period cycle. The example that produces the bizarre path of Figure 9 can also give rise to cycles of two, four, and eight periods as well as periods of some higher powers of two. And the parameter values used to generate Figure 8 can also give rise to cycles of *every integer* period in addition to the bizarre sorts of time paths, as in Figures 9 and 10, which seem to lack any periodic pattern.[8] Furthermore, in every example there is an equilibrium path along which the stock price is constant over time. This is because in all of these examples, the nature of the relationship between $p(t)$ and $p(t+1)$ is similar to the hump-shaped curve shown in Figure 2. This constant time path is indicated by the line marked p^* on the figures.

□ *Animal Spirits and Hemlines*

We now turn to an illustration of the kind of time path that can be generated when consumers are driven by animal spirits. Suppose consumers believe the following maxim:

When hemlines are up, stocks will be up;
when hemlines are down, stocks will be down.

Suppose further that the fashion industry decides randomly when hemlines will be up and when they will be down, perhaps by consulting a different astrologer each period. Even though such randomness has no connection with the tastes, endowments, or productivities of consumers in the model, it turns out that stock prices (and hence interest rates and output) respond to such extraneous randomness.

I now explain how such beliefs, which have no relation to economic fundamentals, can be self-fulfilling. Let the indexes i and j indicate the state of hemlines at dates t and $t + 1$, respectively, and suppose that each index takes the value of 1 or 2, depending on whether hemlines are high or low. In state i, let p_i be the stock price, s_i the demand for shares, $c_1(i)$ and $c_2(i)$ the consumptions of the young and the old, and $l_1(i)$ and $l_2(i)$ the leisure times of the young and the old. Let π_{ij} be the probability that the hemline state at $t + 1$ is j, given that the hemline state at t is i. The young consumer at t maximizes expected utility given the state i at t. This is denoted by $E(u|i)$. Using (1), the expression for expected utility can be written as

$$(8) \qquad E(u|i) = U\big(c_1(i),l_1(i)\big) + \textstyle\sum_j \pi_{ij} V\big(c_2(j),l_2(j)\big).$$

In equation (8), we are simply adding up the utilities in each possible state in the second period of life, weighted by the respective probabilities.

The consumer's budget constraints can be written, by analogy with (2) and (3), as

$$(9) \qquad c_1(i) = w_1[1-l_1(i)] - p_i s_i$$

$$(10) \qquad c_2(j) = w_2[1-l_2(j)] + (p_j+d)s_i.$$

The interpretation of the constraints (9) and (10) is similar to that for (2) and (3).

It is now possible to solve for the consumer's demand for shares. We can then impose the equilibrium condition (4) and solve for the prices p_1 and p_2. (Details are provided in the Appendix.) These prices together with the probabilities π_{ij} determine the possible time paths for the stock price. Such an equilibrium is self-fulfilling, or rational, because the distribution of future prices on the basis of which the consumer determines the demand for shares is in fact the actual distribution of prices that lead to equilibrium between the demand and supply of shares. Thus, the consumer's beliefs are consistent with the actual behavior of equilibrium prices.

Figure 11 shows an example in which the stock price fluctuates randomly between two values, marked p_1 and p_2, with probabilities as noted. The reason for such behavior is the following. If the current state i of hemlines were to be different (say, 2 instead of 1), then the probabilities π_{ij} for the future state j of hemlines will be different. Given the belief held by consumers about the relationship between hemlines and stock prices, the probabilities π_{ij} affect the consumer's expectation of tomorrow's stock price. This influences the consumer's current demand for the stock and hence its current price.

For this result, it is indeed important that the probabilities π_{ij} vary as i varies. That is, the probability distribution of future hemline states must differ if the current hemline state is different. Otherwise, the consumer's expectation of tomorrow's stock price will be independent of the current state and hence so will be the consumer's demand for shares. Consequently, the current equilibrium price will be the same no matter what the current state is. Rational expectations then imply that the stock price must be constant forever.

□ Summary

So far we have seen many examples in which even though there is always a path along which stock prices and other variables are constant, there are also many other equilibrium paths along which stock prices and other macroeconomic variables can exhibit somewhat unusual fluctuations. Therefore, it follows that the economy can exhibit instability even when there is a stable path that is attainable if only consumers would believe in it.

Policy Implications

What implications does this simple stock price model have for consumer welfare and government policy? It turns out that every one of the equilibrium paths we have studied has the property of being *Pareto optimal;* that is, it is not possible to make some consumer better off without hurting some other consumer.[9] Therefore, there is no government policy that will improve everyone's lot. However, this conclusion depends on how seriously we take the assumption of *perfect foresight.* Remember that every one of the equilibrium paths was constructed on the assumption that it was perfectly foreseen by all consumers. If consumers make occasional mistakes in expectations, then the welfare properties of the paths discussed may no longer be true. Consequently, there may exist government policies that enhance the welfare of all consumers.

The perfect foresight assumption may not seem unreasonable if the economy has been moving along a constant path or perhaps along a path with an easily discernible cyclical pattern. Then we may reasonably expect that consumers, by looking at the past behavior of stock prices, will be able to form accurate forecasts of their future behavior, somewhat like the chartists on Wall Street. However, some of the paths we have seen (for instance, those in Figures 9 and 10) are so complex that it is hard to imagine how anyone could form an accurate forecast of the future behavior of stock prices based on past observations.[10] When such forecasting seems difficult, the assumption of rational expectations may be somewhat questionable. At the very least, however, one can argue that the government *ought* to pursue policies that put the economy on a stable path, thereby making it easier for consumers to form accurate forecasts of the future and thus keeping the economy moving along a stable path. The justification for this argument is simply that mistaken expectations are much more likely when the economy is following a highly unstable path.

Do there exist government policies that can eliminate all the highly fluctuating paths we have seen are possible and push the economy inexorably onto a constant path with no fluctuations whatsoever? In the context of the stock price model, there is in fact a fairly simple policy that can achieve this objective: Let the government announce a benchmark stock price p', which is less than w_1, and also levy a tax (or subsidy, if negative) at the proportional rate $[1-p'/p(t)]$ on the value of shares held by the old at each date t (including the initial old). The proceeds of this tax are handed over to the young at t as a lump-sum rebate (or tax, if negative), denoted $\tau(t)$. This policy will alter the budget constraints (2) and (3) as follows:

$$(11) \qquad c_1(t) = w_1[1-l_1(t)] - p(t)s(t) + \tau(t)$$

$$(12) \qquad \begin{aligned} c_2(t+1) &= w_2[1-l_2(t+1)] \\ &\quad + [\,p(t+1) + d\,]s(t) \\ &\quad - [1-p'/p(t+1)]p(t+1)s(t) \\ &= w_2[1-l_2(t+1)] + (p'+d)s(t). \end{aligned}$$

Along an equilibrium path, the rebate $\tau(t)$ must satisfy the following relationship:

$$(13) \quad \tau(t) = p(t) - p'.$$

Equation (13) follows because in equilibrium the quantity of shares sold is unity, and hence the value of shares sold is $p(t)$. Therefore, taxes paid must be $p(t)[1-p'/p(t)]$, which equals $[\,p(t)-p'\,]$.

