A Unilateral Pricing Policy and the Stackelberg Equilibrium

Kazuhiro Ohnishi*

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Abstract

Cooper (1986) examined the equilibrium of the retroactive most-favored-customer pricing policy by using a two-period duopoly model. He showed that the most-favored-customer policy enables both firms to offer higher prices and to enjoy higher profits. Neilson and Winter (1992) showed that even if one firm in a price-setting duopoly adopts the most-favored-customer policy, the equilibrium does not coincide with the Stackelberg solution. This paper introduces a pricing policy by using a one-period two-stage model and shows that if one firm in a price-setting duopoly adopts this policy, then the equilibrium coincides with the Stackelberg solution.

Keywords: Most-favored-customer pricing policy, Stackelberg equilibrium

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* Corresponding author. Name: Kazuhiro Ohnishi (Osaka University, Ph.D.). Address: Plesanthouse 106, 5-20-5 Sakura, Minoo, Osaka 561-0041, JAPAN. Phone / fax: (81) 72-722-8638. E-mail: ohnishi@e.people.or.jp
1. Introduction

In today’s markets, many firms compete keenly day and night. If a firm reduces its price, then the demand and profits of its rivals diminish. Therefore, the rivals will counter by reducing their prices. This kind of price-cutting competition is good for consumers, but bad for the firms. Therefore, the firms will try to avoid price-cutting competition.

Cooper (1986) has examined the retroactive most-favored-customer policy as a practice facilitating coordination in a two-production-period model of a price-setting duopoly. The most-favored-customer policy is a price-protection contract of a firm toward its customers, wherein the firm guarantees to rebate its first period customers if its second period price is below its first period price. In the two-period most-favored-customer duopoly model based on Cooper (1986), each firm is to select a strategy that maximizes the undiscounted sum of its profits in the two periods, given the strategy of the other firm. Consequently, by reducing each firm’s incentive to reduce its price, the most-favored-customer policy enables both firms to offer higher prices and to enjoy higher profits.

Neilson and Winter (1992) have also examined the equilibrium when one firm in a price-setting duopoly adopts the most-favored-customer pricing policy in the two-production-period model. They have shown that the unilateral most-favored-customer pricing policy generates an equilibrium price that is higher than the Bertrand price, but that is lower than the Stackelberg leader price.

We introduce a pricing policy in a one-production-period model. This policy is that the firm agrees to make donations to nations or to charities for social services if the firm lowers its price in the future. Therefore, we call this policy a donative most-favored-nation policy (MFNP). We examine a one-period two-stage duopoly model in which each
firm conducts profit maximization behavior in the period. Then, we show that if one firm in a price-setting duopoly adopts MFNP, the equilibrium coincides with the Stackelberg solution.

The remainder of this paper is organized as follows. In Section 2, we formulate the model of MFNP. Section 3 shows that the equilibrium of the model coincides with the Stackelberg solution. Finally, Section 4 contains concluding remarks.

2. The Model

In this section, we formulate the one-production-period model of MFNP. There are two firms, denoted 1 and 2. For the remainder of this paper, when $i$ and $j$ are used to refer to firms in an expression, they should be understood to run from 1 and 2 with $i \neq j$. The duopolists produce differentiated goods in an effort to serve a single market. Each firm conducts profit maximization behavior in the period. There is no possibility of entry or exit. Firm $i$’s profit is

$$\Pi_i(p_i, p_j) = (p_i - c_i)Q_i(p_i, p_j),$$

(1)

where $p_i \in [0, \infty)$ is firm $i$’s price per unit, $c_i \in [0, \infty)$ is firm $i$’s constant marginal cost for output, and $Q_i : \mathbb{R}_+^2 \to \mathbb{R}_+$ is firm $i$’s demand function.