It is possible to show that under such a policy, the only possible equilibrium path for the stock price (and hence for the interest rate and output) is a constant one. (See the Appendix for details.) The reason for this is as follows. Since the government taxes away any excess of $p(t+1)$ above the benchmark price p' [or subsidizes the difference if $p(t+1)$ falls short of p'], the consumer is, in effect, faced with a future price that is always equal to p'. Consequently, the consumer's current demand for shares depends on p' but not on $p(t+1)$. Therefore, the current equilibrium price $p(t)$ also depends on p' only and is hence constant over time. This simple policy, therefore, eliminates the possibility of all fluctuations and leads the economy onto a stable path. In addition, it is possible to choose the benchmark price p' in order to ensure that the equilibrium path is Pareto optimal.

The policy just described should be viewed with caution, however. Even though it works for the simple stock price model, it may not work for a more complex model with more assets, uncertainty, and capital accumulation. In practice, the policy is likely to be very difficult to define and implement and may also have undesirable side effects on risk taking and investment. To judge the overall desirability of such a policy, these potential ill effects would have to be weighed against the possible benefits from a stabilized economy and improved forecasting.

A Model of Frictional Unemployment

We now turn to the second model chosen to illustrate intrinsic fluctuations and the role of animal spirits—a model of frictional unemployment.

The concept of *frictional unemployment* plays an important role in policy discussions in government and the media. Frictional unemployment represents unemployment resulting from the imperfect matching of workers and employment opportunities. The *natural* rate of unemployment represents the normal level of frictional unemployment and is taken as the benchmark for full employment. It is often said that in the 1960s, full employment corresponded roughly to a natural rate of unemployment between 3 and 4 percent, while in the 1970s the natural rate of unemployment increased to around 6 percent. This is considered relevant for aggregate demand policies because it is thought that any attempt to keep the unemployment rate below the natural rate will only result in spiraling inflation. In spite of this, most models of business fluctuations eschew any attempt to explain the determination of frictional unemployment and instead focus on explaining the characteristics of fluctuations around the natural rate of unemployment. In contrast, I show here that an explicit attempt to model frictional unemployment leads to some very surprising results and some important policy implications.

The model discussed consists of a large number of producer-traders who can only trade bilaterally, if at all. I show that because of this decentralization, there may be several stationary equilibria in some of which employment and output are higher and many people are better off (and none is worse off) than in others. Which of these equilibria obtains depends on whether the expectations of the producer-traders are optimistic or pessimistic. In addition, there may be fluctuations in employment and output due to changing moods of optimism and pessimism. The model is a simplified version of Diamond's (1984).[11]

An Island Economy

Consider a hypothetical economy in which there are a large number of individuals scattered all over a large number of islands, one person per island. Each individual has the opportunity to produce one unit of a specialized good which is of no use to the producer but is desired by all the other persons. Therefore, each person would like to be able to exchange the good produced (if that person chooses to produce) for the product of another person. This setup is designed to capture the notion that in large, modern industrial economies, people develop specialized skills which are, for the most part, of no use to themselves. Instead, these skills (or goods produced with them) are sold to others and the proceeds are used to purchase goods produced by others.

Assume that the cost of production, measured in units of foregone utility u, is different for different people and varies between u_1 and u_2, where $0 < u_1 < u_2 < \infty$. Let the distribution function $G(u)$ denote the fraction of people whose costs of production are no higher than u. If an individual chooses to produce, then that person must engage in a search for other producers (across the many islands) in order to trade. Assume that each person can visit only one other island and that the probability of running into a producer (as opposed to a nonproducer) is π. Also assume that each unit of the good yields a utility of u^* when traded. Therefore, if a producer is successful in meeting a trading partner, then each of them receives utility u^*. If a producer is unsuccessful in meeting a trading partner, then the producer receives zero utility, since the product is useless to its maker.[12]

It is now easy to describe an individual's decision regarding whether or not to engage in production. Intuitively, if the probability of meeting another producer π is sufficiently large relative to the cost of production u, then it pays to produce. More formally, the following condition describes the production decision:

$$(14) \quad \begin{array}{l} \text{If } \pi u^* \geq u, \text{ then produce;} \\ \text{if } \pi u^* < u, \text{ then do not produce.} \end{array}$$

In (14), πu^* is the expected benefit (utility) from producing and u is the cost. It follows that the fraction of producers (and also the per capita output) y is given by

$$(15) \quad y = G(\pi u^*).$$

Assume also that $u_2 < u^*$. This assumption has the following implication. If producers could costlessly communicate and trade with each other, then the best situation is one in which everyone produces and trades. Such a situation might arise if all trade took place in a centralized market with everyone present. In this case it pays for even the producer with the highest production costs to produce, and therefore per capita output will be at its maximum possible level of one. In this model the lack of communication and hence coordination among the many producer-traders is the *friction* which prevents a costless centralized market from arising. We will see that because of this friction, it will not be possible to attain the maximum possible per capita output. In fact, the situation could be considerably worse.

Next I need to describe how the probability of a successful match between producers is related to the decisions of all the people. It is intuitively clear that if either all persons or all but one person decide not to produce and seek out trading partners, then the probability π is zero. If everyone decides to produce and seek out trading partners, then the probability of a successful match will be high.[13] Therefore, in general, there is an increasing relationship between the fraction of people who decide to produce and the probability of a match. This is described by the increasing function $f(y)$ as follows:

$$(16) \qquad \pi = f(y).$$

Equilibria

It is now easy to describe the determination of the equilibrium values of π and y. Figure 12 graphs the two relationships between π and y as given by equations (15) and (16). Equation (15) is marked by G, while (16) is marked by f. By virtue of my assumptions, both functions are increasing.[14] Any intersection of the two curves gives an equilibrium pair (π, y). This pair has the property that given the probability of a match π, a fraction y of people find it profitable to produce; and given the fraction of producers, each person's expectation of the probability of a successful match is accurate. We see that in Figure 12 there are three possible equilibrium pairs (π, y), marked low, middle, and high.

There are two remarkable features of this simple model of production and trading. The first is that there may be several equilibria which are distinguished by varying levels of output and trade, depending on the expectations of producers regarding trading opportunities. If expectations are optimistic, so that people think the probability of successfully consummating trade is high, then many people will be induced to produce and seek out partners. This in turn leads to a high probability of success, thereby justifying the optimistic beliefs. This corresponds to the high equilibrium in Figure 12, indicating a high level of output and trade. If people have pessimistic expectations of being able to trade, then few will be induced to produce and look for trades. This in turn leads to a low probability of a successful match, thereby justifying the pessimism. In Figure 12 this is indicated by the low equilibrium, for low (in this case, zero!) output and trade.

Also shown in Figure 12 is a middle equilibrium outcome which, however, is unstable. This is because if some nonproducers become slightly more optimistic than at the middle outcome, then they will choose to produce, which increases the probability of a match for everyone sufficiently that even more nonproducers will choose to produce, and so on, until the high equilibrium is reached. Conversely, if some producers become slightly more pessimistic than at the middle outcome, then they will choose not to produce, which decreases the probability of a match sufficiently for the remaining producers so that more producers will stop production, and so on, until the low equilibrium is reached and the economy shuts down. The situation of the low equilibrium economy resembles that of a depression economy.

In fact, the three equilibria marked in Figure 12 are not the only equilibria for this economy. There also exist many other equilibria characterized by fluctuations in which output and employment are forever shifting between the high and low equilibria. For instance, suppose people believe that when sunspot activity is high the economy will be in the good (high equilibrium) state and when sunspot activity is low the economy will be in the bad (low equilibrium) state. That is, people become optimistic or pessimistic depending on whether sunspot activity is high or low. Then indeed it will be the case that the state of the economy will fluctuate between the high and the low equilibria precisely in tune with sunspot activity! These fluctuations will be just like the ones for the stock price model's economy, as depicted in the hemline example of Figure 11, in which people were driven by animal spirits bearing no relation to economic fundamentals.[15]

A second remarkable feature of this hypothetical economy is that some people are unambiguously better off and no one is worse off (in terms of expected utility) at the high equilibrium than at the low one, yet there is no market mechanism that can move the economy out of the low equilibrium and toward the high. Specifically, all those who are producing at the high equilibrium are better off than they were at the low one (or they would not be producing), and those who are not producing at the high equilibrium are no worse off than at the low.[16]

Policy Implications

Is there a government policy that can get the economy out of the doldrums at the low equilibrium and move it permanently to the better equilibrium with high employment and output? In fact, it is possible to suggest such a policy in the context of the island economy.

Consider a production subsidy equal to u'/u^* units of the good, where u' is just slightly larger than u_1. Suppose that this subsidy is financed by a sales tax of σ levied on successful trades. This policy changes condition (14) to

$$(17) \qquad \begin{array}{l} \text{If } (1-\sigma)\pi u^* + u' \geq u, \text{ then produce;} \\ \text{if } (1-\sigma)\pi u^* + u' < u, \text{ then do not produce.} \end{array}$$

Equation (15) describing the fraction of people who choose to produce (and also the per capita output) gets modified to

$$(18) \qquad y = G\big((1-\sigma)\pi u^* + u'\big).$$

Equation (16) continues to describe the probability of a successful match as a function of the fraction of producers.

In Figure 13 the relation between π and y described by equation (18) has been superimposed on the previous relations described by equations (15) and (16) and shown in Figure 12. The new curve, indicated by $\hat{G}$, has a positive intercept on the horizontal axis, unlike G of Figure 12. This is because even if the probability of a successful match is zero, a positive fraction of producers (those with production costs between u_1 and u') will find it profitable to produce in order to collect the subsidy. However, the new curve $\hat{G}$ must pass through the same high equilibrium point. This is because in equilibrium the sales taxes collected must be just sufficient to pay for the production subsidies. This requires that the following relationship hold:

$$(19) \qquad \sigma\pi y = u'y/u^*.$$

When we substitute equation (19) in (18), we see that it reduces to equation (15) at equilibrium, which shows that the new equilibrium according to equations (17), (18), and (19), is the same as the high one. However, we see that whereas in Figure 12 there are three possible equilibria, in Figure 13 the high equilibrium is the only one. The low depression equilibrium in Figure 12 is no longer a possible equilibrium in Figure 13. This is because even under the most pessimistic assumptions regarding trading opportunities, a positive fraction of people will produce and look for trading partners. Therefore, such grossly pessimistic expectations are incompatible with equilibrium, and the only equilibrium is the one corresponding to optimistic expectations. Thus, such a production subsidy financed by a sales tax can move the economy to a better and higher level of output.

It should also be noted that because the equilibrium under such a policy is unique, there cannot be any fluctuations in output and employment resulting from changing moods of optimism and pessimism. Therefore, such a policy, in addition to making it possible to achieve a better and higher level of output, also eliminates fluctuations and leads the economy onto a stable path.

This policy conclusion needs to be qualified because of the friction in the model. The policy conclusion depends very critically on there being some external entity (say, a *government*) which is outside the economic system of producer-traders and which is able to impose taxes and distribute subsidies. Indirectly, the government is performing a coordinating role by moving goods across people and islands costlessly via taxes and subsidies—a role which the islanders are, by assumption, unable to perform for themselves. In the absence of such an external entity, it is not at all clear whether such policies are even feasible and whether there exist any feasible policies that can improve matters. Therefore, the fact that an economy is in a bad equilibrium state may not necessarily imply that anything can be done about it.

Conclusion

I now summarize what I think economists are learning by studying the sorts of models I have described in this paper. I should emphasize that this is a tentative report on a relatively new and ongoing research program rather than a definitive judgment of a ripe old one. The important points seem to be the following.

Most business cycle models explain fluctuations in economic variables as resulting from the effects of taste and technology shocks continually impinging on the economy. While some of these models are able to explain some of the qualitative and quantitative features of observed business fluctuations, there are many phenomena that they have difficulty explaining or for which explanations based on taste or technology shocks strain credibility. Some of these phenomena include the high degree of volatility of the financial markets, the great sensitivity of these markets to apparently unrelated events, and deep depressions like the one in 1929.[17]

These considerations suggest that perhaps even in the absence of any taste or technology shocks hitting the economy and even when the environment is completely stationary, the economy might be unstable and exhibit fluctuations. As Keynes argued, the economy might be driven by investors' animal spirits, which need bear no relation to economic fundamentals. Further, the economy might simply become stuck in a situation of low employment and output, with market forces being powerless to move the economy to a better situation of higher employment and output.

I have shown by means of two examples that it is not at all difficult to construct simple model economies that exhibit the above properties. The stock price model generates a variety of periodic and aperiodic paths for the stock price as well as paths driven by purely extraneous shocks having no relation to fundamentals. The frictional unemployment model seems to capture to some extent the cycle of pessimism followed by the breakdown of market interactions followed by more pessimism—a cycle that may be an integral part of severe depressions. I have also shown that in each of these models there exist appropriate government policies that, although subject to some important qualifications, are capable of eliminating fluctuations. Additionally, in the frictional unemployment model such policies can lift the economy out of a state of low output and move it to a better state with higher output.

I therefore conclude that there are important advances in understanding to be gained by further study of models of intrinsic fluctuations.

Appendix
More About the Models

This Appendix provides the details of solving the stock price model and explains the simulation method used to generate time paths for the stock price. I also explain how my exposition of the stock price model and the frictional unemployment model differs from the models on which they are based.