Firm 1 unilaterally adopts MFNP.1 The two stages of the model run as follows. In the

1 We suppose that only firm 1 adopts MFNP in order to show the purpose of this paper: If one firm in a price-setting duopoly adopts MFNP, then the equilibrium coincides with the Stackelberg solution.
first stage, firm 1 chooses a price \( r_i \in [0, \infty) \) and a number \( z_i \in [0, \infty) \), and advertises that if it sells goods to its customers at a lower price \( p_i \) than \( r_i \), then it will subscribe the amount of \( z_i \) times the difference \( (r_i - p_i) \) to nations or to charities for social services.

At the end of the first stage, firm 2 observes this behavior of firm 1. In the second stage, each firm \( i \) chooses an actual price \( p_i \). At the end of the second stage, the market opens and each firm \( i \) sells its output at the price \( p_i \). If \( p_i < r_i \), then firm 1 subscribes the amount \( (r_i - p_i)z_i \) to nations or to charities for social services.

Therefore, firm 1’s profit changes as follows:

\[
\Pi_i^\mu(r_i, z_i, p_i, p_2) = \begin{cases} 
\Pi_i(p_i, p_2) & \text{if } p_i \geq r_i \\
\Pi_i(p_i, p_2) - (r_i - p_i)z_i & \text{if } p_i < r_i .
\end{cases} \tag{2}
\]

On the other hand, since firm 2 does not offer MFNP, its profit function does not change.

If firm 1 does not adopt MFNP, then firm 1’s best reply function is given by

\[
R_i^r(p_2) = \arg \max_{\{p_i \geq 0\}} \Pi_i(p_i, p_2) . \tag{3}
\]

If firm 1 adopts MFNP and subscribes the amount \( (r_i - p_i)z_i \), then a usual way to illustrate its best reply function is

\[
R_i^\mu(p_2) = \arg \max_{\{p_i \geq 0\}} (\Pi_i(p_i, p_2) - (r_i - p_i)z_i) . \tag{4}
\]

Therefore, if firm 1 offers MFNP, then its best response changes as follows:

\[
R_i^\mu(p_2) = \begin{cases} 
R_i(p_2) & \text{if } p_i > r_i \\
r_i & \text{if } p_i = r_i \\
R_i^r(p_2) & \text{if } p_i < r_i .
\end{cases} \tag{5}
\]

Firm 1’s marginal profit exhibits a discontinuity at the level equal to \( r_i = p_i \) as a result of MFNP. On the other hand, since firm 2 does not offer MFNP, its reaction function is given by
\[ R_2(p_1) = \arg \max_{(p_1, p_2)} \Pi_2(p_1, p_2). \] (6)

Now, we assume that the following conditions are satisfied.

**Assumption 1.** \( Q_i \) is twice continuously differentiable, \( \frac{\partial Q_i}{\partial p_i} < 0 \) (downward-sloping demand), and \( \frac{\partial Q_i}{\partial p_j} > 0 \) (substitute goods).

**Assumption 2.** \( \frac{\partial^2 \Pi_i}{\partial p_i \partial p_j} > 0 \) (strategic complements).²

**Assumption 3.** \( \frac{\partial^2 \Pi_i}{\partial p_i^2} + \frac{\partial^2 \Pi_i}{\partial p_i \partial p_j} < 0 \) (stability).

These assumptions are standard in Bertrand models.³ Assumption 2 states that the firms’ reaction curves slope upward in price space, i.e., strategic complements.

Both firms choose their prices simultaneously and independently. Given \( p_j \), firm \( i \) maximizes its profit \( \Pi_i(p_i, p_j) \) with respect to \( p_i \).

The Bertrand equilibrium is defined as a pair \( (p_1^a, p_2^a) \) of price levels: \( p_1^a \in R_1(p_2^a) \) and \( p_2^a \in R_2(p_1^a) \).

We assume that the price-setting model has a unique Bertrand equilibrium and that both

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² For details of strategic complements, see Bulow, Geanakoplos and Klemperer (1985).