The Stock Price Model

I assume the following form for the utility function in equation (1) of the text:

$$(\text{A1}) \qquad u = c_1(t)^{\alpha_1} l_1(t)^{1-\alpha_1} + \{\beta[c_2(t+1)^{\alpha_2} l_2(t+1)^{1-\alpha_2}]^{1-\mu}/(1-\mu)\}.$$

I assume that $0 < \alpha_1 < 1$, $0 < \alpha_2 < 1$, $\beta > 0$, and $\mu > 0$, but that $\mu \neq 1$. If $\mu = 1$, the second term in (A1) should be replaced by

$$\beta[\alpha_2 \ln c_2(t+1) + (1-\alpha_2) \ln l_2(t+1)].$$

Here I note some of the differences between my model and the ones of Grandmont (1985) and Azariadis (1981). The main difference is that the asset in their models pays a zero dividend forever, rather than a positive dividend. One may think of their asset as corresponding to cash. In addition, my specification of the utility function is a special case of that of Grandmont (1985). If I set α_1 to zero and α_2 to unity (so that people consume only leisure when young and only the consumption good when old), then my specification of the utility function becomes a special case of that of Azariadis (1981). Grandmont (1985) analyzes only deterministic fluctuations, like the ones generated in Figures 3–10, where there is no uncertainty about the time path of prices. Azariadis (1981) analyzes fluctuations, like the hemline example in Figure 11, which are generated by extraneous uncertain events that have no connection to tastes or technology.

Consumer Choices and Equilibrium

I now analyze the consumer's choices of lifetime consumptions, leisure times, and the quantity of shares to buy, given the current stock price and the expected future price.

First, the consumer will equate the marginal rate of substitution between leisure time and consumption in each period of life to the corresponding opportunity cost of leisure time. The opportunity cost of leisure time is w_1 when the consumer is young and w_2 when old. This leads to the following relationships:

(A2) $\quad (1-\alpha_1)c_1(t)/\alpha_1 l_1(t) = w_1$

(A3) $\quad (1-\alpha_2)c_2(t+1)/\alpha_2 l_2(t+1) = w_2.$

Second, the consumer will equate the marginal rate of substitution between consumption at t and consumption at $t+1$ to the gross expected rate of return on the stock. This yields

(A4) $$(\alpha_1/\beta\alpha_2)[l_1(t)/c_1(t)]^{1-\alpha_1}[c_2(t+1)^{\alpha_2} l_2(t+1)^{1-\alpha_2}]^{\mu} \times [c_2(t+1)/l_2(t+1)]^{1-\alpha_2} = [\,p^e(t+1) + d\,]/p(t).$$

We may now substitute for $l_1(t)$ and $l_2(t+1)$ from (A2) and (A3) into equations (2) and (3) of the text to obtain the following simplified expressions for the consumer's budget constraints:

(A5) $\quad c_1(t) = \alpha_1[w_1 - p(t)s(t)]$

(A6) $\quad c_2(t+1) = \alpha_2\{w_2 + [\,p^e(t+1) + d\,]s(t)\}.$

Next we may substitute for $l_1(t)$ and $l_2(t+1)$ from (A2) and (A3), and $c_2(t+1)$ from (A6) into (A4) to obtain

(A7) $$\{w_2 + [\,p^e(t+1) + d\,]s(t)\}^{\mu} = A[\,p^e(t+1) + d\,]/p(t).$$

Equation (A7) determines the demand for shares in terms of $p(t)$ and $p^e(t+1)$. The coefficient A in (A7) is given by

(A8) $$A = \beta[\alpha_1 w_1/(1-\alpha_1)]^{1-\alpha_1}[\alpha_2 w_2/(1-\alpha_2)]^{(1-\alpha_2)(\mu-1)} \div \alpha_1\alpha_2^{\mu-1}.$$

It may be verified from equation (A7) that the demand for shares is decreasing in the current price $p(t)$. Now substitute equations (4) and (5) in (A7) to get the following relationship between $p(t)$ and $p(t+1)$:

(A9) $$p(t) = f\big(p(t+1)\big) \equiv A[\,p(t+1) + d\,]/[\,p(t+1) + d + w_2]^{\mu}.$$

The graph of $p(t)$ against $p(t+1)$ will be hump shaped (as in Figure 2) provided $\mu > 1$ and $w_2 > (\mu-1)d$. Any time path for $p(t)$ that satisfies (A9) for all t constitutes a perfect foresight or rational expectations equilibrium.

Output and the Stock Price

A simple relationship between total output and the stock price can be obtained as follows. From equations (2)–(5) we have

(A10) $$c_1(t) + c_2(t) = w_1[1-l_1(t)] + w_2[1-l_2(t)] + d = y(t).$$

Substituting from equations (A5), (A6), (4), and (5) into equation (A10), we obtain the following linear relationship between $y(t)$ and $p(t)$:

(A11) $\quad y(t) = \alpha_1 w_1 + \alpha_2(w_2+d) + (\alpha_2-\alpha_1)p(t).$

Parameter Values and Simulation Method

I now describe the choice of parameter values and the method of simulation used to produce the intrinsic fluctuations shown in Figures 3–11. Except for Figure 10, I chose these values: $\alpha_1 = ¼$, $\alpha_2 = ½$, $w_1 = 50$, and $d = 0.01$. The parameter μ was varied from 2 to 20 in steps of one half. The parameters w_2 and β were chosen indirectly as follows: Let $\bar{p}$ be the maximum value of $f(p)$ and let p_m be the value of p at which $f(\cdot)$ attains its maximum. These values are illustrated in the accompanying figure. The value of p_m may be found by setting the derivative of $f(\cdot)$ equal to zero and solving for p. This yields

(A12) $\quad p_m = [w_2/(\mu-1)] - d$

(A13) $\quad \bar{p} = A/[\mu^{\mu}(p_m+d)^{\mu-1}].$

We may now substitute for w_2 and A from (A12) and (A13) into (A9) and express the function $f(\cdot)$ in terms of the parameters p_m, $\bar{p}$, μ, and d. I chose $p_m = 1$ and $\bar{p} = 2\mu + 1$. The implied values of w_2 and β may now be found using (A12), (A13), and (A8). Figure 10 was generated using the same parameter values as above, with the following exceptions: $d = 0.001$, $\mu = 15.0$, and $\bar{p} = 10.0$. Note that the values of $\bar{p}$ and p_m are chosen such that $\bar{p} > p_m$. That is, the hump occurs to the left of the 45-degree line. Equivalently, the curve cuts the 45-degree line at p^* with a negative slope. This is crucial in order to be able to generate fluctuations.