³ Similar assumptions are used in many papers of Bertrand competition such as Cheng (1985), Cooper (1986), Neilson and Winter (1992), and Aguirre (2000). See also Friedman (1977, pp. 50-55).
prices and outputs are positive in the equilibrium. In this paper, we discuss a subgame perfect equilibrium. Therefore, we state the equilibrium outcome of the one-period two-stage model. In the second stage, both firms’ prices and profits are decided in a Bertrand fashion. In the first stage, firm 1 decides whether or not to adopt MFNP. Let $p^M$ denote the set of equilibrium prices when firm 1 unilaterally adopts MFNP, and let $p^N$ denote the set of equilibrium prices without MFNP. A pair $(p_1^M, p_2^M) \in p^M$ of price levels is a subgame perfect Bertrand equilibrium if and only if $\Pi_1(p_1^M, p_2^M) > \Pi_1(p_1^N, p_2^N)$.

3. The Stackelberg Solution

In this section, we discuss the equilibrium of the model. First of all, we state the Stackelberg equilibrium. Firm $i$ chooses $p_i$ and firm $j$ chooses $p_j$ after observing $p_i$. Given $p_j$, firm $i$ maximizes its profit $\Pi_i(p_i, R_j(p_i))$ with respect to $p_i$.

The Stackelberg equilibrium is a pair $(p_i^S, p_j^S)$ of price levels:

$$p_i^S = \max_{p_i \geq 0} \Pi_i(p_i, R_j(p_i)) \quad \text{and} \quad p_j^S = R_j(p_i^S),$$

where firm $i$ is the leader and firm $j$ is the follower.

Now, we consider the following proposition.

**Lemma 1.** Suppose the game with no MFCP. Under Assumptions 1-3, firm 1’s Stackelberg leader price exceeds its Bertrand price.
Proof. If firm 1 is a Stackelberg leader, its Stackelberg leader price satisfies the first order condition:
\[
\frac{\partial \Pi_1}{\partial p_1} + \frac{\partial \Pi_1}{\partial p_2} \frac{\partial R_2}{\partial p_1} = 0.
\]
Since the model is the case of strategic complements in which goods are substitutes, the signs of \(\partial \Pi_1 / \partial p_2\) and \(\partial R_2 / \partial p_1\) both are plus. To satisfy the first order condition, \(\partial \Pi_1 / \partial p_1\) must be minus. Hence, firm 1’s Stackelberg leader price exceeds its Bertrand price. Q.E.D.

Lemma 2. Under Assumptions 1-3, if firm 1 adopts MFNP, then in equilibrium \(r_1 = p_1\).

Proof. First, consider the possibility that \(r_1 > p_1\) in equilibrium. From (1) and (2), firm 1’s profit is \((p_1 - c_1)Q_1(p_1, p_2) - (r_1 - p_1)z_1\), and firm 1 must subscribe \((r_1 - p_1)z_1\). Here, firm 1’s profit increases because reducing \(r_1\) to \(p_1\) decreases its subscription cost, and the equilibrium does not change in \(r_1 \geq p_1\). Hence, firm 1’s profit must be higher when \(r_1\) reduces to \(p_1\).

Next, consider the possibility that \(r_1 < p_1\) in equilibrium. It is impossible for firm 1 to change its price in equilibrium because such a strategy is not credible. That is, MFCP does not function as a strategic commitment. Q.E.D.

Both firms’ reaction curves are illustrated in Figure 1. In this figure, \(R_1\) is firm 1’s reaction curve when firm 1 does not adopt MFNP, \(R_2\) is firm 2’s reaction curve, and \(R_1^M\) (the kinked bold dotted line) is firm 1’s reaction curve when firm 1 adopts MFNP. From (5), if firm 1 adopts MFNP, its best response is shown by the kinked bold dotted
line $R_i^M$ in the figure. Since the model of this paper is the case of strategic complements in which goods are substitutes, each firm’s Stackelberg equilibrium price exceeds its Bertrand equilibrium price, i.e., the Stackelberg point $S$ is to the right of the Bertrand point $N$ on $R_2$. If $r_i > p_i$, then firm 1 subscribes the amount $(r_i - p_i)z_i$ to charities or to nations for social services, and its marginal profit decreases by $(r_i - p_i)z_i$. Therefore, firm 1’s reaction curve shifts to the right. Since $r_i$ and $z_i$ can take any values of zero and above, $R_i^M$ defined by (5) can shift to any point to the right of the Bertrand point $N$ on $R_2$ in Figure 1. Each firm’s equilibrium price is equal to or higher than the Bertrand price as a result of MFNP. Therefore, the equilibrium can occur at the appropriate point at and to the right of $N$ on $R_2$.