Figures 3–9 were generated by iterating backward using the relationship between $p(t)$ and $p(t+1)$ given by equation (A9). That is, I started with a terminal value of the stock price and worked backward to find the values of the stock price at earlier dates. Figure 10, however, was generated by iterating forward. This procedure has to be used with care. As the appendix figure shows, there are two possible values of $p(t+1)$ for some values of $p(t)$. Which value of $p(t+1)$ to choose may depend on whether there exists some value of $p(t+2)$ that can follow $p(t+1)$ and whether there is some value of $p(t+3)$ that can follow $p(t+2)$, and so on. For instance, if $p(t)$ is too small, then for whichever value of $p(t+1)$ we pick, there will be no value of $p(t+2)$ that can follow it. If $p(t)$ is somewhat larger, then only the larger of the two values of $p(t+1)$ can be chosen. However, if $p(t)$ is sufficiently large, then either of the two values of $p(t+1)$ is a legitimate choice. In generating Figure 10, this type of situation was resolved by selecting randomly between the two values.

Note that the backward iteration time path in Figure 9 can be extended indefinitely into the future by starting with the terminal price and using the forward iteration procedure that generated Figure 10. As noted in the previous paragraph, to do this it is, of course, necessary that the terminal price be not too low. Therefore, the time path in Figure 9 does indeed constitute a legitimate equilibrium time path that satisfies (A9) for all t.

Solving the Hemline Example

I now show how to solve the hemline example presented in the text (and depicted there in Figure 11). Substitute from equations (1) and (A1) into equation (8) to get the following expression for expected utility:

(A14) $$E(u\,|\,i) = c_1(i)^{\alpha_1} l_1(i)^{1-\alpha_1} + \Big\{\beta\sum_{j=1}^{2}\pi_{ij}[c_2(j)^{\alpha_2} l_2(j)^{1-\alpha_2}]^{1-\mu}/(1-\mu)\Big\}.$$

In deriving (A14), it is implicitly assumed that the young consumer at date t is born *after* the current state i is realized. In the contrary case, equation (A14) would have to be modified by also adding up the utilities in each state when young, weighted by the respective probabilities. In addition, we would have to recognize the possibilities for risk sharing between the young and the old, which will alter the budget constraints (9) and (10). By assuming that the young consumer is born after the current state is realized, we rule out such risk-sharing arrangements. This assumption leads to (A14) and the budget constraints (9) and (10). The assumption is indeed very crucial because in the contrary case it can be shown that it is *impossible* for stock prices to fluctuate in response to extraneous events like hemlines or sunspots. For a demonstration of this statement, see Azariadis 1981.

I now analyze in several steps the consumer's choice problem. As before, the consumer equates the marginal rate of substitution between leisure and consumption in each period and in each state to the corresponding opportunity cost. This yields the following conditions, analogous to (A2) and (A3):

$$\text{(A15)} \quad (1-\alpha_1)c_1(i)/\alpha_1 l_1(i) = w_1$$

$$\text{(A16)} \quad (1-\alpha_2)c_2(j)/\alpha_2 l_2(j) = w_2.$$

Now substitute equations (A15) and (A16) into equations (A14), (9), and (10) to simplify them as follows:

$$\begin{aligned}\text{(A17)} \quad E(u\,|\,i) = {} & [(1-\alpha_1)/\alpha_1 w_1]^{1-\alpha_1}c_1(i) \\ & + \Big\{\beta[(1-\alpha_2)/\alpha_2 w_2]^{(1-\alpha_2)(1-\mu)} \\ & \times \textstyle\sum_{j=1}^{2}\pi_{ij}c_2(j)^{1-\mu}/(1-\mu)\Big\}\end{aligned}$$

$$\text{(A18)} \quad c_1(i) = \alpha_1(w_1 - p_i s_i)$$

$$\text{(A19)} \quad c_2(j) = \alpha_2[w_2 + (p_j+d)s_i].$$

We can now substitute (A18) and (A19) in (A17) and maximize expected utility by choice of s_i. This leads to the following condition:

$$\text{(A20)} \quad p_i = A\sum_{j=1}^{2}[\pi_{ij}(p_j+d)]/[w_2 + (p_j+d)s_i]^{\mu}.$$

We may now substitute the equilibrium condition (4) in (A20) to obtain

$$\begin{aligned}\text{(A21)} \quad p_i &= A\sum_{j=1}^{2}[\pi_{ij}(p_j+d)]/(w_2 + p_j+d)^{\mu} \\ &= \textstyle\sum_j \pi_{ij} f(p_j)\end{aligned}$$

for $i = 1, 2$, where $f(\cdot)$ is the same function as in (A9).

We thus have two equations in the two unknowns, p_1 and p_2. Note that there is always a solution in which p_1 and p_2 both equal p^*. When p_1 equals p_2, the two equations in (A21) collapse to a single equation because the sum of probabilities ($\pi_{i1}+\pi_{i2}$) must be unity for each i. The resulting equation is the same as equation (A9) with $p(t)$ equal to $p(t+1)$, and the solution is p^*. This solution corresponds to the case where the stock price is unaffected by people's belief about hemlines and the stock market. If we can find probabilities π_{ij} such that there is a solution in which p_1 and p_2 are different, then we have an example where the stock price responds to "rational" animal spirits.

Such an example can be constructed as follows. First, substitute $\pi_{12} = 1 - \pi_{11}$ and $\pi_{21} = 1 - \pi_{22}$ in equation (A21) and solve for π_{11} and π_{22} to obtain

$$\text{(A22)} \quad \pi_{11} = [f(p_2)-p_1]/[f(p_2)-f(p_1)]$$

$$\text{(A23)} \quad \pi_{22} = [p_2-f(p_1)]/[f(p_2)-f(p_1)].$$

I look for a solution such that $p_1 > p^* > p_2$ and such that the points $(p_1, f(p_1))$ and $(p_2, f(p_2))$ lie on the downward-sloping branch of the curve $f(\cdot)$. It follows that we must have $f(p_2) > f(p_1)$. (See the appendix figure for an illustration.) Since the probabilities π_{11} and π_{22} must each be between zero and one, we require that p_1 and p_2 satisfy the following conditions:

$$\text{(A24)} \quad f(p_1) < p_1 < f(p_2)$$

$$\text{(A25)} \quad f(p_1) < p_2 < f(p_2).$$

The appendix figure shows two values, p_1 and p_2, that satisfy the two inequalities. The associated probabilities π_{ij} can be calculated from (A22) and (A23).

For the examples presented here, it is important that the slope of the curve at p^*, shown in the appendix figure, be negative and greater than one in absolute value in order to generate periodic cycles other than the constant time path corresponding to p^*. This slope condition is also crucial for generating the hemline example of Figure 11. Otherwise, the inequalities (A24) and (A25) cannot be met. In fact, it turns out that for the type of model presented here, such a hemline equilibrium will exist *if and only if* there exists a two-period cycle such as the one generated in Figures 3 and 4 (see Azariadis and Guesnerie 1986). A heuristic argument for the *if* part of this statement can be made as follows. A two-period cycle corresponds to having π_{11} and π_{22} each equal to zero. Therefore, it will generally be possible to find differing values for p_1 and p_2 if π_{11} and π_{22} are both positive but small. The *only if* part is not generally true. For example, if the $f(\cdot)$ function has a slope that is positive and greater than one at p^* (this can never happen in the present model), then there cannot be a two-period cycle. However, it is possible to find differing values for p_1 and p_2 and values for the probabilities π_{11} and π_{22} that satisfy equations (A22) and (A23).

As noted in the text, it is also important that the probabilities π_{ij} depend on i. Otherwise, the only solution to (A21) is $p_1 = p_2 = p^*$. This follows because the right-hand side of (A21) is then independent of i.

The Tax/Subsidy Policy

I now analyze the tax/subsidy policy described in the text. The consumer's choices lead to the same conditions as before, namely, equations (A2), (A3), and (A4), except that $p^e(t+1)$ is replaced by p'. This is because the after-tax gross rate of return on the stock is given by $(p'+d)/p(t)$. As before, we may substitute for $l_1(t)$ and $l_2(t+1)$ from (A2) and (A3), $s(t)$ from (4), and $\tau(t)$ from (13) into equations (11) and (12) to obtain

$$\text{(A26)} \quad c_1(t) = \alpha_1(w_1 - p')$$

$$\text{(A27)} \quad c_2(t+1) = \alpha_2(w_2+p'+d).$$

Next, we may substitute for $l_1(t)$ and $l_2(t+1)$ from (A2) and (A3), and $c_2(t+1)$ from (A27) into equation (A4) and replace $p^e(t+1)$ by p' to get the following version of equation (A9):

$$\text{(A28)} \quad p(t) = A(p'+d)/(p'+d+w_2)^{\mu}.$$

This proves that the equilibrium stock price will be constant over time. The equilibrium price under such a policy need not equal the benchmark price p'. This will happen only when p' is the same as p^*, where p^* is the price depicted in the appendix figure. This follows from equations (A9) and (A28), and the figure. Further, if the government announces p^* as the benchmark price, then it can be seen from equation (13) that along the equilibrium path there will be no taxes or rebates.

The Frictional Unemployment Model

Here I explain in some detail the difference between Diamond's (1984) model and my simplified version of it. As stated in footnote 11, Diamond's model is dynamic since he allows the good to be stored. However, no more than one unit of the good may be stored; therefore, production cannot be resumed until the current inventory is sold. Thus, at any given time, the economy consists of some people who hold a unit in inventories and cannot produce any more until they have sold it and of others who have zero inventories and can produce. Further, over time a given individual may receive a variety of production opportunities which may differ in cost. The individual may, therefore, choose either to take advantage of the current production opportunity or to wait for a better (less costly) one. This makes the decision to produce a more complicated dynamic problem, and thereby makes the derivation of the *G* curve in Figure 12 more difficult.

[1] For a recent example of one such model, see Prescott 1986. The fluctuations in Prescott's model are driven by shocks to technology.

[2] Expectations are said to be *rational* if beliefs regarding possible future events are (probabilistically) correct, that is, verified by the actual future course of events. In a world without uncertainty, this amounts to having perfect foresight regarding future developments.

[3] It should be clear that allowing for taste or technology shocks would only magnify the fluctuations.

[4] This may be viewed as capturing Keynes' notion of animal spirits. Fluctuations resulting from such beliefs are often referred to as *sunspot* fluctuations (see Cass and Shell 1983).

[5] Models exhibiting these features have been studied extensively by many people, among whom the following are prominent: Costas Azariadis (1981), David Cass and Karl Shell (1983), and Jean-Michel Grandmont (1985).

[6] Models of this type were pioneered and studied by Peter Diamond (1984).

[7] The mathematical details of solving the model are given in the Appendix, where I also note the (very minor) differences between my exposition and the models of Grandmont (1985) and Azariadis (1981).

[8] The variety of different periodic cycles that can exist simultaneously was discovered by the Russian mathematician A. N. Sarkovskii and systematized in a beautiful mathematical theorem. See Grandmont 1985 (pp. 1019–20) for a more detailed explanation.

[9] This property is named after the Italian economist and sociologist Vilfredo Pareto (1848–1923). The converse of this property, that it is possible to improve someone's welfare without hurting anyone else, is known as *Pareto nonoptimality.* In this case it would generally be possible to find government policies that would make everyone better off.

[10] This is only partially true in the present model because of its very simple structure. For instance, one can use past data on stock prices to plot the current price against the future price, as in Figure 2. In a more complex model such simple procedures will no longer be useful.

[11] The main difference between Diamond's model and my simplified version is that his is dynamic, since he allows production to be stored as inventories, whereas I assume that production is nonstorable; hence, my version is static. See the Appendix for a fuller discussion of the differences.

[12] I also assume that production must occur prior to trade and that no production is possible once trade starts. This assumption rules out the possibility that someone who initially chose not to produce might wish to produce after encountering another producer. This corresponds to the real-world feature that most production is not for immediate sale but for inventory, with sales occurring subsequently out of inventory.

[13] The probability of a successful match need not be one even in this case when everyone decides to produce. Imagine that there are two producers on two islands. If each producer decides with equal chance either to stay home or to go to the other island, then there is only a 50–50 chance that the two will meet.

[14] Intuitively, the curve marked *G* must be increasing because as π goes up the expected utility of producing and trading goes up. This increase in expected utility induces more people to undertake production, thereby increasing output.

[15] Here is another illustration of Keynes' idea of self-fulfilling animal spirits.

[16] This feature is in sharp contrast to the traditional economic model of perfect competition as described by, say, Debreu (1959). All of the equilibria in the Debreu model are Pareto optimal. Therefore, in that model it is impossible for one equilibrium to dominate another, in the sense that some consumers are better off and none is worse off.

[17] For instance, Keynesians like Franco Modigliani have ridiculed neoclassical economists by saying that the only way to explain the Great Depression on the basis of neoclassical theories is to attribute it to a "severe attack of contagious laziness!" (Modigliani 1977, p. 6).

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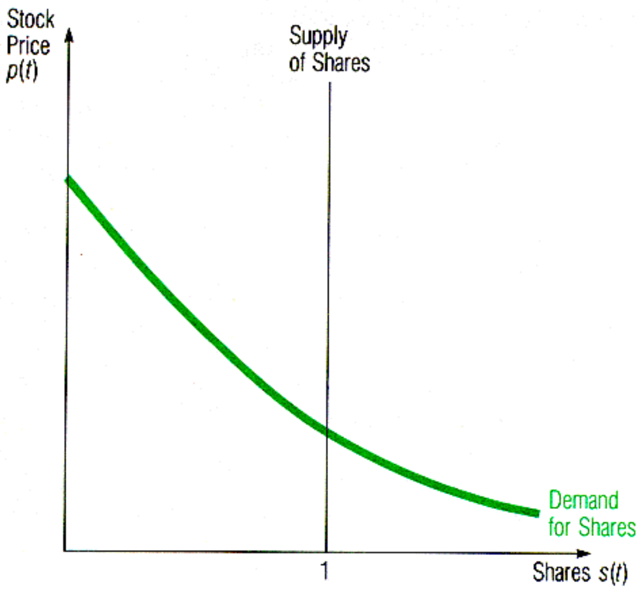
Figure 1