The Stackelberg equilibrium is achieved as follows. The equilibrium concept is the subgame perfect Nash equilibrium, and all information in the model is common knowledge. In the first stage, firm 1 chooses $r_1^s$ and $z_1^s$ corresponding to the Stackelberg point $S$ on $R_2$ and adopts MFNP. Firm 1’s reaction curve is kinked at the level equal to $r_1^s$, and firm 1’s and firm 2’s reaction curves intersect at the Stackelberg point $S$ where firm 1 is the leader and firm 2 is the follower. In the second stage, the equilibrium is decided in the Bertrand fashion. Firms 1 and 2 choose the Stackelberg leader price $p_1^s$ and the Stackelberg follower price $p_2^s$, respectively. Thus, the equilibrium occurs at the Stackelberg point $S$. This result is summarized in the following proposition.

**Proposition 1.** Under Assumptions 1-3, if firm 1 unilaterally adopts MFNP, then there exists an equilibrium in which firm 1’s price coincides with the Stackelberg leader one. At equilibrium, firm 1 earns the Stackelberg leader profit.
Proof. From Lemma 1, we see that firm 1’s Stackelberg leader price exceeds its Bertrand price. We can rewrite (4) as

\[ R^*_1(p_2) = \arg \max_{(p_1 \geq 0)} ((p_1 - c_1)(Q_1(p_1, p_2) + z_1) - (r_1 - c_1)z_1). \]

Here, the second term is irrelevant as far as marginal choices are concerned, and we can view it as if firm 1 faced demand \( Q_1(p_1, p_2) + z_1 \). Let \( z_1 \) be a variable which can take any value of zero and above. Therefore, firm 1’s price rises according to the value of \( z_1 \). Let \( r_1 \) be also a variable which can take any value of zero and above. The equilibrium concept of the model is the subgame perfect Nash equilibrium, and all information is common knowledge. In the second stage, the equilibrium is decided in the Bertrand fashion. In the first stage, firm 1 selects \( r_1 \) and \( z_1 \) corresponding to a point at which its own profit is highest by considering firm 2’s reaction function. Hence, firm 1’s equilibrium price coincides with its Stackelberg leader one. From Lemma 2, we see that firm 1’s actual price is equal to \( r_1 \) in equilibrium. Thus, firm 1 earns the Stackelberg leader profit. Q.E.D.

4. Concluding Remarks

We have analyzed the one-period two-stage model of MFNP and have found that if firm 1 unilaterally adopts MFNP, then there exists an equilibrium in which firm 1’s equilibrium price coincides with the Stackelberg leader one and firm 1 earns the Stackelberg leader profit. At this time, it is clear that firm 2’s equilibrium price coincides with the Stackelberg follower one and firm 2 earns the Stackelberg follower profit.
The model of this paper is the case of strategic complements in which goods are substitutes. Therefore, if firm 1 adopts MFNP, then firm 2’s profit also increases. As a result, we can see that MFNP also facilitates tacit collusion like the most-favored-customer pricing policy based on Cooper (1986) and others. In this paper, we have examined a one-shot game. We will also study the equilibrium outcomes of various dynamic games with MFNP in the future.
References


Figure 1. Firm 1’s reaction curve changes from $R_1$ to $R_1^M$ (the kinked bold dotted line) by MFNP.