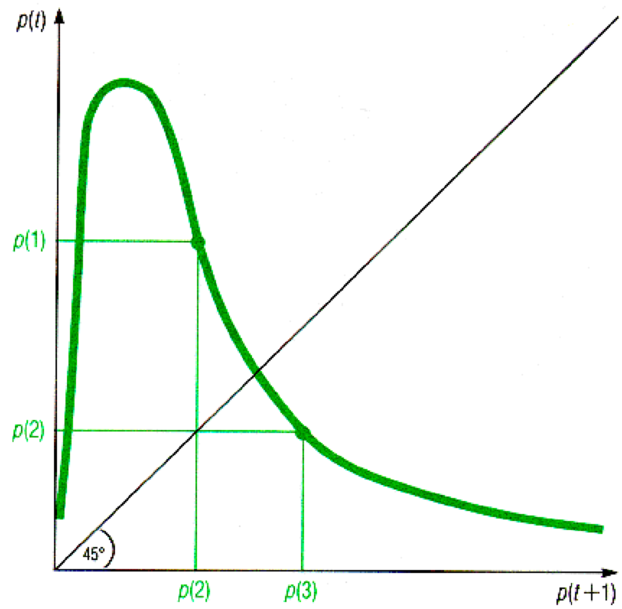
How the Stock Price is Determined in the Model*



*The position of the demand curve depends on the expected future price $p^e(t+1)$.

Figure 2

The Relationship Between Today's and Tomorrow's Stock Price

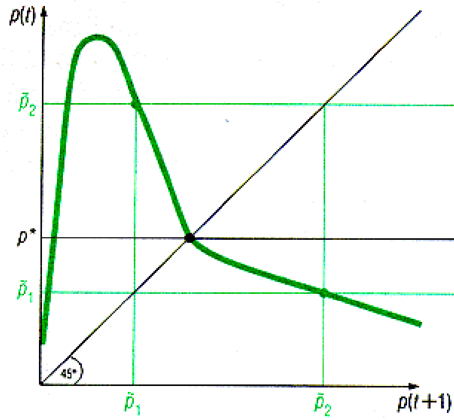


Figures 3–8

Some Periodic Cycles Generated by the Stock Price Model*

Relationships Between Today's and Tomorrow's Stock Price

Figure 3 When $\mu = 4.0$



Equilibrium Time Paths for the Stock Price

Figure 4 A Two-Period Cycle

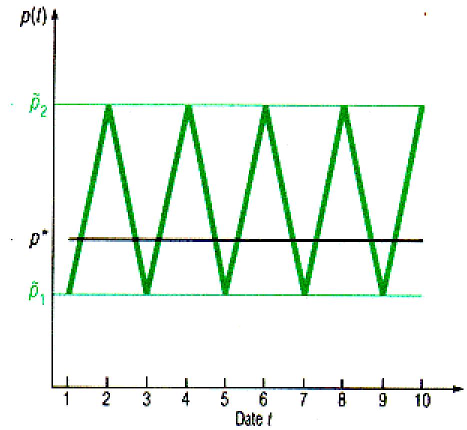


Figure 5 When $\mu = 6.0$

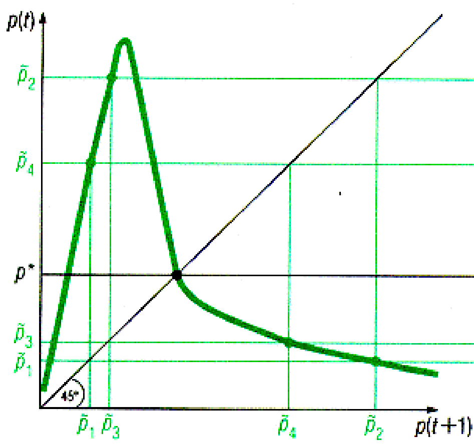


Figure 6 A Four-Period Cycle

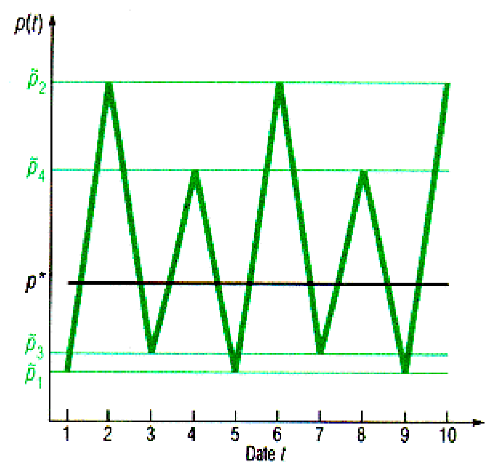


Figure 7 When $\mu = 11.0$

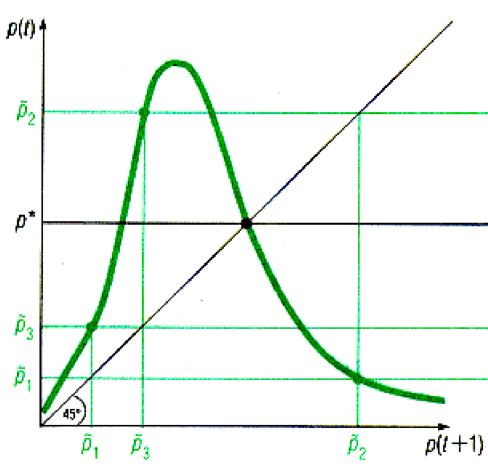
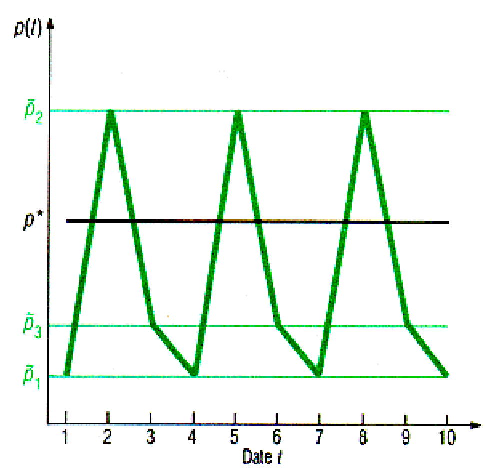


Figure 8 A Three-Period Cycle



*These figures are based on computer simulations. For details of the parameter values and simulation method used, see the Appendix.

Figures 9 and 10

Some Bizarre Time Paths for the Stock Price*

Figure 9 When $\mu = 7.5$

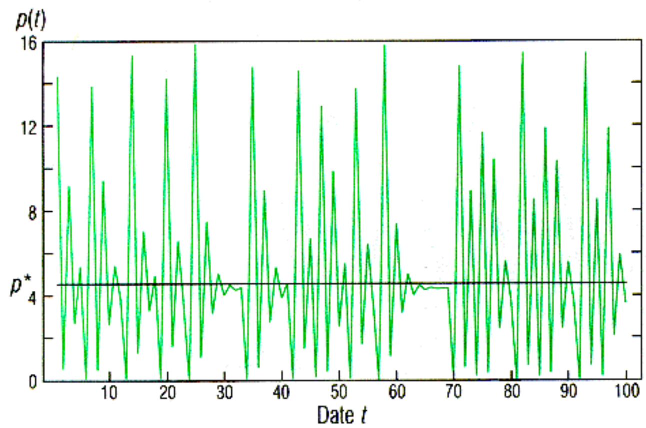
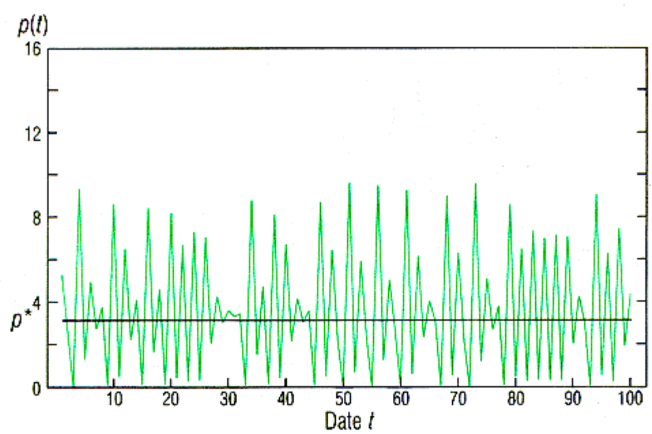


Figure 10 When $d = 0.001$, $\bar{p} = 10.0$, and $\mu = 15.0$



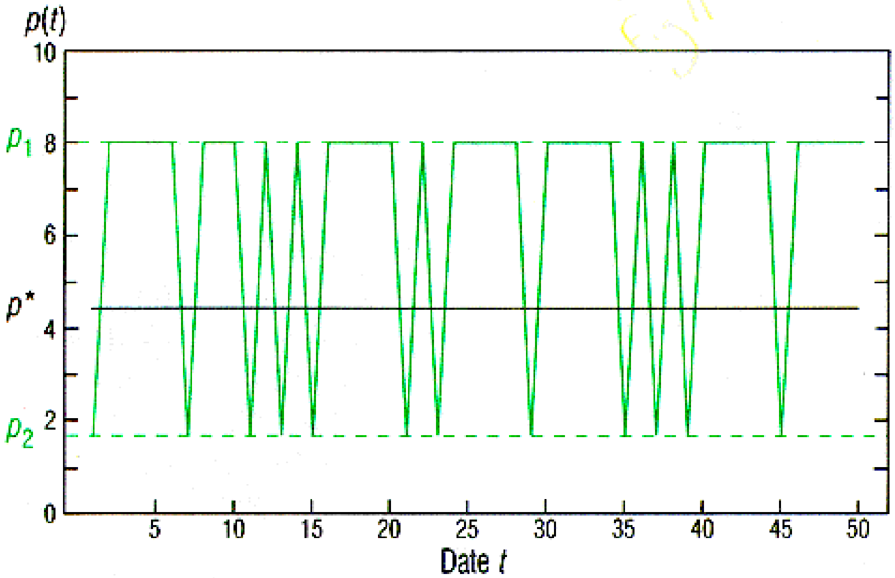
*These figures are reproduced from actual computer simulations. For details of the parameter values and simulation method used, see the Appendix.

Figure 11

The Hemline Example

An Equilibrium Time Path Generated When Consumers Believe That Movements in Stock Prices and Hemlines Correspond*

($\mu = 8.0$)



Probability Matrix:

$$\begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} = \begin{bmatrix} .49 & .51 \\ .95 & .05 \end{bmatrix}$$

π_{ij} = probability that the stock price is p_j tomorrow, given that it is p_i today, for i = 1, 2.

*This figure is reproduced from an actual computer simulation. For details of parameter values and simualtion method used, see the Appendix.

Figure 12

Three Equilibria for the Frictional Unemployment Model

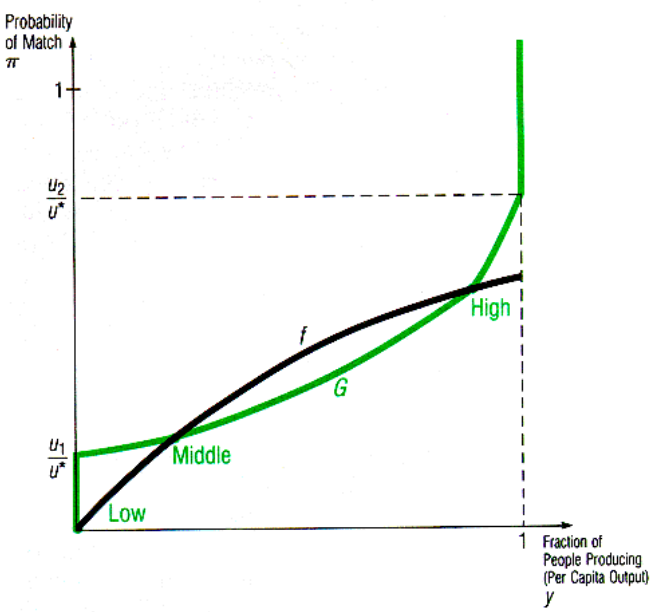
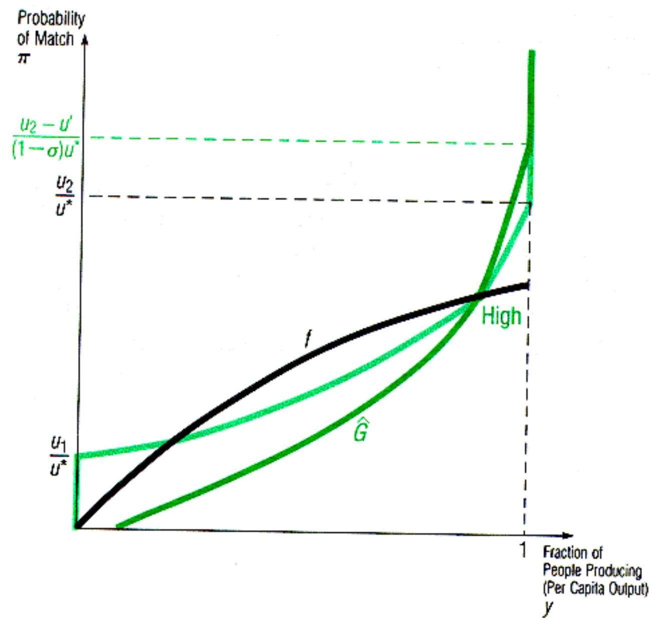


Figure 13

A Policy That Produces Only a High Equilibrium



Illustrating the Choices for Paramater Values and the Prices in the Hemline Example